THE DEMAND DISRUPTION MANAGEMENT PROBLEM FOR A SUPPLY CHAIN SYSTEM WITH NONLINEAR DEMAND FUNCTIONS*

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Abstract

This paper addresses the problem of handling the uncertainty of demand in a one-supplier-one-retailer supply chain system. Demand variation often makes the real production different from what is originally planned, causing a deviation cost from the production plan. Assume the market demand is sensitive to the retail price in a nonlinear form, we show how to effectively handle the demand uncertainty in a supply chain, both for the case of centralized-decision-making system and the case of decentralized-decision-making system with perfect coordination.

Keywords: Supply chain management, disruption management, demand uncertainty

1. Introduction

Uncertainty plays an important role in the modern supply chain systems. Handling uncertainty in an efficient and effective way is becoming more and more important to the success of supply chain management. Traditionally, uncertainty is studied by stochastic models with some appropriate probability assumptions. In this paper, we present an alternative model, the so-called disruption management, to approach the demand uncertainty.

Generally speaking, disruption management studies the situation where an operational plan has to be made before the uncertainty is

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resolved, and deviation costs will be occurred for revising the operational plan in its execution period with the resolution of the uncertainty. In the supply chain model considered in this paper, a production plan (the quantity of the product) has to be made before the selling season based on market estimations while the real market demand is yet unknown. When the selling season comes, the real market demand is known. Then it may be necessary to adjust the production plan (producing more or less) to respond to the resolved market demand. Very often, we need to consider the case that the adjustment to the original production plan will cause deviation costs.

The supply chain model contains one supplier and one retailer, where the supplier produces one type of product and sells the product to the retailer by a wholesale price, the retailer then sells the product to an open market by a retail price. Both the supplier and the retailer are independent decision makers seeking for maximizing their individual profits. There are two cases of the sequence of decisions, static case and dynamic case. In both cases, we assume the supplier and the retailer have the same information about the demand.

The static case (deterministic demand) can be described as a Stackelberg game led by the supplier and followed by the retailer. In this context, the supplier first declares a wholesale policy; the retailer then decides the quantity to purchase from the supplier and the retail price to set. Finally, the supplier needs to produce the quantity that the retailer orders. Due to the well-known double marginalization phenomenon, if the retailer and the supplier make decisions independently, the total supply chain profit may be less than the case if there is a central decision maker who seeks to maximize the profit of the whole supply chain. Therefore, a coordination scheme is needed so that both the supplier and the retailer can make decisions in a cooperative way. While there are many possible coordination schemes, a commonly used way is by a wholesale quantity discount policy.

In the dynamic case, there is an additional step for the supplier to make a production plan before the demand is known. After the demand is known, the supplier and the retailer will play the above Stackelberg game. It should be noted that in this case, if the retailer places an order that is different from the production plan of the supplier, the supplier has to bear additional costs to this deviation, either producing more or handling the leftover inventory.

The purpose of demand disruption management studied here is to model and optimize the situation of the above dynamic case. The problem is originally studied by Qi et al. (2002(a)) where more detailed background and motivations of the model can be found. For related supply chain literatures, we refer to Jeuland and Shugan (1983), Weng (1995), Chen et al. (2001), Milner and Rosenblatt (2002), and Boyaci and Gallego (2002). In the model of Qi et al., they assume that the market demand is a downside linear function of the retail price, $d = D - kp$, where D is the maximum market scale, p is the retail price, k is a coefficient of price sensitivity, and d is the real demand under retail price p. While such a linear demand function is used in many literatures, another popular, or maybe more realistic, demand function is in a nonlinear form such as $d = Dp^{-2k}$ (k>=1)(For example, Weng 1995). As pointed by many researchers, the results obtained from a linear demand function may not be able to be directly applied to the case of a nonlinear demand function. To this reason, we will investigate the case of the nonlinear demand function of $d = Dp^{-2k}$ in this paper. We will show how and to what extent of the results of a linear model can be applied to the nonlinear case, and what new findings we may have.

Before going into technique details of the model, we would like to give a brief review of the researches on disruption management. The concept of disruption management appears in airline flight and crew scheduling problems (for examples, Thengvall et al. 2000, Clausen et al. 2001), and has been successfully applied for many years (Yu et al. 2002). Recently, people begin to study disruption management in many other areas, such as production planning (Yang et al. 2001), machine scheduling (Qi et al. 2002(b)), and supply chain management (Qi et al. 2002(a)). It is anticipated that disruption management is becoming a more and more important, both in theoretical researches and practical applications.

The rest of the paper is organized as follows. In Section 2 we formally define the model and discuss its coordination policy in the static case. The centralized decision making policy with demand disruptions will be given in Section 3. In Section 4, we consider coordination mechanisms under the case of de-centralized decision-making. We illustrate our results in Section 5 by some numerical examples. Section 6 concludes this paper.

2. Deterministic Model

We first introduce the supply chain model in which the price-demand function is deterministic and known. The supplier manufactures a product that is purchased by the retailer who then sells it to consumers. In the Stackelberg game, both players are independent decision makers seeking to maximize their individual profit where the supplier declares a wholesale policy first and the retailer makes his decisions of placing orders.

