SUPPLY CHAIN DECISION-MAKING AND COORDINATION UNDER PRICE-DEPENDENT DEMAND*

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Abstract

This paper studies the decision-making and coordination of supply chain (SC) considering the effect of price-dependent demand. By assuming demand decreases as the price increases, we analyse the impacts of the dependence on the SC in three different models: decentralized without coordination, centralized coordination and decentralized with coordination by revenue sharing contract. The existence of the best solution in the different models is proved, and the performance of revenue sharing coordination SC is similar to the centralized one. We find that the more evidently the price affects the demand, the more revenue sharing contract improves the performance of SC. The dependence affects the decision-making and the parameter setting of revenue sharing contract is also found.

Keywords: Supply chain coordination, price-dependent demand, decision making, revenue sharing

1. Introduction

Research on supply chain management (SCM) mainly focused on the decision-making and coordination of supply chains (SCs). There are two types of SC control models: centralized and decentralized. A centralized model involves the existence a unique decision-maker in the SC, who should possess all the information on the whole SC that is relevant to make decision as well as the contractual power to have such decisions implemented. The centralized control assures the system efficiency.

However, because both conditions are

difficult to be verified, the decentralized model is more realistic. This involves the existence of several decision makers pursuing different objectives, possibly conflicting among each other. In fact, a behavior that is locally rational could be globally inefficient (Whang 1995). Coordination mechanisms are then necessary so as to have local decision-makers pursue channel coordination. Such coordination mechanisms include the SC contracts, which formally rule the transactions between the SC actor's decisions coherent among each other.

Therefore, SC contracts allow two main

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objectives to be achieved: one is to increase the total SC profit so as to make it closer to the profit resulting from a centralized control; the other is to share the risk among the SC partners.

Different models of SC contracts have been developed in literatures. They include the quantity flexibility contracts, the backup agreements, the return policies, the incentive mechanisms, the revenue sharing (RS) contracts, the allocation rules, and the quantity discounts. In this paper, we will focus on the RS contracts.

But most of these models address the problem of decision-making and coordinating SCs with price-independent demand. In fact, most of SCs are price-dependent demand. Especially a series of high technology products, such as the mobile phone, PC, due to rapid development of technology, sufficient market competition and high substitutability, the price often affect the quantity of selling. It makes SC more difficult to make decision and be coordinated.

As the aspect of decision-making with price-dependent demand, some scholars have researched the problem. Abad (1988(a), 1988(b)) considers optimal lot size and selling price by assuming demand is a function of the selling price. He also presented a method for finding the optimal price and lot-size when the supplier offers all-unit quantity discounts and demand is assumed to be a decreasing function of price.

Timothy and Dinesh et al. (1997) incorporate quantity and freight discounts in inventory decision making when demand depend on price, and develop an algorithm to determine the optimal lot size and price for a class of demand function, including constant price-elasticity and linear demand. Kali and Gongyun (2001) examine a two stage of product choice while consumers' demand is linear in price.

Datta and Paul (2001) analyzed a multi-period EOQ (Economic Order Quantity) model with stock-dependent and price-sensitive demand rate. Jinn-Tsair Teng and Chun-Tao Chang (2005) consider the EPQ (Economic Production Quantity) problem for deteriorating items when the demand rate depend not only the on-display stock level but also the price per unit.

Although the studies above all take price-dependent demand into account, they didn't consider the decision-making problem from the view of a SC system.

As the aspect of coordination of SC by RS, Cachon (2005) demonstrates that RS could coordinate a SC with a single retailer and arbitrarily allocate the SC's profit, after comparing RS contract to a number of other SC contracts such as buy-back contracts. price-discount contracts. He also denotes that RS contract always depends on the order quantity and price the retailers set, without considering the influence of the retailers' price upon demand. It is possible that the RS contract does not coordinate a SC with demand that depends on retail effort.

Ilaria and Pierpaolo (2004) propose a model of three-stage SC, which is based on the Cachon's two-stage model, and the model could improve the profits of all the actors by tuning the contract parameters. Elisabetta and Francesca (2005) study the incentive of regulated firms to acquire costly information under price cap regulation by RS plans.

The studies mentioned above are without considering the influences of the prices upon demands.

Different from the two situations, Styaveer and Jean-Marie (2005) analyze the inventory coordination problem by assuming that customer demand depends upon the retail price. But they optimize centralized SC's profit and allocate the profit among partners proportional according to their investment risk.

