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Technical Note

FUZZY MULTI-LEVEL WAREHOUSE LAYOUT PROBLEM: NEW MODEL AND ALGORITHM*

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Abstract

This paper deals with a multi-level warehouse layout problem under fuzzy environment, in which different types of items need to be placed in a multi-level warehouse and the monthly demand of each item type and horizontal distance traveled by clamp track are treated as fuzzy variables. In order to minimize the total transportation cost, chance-constrained programming model is designed for the problem based on the credibility measure and then tabu search algorithm based on the fuzzy simulation is designed to solve the model. Some mathematical properties of the model are also discussed when the fuzzy variables are interval fuzzy numbers or trapezoidal fuzzy numbers. Finally, a numerical example is presented to show the efficiency of the algorithm.

Keywords: Multi-level warehouse layout problem, fuzzy variable, tabu search algorithm

1. Introduction

As we know, in most manufactory, above all in large-scale manufactory, warehouse operation is an important part of a manufacturer's material handling operation. Effective warehouse planning can not only reduce material handling cost but also increase productivity. Hence, it is very important for the manufacturer to make the best plan to manage the warehouse.

The warehouse layout problem concerns how to place different items into the warehouse so that people resources, equipments and space are utilized efficiently. In other words, it is necessary to seek the best arrangement of items within the available area in order to operate efficiently and cost effectively. This problem was received much attention by many researchers recently. For more details, interested readers may refer to Zhang, Xue *et al.* (2000), Kusiak and Heragu (1987), Lai, Xue *et al.* (2002), Larson, March *et al.* (1997), and so on. In practice, the warehouse does not always have only one floor, and multi-level warehouse can be constructed for saving land in some

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manufactories. For such case, the warehouse layout problem turns more complex. Zhang and Lai (2000) investigated a multi-level warehouse layout problem and then designed an algorithm for the problem. In addition, Meller and Bozer (1997) studied a multi-floor facility layout problem to minimize the cost. It is necessary to point out that the layout problems studied in the above articles are all under certain environment, i.e., the coefficients in these problems are all fixed quantities. But in the real life, some coefficients are not necessarily fixed due to the uncertainty of the real decision-making system. Thus the methods investigated in certain environment will be invalid. In order to solve the problem under uncertain environment, Zhang, Lai *et al.* (2002) studied a fuzzy multi-level warehouse layout problem and constructed fuzzy expected value model. But no numerical example was given to show the efficiency of the algorithm in their works.

In this paper, we shall consider fuzzy multi-level warehouse layout problem from another point of view. A new model and algorithm are presented to seek the best storage plan of the problem. A numerical example is also given to show the application of model and algorithm.

2. Multi-Level Warehouse Layout Problem

The multi-level warehouse layout problem concerns how to place different items into the warehouse so that the total transportation cost is minimized. The warehouse has several floors and only one exit on the first floor. An elevator, located beside the exit, is employed to transfer items between the stored floors and the ground. When items arrive, they are unloaded beside the exit of the warehouse. Then warehouse keeper makes the storage plan. Clamp trucks and the elevator will then transport the items to their locations and the warehouse inventory record will be updated. When an order arrives, the reverse process will be carried out. Because the warehouse is multi-level, both horizontal and vertical costs need to be considered in the process of making layout.

In the following, we shall list characteristics and constraints in the problem we are considering.

For each type of items, there is a demand for every month. Generally, the demand is not always fixed due to the uncertainty of the real life. It may vary during the different periods. If the previous data are enough, we may treat it as a random variable by statistics ways. When there are no enough data to be used, we always treat the monthly demand as a fuzzy variable in practice. Also, the membership function of fuzzy variable can be obtained by means of experience evaluation. In this paper, we assume that the monthly demand is a fuzzy variable. There is also an inventory requirement associated with each type of item to offset its demand variation. The inventory requirement is assumed to be known in advance. It is used to decide the space needed for each type of items.