We assume the market demand Q (or the production quantity of the supplier) is a decreasing function of the retail price p such as $Q = Dp^{-2k}$ with $k \geq 1$, or equivalently

$$p = (\frac{D}{Q})^{1/2k}$$

Suppose the unit production cost of the supplier is a constant c. Then the profit of the supply chain can be written as

$$\bar{f}(Q) = Q[(\frac{D}{Q})^{1/2k} - c] \qquad (1)$$

It is easy to see that $\bar{f}(Q)$ is strictly concave over Q, so there must be a unique optimal point $\overline{Q}$ to maximize the supply chain's total profit. By the first order condition, we have that the supply chain profit is maximized at

$$\overline{Q} = D(\frac{2k-1}{2ck})^{2k}$$

with optimal retail price

$$\overline{p} = \frac{2ck}{2k-1}$$

and the maximum supply chain profit is

$$\bar{f}_{\max}^{SC} = \frac{Dc}{2k-1}(\frac{2k-1}{2ck})^{2k}$$

When the supply chain profit is maximized, how to share the maximum profit of the supply chain between the supplier and the retailer and how to make incentives to the retailer so that the retailer would like to place an order of $\bar{Q}$ and accordingly set the retail price to $\bar{p}$ are two important issues. The problem is called supply chain coordination. In this study, we introduce a wholesale quantity discount policy to coordinate the supply chain.

Suppose $\bar{f}^S$ is the profit that the supplier wants to earn. Then $\bar{f}^S$ can be written as $\bar{f}^S = \eta \bar{f}_{\max}^{SC}$ with $0 < \eta < 1$, where η is the ratio of total supply chain profit that the supplier takes. The following theorem indicates that the supplier can set an all-unit quantity discount to induce the retailer to place a "right" order and set a "right" retail price so that the supplier's goal of profit and the maximum supply chain profit can be achieved.

An all-unit quantity discount policy, denoted by $AQD(w_1,w_2,q_0)$ with $w_1>w_2$, works as follows. If the retailer orders $Q < q_0$, the unit wholesale price is w_1. If the retailer orders $Q \geq q_0$, the unit wholesale price becomes w_2.

Theorem 1 *For $\bar{f}^S = \eta \bar{f}_{\max}^{SC}$, $0 < \eta < 1$, the supply chain can be coordinated under the quantity discount policy $AQD(\bar{w}_1, \bar{w}_2, \bar{Q})$ where w_1 is large enough and*

$$\bar{w}_2 = c + \frac{c\eta}{2k-1}.$$

Proof. Suppose the retailer wants to take the wholesale price $\bar{w}_2$, then he needs to order no less than $\bar{Q}$ from the supplier. His profit can be written as

$$f_1^r(Q) = Q[(\frac{D}{Q})^{1/2k} - \bar{w}_2] \quad \text{with } Q \geq \bar{Q},$$

which is maximized at

$$Q_1 = D(\frac{2k-1}{2ck})^{2k}(\frac{c}{\bar{w}_2})^{2k} = \bar{Q}(\frac{c}{\bar{w}_2})^{2k} < \bar{Q}$$

So the retailer cannot maximize his profit by ordering Q_1 and taking the price $\bar{w}_2$. Because $f_1^r(Q)$ is a concave function and its feasible domain is $Q \geq \bar{Q}$, the retailer should order $\bar{Q}$ to maximize his profit if he wants to take the price $\bar{w}_2$, and his profit is

$$\begin{aligned} f_1^r(\bar{Q}) &= \bar{Q}[(\frac{D}{\bar{Q}})^{1/2k} - \bar{w}_2) \\ &= \bar{Q}[(\frac{D}{\bar{Q}})^{1/2k} - c - \eta\frac{c}{2k-1}] \\ &= \bar{f}_{\max}^{SC} - \eta\bar{Q}(\frac{2ck}{2k-1} - c) \\ &= (1-\eta)\bar{f}_{\max}^{SC} \end{aligned}$$

If the retailer orders less than $\bar{Q}$ and takes the unit wholesale price $\bar{w}_1$, his profit can be written as

$$f_2^r(Q) = Q[(\frac{D}{Q})^{1/2k} - \bar{w}_1] \quad \text{with } Q < \bar{Q},$$

which is maximized at

$$Q_2 = \bar{Q}(\frac{c}{\bar{w}_1})^{2k}.$$

With a sufficient large $\bar{w}_1$, we have that

$$f_2^r(Q_2) = \bar{Q}(\frac{c}{\bar{w}_1})^{2k}[(\frac{D}{\bar{Q}})^{1/2k}(\frac{\bar{w}_1}{c}) - \bar{w}_1] < f_1^r(\bar{Q})$$

Therefore the retailer would like to order $\bar{Q}$ to maximize his own profit. As a result, the supplier's desired profit is achieved and the maximum supply chain profit is also obtained.

■

We now use the following analysis to

illustrate the importance of supply chain coordination. To do so, we suppose the supplier uses a linear wholesale policy with the unit wholesale price of w. Then the retailer's profit function can be written as

$$f^r(p) = (p-w)Dp^{-2k} \quad (2)$$

and the supplier's profit function is

$$f^S(w) = (w-c)Dp^{-2k} \quad (3)$$

In the Stackelberg game, the supplier can determine his optimal wholesale price w assuming that he knows the retailer's optimal reaction with a given w.