In this paper, we will study the decision-making and coordination of the SC considering price-dependent demand in three different models: decentralized without coordination (DN), centralized coordination (CC) and RS coordination (RSC), and analyze the impacts of the dependence on the SC. The rest of this paper is organized as follows. In section 2 we introduce the notation and assumption required in the context. Section 3 presents the model of DN. The CC model is established as benchmark in section 4, and the RSC model is presented in section 5. A numerical example and some analysis are proposed in section 6. Lastly, we draw some conclusions.

2. Notation and Assumptions

Consider a two-stage SC in which a supplier S provides a single product to a retailer R, who sets the price of product and serves the market demand. The demand x is random during the selling period, with known expectation μ , probability density function f(x) and cumulate distribution function F(x). Since the market is competitive, customer demand disappears as the price increases. In this paper, we introduce a function k(p) > 0, which gives the fraction of expected customers that are ready to buy the product at price p. In other words, the quantity sold by the retailer is k(p)x. Here we make an assumption that k(p) is a decreasing and

concave function, that is k'(p) < 0, k''(p) < 0.

The wholesale price per unit charged by supplier is w, product cost per unit is c. Retailer order quantity is q. If the sale quantity exceeds q, that is x > q/k(p), the retailer's ordering cost per unit is $\tilde{w} = w + \Delta w$, supplier's is $\tilde{c} = c + \Delta c$, product cost per unit where Δw , Δc are additional per unit wholesale price and product cost respectively. We assume that customers can wait for some time to get their desired configuration at price $\tilde{p} = p - g$ because the producing cycle is short, where g is compensation for delayed delivery paid by retailer.

The decision variables are: order quantity q and price p, that is (q, p). The SC model is show in Figure 1.





The superscripts and subscripts in context, DN, CC and RSC stand for decentralized without coordination, centralized coordination and RS coordination respectively. T, R and S are for the whole SC system, retailer and supplier.

3. DN Model

In DN SC system, the profit function of retailer, supplier and the whole system are, respectively,

$$\Pi_{R}^{DN} = \int_{0}^{\frac{q}{k(p)}} \left[pk(p)x - wq \right] f(x) dx$$

$$+ \int_{\frac{q}{k(p)}}^{\infty} \left[(p-w)q + (\tilde{p} - \tilde{w}) \right]$$

$$\bullet (k(p)x-q) f(x)dx$$

$$= (\tilde{w}+g)\Gamma(q,p) + (\tilde{p} - \tilde{w})k(p)\mu$$

$$+ (\Delta w + g)q$$
(1)

$$\Pi_{S}^{DN} = \int_{0}^{\frac{q}{k(p)}} \left[(w-c)q \right] f(x) dx$$

+
$$\int_{\frac{q}{k(p)}}^{\infty} \left[(w-c)q + (\tilde{w}-\tilde{c})(k(p)x-q) \right] f(x) dx$$

=
$$(\tilde{w}-\tilde{c})(k(p)\mu - \Gamma(q,p)) - (\Delta w - \Delta c)q \quad (2)$$

$$\Pi_{T}^{DN} = \Pi_{R}^{DN} + \Pi_{S}^{DN}$$

$$= (\tilde{c} + g)\Gamma(q, p) + (\tilde{p} - \tilde{c})k(p)\mu + (\Delta c + g)$$
(3)
where $\Gamma(q, p) = \int_{0}^{\frac{q}{k(p)}} [k(p)x - q]f(x)dx$

In DN system, the retailer makes its decision independently, so the supplier and whole system's profit is decide by the retailer's decision.

Theorem 1 In DN SC, retailer's profit function Π_R^{DN} is concave of (q, p).

