MODELING JUMPS IN RETURNS OF FINANCIAL ASSETS AS M4 PROCESSES: MEASURED EXCHANGE RATE EXPOSURE OF ASIAN EQUITY PORTFOLIO

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Abstract

Previous work on the exposure of equity markets to exchange rate risk, surprisingly, found stock returns were not significantly affected by exchange rate fluctuations. In this paper, we examine the relation between China, Japan and USA MSCI (Morgan & Stanley Capital International) daily equity index returns and SAFE (State Administration of Foreign Exchange) exchange rate returns of Chinese RMB and Japanese Yen in US dollar. We find a significant relation between Asian foreign equity stock returns and Chinese RMB and Japanese Yen exchange rate returns. This article incorporates foreign exchange values as partial determinants of Asian foreign equity market returns and suggests that currency risk is of hedging concern to investors with implications for portfolio management. We implement our result in portfolio's CaR determination under VaR constraints.

Keywords: Exchange rate exposure, extreme value theory, M4 process

1. Introduction

An early analysis of exchange rate changes and share prices found that the general stock market index tended to move up (down) immediately after a devaluation (revaluation) of the local currency (Giddy 1974). It is surprising that previous researches (Jorion 1990; Amihud 1993; Bodnar and Gentry 1993) find that no contemporaneous relationship between exchange rate changes and excess equity market returns of largest U.S. exporting firms. Recently, it has argued that there is a significant contemporaneous correlation between exchange rate change and equity markets excess returns (Allaynnis 1996). Also, compared to investments in domestic assets, fluctuating exchange rates represent an additional risk factor for investors who want to diversify their portfolio internationally (He and Ng 1998; Aguiar 2001; Forbes 2002). Then, it is important to deal with a potential existence of exchange rate exposure of international equity stock portfolios by taking the viewpoint of international investors in China, Japan and USA.

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This can be by using extreme value theory for the dependence analysis between equity excess returns and exchange rate excess return falls for more accurate measure of risk.

In many circumstances, extremal observations appear to be clustered in time. Neither univariate nor multivariate extreme value theory is adequate to describe this kind of clustering of extreme events in a time series. By using M4 (multivariate maxima of moving maxima process) model, our goal is to model downside multivariate financial time series MSCI equity indexes returns and SAFE exchange rate returns, particularly jumps in returns, as M4 processes and then use the estimated model to determine exchange rate exposure of the equity portfolio composed of 3 markets: China Index, Japan Index and USA Index. Our contribution aims at providing tools for going one step further: is there a way of understanding so called downside exchange rate exposure of equity markets.

The paper proceeds in the following way. In Section 2, we present the theoretical foundations of exchange rate return and equity stock market return combination. Section 3 briefly describes the data used in the analysis and gives some important details on the SAFE Stock Exchange. We start by a short introduction of M4 model and then deal by the model selection and its estimation. We end by its application in Portfolio Capital-at-Risk determination in section 4. We evaluate also empirical arguments on the potential existence of exchange rate exposure of international equity stock portfolios by taking the viewpoint of international investors in China, Japan and USA. Section 5 provides a conclusion of our work.

2. Currency Market return and Equity Stock Market Return Combination

A foreign equity option is an option on a foreign asset where the strike price is specified in either domestic or foreign currency and the payoff at expiration is valued in domestic currency. To compute return on a foreign equity for a Chinese based investor (in perspective of dollar investor in China), let E_t represent the initial purchase price of the equity in foreign currency terms (in dollar). And let S_t represent the spot exchange rate, in RMB/FC terms (FC means foreign currency), on the purchase date. The product, $E_t S_t$ is the Chinese purchase prices of the foreign equity. After one period, the value of the equity is $\tilde{E}_{t+1}$ representing the initial equity price plus the price change over the period, $\tilde{\Delta}_{t+1}$, plus dividends, D_{t+1}. So, the value of the equity after one period in RMB terms is $\tilde{E}_{t+1}\tilde{S}_{t+1}$, where $\tilde{E}_{t+1} = E_t + \Delta_{t+1} + D_{t+1}$. The continuous rate of return on the foreign equity measured in RMB and on an unhedged basis is:

$$\begin{aligned} \tilde{R}_{\mathrm{RMB}} &= \ln\left(\tilde{E}_{t+1}\tilde{S}_{t+1} \big/ E_t S_t\right) \\ &= \ln\left(\tilde{E}_{\boldsymbol{t+1}} \big/ E_t\right) + \ln\left(\tilde{S}_{t+1} \big/ S_t\right) \\ &= \tilde{E}_{\mathrm{FC}} + \tilde{S}_{\mathrm{RMB,FC}} \end{aligned} \tag{1}$$

Equation (1) shows that the unhedged RMB return on the foreign equity has two pieces: the return on the equity shares in foreign currency terms plus the return on the foreign currency used to buy the shares. Both terms in equation (1) may be greater than or less than zero. The return on a foreign equity measured in RMB has an additional source of uncertainty, namely the foreign exchange gain or loss. The variance of the returns in equation (1) reflects the variance