The warehouse considered in this paper has several floors to place items. For the convenience of storage and transportation, each floor space is divided into cells with the same dimension, that is, the physical size of each cell (length and width) are the same (e.g. 8m×8m). Some of the cells are used to place items and the others may be reserved for special purposes. We allow an aisle (e.g. 2m width) between the storage cells for the movement of clamp trucks (the actual area for storage in each cell is 7m×7m). Furthermore, for the convenience of working of trucks, it is assumed that one cell can hold at most two types of items.

It is reasonable that items of the same type be stored in the same cell unless the inventory requirement exceeds the capacity of a cell. For such case, this type of items will occupy more than one cell, which is called a *split item* type. When an item type splits, we stipulate that at most one cell can be partially occupied by items of the split type. That is, the items of this type are divided into different parts, in which amount of at most one part is less than the capacity of the cell. For convenience, we will treat different parts as several independent item types. The total demand and inventory requirement of a split item type will be divided proportionally among its different parts. In this paper, we assume that the above split has already been accomplished. So, the inventory requirement of each type of items is no more than the capacity of a cell.

In the process of making layout, for the convenience of working of tracks, the items are divided into different classes according to some characteristics. Items of different classes can not be stored in the same cell. However, one cell may store more than one item type as long as they belong to the same class. There is only one exit in the warehouse, which is on the first floor. The distance of each cell from the exit can be divided into two parts, i.e., *horizontal distance* and *vertical distance*. Usually, we use the following way to measure the horizontal distance between the center of a cell and the elevator: this distance can be regarded as the length of the shortest rectilinear structure along the aisle. For instance, as shown in Figure 1, the cell *B* denotes the position of the elevator, then the horizontal route for clamp trucks from the elevator to the cell *A* is shown as the bold segments between *A* and *B* along the aisle.

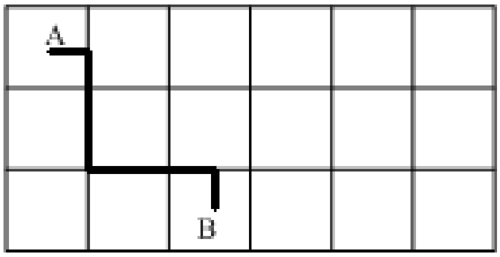

Figure 1 The horizontal distance

In fact, it is impossible for the clamp trucks to travel along the bold segments strictly, so the distance traveled by clamp trucks is imprecise in nature. Hence, in our problem, it is reasonable that the distance traveled by clamp trucks between each cell and the elevator be treated as a fuzzy variable. In addition, since the elevator is located beside the exit, the vertical distance from the exit is regarded as the height of this floor, which is a fixed quantity.

The warehouse has enough capacity to store all items. The objective in the problem is to minimize the total transportation cost for storage, which is the sum of the transportation cost of each item type. The transportation cost of a given item type can be divided into two parts: *horizontal transportation cost* and *vertical transportation cost*. Horizontal transportation cost is the horizontal unit transportation cost multiplied by amount of the horizontal transportation, where amount of the horizontal transportation is supposed to be the monthly demand of item type multiplied by the horizontal distance traveled. Similarly, vertical transportation cost is the vertical unit transportation cost multiplied by amount of the vertical transportation, where amount of the vertical transportation is supposed to be the monthly demand of item type multiplied by the vertical distance traveled.

This paper assumes that both the monthly demand of each item type and the distance from each cell to the elevator are fuzzy variables. Thus the horizontal transportation cost and the vertical transportation cost of a given type of items are both fuzzy variables. Then the total transportation cost of the items is a fuzzy variable. In fact, it is difficult to rank fuzzy objective values during the process of seeking the best storage plan. Enlightened by the idea of using chance-constrained programming to treat fuzzy programming proposed by Liu (2002), in the following section, we shall construct chance-constrained programming model for the problem.

3. Mathematical Model

In the following, we shall introduce some knowledge of credibility theory briefly. Possibility theory was proposed by Zadeh (1978). Recently, many researchers further developed this theory such as Dubois and Prade (1988), Liu (2002), Liu and Liu (2002).