For (2), using the first order condition $(f^r(p))' = 0$, we get the optimal retail price

$$\tilde{p} = \frac{2wk}{2k-1}$$

and the optimal ordering quantity

$$\tilde{Q} = D(\frac{2k-1}{2wk})^{2k}$$

Thus $f^S(w)$ becomes

$$f^S(w) = (w-c)D(\frac{2k-1}{2wk})^{2k} \quad (4)$$

According to the retailer's pricing policy, the supplier can decide the wholesale price to maximize his own profit in (4). Similarly, it can be shown (4) is maximized at $\tilde{w} = \frac{2ck}{2k-1}$

As a consequence, the real retail price and the order quantity are respectively $\tilde{p} = c(\frac{2k}{2k-1})^2$ and $\tilde{Q} = Dc^{2k}(\frac{2k-1}{2ck})^{4k}$, and the maximum profits of the retailer and the supplier are respectively

$$\tilde{f}^r = \frac{Dc^{2k}}{2k-1}\left(\frac{2k-1}{2ck}\right)^{4k-1},$$

$$\tilde{f}^S = \frac{Dc^{2k+1}}{2k-1}\left(\frac{2k-1}{2ck}\right)^{4k},$$

and the total profit of the supply chain is

$$\tilde{f} = \frac{(4k-1)Dc^{2k}}{2k(2k-1)}\left(\frac{2k-1}{2ck}\right)^{4k-1}$$

which can be shown less than the maximum profit $\bar{f}^{SC}_{\max}$ under the coordination mechanism $AQD(w_1, w_2, q_0)$. This indicates the importance of supply chain coordination.

3. Centralized Decision Making with Demand Disruptions

In Section 2, we discuss the supply chain model as a static model where the demand function is assumed deterministic and accurately known. In reality, the real demand function may be different from what has been estimated in the planning period. In this section, we consider the case of market scale change, where the resolved market scale becomes $D + \Delta D$, with a positive ΔD representing an increased market, and a negative ΔD representing a decreased market. Thus the demand function becomes $d = (D + \Delta D)p^{-2k}$ with $D + \Delta D > 0$.

We start with the discussion on the case that there is a central decision maker in the supply chain who seeks to maximize the total profit of the supply chain. Let Q be the real demand under the new demand function with probably a new retail price p, where

$$p = (\frac{D+\Delta D}{Q})^{1/2k}$$

Because of the variety of market scale, there may be a production deviation $\Delta Q = Q - \overline{Q}$. If $\Delta Q < 0$, there is some leftover inventory which has to be sold to a secondary market with a very low price. If $\Delta Q > 0$, more products than the planned $\overline{Q}$ have to be produced to meet the unplanned demand increase. In either case, the demand disruption will cause disruptions to the original production plan and certain extra costs and efforts beyond the planned resources may be required, and should be included when making the new price and production decisions.

The supply chain profit with the production deviation cost considered under demand disruption can be written as

$$f(Q) = Q[(\frac{D+\Delta D}{Q})^{1/2k} - c] - \lambda_1(Q-\overline{Q})^+ - \lambda_2(\overline{Q}-Q)^+ \quad (5)$$

where $\lambda_1, \lambda_2 > 0$ are marginal extra costs of increased and decreased production from the original plan respectively, and

$$(x)^+ = \max\{x, 0\}.$$

For their practical meanings, λ_1 is the unit extra production cost more than what has been planned, and λ_2 is the unit cost of handling the leftover inventory less than what has been planned. We assume $\lambda_2 < c$, i.e., the leftover inventory can be sold on a secondary market and has some salvage value.

In order to investigate how demand disruptions influence production plan, we have the following lemma.

Lemma 1 *Suppose f (Q) in (5) is maximized at an optimal ordering quantity* Q^*. *Then*

1)when $\Delta D > 0$, *we have* $Q^* \geq \overline{Q}$, *and*

2)when $\Delta D < 0$, *we have* $Q^* \leq \overline{Q}$.

Lemma 1 says that when the market scale increases, the production quantity cannot be decreased; when the market scale decreases, the production quantity cannot be increased. This is consistent with the common intuition. The proof is omitted because it is similar to the analysis in Qi et al. (2002(a)).

From Lemma 1, when $\Delta D > 0$, we can reduce the objective function of the problem to

$$f_1(Q) = Q[(\frac{D+\Delta D}{Q})^{1/2k} - c] - \lambda_1(Q-\overline{Q}) \quad (6)$$

with the constraint of $Q \geq \overline{Q}$. Using the first order condition $f_1'(Q) = 0$, we have

$$Q_1 = (D+\Delta D)\left(\frac{2k-1}{2(c+\lambda_1)k}\right)^{2k} \quad (7)$$

For $f_1(Q)$ in (6), we see that it is strictly concave with the constrain of $Q \geq \overline{Q}$. Therefore, if $Q_1 \geq \overline{Q}$, or equivalently,

$$\Delta D \geq D\left[(1+\frac{\lambda_1}{c})^{2k} - 1\right]$$

$f_1(Q)$ in (6) is maximized at Q_1; if $Q_1 \leq \overline{Q}$, or

$$0 < \Delta D \leq D\left[(1+\frac{\lambda_1}{c})^{2k} - 1\right]$$

$f_1(Q)$ is maximized at $\overline{Q}$ because of concavity.

Therefore when $\Delta D > 0$ we have two cases,

Case 1. $\Delta D > D\left[\left(1+\frac{\lambda_1}{c}\right)^{2k} - 1\right]$, and

Case 2. $0 < \Delta D \leq D\left[\left(1+\frac{\lambda_1}{c}\right)^{2k} - 1\right]$

The optimal production quantity is

$$Q^* = \begin{cases} Q_1^* = (D+\Delta D)\left(\frac{2k-1}{2(c+\lambda_1)k}\right)^{2k}, & \text{for Case 1} \\ Q_2^* = D\left(\frac{2k-1}{2ck}\right)^{2k}, & \text{for Case 2} \end{cases}$$

We see that the original plan should not increase unless the increase of the market scale ΔD is large enough. On the contrary, the optimal retail price p^* always has a chance to increase as long as the market scale increases.