Table 1 Currency market return and equity stock market return combination

		Currency Market Returns	
		Negative	Positive
Stock Market Returns	Negative	Equity Stock Market Prices ↓ Spot exchange rate FX ↓ (A)	Equity Stock Market Prices ↓ Spot exchange rate FX (D) ↑
	Positive	Equity Stock Market Prices ↑ Spot exchange rate FX ↓ (C)	Equity Stock Market Prices ↑ Spot exchange rate FX (B) ↑

of each term and the covariance between the returns on the foreign equity and the returns on spot foreign exchange (FX) or:

$$\sigma^2\left(\tilde{R}_{\text{RMB}}\right) = \sigma^2\left(\tilde{E}_{\text{FC}}\right) + \sigma^2\left(\tilde{S}_{\text{RMB,FC}}\right) + 2\,Cov\left(\tilde{E}_{\text{FC}};\tilde{S}_{\text{RMB,FC}}\right) \quad (2)$$

As a theoretical matter, the covariance term can be either positive or negative and represents the sensitivity of share returns to exchange rate change or adopts as a measure of exposure for international investors to exchange risk.

A positive covariance implies that the value of foreign equity tends to fall or rise along with the value of foreign currency as shown in cells A and B in table 1. A negative covariance implies that the value of foreign equity falls (rises) when foreign currency appreciate (depreciate) as shown in cells C and D in table 1.

As an empirical matter, correlation of spot exchange rate change and aggregate stock indexes was always considered fairly small by international investors. The upper view on Chinese RMB can be considered in the case Japanese Yen. We are interested in the falls co-movement (case (A) in the table 1) of these currencies exchange rate in U.S. dollar term and their respective unhedged foreign equity returns. The linkage in falls (covariance will be replaced by tail dependence parameter) of the currencies and foreign equity will be study by using the *multivariate maxima of moving maxima* process analysis, or M4 for short.

3. Multivariate Maxima of Moving Maxima Process

Suppose we have stationary multivariate time series X_{id}, where i is time and d indexes a component of the process. By transforming each X_{id} into a unit Fréchet random variable Y_{id}, the characterization of Y_{id} as max-stable processes was provided by Deheuvels (1983) and extended to M4 by Smith & Weissman (1996), with the practical representation

$$Y_{id} = \max_{1\le l\le L_d} \max_{-K_{1ld}\le l\le K_{2ld}} a_{l,k,d} Z_{l,i-k}, \quad -\infty < i < \infty, d = 1,\ldots,D, \quad (3)$$

where $\{Z_{l,i}, l = 1,2,\ldots,l_d, -\infty < i < \infty\}$ are independent unit Fréchet random variable, L_d (number of signatures pattern), K_{1l_d}, K_{2l_d} (moving range) are finite and $a_{l,k,d} \ge 0$ satisfy $\sum_{l=1}^{L_d} \sum_{K=-K_{1ld}}^{K_{2ld}} a_{l,k,d} = 1$, for each d.

3.1 Empirical Model Selection

We consider China, Japan and USA MSCI standard price returns from August 1998 to July 2003. Since standardization of data may remove the jumps in volatilities and transformation of data to unit Fréchet margins is a condition for M4 process modeling, they are applied in our analysis.

MSCI Equity Indices, calculated since 1969, are used around the world to measure the performance of international securities. It reflects approximately 80 percent of the world's equity market capitalization. Index level data is available from MSCI's web site at www.msci.com.

The exchange rate data are these of State Administration of Foreign Exchange (SAFE). Through these data the financial academics researchers argued the stability of RMB pegged to US dollar since 1994. From the original data we find the exchange of US dollar in Japanese Yen. The SAFE data set are provided by http://www.safe.gov.cn on daily RMB exchange rate.

In order to apply M4 modeling we need to determine the order of moving range i.e. K_{1l_d} and K_{2l_d} and the number of signature patterns Ld. This is the importance of empirical selection of the model. The order of moving range can be regarded as a measure of asymptotic dependence – or extremal dependence, tail dependence – among a sequence of random variables, while the number of signature patterns tells how many clustered extremal moving patterns existing in each process.

To determine the orders of moving ranges, the numbers of signature patterns, and the dependence structure, we study the empirical estimates of tail dependence indexes between random variables using empirical counts.

First, based on the properties that an M4 process appears to have clustered observations when an extreme observation occurs, we check those observed values that are larger than a certain threshold. We look at the counts of negative daily returns on the unit Fréchet scale in different ways. We count the days when the negative returns of a single stock were below a certain threshold on two or more consecutive days. We summarize the counts information within each range in Table 2. When a threshold value of 19.496 (equal to 95% standard Fréchet quantile) is used, we find that the maximal range of consecutive days from which the jumps in returns are over the threshold value are 5, 2, 3, 5, 4 days for China index, RMB exchange rate, Yen exchange rate, Japan index and USA index respectively. The dependence in series of data is high for USA equities data.

Second, the counts of days that two different returns are below the threshold value are summarized in Table 3. It clearly suggests tail dependencies between jumps in returns from any two different stocks. The number of days that at lest one the five returns are over the threshold value in any day is 367. No day where the five returns are simultaneously over the threshold. The dependence between pairs of series is high for the couples (China, RMB/USD), (Japan, Yen/USD) and (China, Japan). This means degree of lag-1 dependence between Asian equity stocks markets and their Exchange rate is not neglect.