Proof. Observe the Hessian matrix

$$H^{DN}(q,p) = \begin{bmatrix} \frac{\partial^2 \Pi_R^{DN}}{\partial q^2} & \frac{\partial^2 \Pi_R^{DN}}{\partial q \partial p} \\ \frac{\partial^2 \Pi_R^{DN}}{\partial p \partial q} & \frac{\partial^2 \Pi_R^{DN}}{\partial p^2} \end{bmatrix}.$$

where $\frac{\partial^2 \Pi_R^{DN}}{\partial q^2} = -\frac{g + \tilde{w}}{k(p)} f\left(\frac{q}{k(p)}\right)$
 $\frac{\partial^2 \Pi_R^{DN}}{\partial p^2} = 2k'(p)\mu + (\tilde{p} - \tilde{w})k''(p)\mu$

$$+ \left(g + \tilde{w}\right) \left\{ k''(p) \int_{0}^{\frac{q}{k(p)}} xf(x) dx \\ - \frac{q^{2} \left[k'(p)\right]^{2}}{k^{3}(p)} f\left(\frac{q}{k(p)}\right) \right\}$$

$$\frac{\partial^{2} \prod_{R}^{DN}}{\partial q \partial p} = \frac{\left(g + \tilde{w}\right) qk'(p)}{k^{2}(p)} f\left(\frac{q}{k(p)}\right)$$

$$\frac{\partial^{2} \prod_{R}^{DN}}{\partial p \partial q} = \frac{\left(g + \tilde{w}\right) qk'(p)}{k^{2}(p)} f\left(\frac{q}{k(p)}\right).$$

Because $\frac{\partial^2 \Pi_R^{DN}}{2^{2}} \le 0$

$$\frac{\partial^2 \Pi_R^{DN}}{\partial p^2} \leq -\frac{\left(g + \tilde{w}\right) q^2 \left[k'(p)\right]^2}{k^3(p)} f\left(\frac{q}{k(p)}\right),$$

we can demonstrate all the leading principal minors of $H^{DN}(q,p)$ are satisfied $(-1)^m \det H^m \ge 0$. So the retailer's profit function Π_R^{DN} is concave of (q,p). • Lemma 1 Decision variables set (q^{DN}, p^{DN}) which optimizes the retailer's profit in DN SC exists, and (q^{DN}, p^{DN}) satisfies the following formula:

$$\begin{cases} -\left(g+\tilde{w}\right)F\left(\frac{q^{DN}}{k\left(p^{DN}\right)}\right) + \Delta w + g = 0 \\ \left(g+\tilde{w}\right)A\left(q^{DN}, p^{DN}\right) + \mu B\left(p^{DN}\right) = 0 \end{cases}$$
(4)

where

$$A(q, p) = k'(p) \left[\int_{0}^{\frac{q}{k(p)}} xf(x) dx - \mu \right]$$
$$B(p) = k(p) + pk'(p).$$

Proof. From the theorem 1 and basic property of the constrained optimal question:

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$$\begin{cases} \frac{\partial \Pi_R^{DN}}{\partial q} = -(g + \tilde{w}) F\left(\frac{q}{k(p)}\right) + \Delta w + g = 0\\ \frac{\partial \Pi_R^{DN}}{\partial p} = (g + \tilde{w}) A(q, p) + \mu B(p) = 0 \end{cases}$$

Lemma 2 can be gained.

According to the theorem 1 and lemma 1, the retailer make optimal decision (q^{DN}, p^{DN}) according it's own profit function, so the optimal profit of retailer, supplier and the whole system are, respectively,

$$\Pi_{R}^{DN} = (g + \tilde{w})\Gamma(q^{DN}, p^{DN}) + (\Delta w + g)q^{DN} + (p^{DN} - g - \tilde{w})k(p^{DN})\mu^{DN}$$
(5)

$$\Pi_{S}^{DN} = \left(\tilde{w} - \tilde{c}\right) \left(k \left(p^{DN} \right) \mu - \Gamma \left(q^{DN}, p^{DN} \right) \right) - \left(\Delta w - \Delta c \right) q^{DN}$$
(6)

$$\Pi_{T}^{DN} = (\tilde{c} + g) \Gamma(q^{DN}, p^{DN}) + (\Delta c + g) q^{DN} + (p^{DN} - g - \tilde{c}) k(p^{DN}) \mu$$
(7)

4. CC Model

In CC model, we get the total profit Π_T^{CC} of SC is:

$$\Pi_T^{CC} = \int_0^{\frac{q}{k(p)}} \left[pk(p)x - cq \right] f(x) dx$$

+
$$\int_{\frac{q}{k(p)}}^{\frac{q}{k(p)}} \left[(p-c)q + (\tilde{p} - \tilde{c}) \right] (k(p)x - q) f(x) dx$$

=
$$(\tilde{c} + g)\Gamma(q, p) + (\tilde{p} - \tilde{c})k(p)\mu + (\Delta c + g)q$$

Theorem 2 The total profit Π_T^{CC} of CC SC is concave of (q, p).

