COMPARISON OF MAX-MIN APPROACH AND NN METHOD FOR RELIABILITY OPTIMIZATION OF SERIES-PARALLEL SYSTEM

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Abstract

Two heuristics, the max-min approach and the Nakagawa and Nakashima method, are considered for the redundancy allocation problem with series-parallel structure. The max-min approach can formulate the problem as an integer linear programming problem instead of an integer nonlinear problem. This paper presents a comparison between those methods from the standpoint of solution quality and computational complexity. The experimental results show that the max-min approach is superior to the Nakagawa and Nakashima method in terms of solution quality in small-scale problems, but analysis of computational complexity shows that the max-min approach is inferior to other greedy heuristics.

Keywords: Max-min approach, heuristics, redundancy allocation, reliability optimization

1. Introduction

The redundancy allocation problem is one of the representative problems in reliability optimization. A variety of system structures and methodologies for the problem have been considered by numerous researchers (Kuo Hwang and Tillman 1978, Kuo and Prasad 2000, Kuo Prasad and Tillman 2001). Among various system structures, the series-parallel system, which consists of a series of connected subsystems whose components are connected in parallel, received the most extensive attention (Chern 1992, Coit and Smith 1996, Kuo Prasad and Tillman 2001, Levitin 2000, Prasad and Raghavachari 1998, Ramirez-Marquez Coit and Konak, Rubinstein Levitin Liniaski and Ben-Haim 1997, Sung and Cho 1999). The problem can be generally formulated as an integer nonlinear programming problem (INLP), and its computational complexity is very high, in fact, NP-hard (Chern 1992).

The system considered in this paper is more complicated than a general series-parallel system. In a general system, there is only one kind of component and one kind of redundancy in the subsystems, but the system under study has several options for kinds of components and

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redundancies in the subsystems (Coit and Smith 1996). The reliability of the system can be improved by appropriately adjusting the number of redundancies for each component. The objective of the problem is to find the best redundancy allocation for maximizing the overall system reliability under the given constraints.

There is as yet no polynomial algorithm for NP-hard problems; thus various exact solution methods and heuristics have been suggested for the redundancy allocation problem with series-parallel structure (Kuo Hwang and Tillman 1978, Kuo and Prasad 2000, Kuo Prasad and Tillman 2001). Exact methods yield global optimal allocations, but they are very time consuming. This is the reason why various heuristics are developed and preferred for large size real-world applications. Although heuristic approaches do not guarantee the global optimality of a solution, they can provide satisfying results with quite short computation times.

Most heuristics, such as the steepest ascent method using a sensitivity factor, boundary region search, increasing the redundancy under a minimum path, and meta-heuristics, are based on some intuitions on the modeled problem itself. On the other hand, the max-min approach, which was recently developed by Ramirez-Marquez, Coit, and Konak, is based on reformulation of the problem. This approach formulates the problem as a mixed integer linear programming problem (MILP) instead of an integer nonlinear programming problem (INLP) as was done by past methods. The reformulated problem can then be solved by readily available computational tools and well-developed mixed integer linear programming methods, such as branch-and-bound and LP relaxation.

The objective of the redundancy allocation problem is to maximize the overall system reliability. On the other hand, the objective of the reformulated problem using the max-min approach is to maximize the minimum subsystem reliability. Because the objective functions of the original problem and the reformulated problem are not identical, the solution given by the max-min approach may not be globally optimum. A solution of the original problem can be obtained by repeatedly solving a series of max-min MILP where the minimum subsystem reliability is fixed to create the next max-min MILP to be solved, and so on.

In the Ramirez-Marquez, Coit, and Konak paper, results are presented for three different examples but compared to the genetic algorithm for only one example. Because the number of tested problems is small, it is insufficient to show that this approach is superior to other heuristics. The purpose of our paper is to show that the max-min approach is indeed better than other heuristics in terms of solution quality and computation time.

To compare the performance of the max-min approach, we chose a well-known greedy heuristic, which is a general version (Kuo Hwang and Tillman 1978) of the Nakagawa and Nakashima method (Nakagawa and Nakashima 1977), and an exact solution method, the lexicographic search method developed by Prasad and Kuo (2000). The solution with the Nakagawa and Nakashima method is obtained by iteratively adding redundancy to the component that has the greatest sensitivity factor value. The sensitivity factor is computed by multiplying the difference in improvement in system reliability by the relative consumed resources. The lexicographic method is a total enumeration method that searches all feasible solutions of the problem in lexicographical order, where incumbent upper and lower bounds on the objective value and constraints are computed to reduce the search effort.