Third, we count the days when two different stock negative returns both were below a certain threshold on two consecutive days. We summarize the counts information in Table 4. The lag-2 dependence is high for the couple (China, RMB/USD), (Japan, Yen/USD) and (USA, China). From table 2, 3, 4, the empirical estimates of lag-k tail of dependence indexes can be computed by dividing counts for each range by total number of days that exceedances occur. The observations with moving range over more than 2 days are a very small portion, so it is reasonable to use moving range of order 2 in our M4 process modeling. All the tables suggest that an M4 process fitting may be a good choice for the transformed returns data that have unit Fréchet scale. It might be safe to say that Table 2, 3, 4 suggest that a model of time dependence range of order 2 and at least 4 signature patterns for all five indexes data.

We fit five transformed time series data using the following model for each data:

$$Y_{id} = \max \begin{bmatrix} a_{1,0,d} Z_{1,i}, a_{1,1,d} Z_{1,i+1} \\ a_{2,0,d} Z_{2,i}, a_{2,1,d} Z_{2,i+1} \\ a_{3,0,d} Z_{3,i}, a_{3,1,d} Z_{3,i+1} \\ a_{4,0,d} Z_{4,i}, a_{4,1,d} Z_{4,i+1} \end{bmatrix}, \qquad (4)$$

$$d = 1, 2, 3, 4, 5$$

This model is simple one and more complicated structures can be adopted, see for instance Zhang (2002).

Table 2 Numbers of days, negative returns are below threshold value in consecutive days.

	1	2	3	4	5
China	65	21	8	3	1
RMB/USD	89	8			
Yen/USD	49	7	1		
Japan	70	14	7	3	1
USA	66	11	3	1	

Table 3 Number of day, two different negative returns are below the threshold value simultaneously

Range $k = 1$	China	RMB/USD	YEN/USD	Japan	USA
China	65	10	1	9	3
RMB/USD		89	1	5	2
YEN/USD			49	3	3
Japan				70	3
USA					66

Table 4 Number of day, two different negative returns are below the threshold value simultaneously during two days consecutives

Range $k = 2$	China	RMB/USD	YEN/USD	Japan	USA
China	8	1	1	0	1
RMB/USD		8	0	0	0
YEN/USD			7	0	0
Japan				3	0
USA					8

3.2 Model Estimation and Determination of Tail Dependence Index

We assume the marginal distribution is unit Fréchet, which can be by methods of univariate extreme value theory (Coles 2001). Based on the joint empirical distribution function, we estimate all parameters a_{lkd}. Following Zhang (Zhang and Smith 2004) and using the special M4 structure (4), these parameters can be estimated by using the follow system:

$$\begin{cases} \hat{b}_d\left(x_{jd}\right) = \sum_{l=1}^{L_d}\left[\hat{a}_{l,K_{2ld}} + \max\left(\hat{a}_{l,K_{2ld}-1}, \frac{\hat{a}_{l,K_{2ld},d}}{x_{jd}}\right) + \max\left(\hat{a}_{l,K_{2ld}-2}, \frac{\hat{a}_{l,K_{2ld}-1,d}}{x_{jd}}\right) + \max\left(\hat{a}_{l,K_{2ld}-3}, \frac{\hat{a}_{l,K_{2ld}-2,d}}{x_{jd}}\right) + \ldots + \max\left(\hat{a}_{l,-K_{2ld}}, \frac{\hat{a}_{l,-K_{1ld}+1,d}}{x_{jd}}\right) + \hat{a}_{l,-K_{1ld}}\right] \\ j=1,\ldots,8;\ d=1,\ldots,5. \\ \hat{b}_{1d'}\left(x'_{j'd'}\right) = \sum_{l=1}^{\max(L_1,l_{d'})} \sum_{m=1-\max(K_{2l1},K_{2ld'})}^{1+\max(K_{1l1},K_{1ld'})} \max\left(\hat{a}_{l,1-m,1}, \frac{\hat{a}_{l,1-m,d'}}{x'_{j'd'}}\right) \\ j'=1,\ldots,8;\ d'=2,\ldots,5. \end{cases}$$

The estimation processed by:

1. estimate marginal distributions by Generalized Pareto Distribution (GPD);
2. transform each marginal exceedance in standard Fréchet distribution;
3. select multivariate standard Fréchet distribution exceedances over a fixed standard Fréchet distribution quantile;
4. chose number of signature pattern L_d and the moving range K_{1ld}, k_{2ld};
5. estimate the M4 distribution parameters and determine tail dependence parameters.

Following the process describe above, we compute x_{jd} and $x'_{j'd'}$ listed in table 5(a):

and their corresponding $b_d(x)$ and $b_{1d'}(x)$ are computed by (Zhang 2004):

$$\hat{b}_d(x) = -\log\left(\frac{1}{n_u}\sum_{i=1}^{n_u} I_{\{Y_{id}\le u, Y_{i+1,d}\le u+x\}}\right), d = 1,\ldots,5$$

$$\hat{b}_{1d'}(x) = -\log\left(\frac{1}{n_u}\sum_{i=1}^{n_u} I_{\{Y_{i1}\le u, Y_{id'}\le u+x\}}\right), d' = 2,\ldots,5$$

where (n_u number of threshold u exceedances) The results b_d, $b_{1d'}$ are in table 5(b) and the estimated parameters of the model (4) are summarized in panel A of table 6.