Proof. It can be easily proved according to the proof of theorem 1.

Lemma 2 Decision variables set (q^{CC}, p^{CC}) which optimizes the whole SC profit in CC SC exists, and (q^{CC}, p^{CC}) satisfies following formula:

$$\begin{cases} -(g+\tilde{c})F\left(\frac{q^{CC}}{k(p^{CC})}\right) + \Delta c + g = 0\\ (g+\tilde{c})A(q^{CC}, p^{CC}) + \mu B(p^{CC}) = 0 \end{cases}$$
(9)

Proof. It can be easily proved according to the proof of lemma 1.

In CC model, system makes decision according to the formula (8), so the optimal profit of retailer, supplier and the whole system are, respectively,

$$\Pi_{R}^{CC} = (\tilde{w} + g) \Gamma(q^{CC}, p^{cC}) + (p^{CC} - g - \tilde{w}) k(p^{CC}) \mu + (\Delta w + g) q^{CC}$$
(10)
$$\Pi_{S}^{CC} = (\tilde{w} - \tilde{c}) (k(p^{CC}) \mu - \Gamma(q^{CC}, p^{CC})) - (\Delta w - \Delta c) q^{CC}$$
(11)

$$\Pi_T^{CC} = (\tilde{c} + g) \Gamma(q^{CC}, p^{CC}) + (p^{CC} - g - \tilde{c}) k(p^{CC}) \mu + (\Delta c + g) q^{CC}$$
(12)

5. RSC Model

We extend Cachon's (2005) RS contract model to three parameters ϕ , w', $\Delta w'$, which are confirmed by all members of SC. ϕ is the quota of retail revenue that the retailer keeps while giving the rest $\overline{\phi} = 1 - \phi$ to the supplier, i.e., given retail revenues R(q, p), the retailer transfer $\overline{\phi}R(q, p)$ to the supplier and retains the remaining $\phi R(q, p)$.

With RS contract $(\phi, w', \Delta w')$, R(q, p) is:

$$R(q, p) = \int_{0}^{\frac{q}{k(p)}} \left[pk(p)x \right] f(x) dx$$
$$+ \int_{\frac{q}{k(p)}}^{\infty} \left[pq + p(k(p)x - q) \right] f(x) dx$$

JOURNAL OF SYSTEMS SCIENCE AND SYSTEMS ENGINEERING

$$= pk(p)\mu \tag{13}$$

Retailer's cost C_R is:

$$C_{R} = \int_{0}^{\frac{q}{k(p)}} w'qf(x) dx$$
$$+ \int_{\frac{q}{k(p)}}^{\infty} \left[w'q + (\tilde{w}' + g)(k(p)x - q) \right] f(x) dx$$
$$= (\tilde{w}' + g)(k(p)\mu - \Gamma(q, p)) - (\Delta w' + g)q \quad (14)$$

So the profit function of retailer, supplier and the whole system are, respectively,

$$\Pi_{R}^{RSC} = \phi R(q, p) - C_{R}$$

$$= (\tilde{w}' + g) \Gamma(q, p)$$

$$+ (\phi p - g - \tilde{w}') k(p) \mu + (\Delta w' + g) q$$
(15)
$$-\frac{q}{2}$$

$$\Pi_{S}^{RSC} = \overline{\phi} R(q, p) + \int_{0}^{k(p)} \left[(w'-c)q \right] f(x) dx$$

$$+ \int_{\frac{q}{k(p)}}^{\infty} \left[(w'-c)q + (\tilde{w}'-\tilde{c}) \right] (w'-c)q + (\tilde{w}'-\tilde{c}) dx$$

$$\bullet \left(k(p)x-q \right) f(x) dx$$

$$= \left(\overline{\phi} p + \tilde{w}'-\tilde{c} \right) k(p)\mu - \left(\Delta w' - \Delta c \right) q$$

$$- \left(\tilde{w}'-\tilde{c} \right) \Gamma(q, p)$$

$$\Pi_{T}^{RSC} = \Pi_{S}^{RSC} + \Pi_{R}^{RSC}$$

$$= \left(g + \tilde{c} \right) \Gamma(q, p) + \left(\tilde{p} - \tilde{c} \right) k(p) \mu + \left(\Delta c + g \right) q$$

(17) **Theorem 3** The retailer's profit function Π_R^{RSC} in RSC SC is concave of (q, p).