Sections 2 and 3 briefly present the procedures of the max-min approach, the Nakagawa and Nakashima method (NNK), and the lexicographic search method (LSM) and their application to two numerical examples. In section 4, the performance of the two heuristics are investigated and analyzed based on experimental simulation results with regard to associated performance criteria. Finally, conclusions are presented in Section 5.

Notation

s	number of subsystems
m_i	number of available component choices for subsystem i
i	index for subsystems, $i = 1,\ldots, s$
j	index for component choices for subsystem i, $j = 1,\ldots, m_i$
$\mathbf{x}$	$(x_{11},\ldots, x_{1m_1},\ldots, x_{ij},\ldots, x_{s1},\ldots, x_{sm_i})$
$\mathbf{x}_i$	$(x_{i1}, x_{i2},\ldots, x_{ij},\ldots, x_{im_i})$
$\mathbf{x}_0$	initial redundancy allocation
x_{ij}	number of components in subsystem i and component choice j
C, W	maximum allowable system cost and weight consumed
r_{ij}	reliability of component choice j in subsystem i
c_{ij}, w_{ij}	cost and weight of component choice j in subsystem i
$R_s, R_s{*}$	overall system reliability and optimal system reliability
$R_i(\mathbf{x}_i)$	reliability of subsystem i

2. Algorithms and Procedures

2.1 Max-Min Approach by Ramirez-Marquez et al.

A series-parallel system fails if anyone of its subsystems fails. Because the overall system reliability of a series-parallel system is always greater than, or equal to, any of its subsystem reliabilities, increasing the reliability of the least reliable subsystem improves the system reliability. The max-min approach based on Ramirez-Marquez et al. reformulates the objective function on the basis of this fact. The objective of the max-min approach is to maximize the minimum subsystem reliability instead of maximizing system reliability as is the case in the general redundancy allocation problem.

The general redundancy allocation problem and the reformulated max-min problem can be formulated as P and P1, respectively:

P

$$\max_{\mathrm{x}} R_S = \max_{\mathrm{x}} \prod_i R_i(\mathrm{x}_i)$$

subject to

$$\sum_i \sum_j c_{ij} x_{ij} \le C,$$
$$\sum_i \sum_j w_{ij} x_{ij} \le W,$$
$$x_{ij} \in Z_+ .$$

P1

$$\max_{\mathrm{x}} R_S = \max_{\mathrm{x}} \{\min_i R_i(\mathrm{x}_i)\}$$

subject to

$$\sum_i \sum_j c_{ij} x_{ij} \le C,$$
$$\sum_i \sum_j w_{ij} x_{ij} \le W,$$
$$x_{ij} \in Z_+ .$$

where $Z+$ denotes a set of nonnegative integers. Since each subsystem has parallel structure, problem P1 can be equivalently written as problem P2:

P2

$$\max_{x}\{\min_{i}\{1-\prod_{j}(1-r_{ij})^{x_{ij}}\}\}$$

subject to

$$\sum_{i}\sum_{j}c_{ij}x_{ij} \le C$$
$$\sum_{i}\sum_{j}w_{ij}x_{ij} \le W$$
$$x_{ij} \in Z_{+}$$

Let $z=\min_{i}\{1-\prod_{j}(1-r_{ij})^{x_{ij}}\}$, then P2 can be transformed to problem P3 as follows:

P3

$$\max_{x} z$$

subject to

$$\sum_{i}\sum_{j}c_{ij}x_{ij} \le C$$
$$\sum_{i}\sum_{j}w_{ij}x_{ij} \le W$$
$$(1-\prod(1-r_{1j})^{x_{1j}}) \ge z$$
$$\vdots$$
$$(1-\prod(1-r_{sj})^{x_{sj}}) \ge z$$
$$x_{ij} \in Z_{+}$$

Now, P3 is an integer nonlinear programming problem which is also hard to solve. However, it can be easily transformed to an equivalent mixed integer linear programming problem. Simply defining $\gamma_{ij}=-\ln(1-r_{ij})$ can lead to the following problem:

P4

$$\max_{x} z$$

subject to

$$\sum_{i}\sum_{j}c_{ij}x_{ij} \le C$$
$$\sum_{i}\sum_{j}w_{ij}x_{ij} \le W$$
$$\sum_{i}\gamma_{ij}x_{ij} \ge z \quad \text{for } i=1,2,\cdots,s$$
$$x_{ij} \in Z_{+}$$

With the above procedures, an equivalent MILP formulation can be obtained through transformations. Thus, the problem can be solved by efficient MILP algorithms and software such as AMPL (with CPLEX as its solver which has a built-in simplex method, interior-point method, and branch-and-bound algorithm, etc.).