If ξ is a fuzzy variable with membership function $\mu(x)$, then the possibility of fuzzy event $\xi \le r$ is defined as

$$\mathrm{Pos}\{\xi \le r\} = \sup_{x \le r} \mu(x). \tag{1}$$

The necessity of a fuzzy event is defined as the impossibility of the opposite event. Thus necessity measure of fuzzy event $\xi \le r$ is defined as

$$\mathrm{Nec}\{\xi \le r\} = 1 - \mathrm{Pos}\{\xi > r\} = 1 - \sup_{x > r} \mu(x). \tag{2}$$

In 2002, Liu and Liu (2002) presented the concept of credibility measure. The credibility measure of fuzzy event $\xi \le r$ is defined as the average of its possibility measure and necessity measure. Then,

$$\mathrm{Cr}\{\xi \le r\} = \frac{1}{2}(\mathrm{Pos}\{\xi \le r\} + \mathrm{Nec}\{\xi \le r\}). \tag{3}$$

By using the above definition, we can calculate the credibility measures for some special fuzzy events. Let $\xi = [a, b]$ be an interval fuzzy number. Then we have

$$\mathrm{Cr}\{\xi \le r\} = \begin{cases} 0, & \text{if } r < a \\ \frac{1}{2}, & \text{if } a \le r < b \\ 1, & \text{if } r \ge b. \end{cases} \tag{4}$$

Similarly, for a trapezoidal fuzzy number $\xi = (a_1, a_2, a_3, a_4)$, we have

$$\mathrm{Cr}\{\xi \le r\} = \begin{cases} 0, & \text{if } r < a_1 \\ \frac{r - a_1}{2(a_2 - a_1)}, & \text{if } a_1 \le r < a_2 \\ \frac{1}{2}, & \text{if } a_2 \le r < a_3 \\ \frac{r + a_4 - 2a_3}{2(a_4 - a_3)}, & \text{if } a_3 \le r < a_4 \\ 1, & \text{if } r \ge a_4. \end{cases} \tag{5}$$

In order to rank fuzzy variables, Liu (2002) presented the definition of $\alpha-$critical value, which is defined as

$$\xi_{\inf}(\alpha)=\inf\{r \mid \mathrm{Cr}\{\xi \le r\} \ge \alpha\}, \tag{6}$$

where $\alpha \in (0,\ 1]$ is a credibility confidence level.

In order to model the multi-level warehouse layout problem, we introduce the following notations for the parameters:

- $j \in \{1,\ 2,\ ...,\ J\}$ is the index for item types;
- $l \in \{1,\ 2,\ ...,\ L\}$ is the index for item classes;
- $i \in \{1,\ 2,\ ...,\ I\}$ is the index for levels of the warehouse;
- $k \in \{1,\ 2,\ ...,\ K_i\}$ is the index for available cells of level i;
- R_l is the set of item types belonging to class l;
- $\tilde{Q}_j$ is the monthly demand of item type j and a fuzzy variable;
- $\tilde{D}^h_{ik}$ is the horizontal distance from cell k of level i to the elevator and a fuzzy variable;
- D^v_i is the vertical distance from the stored floor i to the exit;
- H_j is the horizontal unit transportation cost for item type j;
- V_j is the vertical unit transportation cost for item type j;
- C is the storage capacity of a cell;
- S_j is the inventory requirement of item type j.

The solutions of the warehouse layout problem are represented by the following decision variables:

$$x_{jik}=\begin{cases}1, & \text{if item type } j \text{ is assigned to} \\ & \text{cell } k \text{ of level } i \\ 0, & \text{otherwise}\end{cases}$$

$$y_{lik}=\begin{cases}1, & \text{if cell } k \text{ of level } i \text{ is used for} \\ & \text{items of class } l \\ 0, & \text{otherwise,}\end{cases}$$

where j=1, 2, …, J; l=1, 2, …, L; i=1, 2, …, I; k=1, 2, …, K_i.