Proof. Observe the Hessian matrix

$$H^{RSC}(q,p) = \begin{bmatrix} \frac{\partial^2 \Pi_R^{RSC}}{\partial q^2} & \frac{\partial^2 \Pi_R^{RSC}}{\partial q \partial p} \\ \frac{\partial^2 \Pi_R^{RSC}}{\partial p \partial q} & \frac{\partial^2 \Pi_R^{RSC}}{\partial p^2} \end{bmatrix}$$

where

$$\frac{\partial^2 \Pi_R^{RSC}}{\partial q^2} = -\frac{\left(\tilde{w}' + g\right)}{k(p)} f\left(\frac{q}{k(p)}\right)$$

JOURNAL OF SYSTEMS SCIENCE AND SYSTEMS ENGINEERING

$$\frac{\partial^2 \Pi_R^{RSC}}{\partial p^2} = 2\phi k'(p) \mu + (\phi p - g - \tilde{w}') k''(p) \mu$$
$$+ (\tilde{w}' + g) k''(p) \int_0^{\frac{q}{k(p)}} xf(x) dx$$
$$- \frac{q^2 \left[k'(p) \right]^2}{k^3(p)} f\left(\frac{q}{k(p)}\right)$$
$$\frac{\partial^2 \Pi_R^{RSC}}{\partial q \partial p} = \frac{(\tilde{w}' + g) q k'(p)}{k^2(p)} f\left(\frac{q}{k(p)}\right)$$
$$\frac{\partial^2 \Pi_R^{RSC}}{\partial p \partial q} = \frac{(\tilde{w}' + g) q k'(p)}{k^2(p)} f\left(\frac{q}{k(p)}\right).$$

Because

$$\begin{aligned} &\frac{\partial^2 \Pi_R^{RSC}}{\partial q^2} \leq 0\\ &\frac{\partial^2 \Pi_R^{RSC}}{\partial p^2} \leq -\frac{\left(\tilde{w}' + g\right) q^2 \left[k'(p)\right]^2}{k^3(p)} f\left(\frac{q}{k(p)}\right), \end{aligned}$$

we can demonstrate all the leading principal minors of $H^{RSC}(q, p)$ are satisfied $(-1)^m \det H^m \ge 0$. So the retailer's profit function Π_R^{RSC} is concave of (q, p). **Lemma 3** Decision variables set (q^{RSC}, p^{RSC}) which optimizes retailer's profit in RSC SC exists, and (q^{RSC}, p^{RSC}) satisfies following formula:

$$\begin{cases} -\left(\tilde{w}'+g\right)F\left(\frac{q^{RSC}}{k\left(p^{RSC}\right)}\right) + \left(\Delta w'+g\right) = 0 \\ \left(\tilde{w}'+g\right)A\left(q^{RSC}, p^{RSC}\right) + \phi\mu B\left(p^{RSC}\right) = 0 \end{cases}$$
(18)

Proof. It can be easily proved according to the proof of lemma 1.

In RSC model, retailer make it's decision according to the formula (15), so the optimal profit of retailer, supplier and the whole system are, respectively,

$$\Pi_{R}^{RSC} = (\tilde{w}' + g) \Gamma(q^{RSC}, p^{RSC}) + (\Delta w' + g) q^{RSC} + (\phi p^{RSC} - g - \tilde{w}') k(p^{RSC}) \mu$$
(19)

$$\Pi_{S}^{RSC} = -(\tilde{w}' - \tilde{c}) \Gamma(q^{RSC}, p^{RSC}) - (\Delta w' - \Delta c) q^{RSC} + (\bar{\phi} p^{RSC} + \tilde{w}' - \tilde{c}) k(p^{RSC}) \mu$$
(20)

$$\Pi_{T}^{RSC} = (g + \tilde{c}) \Gamma(q^{RSC}, p^{RSC}) + (\Delta c + g) q^{RSC} + (p^{RSC} - g - \tilde{c}) k(p^{RSC}) \mu$$
(21)

Theorem 4 *RS* contract $(\phi, w', \Delta w')$ can coordinate SC system.