2.2 The Nakagawa-Nakashima Method (1977)

Nakagawa and Nakashima (1977) presented a heuristic method for optimizing redundancy allocation in series systems, and subsequently Kuo et al. (1978) extended the method for general reliability systems by modifying the sensitivity factor. The solution is obtained by repeatedly adding redundancy to a more reliable candidate with the greatest value of the 'weighted sensitivity function' at each iteration. The weighted sensitivity function is the product of a quantity obtained as a function of the object function and a quantity obtained as a function of the constraints. A balancing coefficient, α, controls the balance between these two quantities. The best solution is selected from different results using multiple values of α. Nakagawa and Nakashima suggested α= 0, 0.1, 0.2,…, 0.9, 1, 1/0.9, 1/0.6, 1/0.3. The best of the 14 solutions yielded by the algorithm is taken as the final solution.

This algorithm starts with an initial feasible allocation $\mathbf{x}_0$. Choosing the initial allocation is critical since the method involves a greedy heuristic. Once the initial allocation is chosen, each value of the solution is greater than, or equal to, the one of initial allocation. In addition, the considered problem has a number of options for each subsystem; that is, there is more than one combination of initial allocations. It is possible that some initial feasible allocations might be ignored which could actually achieve higher system reliability. The more limited the resource is, the more important it is to choose a proper initial point.

In this paper, the following procedure is used to determine the initial allocation:

Step 1. compute $\Delta x_{ij} = \min\{C/cij, W/wij\}$;

Step 2. let $l = ij$ and $L(\mathbf{x}) = \{l : \Delta x_l \geq 1\}$;

Step 3. compute

$F_l = r_l\, [\alpha\Delta x_l + (1-\alpha)\min_{k\in L(\mathbf{x})} \Delta x_k]$ for $l \in L(\mathbf{x})$;

Step 4. let $x_l = 1$ and $L(\mathbf{x}) = L(\mathbf{x}) \setminus \{p \in$ subsystems which indludes $l\}$;

Step 5. if $\mathbf{x}$ is a feasible solution, stop. Otherwise, go to *Step 3*.

2.3 Lexicographic Search Method

The lexicographic search method is an efficient implicit enumeration method based on a lexicographic search using bounds on the objective value. There are two control values, current component and current allocation in the procedure. A new redundancy is repeatedly added to the current component lexicographically until the current allocation is the largest lexicographical point. During the procedure, if the current allocation is infeasible, the current component changes to the next component because increasing any redundancy to the allocation generates an infeasible allocation due to the system is coherent. To see the detailed procedure and examples, refer to Prasad and Kuo (2000).

To guarantee the optimality of a solution, the method requires a proper initial allocation that is the smallest point in the lexicographical order. In general, the lower bound of the variables is chosen as a reasonable initial point for reliability allocation problems. Although the initial allocation is proper, it is inefficient to solve the considered problem because the initial allocation, that is, $\mathbf{x}_0 = (0, 0,\ldots,0)$, generates zero system reliability. In order to make any allocation meaningful, at least one component has to be used in each subsystem of the series-parallel system. To solve the problem efficiently, we add the following procedure: if any subsystem has zero reliability at the current allocation, the value of the last component must be one in the subsystem. For example, consider a three-stage system with two alternative components in each stage. If the current allocation is $\mathbf{x} = (0, 0, 3, 0, 0, 0)$, the above procedure automatically changes the allocation into $\mathbf{x}_0 = (0, 1, 3, 0, 0, 1)$.

3. Numerical Examples

Example 1

This simple example demonstrates the weakness of the max-min approach as well as the procedure. Consider a series system with three subsystems, each having one choice of component. Redundancy can be added to each subsystem as long as the total cost does not exceed a cost limit of 19. The costs and reliabilities of the components are $\mathbf{c} = (4, 3, 6)$ and $\mathbf{r} = (0.6, 0.61, 0.8)$, respectively. Since a series system is not meaningful unless each subsystem has at least one component, it is reasonable to set $\mathbf{x}_0 = (1, 1, 1)$ and $\boldsymbol{R_i} = (0.6, 0.61, 0.8)$. Then the residual cost is 6, which allows applying the max-min approach to add one redundant component to the first subsystem that has minimum reliability. The final results are $R_s = 0.4099$, $\mathbf{x} = (2, 1, 1)$, $\boldsymbol{R_i} = (0.84, 0.61, 0.8)$, and the residual cost is 2. On the other hand, the optimal redundancy allocation is $\mathbf{x}^* = (1, 3, 1)$ which makes $R_s = 0.4515$, $\boldsymbol{R_i} = (0.6, 0.94, 0.8)$, and the residual cost 0.