Here, we deal with the determination of dependence tail index. Sibuya (1960) introduces the concept of the asymptotic independence of a bivariate random vector with identical marginal distribution. de Haan and Resnick (1977) extend it to case of multivariate variables and Zhang (2002) extends last one to lag-k tail dependencies among a sequence of random variables, defining that a sequence of sample $\{Y_1, Y_2, ..., Y_n\}$ is called lag-k tail dependent if

$$\lambda_k = \lim_{u \to x_F} \Pr(Y_1 > u \mid Y_{k+1} > u) > 0,$$
$$\lim_{u \to x_F} \Pr(Y_1 > u \mid Y_{k+j} > u) = 0, j > 1 \tag{5}$$
$$x_F = \sup\{x \in \mathbb{R} : \Pr(Y_1 \le x) < 1\}.$$

λ_k is called the lag-k tail dependence index. If $\lambda_k > 0$, then Y_1 and Y_2 are called tail dependent, otherwise the two random variables are called tail independent. Hence from (3) the extremal dependence index $\lambda_{dd'}$ between Y_{id} and $Y_{id'}$ is:

$$\lambda_{dd'} = 2 - \sum_{l=1}^{\max(L_d, L_{d'})} \sum_{m=m_1}^{m_2} \max(a_{l,1-m,d}, a_{l,1-m,d'})$$

$$m_1 = 1 - \max(K_{2ld}, K_{2ld'}); m_2 = 1 + \max(K_{1ld}, K_{1ld'}).$$

Similar $\{Y_{id}\}$ is lag-k tail dependent and its index is:

$$l\lambda_{dk} = 2 - \sum_{l=1}^{L_d} \sum_{m=1-K_{2ld}}^{1+k+K_{1ld}} \max(a_{l,1-m,d}, a_{l,1+k-m,d'})$$

where $k = \max_{1 \le l \le Ld} \left(K_{1ld} + K_{2ld}\right)$.

The computed lag-2 interval of tails dependence indexes between China negative returns and others negative returns are in table 6, panel B column 2. These numbers of lag-2 tail dependence means that China negative returns falls below the specified threshold value (95th percentile of the data was used) consecutively two day given that others negative returns have fall below the specified threshold value consecutively two days. These results are similar to their corresponding empirical tail indexes 1/8, 1/8, 0/8, 1/8 computed from tables 3, 4.

The computed lag-2 interval of tail dependence indexes among china, RMB/USD, Yen/USD, Japan, and USA negative returns series are on the diagonal of table 6, panel B. These numbers quantify tail indexes that China equity, RMB/USD, Yen/USD, Japan equity and USA equity negative returns falls below the specified threshold value (95th percentile of the data was used) are lag-2 dependent and these numbers are larger than their corresponding empirical tail indexes 8/65, 8/89, 7/49, 3/70, 8/66, computed from table 2. That means also there is more lag-2 tail dependence within each series that an empirical computation cannot reflect.

Table(a) Estimated value of X_{jd} and $X'_{j'd'}$ for d=1,...,5 and d'=2,...,5

X_{i1}	99.31	35.09	5.21	7.36	3241.3	1081	273.63	91.47
X_{i2}	31.18	92.50	5.12	12.37	95.00	285	11.14	32.23
X_{i3}	31.18	92.50	5.12	12.37	95.00	285	11.14	32.23
X_{i4}	3.71	3.65	77.35	231.50	373.00	1113	22.38	66.22
X_{i5}	75.89	227.48	920.40	276.17	217.70	653	4.74	9.42
$X'_{i'2}$	4.12	71.48	44.17	5.88	5.728	573	71.48	69.90
$X'_{i'3}$	362.97	326.40	82.59	412.19	52.20	121	1761.30	1741.69
$X'_{i'4}$	95.93	90.89	555.99	512.59	62.50	163	31.74	552.99
$X'_{i'5}$	273.16	256.27	8.25	10.79	29.9	300	65.74	41.93

Table 5(b) Estimated value of b_d and $b_{1d'}$ for d = 1,…,5 and d' = 2,…,5

b_{11}	0.215	0.253	0.322	0.307	0.198	0.198	0.208	0.215
b_{22}	0.341	0.318	0.485	0.372	0.318	0.289	0.372	0.341
b_{33}	0.188	0.150	0.229	0.229	0.150	0.134	0.229	0.185
b_{44}	0.376	0.376	0.246	0.222	0.218	0.212	0.296	0.249
b_{55}	0.232	0.212	0.208	0.202	0.212	0.208	0.341	0.307
b_{12}	0.836	0.467	0.516	0.836	0.376	0.376	0.467	0.467
b_{13}	0.380	0.388	0.442	0.380	0.489	0.412	0.368	0.368
b_{14}	0.400	0.400	0.360	0.360	0.412	0.392	0.489	0.360
b_{15}	0.372	0.372	0.632	0.587	0.467	0.372	0.421	0.437

Table 6 Estimations of parameters in model (4), standard errors in parentheses

	China Equity		RMB/USD		Yen/USD		Japan Equity		US Equity	
Signature	$a_{l,-1,1}$	$a_{l,0,1}$	$a_{l,-1,2}$	$a_{l,0,2}$	$a_{l,-1,3}$	$a_{l,0,3}$	$a_{l,-1,4}$	$a_{l,0,4}$	$a_{l,-1,5}$	$a_{l,0,5}$
Panel A										
1	0.680	0.074	0.261	0.019	0.155	0.025	0.160	0.001	0.200	0.016
	(0.008)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
2	0.003	0.138	0.107	0.133	0.068	0.075	0.072	0.063	0.090	0.098
	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
3	0.003	0.120	0.019	0.021	0.118	0.018	0.109	0.001	0.131	0.013
	(0.001)	(0.008)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
4	0.013	0.152	0.100	0.013	0.058	0.079	0.067	0.066	0.080	0.098
	(0.001	0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)	(0.001)
Panel B										
China	0.472									
RMB/USD	0.177		1.720							
YEN/USD	–0.287				0.323					
Japan	0.169						1.596			
USA	0.146								1.351	