Then we formulate the model of fuzzy multi-level warehouse layout problem as model (9). In this model, the objective is to minimize the $\alpha-$critical value of transportation cost of the items. The first constraint ensures that each item type is assigned to exactly one cell. The second constraint ensures that each cell holds no more than two item types. The third constraint ensures that each cell holds only one class of items. The fourth constraint ensures that the cell capacity is not violated.

In order to compute the objective value conveniently, we shall discuss some crisp equivalents of the objective function.

Theorem 1 *Let* $\tilde{Q}_j=[\tilde{Q}^-_j,\ \tilde{Q}^+_j]$ *and* $\tilde{D}^h_{ik}=[(\tilde{D}^h_{ik})^-,\ (\tilde{D}^h_{ik})^+]$ *be positive interval fuzzy numbers, j=1, 2, …, J; i=1, 2, …, I; k=1, 2, …, K*$_i$*. Then the objective function in model (9) may be rewritten as follows:*

$$f(x,\ \alpha)=\begin{cases}\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}^-_j(\tilde{D}^h_{ik})^-+V_i\tilde{Q}^-_jD^v_i)x_{jik}, & \text{if } 0<\alpha\le 0.5 \\ \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}^+_j(\tilde{D}^h_{ik})^++V_i\tilde{Q}^+_jD^v_i)x_{jik}, & \text{if } \alpha>0.5.\end{cases} \tag{7}$$

Proof. Since $\tilde{Q}_j=[\tilde{Q}^-_j,\ \tilde{Q}^+_j]$ and $\tilde{D}^h_{ik}=[(\tilde{D}^h_{ik})^-,\ (\tilde{D}^h_{ik})^+]$ are positive interval fuzzy numbers, we have

$$\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j\tilde{D}_{ik}^h+V_i\tilde{Q}_jD_i^v)x_{jik}$$
$$=\left[\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^-(\tilde{D}_{ik}^h)^-+V_i\tilde{Q}_j^-D_i^v)x_{jik},\ \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^+(\tilde{D}_{ik}^h)^++V_i\tilde{Q}_j^+D_i^v)x_{jik}\right] \quad (8)$$

which is also an interval fuzzy number. By using the equality (4), we may prove the theorem.

$$\begin{cases} \min\quad f(x,\alpha) \\ =\inf\left\{\bar{f}\mid \mathrm{Cr}\left\{\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j\tilde{D}_{ik}^h+ V_i\tilde{Q}_jD_i^v)x_{jik}\le\bar{f}\right\}\ge\alpha\right\} \\ \text{s.t.}\ \sum_{i=1}^{I}\sum_{k=1}^{K_i}x_{jik}=1,\quad j=1,2,...,J \\ \sum_{j=1}^{J}x_{jik}\le 2,\ i=1,\ 2,...,\ I;\ k=1,\ 2,...,\ K_i \\ \sum_{l=1}^{L}y_{lik}\le 1,\ i=1,\ 2,...,\ I;\ k=1,\ 2,...,\ K_i \\ \sum_{j\in R_l}S_jx_{jik}\le y_{lik}C,\ i=1,\ 2,...,\ I, \\ \quad k=1,\ 2,...,K_i;\ l=1,\ 2,...,L \\ x_{jik}=0\text{ or }1,\ \forall\ i,\ j,\ k,\ y_{lik}=0\text{ or }1,\forall\ i,\ k,\ l. \end{cases} \quad (9)$$