The max-min approach transforms the objective function of the problem into a different objective function. Thus, even though an algorithm finds the optimum for the max-min problem, the solution may not be the optimal solution for the original problem. This is why the max-min method does not find the optimum in this example.

Example 2

Consider a series-parallel system with two or three subsystems. Each subsystem consists of three distinct components in parallel and the components have different reliabilities, cost and weight coefficients. There are two linear constraints representing the cost and weight limits of the system. The problem can be modeled as an ILP shown below:

$$\max R_s(\mathrm{x}) = \prod_{i=1}^{3}\{1-\prod_{j=1}^{3}[1-(1-r_{ij})^{x_{ij}}]\}$$

subject to

$$\sum_i \sum_j c_{ij} x_{ij} \le C$$

$$\sum_i \sum_j w_{ij} x_{ij} \le W$$

$$x_{ij} \in \mathrm{Z}_+$$

Table 1 Data for Example 2

Subsystem i	Design Alternative j								
	1			*2*			*3*		
	r	c	w	r	c	w	r	c	w
1	0.99	4	2	0.95	13	3	0.92	7	5
2	0.98	8	3	0.80	3	3	0.90	3	9
3	0.98	11	4	0.92	5	6	-	-	-

Table 1 shows all of the given data in the example. Note that the number of decision variable is 8, although the number of the subsystem is 3. The available cost and weight consumed, i.e., C and W, are 30 and 17, respectively.

Table 2 Results of Example 2

	Max-min	NNK[a]	NNK[b]	LSM
x	$x_{11} = 2$	$x_{11} = 2$	$x_{13} = 1$	$x_{11} = 2$
	$x_{21} = 1$	$x_{21} = 1$	$x_{21} = 1$	$x_{21} = 1$
	$x_{22} = 1$	$x_{22} = 1$	$x_{22} = 1$	$x_{22} = 1$
	$x_{31} = 1$	$x_{31} = 1$	$x_{33} = 1$	$x_{31} = 1$
$\mathrm{Min}_i(R_i)$	0.9800	0.9800	0.9200	0.9800
Rs	0.9760	0.9760	0.8430	0.9760

a: Initial allocation is (1,0,0,1,0,0,1,0,0)
b: Initial allocation is (0,0,1,0,1,0,0,0,1)

The results of the example are listed in Table 2. The solutions are obtained from three different methods, max-min, NNK, and LSM, where NNK is tested on two distinct initial allocations. All three methods find the optimal solution $\mathbf{x} = (2, 0, 0, 1, 1, 0, 1, 0, 0)$. The solution of the NNK, however, depends on the initial allocation. The results show the importance of initial allocation to the NNK method.

Contrary to Example 1, this example shows the advantage of the max-min method. In this example, the resources are very limited, i.e., the number of iterations of the NNK is only 2. The max-min method seems to have more chance of reaching the optimal solution if the resources are tight. In addition, if the optimal solution of a problem is close to the optimal solution of the relaxed problem which drops all integrality constraints, the objective function of the max-min transformation describes the original objective function well because the gap between the reliability of the subsystems is decreased in a continuous problem. Since many problems have this property, the max-min method is appropriate.

4. Experimental Simulation and Performance Criteria

This section evaluates and compares two heuristics for solving reliability-redundancy problems with the lexicographic search method. Those algorithms are tested on four different types of problems with a series-parallel structure; 100 sets of problems are randomly generated.

4.1 Problems and Parameters

Consider series-parallel systems with s subsystems and 2, 3 or 4 choices of components within each subsystem. Description of the problems and parameters are listed in Table 3. Problem 1 is a small problem with 3 stages and 11 variables. Problem 2 and 3 have the same number of subsystems and design alternatives, but Problem 3 has larger resources in terms of cost and weight than Problem 2. Problem 4 consists of 15 subsystems and 40 variables. Four sets of 100 randomly generated data are needed for the problems. The cost, weight, and reliability of each alternative component are varied in a uniform fashion over specified ranges. Available resources of cost and weight are normally distributed with proper means and variances. Detailed information is summarized in Table 3.

4.2 Performance Criteria

In order to compare the experimental results, the following four absolute and relative performance measures are defined based on (Xu and Kuo 1990).