Follow Smith (Smith 2003), we compute also the multivariate extremal index of the five markets

$$\theta(\tau_1,...,\tau_5)=\frac{\sum_l \max_k \max_d a_{l,k,d}\tau_d(n)}{\sum_l\sum_k \max_d a_{l,k,d}\tau_d(n)} \quad (6)$$

where $\tau_d(n)$ are sequences of threshold defined in Smith and Weisman(1996). As univariate stochastic processes, multivariate extremal index satisfies $0 \le \theta(\tau_1,...,\tau_5) \le 1$ for all $\tau_1,...,\tau_5$. $\theta = 0$, $\theta = 1$, correspond respectively to dependence and independence within series. We obtain that multivariate extremal index is 0.801. This means there is strong lag-2 tail dependence in series of five markets. The level 0.95 chosen in the analysis of our data in fact is lower than the real level given by $0.95^{0.801}$ = 0.96, which corresponds to the obtained result. To end ours dependence analysis, we are now particularly interested in computing a new extreme co-movement measure defined by Zhang and Smith (2004):

$$\lambda(t,T) = \lim_{u \to x_F} \Pr\left(\xi_{t,T,u} \ge 2 \mid \xi_{0,t,u} \ge 1\right) \qquad (7)$$

where $\xi_{s,T,u}$ a random variable corresponds to the maxima of maximal numbers of risk factors which have values beyond certain threshold over a time period e.i.

$$\xi_{s,t,u} = \max_{s \le i \le t} \sum_{d=1}^{5} I_{(y_{id} > u_d)} \qquad (8)$$

This measure quantifies the probability of the extreme co-movements in a future time period given the history that at least one stock had a large jump in price. For bivariate, when $t = T = 0$ (8) is the usual tail dependence function in literature (Embrechts, Lindskog and McNeil 2001). If it is bigger than 0 there is dependence between series otherwise no dependence.

We compute its empirical values based on the real data and compute the empirical extreme co-movement measures. We also compute the empirical extreme co-movement measure for transformed real data. The price downside moving history of the past 10, 20 trading days are used. We summarize the computed empirical results in Table 7. From Table 7, we see that change in each column elements, this phenomenon suggests that, the price history provides further useful information for extreme price movements by their maximum or minimum (in non transformed data) number of falling together under a certain threshold. For minimum of transformed data there are constant rate in their falling together. Then formula (7) is market linkage indicator and it can be estimated by the method of Hartman (Hartman, Straetmans and de Vries, 2001). We will not do the work here; it can be a research goal.

Does the RMB or Yen devaluation change the movement of the Asian equity markets? We answer yes, because there is positive dependence in time and in across markets. The situation like Mexican peso devaluation in late 1994 and early 1995, where capital flight and a loss in confidence in the Mexican economy brought the Mexican stock market down as well, can be in Asian equity markets if their currencies devaluate against USD. This was understood by different counties, which peg their currencies on USD, since Asian financial crisis in 1998.

Consider the topic of risk and return in international equity markets, the calculation of the unhedged returns on foreign equity in currency terms and portfolio risk in domestic and international stocks are crucial to estimate the diversification gains which has been the central motive for international investment.

Here, to measure the risk in international financial positions of exchange rate exposure, we evaluate the link between the equity market

Table7 Computed empirical values from (7) for 10, 20 days

T-t	10 days history data		20 days history data		10 days history transformed data		20 days history transformed data	
	Max	Min	Max	Min	Max	Min	Max	Min
1	0.990859	1.000923	0.985185	1.00188	1.000593	1.007565	1.000801	1.015235
2	0.990859	1.000923	0.985185	1.00094	1.001186	1.007565	1.001402	1.015235
3	0.990859	1.000923	0.985185	1.00094	1.001977	1.007565	1.002203	1.015235
4	0.991773	1.000923	0.985185	1.00094	1.002768	1.007565	1.002803	1.015235
5	0.991773	1.000923	0.985185	1.00094	1.003559	1.007565	1.003604	1.015235
6	0.991773	1.000000	0.986111	1.00094	1.004350	1.007565	1.004405	1.015235
7	0.991773	0.999076	0.986111	1.00094	1.004943	1.007565	1.005206	1.015235
8	0.991773	0.999076	0.986111	1.00094	1.005733	1.007565	1.006007	1.015235
9	0.991773	0.999076	0.986111	1.00094	1.006524	1.007565	1.006808	1.015235
10	0.991773	0.996306	0.987037	1.00094	1.007117	1.007565	1.007609	1.015235
11			0.987037	1.00094			1.00841	1.015235
12			0.987037	1.00094			1.009211	1.015235
13			0.987963	1.00094			1.010012	1.015235
14			0.987963	1.00094			1.010613	1.015235
15			0.987963	1.00094			1.011414	1.015235
16			0.987963	1.00094			1.012215	1.015235
17			0.987963	1.00000			1.013016	1.015235
18			0.987963	0.999058			1.013817	1.015235
19			0.987963	0.999058			1.014618	1.015235
20			0.987963	0.999058			1.015419	1.015235