In practice, we always treat fuzzy data as triangular fuzzy numbers or trapezoidal fuzzy numbers. Let $\xi=(a_1,\ a_2,\ a_3,\ a_4)$ be a trapezoidal fuzzy number. We can see that if $a_2=a_3$, the trapezoidal fuzzy number degenerates to a triangular fuzzy number. In the sense of this fact, triangular fuzzy number is a special case of trapezoidal fuzzy number. Let $\tilde{b}=[b_1,\ b_2]$ be a positive interval fuzzy number. Then we have

$$\tilde{a}\tilde{b}=(a_1b_1,\ a_2b_1,\ a_3b_2,\ a_4b_2) \quad (10)$$

$$\tilde{a}+\tilde{b}=(a_1+b_1,\ a_2+b_1,\ a_3+b_2,\ a_4+b_2). \quad (11)$$

Theorem 2 Let $\tilde{Q}_j=(\tilde{Q}_j^1,\ \tilde{Q}_j^2,\ \tilde{Q}_j^3,\ \tilde{Q}_j^4)$ and $\tilde{D}_{ik}^h=[(\tilde{D}_{ik}^h)^-,\ (\tilde{D}_{ik}^h)^+]$ *be trapezoidal fuzzy number and positive interval fuzzy number, respectively,* j=1, 2,…, J; i=1, 2,…, I; k=1, 2,…, K_i. *Then we have*

$$f(x,\ \alpha)=\begin{cases}2\alpha(r_2(x)-r_1(x))+r_1(x), & \text{if } 0<\alpha\le 0.5\\ (2\alpha-1)r_4(x)+(2-2\alpha)r_3(x), & \text{if } 0.5<\alpha\le 1,\end{cases} \quad (12)$$

where

$$r_1(x)=\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^1(\tilde{D}_{ik}^h)^-+V_i\tilde{Q}_j^1D_i^v)x_{jik} \quad (13)$$

$$r_2(x)=\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^2(\tilde{D}_{ik}^h)^-+V_i\tilde{Q}_j^2D_i^v)x_{jik} \quad (14)$$

$$r_3(x)=\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^3(\tilde{D}_{ik}^h)^++V_i\tilde{Q}_j^3D_i^v)x_{jik} \quad (15)$$

$$r_4(x)=\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^4(\tilde{D}_{ik}^h)^++V_i\tilde{Q}_j^4D_i^v)x_{jik} \quad (16)$$

Proof. By using the equality(10), we may obtain

$$\sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j\tilde{D}_{ik}^h+V_i\tilde{Q}_jD_i^v)x_{jik}=(r_1(x),\ r_2(x),\ r_3(x),\ r_4(x)). \quad (17)$$

Then by using the equality (5), we may prove the theorem.

Theorem 3 Let $\tilde{Q}_j = [\tilde{Q}_j^-, \tilde{Q}_j^+]$ and $\tilde{D}_{ik}^h = ((\tilde{D}_{ik}^h)^1, (\tilde{D}_{ik}^h)^2, (\tilde{D}_{ik}^h)^3, (\tilde{D}_{ik}^h)^4)$ *be positive interval fuzzy number and trapezoidal fuzzy number, respectively,* j=1, 2,…, J; i=1, 2,…, I; k=1, 2,…, K_i. *Then we have*

$$f(x, \alpha) = \begin{cases} 2\alpha(r_2(x) - r_1(x)) + r_1(x), & \text{if } 0<\alpha \le 0.5 \\ (2\alpha-1)r_4(x) + (2-2\alpha)r_3(x), & \text{if } 0.5<\alpha \le 1, \end{cases} \tag{18}$$

where

$$r_1(x) = \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^-(\tilde{D}_{ik}^h)^1 + V_i\tilde{Q}_j^- D_i^v)x_{jik} \tag{19}$$

$$r_2(x) = \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^-(\tilde{D}_{ik}^h)^2 + V_i\tilde{Q}_j^- D_i^v)x_{jik} \tag{20}$$

$$r_3(x) = \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^+(\tilde{D}_{ik}^h)^3 + V_i\tilde{Q}_j^+ D_i^v)x_{jik} \tag{21}$$

$$r_4(x) = \sum_{j=1}^{J}\sum_{i=1}^{I}\sum_{k=1}^{K_i}(H_j\tilde{Q}_j^+(\tilde{D}_{ik}^h)^4 + V_i\tilde{Q}_j^+ D_i^v)x_{jik} \tag{22}$$

Proof. By using equalities (10), (11) and (5), the theorem may be proven easily.