For Case 1, the optimal retail price is

$$p_1^* = \frac{2(c+\lambda_1)k}{2k-1} = \overline{p} + \frac{2\lambda_1 k}{2k-1},$$

and the maximum total profit of the supply chain is

$$f_1^* = \frac{(D+\Delta D)(c+\lambda_1)}{2k-1}\left(\frac{2k-1}{2(c+\lambda_1)k}\right)^{2k} + D\lambda_1\left(\frac{2k-1}{2ck}\right)^{2k}$$

For Case 2, the optimal retail price is

$$p_2^* = \frac{2ck}{2k-1}\left(1+\frac{\Delta D}{D}\right)^{1/2k} = \overline{p}\left(1+\frac{\Delta D}{D}\right)^{1/2k}$$

and the maximum supply chain profit is

$$f_2^* = D\left(\frac{2k-1}{2ck}\right)^{2k}\left[\left(1+\frac{\Delta D}{D}\right)^{1/2k}\frac{2ck}{2k-1} - c\right]$$

Now we consider the case of a negative market change. When $\Delta D < 0$, the objective function is reduced to

$$f_2(Q) = Q[(\frac{D+\Delta D}{Q})^{1/2k} - c] - \lambda_2(\overline{Q} - Q) \quad (8)$$

with the constraint of $Q \le \overline{Q}$. Similarly, we have two cases,

Case 3: $0 > \Delta D > D\left[\left(1-\frac{\lambda_2}{c}\right)^{2k} - 1\right]$, and

Case 4: $\Delta D \le D\left[\left(1-\frac{\lambda_2}{c}\right)^{2k} - 1\right]$.

Using the first order condition $f_2'(Q) = 0$, we have the solution

$$Q_2 = (D+\Delta D)\left(\frac{2k-1}{2(c-\lambda_2)k}\right)^{2k} \quad (9)$$

Thus when $\Delta D < 0$, the supply chain is maximized at

$$Q^* = \begin{cases} Q_3^* = D\left(\frac{2k-1}{2ck}\right)^{2k}, & \text{for Case 3} \\ Q_4^* = (D+\Delta D)\left(\frac{2k-1}{2(c-\lambda_2)}\right)^{2k}, & \text{for Case 4} \end{cases}$$

For Case 3, the optimal retail price is

$$p_3^* = \frac{2ck}{2k-1}\left(1+\frac{\Delta D}{D}\right)^{1/2k} = \overline{p}\left(1+\frac{\Delta D}{D}\right)^{1/2k}$$

and the maximum profit of the supply chain is

$$f_3^* = D\left(\frac{2k-1}{2ck}\right)^{2k}\left[\left(1+\frac{\Delta D}{D}\right)^{1/2k}\frac{2ck}{2k-1} - c\right]$$

For Case 4, the optimal retail price is

$$p_4^* = \frac{2(c-\lambda_2)k}{2k-1} = \overline{p} - \frac{2\lambda_2 k}{2k-1},$$

and the maximum supply chain profit is

$$f_4^* = \frac{(D+\Delta D)(c-\lambda_2)}{2k-1}\left(\frac{2k-1}{2(c-\lambda_2)k}\right)^{2k} - D\lambda_2\left(\frac{2k-1}{2ck}\right)^{2k}$$

Note that for the four cases above, the retail prices are always positive, implying they are feasible in practice. But it cannot guarantee that the retail prices are larger than the unit production cost c for Case 3 and Case 4, implying that the total supply chain profit may be negative if the market scale becomes too small. For Case 3, $p_3^* > c$ is equivalent to

$$\Delta D > D[\left(\frac{2k-1}{2k}\right)^{2k} - 1].$$

If $c \geq 2\lambda_2 k$, the supply chain profit is positive; if $\lambda_2 < c < 2\lambda_2 k$, the profit of the supply chain is negative if and only if

$$\Delta D < D[\left(\frac{2k-1}{2k}\right)^{2k} - 1]$$

For Case 4, it derives from $p_4^* > c$ that $c > 2\lambda_2 k$.

Summarize the above results, we have

Theorem 2 *When the stated demand-price relationship is* $d = (D + \Delta D)p^{-2k}$ *, the total supply chain profit is maximized at*

$$Q^* = \begin{cases} (D+\Delta D)\left(\dfrac{2k-1}{2(c+\lambda_1)}\right)^{2k}, & \text{if} \quad \Delta D > D\left[\left(1+\dfrac{\lambda_1}{c}\right)^{2k} - 1\right] \\ D\left(\dfrac{2k-1}{2ck}\right)^{2k}, & \text{if} \quad D\left[\left(1-\dfrac{\lambda_2}{c}\right)^{2k} - 1\right] \leq \Delta D \leq D\left[\left(1+\dfrac{\lambda_1}{c}\right)^{2k} - 1\right] \\ (D+\Delta D)\left(\dfrac{2k-1}{2(c-\lambda_2)}\right)^{2k}, & \text{if} \quad \Delta D < D\left[\left(1-\dfrac{\lambda_2}{c}\right)^{2k} - 1\right] \end{cases}$$