excess return and financial price (say, spot exchange rate return). The covariance in the formula (2) is sensitivity measure, which is defined as a portfolio's exposure to foreign exchange rate. If the covariance is nonzero, an investment in a portfolio of equities carries an exposure to foreign exchange risk. The investment manager then must determine whether to retain the risk or to hedge it. More generally, an investment manager (or trader) can asses the sensitivity of an entire portfolio to exchange rate, interest rates, and other financial variables. The covariance is a measure to quantify the degree of dependence between risk factors. We adopt the tail dependence index define in (5) and the co-movement measure in (7) and dependence measure in the place of the covariance. These measures are more efficient than covariance in the dependence measure (Giorgio 2004). The results in table 2, 3, 4, 6 give more information on the exposure of Asian equity portfolio to exchange rate risk. It can see from these tables that each country's equity is sensitive to its currency movement. China Equity index is not sensitive to Japanese Yen exchange rate. But it is seen be that Japanese equity is less sensitive to the Japanese Yen. In order to get an insight into the risk of exchange rate exposure reduction, it is potential to deal with hedging on a multi-currency investment. This indicates that for a MSCI China and Japan investor there is some room left for risk reduction by hedging the exchange rate risk on a multi-currency portfolio. We will use the information funded above to measure the limit of the risk of the portfolio composed of these currencies and their corresponding equity markets.

4. Application of M4 in Portfolio's Capital-at-Risk Determination

Not putting all one's eggs in one basket has been a basic concept for a long history if one suspects the basket is not completely secured. In finance, portfolio diversification has been thought as an essential component of modern risk management. Minimize the risk will result in investing money on market variables with smaller risk.

On the other hand, an investor expects to gain the maximum possible returns with his investment. In general, the higher risk corresponds the higher the return. These two investment strategies, low risk and high return, are opposite to each other. Naturally an investor would seek an optimal investment plan with which either to maximize the portfolio mean return such that the estimated risk is not higher than an upper risk limit or to minimize the risk such that the mean return is not lower than a lower mean return limit within a given time period. This is referred as the portfolio problem in the literature.

In today's financial world, Value-at-Risk (VaR) has become the benchmark risk measure. Following the Basle Accord on the Market Risk (Basle Accord 2001) every bank, in more than 150 countries around the world has to calculate its risk exposure for every individual trading desk. The standard method prescribes: estimate the q-quantile of the profit/loss distribution for the next 10 days and q = 1% (or 5%) based on observations of at least 1 year (220 trading days).

A major reason for implementation of VaR methods is determination of capital requirements (CR). Financial regulators determine the CR according to the formula CR = 3* VaR + constant. Individual financial institutions estimate the VaR, from which the CR is calculated. It is well known that, if the banks underestimate the VaR they get penalized by an increase in the multiplicative factor or the additive constant. If, however, the financial institution over estimate the VaR, it presumably gets penalized the shareholders.

Therefore, the frequently asked question is: how does one estimate VaR from financial time series under realistic model assumptions and what is the consequence of VaR as a risk measure based on low or high quantile for portfolio optimization? Hence accurate estimation of the VaR is important. Extreme value (EV) theory method suggests that the factor should be close to one if EV is used for VaR (Danielsson and de Vries 2000). Here our goal is to use M4 estimated model to optimize the portfolio under VaR constraints.

Usually VaR was computed as the limit l such that $\Pr\{L_T \le l\} = q$, where L_T is the loss variable for the period of long T. Conversely, one may fix the limit l and compute the Capital-at-Risk CaR *(T, q, l)* as the amount which can be invested such that the pertaining loss L_T does not exceed the limit l with a given probability q. Here we will determine CaR as a downside risk measure in the place of the VaR and (2).

Let H_0 be an initial market strategy that determines the proportions of the different assets to each other and $P_{t,j}$ the single market price at the time t. Let again $V_{t,j} = H_{0,j}P_{t,j}$ and $V_t = \sum_j V_{t,j}$ be the market values of the single assets and of the portfolio with respect to H_0. The loss/profit variable $L_T = -(V_T - V_0)$ of the portfolio at time T is

$$\begin{aligned} L_T &= V_0 \sum_j w_j \left(1 - \exp\left(-R_{(T,j)}\right)\right) \\ &\approx V_0 \sum_j w_j R_{(T,j)} = V_0 w R'_{(T,j)} \end{aligned} \qquad (9)$$

where $R_{(T)} = (R_{(T,1)}, \ldots, R_{(T,d)})$ is the vector of random T-day log-returns (with changed sign) for the different asset and w a vector of weights which determines the market strategy of the investor at the beginning of the period. Following Reiss and Thomas (Reiss and Thomas, 2001), we compute a constant b such variable L_T for T, belonging to the multiple bH_0 of the initial market strategy, fulfills the equation

$$\Pr\{L_T \le l\} = \Pr\{b.ER \le l\} = q$$

$$ER = \sum_j V_{0,j} \left(1 - \exp\left(\sum_{t \le T} R_{t,j}\right)\right)$$

This holds with $b = l \big/ F^{-1}_{T,H_0}(q)$, where F_{T,H_0} is the distribution function of ER. Therefore

$$\begin{aligned} \mathrm{CaR}(T,q,l) &= bl \\ &= lV_0 \Big/ \left(1 - \exp\left(-F^{-1}_{T,H_0}(q)\right)\right) \end{aligned} \qquad (10)$$

is the Capital-at-Risk at the probability q. Here the difficulty is how to find the accurate distribution F_{T,H_0} .