Proof. By comparing the formula (19) with the formula (21), it is easy to know that RS contract $(\phi, w', \Delta w')$ can make the retailer's profit function an affine transformation of the whole system one. Comparing the formula (9) and (18), and due to theorem 2 and 3, we can know there must exits $(\phi, w', \Delta w')$ make $(q^{RSC}, p^{RSC}) = (q^{CC}, p^{CC})$, that is SC with RS contract $(\phi, w', \Delta w')$ can gain the performance equal to centralized coordination system. Especially, while $w' = \phi c$, $\Delta w' = \phi (\Delta c + g) - g$, the revenue can be allocated arbitrary among retailer and supplier. This completes the proof.

6. Numerical Illustration

We use the following concave function for k(p) (Styaveer 2005):

$$k(p) = 1 - \frac{p^2}{k}, \quad k \ge p^2$$

Where k is a constant that indicate how evident the effect of pricing on demand is, and the smaller k is, the greater effect of pricing upon demand, vice versa.

Probability density function of demand is:

$$f(x) = \begin{cases} 1/400 & if \quad 100 \le x \le 500\\ 0 & otherwise \end{cases}$$

And the other parameters are: c = 4, $\Delta c = 0.5$, w = 6, $\Delta w = 0.8$, g = 0.2, and let $w' = \phi c$, $\Delta w' = \phi (\Delta c + g) - g$ in RSC model.

In this section, CC SC is set as benchmark model and compared with DN, RSC SC.

6.1 The Effects of *k* on the SC's Profit

The whole profit of the SCs with different k in different models are shown in Table 1.

Table 1 The whole profit of different model SC

| k | CC | DN | RSC |
|-------|----------|----------|----------|
| 100 | 407.58 | 237.68 | 407.55 |
| 200 | 842.60 | 711.21 | 842.53 |
| 300 | 1191.69 | 1073.57 | 1191.66 |
| 500 | 1756.47 | 1649.55 | 1756.43 |
| 1000 | 2809.37 | 2711.82 | 2809.33 |
| 10000 | 10676.97 | 10594.65 | 10678.99 |

Table 1 presents that the performance of the RSC SC is close to the CC one with different k; and as the demand becomes more stable the profit in different models increase. This indicates that the dependence of price on demand has effect on the whole system's profit evidently in different model.

Figure 2 shows the relative error of SC's profit between DN, RSC and the CC. It indicates that the more price-dependent demand is, the more evident deviation of DN SC's profit will be, while RSC SC's performance is nearly the same as that of CC one; on the contrary, the profit of DN SC is close to the CC one when demand is less price-dependent. So, the coordination is more effective when demand is more price-dependent, vice versa.





Figure 2 Relative error of DN and RSC profit

Figure 3 The relative error of DN and RSC decision

| | CO | 2 | DN | | RSC | |
|-------|--------|-------|--------|-------|--------|-------|
| k | q | р | q | р | q | р |
| 100 | 72.18 | 7.42 | 57.59 | 9.00 | 73.19 | 7.41 |
| 200 | 84.18 | 9.76 | 112.88 | 11.23 | 85.28 | 9.75 |
| 300 | 88.94 | 11.57 | 133.89 | 12.99 | 90.08 | 11.56 |
| 500 | 93.45 | 14.46 | 153.04 | 15.81 | 94.59 | 14.45 |
| 1000 | 98.02 | 19.78 | 170.50 | 21.06 | 99.12 | 19.77 |
| 10000 | 103.61 | 59.21 | 192.50 | 60.47 | 103.21 | 59.22 |

Table 2 Different model SC's decision

6.2 The Effects of k on the SC's Decision

Table 2 shows the optimal decisions in different models of SC with different k. From Table 2, it is easy to know that the order quantity and pricing are increasing as the impact of price on demand becomes weaker. Figure 3 shows the relative error, taking the value of CC SC for benchmark, with different k, of DN SC and RSC SC. It depicts that the deviation of ordering quantity of DN SC gets greater as the effect of pricing upon demand decreases, while the pricing decision gradually becomes closer to the CC one. It indicates that the decisions with and without coordination are nearly the same when consumers are not sensitive to price, but the

ordering quantity of SC without coordination deviates from the optimal value. RSC SC's decision keeps consistent with that of CC SC, which also implies RS is capable of coordinating the SC effectively.