with the optimal retail price

$$p^* = \begin{cases} \dfrac{2(c+\lambda_1)k}{2k-1}, & \text{if} \quad \Delta D > D\left[\left(1+\dfrac{\lambda_1}{c}\right)^{2k} - 1\right] \\ \dfrac{2ck}{2k-1}\left(1+\dfrac{\Delta D}{D}\right)^{1/2k}, & \text{if} \quad D\left[\left(1-\dfrac{\lambda_2}{c}\right)^{2k} - 1\right] \leq \Delta D \leq D\left[\left(1+\dfrac{\lambda_1}{c}\right)^{2k} - 1\right] \\ \dfrac{2(c-\lambda_2)k}{2k-1}, & \text{if} \quad \Delta D < \left[\left(1-\dfrac{\lambda_2}{c}\right)^{2k} - 1\right] \end{cases}$$

Theorem 2 indicates how to correctly respond to a demand disruption. (1) When the market scale change is small, keeping the original production plan $\overline{Q}$ and revising the retailing price is optimal. This shows that the original production plan has certain robustness under the variable market scale. (2) When the market scale changes large enough, adjusting both production quantity and the retail price becomes necessary. However, though the production quantity change is proportional to the market scale change ΔD, the retail price change is a constant independent to ΔD. For the case of a positive ΔD, this explains the practical phenomenon that the retail price cannot become arbitrarily high no matter how hot a product is. For the case of a negative ΔD, this is consistent with the fact that setting a too low retail price is less profitable than selling the product in a secondary market.

At the end of this section, we make a brief comparison between the model of a linear demand function (Qi et al. 2002(a)) and our model of a nonlinear demand function. We have some common findings such as the optimal solutions for both models have the same structure (the market scale change is categorized into four cases according to its magnitude), and the production plan has certain robustness when the disruption is small. On the other hand, the nonlinear model does reveal some different results, an interesting one of which is that the retail price cannot become arbitrarily high enough.

4. Coordinating the Supply Chain under Demand Disruptions

In this section, we discuss how the supplier should devise a new scheme (a new quantity discount policy) to achieve the supply chain coordination for the disrupted demand. Accordingly, there are four cases.

4.1 Case 1: $\Delta D \geq D\left[\left(1+\frac{\lambda_1}{c}\right)^{2k}-1\right]$

Suppose the profit that the supplier wants to earn is f^S. Recall that

$$f_1^* = \frac{(D+\Delta D)(c+\lambda_1)}{2k-1}\left(\frac{2k-1}{2(c+\lambda_1)k}\right)^{2k} + D\lambda_1\left(\frac{2k-1}{2ck}\right)^{2k}$$

We will specify two cases:

$$f^S \geq \lambda_1 \overline{Q} = \lambda_1 D\left(\frac{2k-1}{2ck}\right)^{2k} \text{ and } f^S < \lambda_1 \overline{Q}.$$

Case4.1.1 $f^S \geq \lambda_1 \overline{Q} = \lambda_1 D\left(\frac{2k-1}{2ck}\right)^{2k}$

Then f^S can be written as

$$f^S = \eta\frac{(D+\Delta D)(c+\lambda_1)}{2k-1}\left(\frac{2k-1}{2(c+\lambda_1)}\right)^{2k} + \lambda_1 D\left(\frac{2k-1}{2ck}\right)^{2k} \tag{10}$$

where $0 \leq \eta < 1$.

Theorem 3 *When* $\Delta D \geq D\left[\left(1+\frac{\lambda_1}{c}\right)^{2k}-1\right]$ *and*

$$f^S \geq \lambda_1 D\left(\frac{2k-1}{2ck}\right)^{2k},$$

the supply chain can be coordinated by the all-unit quantity discount policy $AQD(w_1, w_2, Q_1^*)$ *where* w_1 *is large enough and*

$$w_2 = c + \lambda_1 + \eta\frac{c+\lambda_1}{2k-1}$$

Proof. If the retailer takes the wholesale price w_2, he should order more than Q_1^* from the supplier and his profit can be written as

$$f_1^r(Q) = Q[(\frac{D+\Delta D}{Q})^{1/2k} - w_2]$$

which is maximized at

$$Q_1 = (D+\Delta D)(\frac{2k-1}{2w_2 k})^{2k}$$

It can be shown that $Q_1 < Q_1^*$, so the retailer can't maximize his profit by ordering Q_1 and taking the price w_2. By concavity of $f_1^r(Q)$, the retailer would like to order Q_1^* if he takes the price w_2, and his profit is

$$\begin{aligned} f_1^r(Q_1^*) &= Q_1^*[(\frac{D+\Delta D}{Q_1^*})^{1/2k} - w_2) \\ &= Q_1^*[(\frac{D+\Delta D}{Q_1^*})^{1/2k} - (c+\lambda_1)(1+\frac{\eta}{2k-1})] \\ &= (1-\eta)\frac{(D+\Delta D)(c+\lambda_1)}{2k-1}(\frac{2k-1}{2(c+\lambda_1)k})^{2k} \end{aligned}$$

If the retailer orders less than Q_1^*, he has to take the wholesale price w_1 and his profit can be written as

$$f_2^r(Q) = Q[(\frac{D+\Delta D}{Q})^{1/2k} - w_1]$$

with the constraint of $Q < \overline{Q}$, which is maximized at

$$Q_2 = (D+\Delta D)(\frac{2k-1}{2w_1 k})^{2k}$$

Since w_1 is large enough, it is easy to know that

$$f_2^r(Q_2) = Q_2[(\frac{D+\Delta D}{Q_2})^{1/2k} - w_1] < f_1^r(Q_1^*)$$

Therefore the retailer would like to order Q_1^* to maximize his own profit. As a result, the supplier's desired profit and the maximum supply chain profit are achieved.