The variance-covariance approach may under-estimate the risk a financial institution exposed to. So we model the portfolio returns by using a multivariate extreme value distribution function, especially in this present case we adopt M4 processes modeling to compute VaR and determine CaR.

Suppose w_1; w_2; w_3; w_4; w_5 are proportions of stock products in a portfolio, VaR_p is the VaR of the portfolio return $w_1Y_{i1} + w_2Y_{i2} + w_3Y_{i3} + w_4Y_{i4} + w_5Y_{i5}$, $i = 1, \ldots, n$, calculated from

$$\Pr\left(w_1Y_{i1} + w_2Y_{i2} + w_3Y_{i3} + w_4Y_{i4} + w_5Y_{i5} > \mathrm{VaR}_p\right) < \alpha$$

confidence level of 1 – α. = *q (initial value of q = 0.95).* Following Zhang (2002), we determine the $\mathrm{VaR}_p = d$ and individual risk factors $\mathrm{VaR}_1 = d_1$; $\mathrm{VaR}_2 = d_2$; $\mathrm{VaR}_3 = d_3$; $\mathrm{VaR}_4 = d_4$ *and* $\mathrm{VaR}_5 = d_5$ simultaneously by

$$\begin{cases} \max\limits_{d_1>0,d_2>0,d_3>0,d_4>0,d_5>0,} \\ \Pr\left(w_1 Y_{i1} > d_1, w_2 Y_{i2} > d_2, \right. \\ \quad \left. w_3 Y_{i3} > d_3, w_4 Y_{i4} > d_4, w_5 Y_{i5} > d_5\right) \\ \text{such that} \\ \Pr\left(w_1 Y_{i1} + w_2 Y_{i2} + w_3 Y_{i3} \right. \\ \quad \left. + w_4 Y_{i4} + w_5 Y_{i5} \ > \ d\right) \ < \ \alpha \\ \text{and } d = d_1 + d_2 + d_3 + d_4 + d_5 \end{cases} \quad (11)$$

The objective is thought to have the highest probability for all individual risk factors beyond certain values when the portfolio is at the VaR$_p$ and to determine d_1, d_2, d_3, d_4, d_5, and then find d to be unique. Considering the two days period of dependence on five years we find the results in table 8.

An alternative is the method of multivariate portfolio with weights w_i for asset i and correlation ρ_{ij} between assets negative return under VaR$_i$ and VaR$_j$, i , j = 1, …, p, proposed by Longin and Solnik (2001):

$$\mathrm{VaR}_P = \sqrt{\sum_{i,j=1}^{q} \rho_{ij} w_i w_j \mathrm{VaR}_i \mathrm{VaR}_j}. \qquad (12)$$

This method is also an extreme value method, but quadratic and used Pearson correlation of negative returns. We used it to get results in table 9.

The last method is not different to the general method of financial portfolio theory which consider that the higher the risk, the higher the return, because it is quadratic method (Embrechts, Lindskog and McNeil 2001). That means, as traditional risk measurement, for example, the variance-covariance approach; it may under-estimates or over-estimates the risk a financial institution exposed to. So the model we adopt should be more efficient in portfolio risk measure. The reason is that VaR$_p$ in table 8 (= 1.593) is lower than VaR$_p$ in Table 9 (2.703). That means the method using (12) may be under-estimate the risk and presumably penalize the financial institutions.

More, Figure1 (a) plots the expected return against VaR$_p$ and can be seen that when the VaR$_p$ is beyond certain value, the expected return is decreasing. This is different from the general portfolio theory based on the fact that increasing risk factor implies increase return. The figure 1 (b) shows the situation describing the quadratic portfolio implemented from the non-transformed data. It can be seen that, the higher VaR$_p$, the higher of the portfolio return. Therefore, there are a net difference between the graphs obtained from (11) and (12), but all both are extreme value theory methods. The quadratic assumption makes most statistical computations easier, but it may give inaccurate results if it doesn't fit the data and result it can imply a wrong decision. In risk measurement (12) may under estimate the risk as normal distribution, because, extreme falls in equity prices are often joint extremes, in the sense that big falls in equity prices are accompanied by simultaneous big falls in currencies exchange rates. Without loss of generality, we take l = V_0 = 1 in (8). Then the CaR for the five equity stock markets is computed from table 8 for daily data with jump two days range. The portfolio CaR, CaR$_p$, is 1.347 with standard deviation 0.107 for the transformed data using M4 method. The portfolio's CaR is constant when VaR is over t =1.593. That's means the expected shortfall of over VaR estimated will not change.

Table 8 VaR_p and risk factor's VaR, standards errors in parenthesis. Estimated $\hat{\alpha}$ is 0.001 less than 0.005; θ is lag-2 dependence tail index of the portfolio.