4. Algorithm Design

Since there are many fuzzy variables in this model, it is difficult for us to use analytic methods to achieve the best solution of this problem. Enlightened by the idea of using hybrid intelligent algorithm to solve uncertain programming problem proposed by Liu (2002), in this paper, we also design an algorithm, that is, tabu search algorithm based on simulation, to achieve an approximate best solution of this model. In fact, if the fuzzy variables in the problem satisfy the conditions in Theorems 1, 2 or 3, then we may compute the objective value directly for any feasible solution. Otherwise, we may employ fuzzy simulation technique to achieve the approximate objective value. Simulation technique can be referred to Liu (2002). In the following, we shall introduce the design of tabu search algorithm principally. Tabu search algorithm is a kind of artificial intelligence algorithm and an extension of local search algorithm. The modern version of tabu search algorithm is rooted in the work of Glover (1986), Glover (1989), Glover (1990), as a heuristic neighborhood search algorithm for solving complex optimization problems. Applying tabu search algorithm to a particular problem requires embedding in the algorithm a fair amount of relevant knowledge. So before solving our problem, we first consider relevant technical details in this problem.

(1) Solution Representation

For the convenience of representation, we use the number $\sum_{t=1}^{i-1} K_t + k$ to denote the index of the cell k of level i. In computer, the following array is used to represent a solution x:

$$\begin{pmatrix} 1 & 2 & 3 & \cdots & J \\ k_1 & k_2 & k_3 & \cdots & k_J \end{pmatrix} \tag{23}$$

where k_j is a random integer in interval $[1, \sum_{i=1}^{I} K_i]$. The meaning of k_j is that the items of type j will be placed in the cell k_j. If the number of item types in each cell k_j is no more than two, the number of item classes in each cell k_j is no more than one and capacity constraint of cell k_j is not violated, then this is a feasible solution.

(2) Neighborhood Structure

In this paper, the move of a solution x is defined by: only one element in the second row is changed. If k_j is changed to anther integer $k_j^{\prime}$ which is in $[1, \sum_{i=1}^{I} K_i]$, then we obtain a new solution $x^{\prime}$. When $x^{\prime}$ is feasible, the move is called a feasible move and $x^{\prime}$ is called a feasible neighbor. The neighborhood $N(x)$ of a feasible solution x contains all the feasible neighbors made by moves of x.

(3) Tabu Moves

If the cell placing the items of type j is changed from k_j to $k_j^{\prime}$ at iteration t. It will be forbidden to change $k_j^{\prime}$ to k_j again until iteration $t + v$ (where v is tabu tenure). Thus, at the present iteration, the tabu move is denoted by $(j, k_j^{\prime}, k_j)$, which implies forbidding $k_j^{\prime}$ to be changed into k_j.

(4) Aspiration Criterion

Assume that x^* is the best solution so far encountered. When a feasible neighbor x of the present solution made by a tabu move satisfies that the objective value of solution x is better than that of x^*, then solution x can be selected.

Now we shall summarize the procedure of tabu search algorithm as follows.

(5) Tabu Search Algorithm

Step 1. Initialize a feasible solution x randomly. Let *presentsol*=x, *bestsol*=x, then initialize tabu list;

Step 2. Produce the neighborhood *N*(*presentsol*) of *presentsol* in which neighbors are nontabu or satisfy aspiration criterion;

Step 3. Calculate the objective value of elements in *N*(*presentsol*) by simulation technique or analytic methods if possible;

Step 4. Find the best solution x^* in the neighborhood of *presentsol*;

Step 5. If the objective value of solution x^* is better than the objective value of *bestsol*, then let *presentsol*= x^*, *bestsol*= x^*; otherwise, let *presentsol*= x^*;

Step 6. Update the tabu list;

Step 7. Repeat the second to the sixth steps for a given number of times;

Step 8. Output *bestsol*, stop.