We have given the detailed proof of Theorem 3, which shows the general idea of designing a coordination scheme under demand disruptions. For the following cases, we will only present the corresponding theorems and omit the proofs because their proofs take very similar approaches.

Case 4.1.2 $f^S < \lambda_1 \overline{Q}$.

Then f^S can be written as

$$f^S = \eta \lambda_1 D\left(\frac{2k-1}{2ck}\right)^{2k} \text{ where } 0 < \eta < 1 .$$

For this case, we need to introduce another wholesale price policy, the capacitated linear price, denoted by CLP (*w, q*). A CLP works as follows. The supplier charges the retailer a constant unit wholesale price *w*, but the retailer cannot place an order more than *q*.

Theorem 4 *When* $f^S < \lambda_1 \overline{Q}$ *and*

$$f^S = \eta \lambda_1 D\left(\frac{2k-1}{2ck}\right)^{2k},$$

1)if $\eta \ge 1 - \frac{c+\lambda_1}{2k-1}\left(1+\frac{\Delta D}{D}\right)\left(\frac{c}{c+\lambda_1}\right)^{2k}$,

the supply chain can be coordinated by the policy $AQD(w_1, w_2, Q_1^*)$ *, and*

2)if $\eta < 1 - \frac{(c+\lambda_1)}{2k-1}\left(1+\frac{\Delta D}{D}\right)\left(\frac{c}{c+\lambda_1}\right)^{2k}$,

the supply chain can be coordinated by the policy $CLP(w_2, Q_1^*)$ *, where* w_1 *is large enough and*

$$w_2 = \frac{2(c+\lambda_1)k}{2k-1} - (1-\eta)\frac{D\lambda_1}{D+\Delta D}\left(1+\frac{\lambda_1}{c}\right)^{2k}.$$

4.2 Case 2: $0 < \Delta D < D\left[\left(1+\frac{\lambda_1}{c}\right)^{2k} - 1\right]$

Suppose the profit that the supplier wants to earn is

$$f^S = \eta D\left(\frac{2k-1}{2ck}\right)^{2k}[\left(1+\frac{\Delta D}{D}\right)^{1/2k}\frac{2ck}{2k-1} - c]$$
$$0 < \eta < 1 \tag{11}$$

Then we have the following theorem.

Theorem 5 *For the case*

$$0 < \Delta D < D\left[\left(1+\frac{\lambda_1}{c}\right)^{2k} - 1\right]$$

and f^S *defined in (11), we have*

1)when $\eta > \dfrac{(1+\frac{\Delta D}{D})^{1/2k} - 1}{(1+\frac{\Delta D}{D})^{1/2k}\frac{2k}{2k-1} - 1}$,

the supply chain can be coordinated by the policy $AQD(w_1, w_2, \overline{Q})$ *;*

2)when $\eta \le \dfrac{(1+\frac{\Delta D}{D})^{1/2k} - 1}{(1+\frac{\Delta D}{D})^{1/2k}\frac{2k}{2k-1} - 1}$, *the supply chain can't be coordinated by the policy $AQD(w_1, w_2, \overline{Q})$, but it can be coordinated by the policy $CLP(w_2, \overline{Q})$, where w_1 is large enough and*

$$w_2 = c + c\eta[\left(1+\frac{\Delta D}{D}\right)^{1/2k} \frac{2k}{2k-1} - 1].$$

4.3 Case 3: $0 > \Delta D > D\left[\left(1-\frac{\lambda_2}{c}\right)^{2k} - 1\right]$

For this case, the total supply chain profit is positive only when

$$\Delta D > D[\left(1-\frac{1}{2k}\right)^{2k} - 1].$$

And we know that, if $c \ge 2\lambda_2 k$, the total profit of the supply chain will be always positive. When $\lambda_2 < c < 2\lambda_2 k$, the profit of the supply chain will be positive if and only if

$$\Delta D > D[\left(1-\frac{1}{2k}\right)^{2k} - 1].$$

First we investigate the case when the profit of the supply chain is positive, i.e. $c \ge 2\lambda_2 k$, or $\lambda_2 < c < 2\lambda_2 k$ but

$$\Delta D > D[\left(1-\frac{1}{2k}\right)^{2k} - 1].$$

Suppose the profit that the supplier wants to earn is

$$f^S = \eta D\left(\frac{2k-1}{2ck}\right)^{2k} [\left(1+\frac{\Delta D}{D}\right)^{1/2k} \frac{2ck}{2k-1} - c]$$
$$0 < \eta < 1 \tag{12}$$

Similar to Theorem 5, we have the following results.

Theorem 6 *For the case*

$$0 > \Delta D > D[\left(1-\frac{\lambda_2}{c}\right)^{2k} - 1]$$

and f^S defined in (12), the supply chain can be always coordinated by the policy $AQD(w_1, w_2, \overline{Q})$, where w_1 is large enough and

$$w_2 = c + \eta[(1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} - c].$$

For the case of

$$D[\left(1-\frac{1}{2k}\right)^{2k} - 1] > \Delta D > D[\left(1-\frac{\lambda_2}{c}\right)^{2k} - 1],$$

the coordination policy in Theorem 6 doesn't work because

$$p_3^* - w_2 = (1-\eta)[(1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} - c] < 0.$$

In this case, the retailer would order nothing because he cannot make any profit. If the retailer does so, the supplier will have to put all his planned production $\overline{Q}$ to the secondary market with a negative profit of $-\lambda_2 \overline{Q}$. In order to reduce his own loss, the supplier would like to take certain coordination policy and give the retailer some incentives so that the retailer can make some profit and then the supplier can reduce his own loss too.