$1-\alpha$ =0.95	$VaR_p = d$	$VaR_1 = d_1$	$VaR_2 = d_2$	$VaR_3 = d_3$	$VaR_4 = d_4$	$VaR_5 = d_5$
$\theta = 0.801$	1.593	1.148	0.189	0.005	0.244	0.007
$\hat{\alpha}$ =0.001	(0.491)	(0.660)	(0.167)	(0.005)	(0.363)	(0.016)

Table 9 Estimation of portfolio VaR and the Risk factor's VaR by formula (12)

VaR from univariate	$VaR_p = d$	$VaR_1 = d_1$	$VaR_1 = d_2$	$VaR_3 = d_3$	$VaR_4 = d_4$	$VaR_5 = d_5$
	2.703(3.533)	0.779	4.733	2.546	1.033	1.307
Univariate tail index and the corresponding level		$\alpha_1 = 0.962$	$\alpha_2 = 0.962$	$\alpha_3 = 0.955$	$\alpha_4 = 0.954$	$\alpha_5 = 0.963$
		$\theta_1 = 0.963$	$\theta_2 = 0.74$	$\theta_3 = 0.900$	$\theta_4 = 0.913$	$\theta_5 = 0.732$

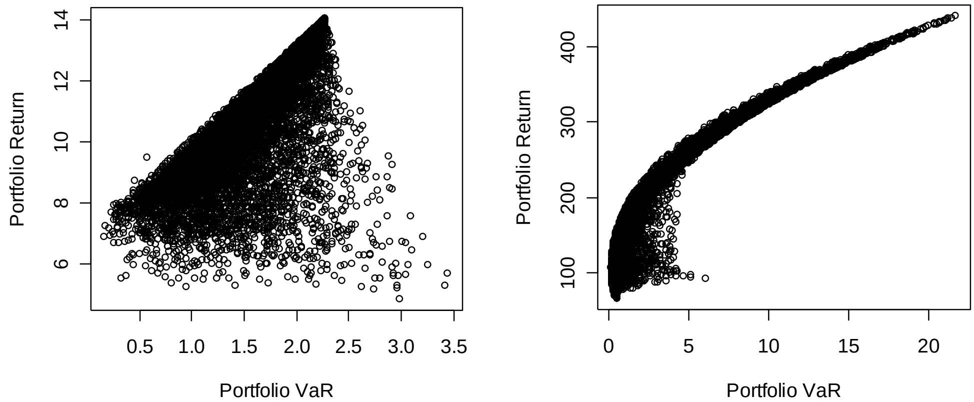

Figure 1 (a) Portfolio Optimization Using M4 Model (b) Portfolio Optimization Using Formula (12)

5. Discussions

The use of M4 processes to the modeling of financial time series data is new modeling method. It can efficiently model the extreme observations of multivariate time series that are both inter-serially and temporally dependent. Our estimation used only the function $b_1 d'(x)$. For more precision in estimation we need to consider functions $b_{dd'}(x)$ (Zhang and Smith 2004). The extremal co-movement measure $\lambda(t,T)$ is useful in risk management.

We compute VaR and optimize the portfolio under VaR constraints. And from the compute VaR we deduce the portfolio CaR. By comparing with traditional assumptions of normality of underlying distribution, these results provide more information to risk managers who may be most interested in the situation when an extreme downside price movement occurs.

Equity markets bring together buyers and sellers in a regulatory climate shaped by policymakers. While most countries permit both inward and outward investment, certain barriers remain that inhibit the flow of portfolio investments across borders. Some of these barriers are economic: exchange rate risk, transaction costs, and transaction taxes. It is evident that international investors have increased asset holdings in foreign countries, particularly in the fast growing Asian economies, for the purpose of enhancing profitability. However, this requires investors to effectively manage both equity price risk and exchange rate risk.

We examine the relation between China, Japan and USA MSCI (Morgan & Stanley Capital International) daily equity index returns and SAFE (State Administration of Foreign Exchange) exchange-rate changes of Chinese RMB, Japanese Yen respectively in U.S. dollar. We find a significant relation between stock returns and Chinese RMB, Japanese Yen exchange- rate fluctuations. This article incorporates foreign exchange values as partial determinants of equity returns. The currency exposure analyses suggests that currency risk is of hedging concern to investors with implications for corporate and portfolio management.

The co-movement of Japanese and Chinese foreign equities markets with their currency exchange rate against U.S. dollar is because they are pegging their currency to the U.S. dollar by intervening and buying dollars. Hence they are keeping their currencies artificially low. U.S. manufacturers contend that the Japanese Yen and Chinese RMB are undervalued, making their products cheaper for American consumers and U.S. products more costly for the Japanese and Chinese. That why the President of U.S. G. Bush said “China and Japan should stop intervening in the currency markets to give themselves an unfair trade advantage.” in an interview with Asian journalists during Asia-Pacific Economic Cooperation forum, which runs from October 20 to October 23 - 2003 in Bangkok (www.yahoo.com 15/10/2003). Although, our results suggest that, the interdependence of equity and currency markets in Asian countries becomes an important factor in determining exposure of exchange rate. And one of the major risks for Asia would be the sustainability of the US economic recovery. As the Asian equities dependent on the US market for export contracts, a growing US market may influences Asia economies. The currency exposure analyses, suggests that currency risk is of hedging concern to investors with implications for corporate and portfolio management. The construction of currency-hedged returns on foreign equities in Asian countries by extreme value theory analysis can be a future sight of research.

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