7. Numerical Example

In this section, we shall present a numerical example to show the application of the algorithm. Suppose that 15 kinds of items need to be placed in a two-level warehouse. The items are stored in different kinds of cubes and the same items are stored in the same kind of cubes. The lengths of the cubes are listed as follows (unit meter): 1.00, 1.10, 1.20, 1.25, 1.25, 1.45, 1.45, 1.50, 1.65, 1.70, 1.70, 1.75, 1.85, 1.95, 2.00. Here the *i*th number represents the length of the cubes containing the *i*th items.

The warehouse has two floors, and the available area of each floor is 72×24 square meters. According to the length of the cubes, each floor space is sub-divided into cells of the same dimension (8m×8m). Only 52 cells can be used to stack the items. Figure 2 is the sketch map of available cells.

Since we need an aisle (at least 4 meters width) between the storage cells for the clamp trucks movement, the actual area for storage in a cell is 6m×6m.

The horizontal distance between the center of each cell and exit is supposed to be an interval fuzzy variable. To estimate it, we use the following way: we first measure the distance according to the shortest rectilinear structure along the aisle, and get a corresponding value x, then the interval fuzzy distance is determined as $[x-2, x+2]$. The vertical distance is 10 meters.

We divide these cubes into 3 classes according to their length. The range of length of cubes in the first class is in [1.0, 1.2], the second class in (1.2, 1.5] and the third class in (1.5, 2.0]. Thus the items' indexes in the first class are {1, 2, 3}, in the second class are {4, 5, 6, 7, 8} and in the third class are {9, 10, 11, 12, 13, 14, 15}. Assume that these cubes will be placed in columns, each column has five cubes. So the capacity of each cell for the first class items is 25 columns, the second class items 16 columns and the third class items 9 columns. And horizontal transportation cost of per column for the first class is 0.23, the second 0.27 and the third 0.33. The vertical transportation cost of per column for the first class is 0.30, the second 0.32 and the third 0.35.

Inventory requirements (column) of items respectively are 18, 23, 9, 16, 12, 16, 9, 7, 8, 9, 5, 4, 9, 7, 9.

Table 1 gives the monthly demands of each item type, which are triangular fuzzy numbers.

In this problem, we set the credibility confident level $\alpha = 0.9$. In order to seek the optimal layout plan, we run tabu search algorithm in a personal computer. No matter what tabu tenure is employed, we always can obtain the same approximate optimal objective value. On the condition that tabu tenure=7, after 8 iterations, we get the approximate best solution, and the optimal objective value is 5804.76. The corresponding inventory strategy is shown in Figure 3 and Figure 4. The numbers in each cell represent that these types of items should be placed in this cell.

c_1^i	c_2^i	c_3^i	c_4^i	c_5^i	c_6^i	c_7^i	c_8^i	c_9^i
c_{10}^i	c_{11}^i	c_{12}^i	c_{13}^i	c_{14}^i	c_{15}^i	c_{16}^i	c_{17}^i	c_{18}^i
c_{19}^i	c_{20}^i	c_{21}^i	c_{22}^i	Exit	c_{23}^i	c_{24}^i	c_{25}^i	c_{26}^i

Figure 2 Sketch map of available cells on floor *i*

Table 1 Monthly demand (column)

Item Type	Monthly Demand	Item Type	Monthly Demand	Item Type	Monthly Demand
1	(45,50,55)	6	(85,90,95)	11	(38,40,42)
2	(25,30,35)	7	(45,50,55)	12	(47,50,53)
3	(75,80,85)	8	(55,60,65)	13	(75,82,83)
4	(75,80,85)	9	(68,71,80)	14	(75,82,88)
5	(55,60,65)	10	(75,78,80)	15	(57,59,63)

			4	5	6			
		11,12	13	14	15	2		
			9	Exit	10			

Figure 3 Storage map on the first floor

				7, 8				
			1	Exit	3			

Figure 4 Storage map on the second floor

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