Consider the wholesale pricing from the retailer's perspective. Suppose the desired profit of the retailer is

$$f^r = -\mu D\left(\frac{2k-1}{2ck}\right)^{2k} [\left(1+\frac{\Delta D}{D}\right)^{1/2k} \frac{2ck}{2k-1} - c]$$
$$\mu > 0 \tag{13}$$

Then the supplier's profit is

$$f^S = f_3^* - f^r$$
$$= (1+\mu) D\left(\frac{2k-1}{2ck}\right)^{2k} [\left(1+\frac{\Delta D}{D}\right)^{1/2k} \frac{2ck}{2k-1} - c] \tag{14}$$

In fact, if the retailer asks to earn too much, i.e. μ is too large, the supplier would like to sell all the $\overline{Q}$ products at the secondary market. Therefore the condition $f^S > -\lambda_2 \overline{Q}$ should be met, which turns out to be

$$\mu < -\frac{\lambda_2 + (1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} - c}{(1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} - c}.$$

Theorem 7 *For the case*

$$D[\left(1-\frac{1}{2k}\right)^{2k} - 1] > \Delta D > D[\left(1-\frac{\lambda_2}{c}\right)^{2k} - 1]$$

and f^r *defined in (13), we have that*

1)when $0 < \mu \le -\frac{(1+\frac{\Delta D}{D})^{1/2k} \frac{1}{2k-1}}{(1+\frac{\Delta D}{D})^{1/2k} \frac{2k}{2k-1} - 1}$,

the supply chain can be coordinated by the policy $AQD(w_1, w_2, \overline{Q})$;

2)when

$$-\frac{(1+\frac{\Delta D}{D})^{1/2k} \frac{1}{2k-1}}{(1+\frac{\Delta D}{D})^{1/2k} \frac{2k}{2k-1} - 1} < \mu < -\frac{\lambda_2 + (1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} - c}{(1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} - c},$$

the supply chain can be coordinated by the policy $CLP(w_2, \overline{Q})$;

3)when $\mu \ge -\frac{\lambda_2 + (1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} - c}{(1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} - c}$,

the supplier would like to sell all the $\overline{Q}$ *products at the secondary market, where* w_1 *is large enough and*

$$w_2 = (1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} + \mu[(1+\frac{\Delta D}{D})^{1/2k} \frac{2ck}{2k-1} - c]$$

4.4 Case 4: $\Delta D < D\left[\left(1-\frac{\lambda_2}{c}\right)^{2k} - 1\right]$

Recall that the maximum supply chain profit for this case is

$$f_4^* = \frac{(D+\Delta D)(c-\lambda_2)}{2k-1}\left(\frac{2k-1}{2(c-\lambda_2)k}\right)^{2k} - D\lambda_2\left(\frac{2k-1}{2ck}\right)^{2k}$$

We still consider from the retailer's perspective. Suppose the desired profit of the retailer is

$$f^r = \frac{(D+\Delta D)(c-\lambda_2)}{2k-1}\left(\frac{2k-1}{2(c-\lambda_2)k}\right)^{2k} - \mu D\lambda_2\left(\frac{2k-1}{2ck}\right)^{2k} \tag{15}$$

It should be positive, i.e.

$$\mu < \frac{1}{2k-1}(1+\frac{\Delta D}{D})(\frac{c}{\lambda_2}-1)(\frac{c}{c-\lambda_2})^{2k}.$$

And the supplier's profit is

$$f^S = (\mu-1)\lambda_2 D\left(\frac{2k-1}{2ck}\right)^{2k} \tag{16}$$

Note that when $\mu < 1$, the supplier's profit is negative. Similar to Case 3, f^S should satisfy that $f^S > -\lambda_2 \overline{Q}$, i.e., $\mu > 0$. Therefore, if

$$\mu \ge \frac{1}{2k-1}(1+\frac{\Delta D}{D})(\frac{c}{\lambda_2}-1)(\frac{c}{c-\lambda_2})^{2k} \text{ or}$$

$\mu < 0$, the supplier would like to dispose of all his $\overline{Q}$ products rather than supply them to the retailer for sale.

Theorem 8 *For the case*

$$\Delta D \le D[\left(1-\frac{\lambda_2}{c}\right)^{2k} - 1]$$

and f^r defined in (15), we have that

1)when

$$0 < \mu < \frac{1}{2k-1}(1+\frac{\Delta D}{D})(\frac{c}{\lambda_2}-1)(\frac{c}{c-\lambda_2})^{2k},$$

the supply chain can be coordinated by policy $AQD(w_1, w_2, Q_4^*)$, where w_1 is large enough and

$$w_2 = c - \lambda_2 + \frac{\mu \lambda_2 D}{D + \Delta D}\left(\frac{c-\lambda_2}{c}\right)^{2k}.$$

2)when

$$\mu \geq \frac{1}{2k-1}(1+\frac{\Delta D}{D})(\frac{c}{\lambda_2}-1)(\frac{c}{c-\lambda_2})^{2k} \text{ or}$$

$\mu < 0$, the supplier would like to sell all of the $\overline{Q}$ products at the secondary market.