6.3 The Effects of k on RS Contract

Table 3 shows that the retailer's profit increases in DN model with decreasing effect of pricing upon demand. If the SC is coordinated by RS contract, it is necessary to ensure that the retailer's profit in RSC higher than that of the DN one, that is $\phi \ge \prod_{R}^{DN} / \prod_{T}^{RSC}$. Given $w, \Delta w$, Figure 4 shows the increasing trend of ϕ ' lower bound as the *k* increases.

| k | Π_R^{DN} | Π_S^{DN} | Π_T^{DN} | Π_T^{RSC} |
|-------|--------------|--------------|--------------|---------------|
| 100 | 54.44 | 183.24 | 237.68 | 407.55 |
| 200 | 353.84 | 357.37 | 711.21 | 842.53 |
| 300 | 650.25 | 423.32 | 1073.57 | 1191.66 |
| 500 | 1166.46 | 483.09 | 1649.55 | 1756.43 |
| 1000 | 2174.60 | 537.22 | 2711.82 | 2809.33 |
| 10000 | 9978.44 | 616.20 | 10594.7 | 10679.0 |

Table 3 Profits under DN and RSC



Figure 4 The trend of ϕ with k increasing

7. Conclusions

After analyzing the effect of price-dependent demand upon SC decision-making and RS coordination, we can draw the following conclusions:

1) RSC SC can achieve the performance of the CC SC and the profit can be allocated arbitrarily among members in the SC. So RSC can coordinate the SC with price-dependent demand, and the partners in the SC can get win-win condition by selecting appropriate contract parameters.

2) RS contract can improve the performance evidently in those SC system where pricing affect the demand greatly, so it is more important to coordinate the SC which are facing sufficient market competition and under the condition that demand is price-dependent. On the contrary, in the SC where pricing affects demand little, due to small increase of profit made by coordination, it is difficult to build a stable SC;

3) In RSC model, given $w, \Delta w$, it is necessary to increase the value of ϕ , the retailer's revenue sharing proportion, as the *k* increases, to keep the SC stable. So, ϕ is relative to *k*. We make assumption that *k* is a constant that indicate how evident the effect of pricing on demand is. In fact, *k* is a variable in selling period because price or market competition. So, how to confirm *k* is further research.

References

- Abad, P.L. (1988). Determining optimal selling piece and lot size when the supplier offers all-unit quantity discounts. Decision Science, 19: 622-634
- [2] Abad, P.L. (1988). Joint price and lot size determination when supplier offers incremental quantity discounts. Journal of the Operational Research Society, 39: 603-607
- [3] Datta, D.K. & Paul, K. (2001). An inventory system with stock-dependent, pricesensitive demand rate. Production Planning and Control, 12: 13-20
- [4] Iossa, E. & Stroffolini, F. (2005). Price cap regulation, revenue sharing and information acquisition. Information Economics and Policy, 17: 217-230
- [5] Cachon, G. P. & Lariviere, M. A. (2005). Supply chain coordination with revenue-sharing contracts: strengths and limitations. Management Science, 51: 30-44

- [6] Giannoccaro, I. & Pontrandolfo, P. (2004). Supply chain coordination by revenue sharing contracts. International Journal of Production Economics, 89: 131-139
- [7] Teng, J.S. & Chang, C.T. (2005). Economic production quantity models for deteriorating items with price- and stock-dependent demand. Computers & Operations Research, 32: 297-308
- [8] Rath, K. P. & Zhao, G.Y. (2001). Two stage equilibrium and product choice with elastic demand. International Journal of Industrial Organization, 19: 1441-1455
- [9] Petruzzi, N. & Dada, M. (1999). Pricing and the newsvendor problem: a review with extensions. Operations Research, 47: 183-194
- [10]Chauhan, S. S. & Proth, J.M. (2005). Analysis of a supply chain partnership with revenue sharing. International Journal of Production Economics, 97: 44-51
- [11]Seshadri, S. & Mishra, R. (2004).Relationship and contract theory. Industrial Marketing Management, 33: 513-526
- [12] Taylor, T. A. (2002). Supply chain coordination under channel rebates with sales effort effects. Management Science, 48: 992-1007

- [13]Burwell, T. H. & Dave, D. S. et. al. (1997). Economic lot size model for price-dependent demand under quantity and freight discounts. International Journal of Production Economics, 48: 141-155
- [14]Whang, S. (1995). Coordination in operations: A taxonomy. Journal of Operations Management, 12:413-422

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