As a final remark, we would like to point out that the coordination schemes (wholesale quantity discount policy with various parameters) developed in our model again have similar structures as those in the linear demand model (Qi et al. 2002(a)). The contribution of this research shows that the wholesale quantity discount policy can be used as an effective scheme to coordinate supply chain in a large range of situations.

5. Numerical Examples

In this section, we present some numerical examples to illustrate the theoretical results obtained in the previous sections. In particular, we are interested in the effect of the disruption management after various demand changes. Knowing a demand change, the supplier will adopt an appropriate new discount policy to re-coordinate the supply chain. On the contrary, if the supplier does not know the demand change, he will keep the pre-assumed discount policy, and the retailer has the freedom to make any order according to the real market. For these two cases, we will compare the profit difference from the perspective of the supplier, the retailer, and the whole supply chain.

In all the following examples, we assume the originally estimated market scale D=1000, the price sensitive coefficient $k = 1$. Thus the demand function becomes $d = 1000 p^{-2}$. Let the unit production cost $c = \$1$. In the deterministic case, the supply chain profit is maximized at $\overline{Q} = 250$, the optimal retail price is $\overline{p} = \$2$, and the maximum profit of the supply chain is $\bar{f}_{\max}^{SC} = \$250$. For the marginal cost of deviation from the production plan, we assume that λ_1=\$0.1, and λ_2=\$0.2. In other words, producing one unit of the product more than the original plan costs the supplier \$1.1, and handling one unit leftover inventory costs the supplier \$0.2.

Recall that the profit that the supplier wants to earn can be represented by $\bar{f}^S = \eta \bar{f}_{\max}^{SC}$. We fix η = 0.4. Under the assumption of symmetric information, it means that the supplier and the retailer agree to share the total supply chain profit as 4:6.

We have considered various magnitudes of possible disruptions of the market scale, i.e., different ΔD's. We calculate the profit differences of the supplier, the retailer and the supply chain between taking disruption management and keeping the original wholesale quantity discount policy in Table 1. In the table, we list the absolute difference in terms of dollars and the relative percentage difference.

In Table 1, the rows of ΔD=500 and ΔD=300 belong to Case 1, i.e., a large demand change.

Table 1 The Effect of Disruption Management for Different Demand Changes

Disruptions ΔD	Profit difference		
	Supplier	retailer	supply chain
500	\$46.36 (46.36%)	\$−42.82 (−16.32%)	\$3.54 (0.97%)
300	\$28.18 (28.18%)	\$−27.82 (−12.64%)	\$0.36 (0.11%)
100	\$9.76(9.76%)	\$−9.76(−5.60%)	0 (Coordination)
−200	\$−21.11(−21.11%)	\$21.11(21.72%)	0 (Coordination)
−450	\$−51.25 (−51.25%)	\$52.32 (251.39%)	\$1.065 (0.88%)
−550	\$86.25	\$54.375	\$140.625

We can see that the maximum supply chain profit is not achieved without the right coordination. Moreover, without the right coordination, more profit would go to the retailer. By the right coordination scheme, the supplier can take back some profit that he may have lost.

The rows of ΔD=100 and ΔD=−200 belong to Cases 2 and 3, respectively. In these cases of small demand changes, the maximum supply chain profit can be reached under the original discount policy, i.e., the supply chain can be coordinated. However, the profit sharing ratio is not 4:6 as it should be. For a positive demand change, the supplier can get more profit in the new discount policy; while for a negative demand change, the retailer is benefited in the new discount policy.

The rows of ΔD=−450 and ΔD=−550 belong to Case 4, a large negative demand change. Like in Case 1, the new discount policy can gain more profit to the whole supply chain, but now the supplier has to move part of his profit to the retailer in order to keep the profit sharing ratio as 4:6. For the row of ΔD=−550, we do not report the percentage difference. The reason is that keeping the original discount policy becomes infeasible in that the retail price would be lower than the wholesale price, then the retailer would order nothing from the supplier, and finally the supplier has to bear the loss of putting all the planned product in the secondary market. In the new discount policy, the retailer can make some profit by selling the product, and the supplier's loss is reduced.

From the above examples, we can see that the disruption management, i.e., adjusting the wholesale discount policy according to the demand change, has two main advantages, to achieve the maximum supply chain profit, and to correctly allocate profits between the partners.

6. Conclusions

In this paper, we have investigated the demand disruption management problem for a supply chain system. The market demand change may cause the production quantity differing from what has been planned, and changing the production plan will cause a deviation cost that needs to be considered under the demand change. While previous research on this topic assumes the demand is a linear decreasing function of the retail price, this paper has addressed the case that the demand is a nonlinear decreasing function.

Besides systemically analyzing the demand disruption problem with nonlinear demand function, this paper has two additional contributions. First, we show that the idea of handling demand disruptions for linear demand functions can also be applied to the case of nonlinear demand functions by appropriate implementations, which implies that the methodology has some universality in coping with similar circumstances. Second, the nonlinear model does reveal some new findings different from the linear model, which can give a better description of the reality in certain situations.

With more and more people realizing the importance of studying disruptions in supply chain systems, extending the one-supplier-one-retailer model to more complicated and practical models are worthy to do as interesting future works. Currently, we are working on the models with multiple players in the game, and various other possible disruptions.

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