1. optimality rate (*OR*): the percentage of optimal solutions with each algorithm.
2. superior rate (*SR*): the percentage of best solutions with each algorithm.
3. average absolute error rate (*AAER*):

$$AAER = \sum_{j=1}^{100} | R_s^j - R_s^{j*} | / 100,$$

   where $R_s{}^j$ denotes the system reliability of each algorithm on the jth problem and $R_s{}^{j*}$ indicates the optimal solution, i.e., solution of LSM on the problem.
4. average relative error rate (*ARER*):

$$ARER = \sum_{j=1}^{100} | R_s^j - R_s^{j*} | / (100 \times R_s^{j*}).$$

4.3 Computational Results and Discussions

NNK and LSM are programmed in Matlab 6.1, and max-min is coded by AMPL modeling language with CPLEX as its solver. All algorithms are run on a Pentium IV PC with 512KB memory. Table 4 lists the experimental results for the comparison of Max-Min and NNK in terms of above performance criteria.

The experimental results demonstrate that both max-min and NNK can provide an approximate, but not always exact, solution. This is a natural phenomenon because both

Table 3 Structures and Parameters for Problem 1, 2, 3, and 4

Problem	Problem 1	Problem 2	Problem 3	Problem 4
Number of stages	3	5	5	15
Number of choices	(4, 3, 4)	(2, 3, 4, 2, 3)	(2, 3, 4, 2, 3)	(2, 2, 2, 2, 2, 3, 3, 3, 3, 3, 3, 3, 3, 3, 3)
Number of variables	11	14	14	40
Bounds of x_{ij}	[0, 5]	[0,3]	[0, 3]	[1, 3]
C_{ij}	$U(3, 11)$	$U(3, 11)$	$U(3, 11)$	$U(3, 11)$
w_{ij}	$U(1, 5)$	$U(1, 5)$	$U(1, 5)$	$U(1, 5)$
r_{ij}	$U(0.85, 0.99)$	$U(0.70, 0.95)$	$U(0.70, 0.95)$	$U(0.70, 0.99)$
C	$N(21, 2)$	$N(60, 11)$	$N(150, 11)$	$1.4\sum_i \sum_j c_{ij}$
W	$N(9, 2)$	$N(35, 8)$	$N(80, 8)$	$1.4\sum_i \sum_j w_{ij}$

$N(\mu,\sigma)$: Normal distribution with mean μ and standard deviation σ

$U(\alpha, \beta)$: Uniform distribution on interval $[\alpha, \beta]$

Table 4 Computational Results for Problem 1, 2, 3, and 4

	Problem 1		Problem 2		Problem 3		Problem 4	
	Max-min	NNK	Max-min	NNK	Max-min	NNK	Max-min	NNK
OR	78	51	51	35	55	42	25	23
SR	42	8	57	29	36	29	40	48
AAER (10^{-4})	38.1	76	62.6	87.8	0.421	0.601	5	4.6
ARER (10^{-4})	45.8	90.3	74.9	103.39	0.436	0.602	5.1	4.7

algorithms are heuristics. Max-min is superior to NNK on the overall quality of solutions for first three sets of problems. One interpretation of this is that the generated problems, similar to Example 2, are suitable to the max-min approach. Another possibility is that the initial allocation generation procedure of NNK is not efficient enough to reveal the performance of the NNK method.

The results in column 2 and 3 clearly show that the performance of these algorithms depends on the resource limits of the constraints. Max-min is much superior to NNK if the problem has tight resources as in the examples. This implies that the balancing of subsystems' reliabilities is more critical as resources are tightened, and the gap between solutions from max-min and NNK gets smaller when resources are increased. On the other hand, in the last problem which deals with more variables, the performance of NNK becomes slightly better than max-min. This fact implies that while solving problems with more variables and subsystems, balancing of subsystems' reliabilities will not lead the overall system reliability the optimal as often as it does in smaller problems. And NNK method provides better solutions more often when it comes to larger problems. Comparing the results in all four columns, there is another fact should be noticed, that is, the gap between the optimal solution and heuristic solution generally increases as size increases.

An important point to consider here is computational complexity. We adopt heuristic methods to decrease the computation time for difficult problems. A superior algorithm has a shorter computation time as well as a better solution, though there is a trade-off between the two. In our experiment, a direct comparison of computation times is impossible because the algorithms are coded in different languages. However, it is sufficient to consider the computational complexity of both algorithms.

NNK is a greedy heuristic method. The computation to obtain the solution is proportional to the number of iterations and the computation on each iteration. Since the computational complexity of each iteration is polynomial and the number of iterations of NNK is the same as the path length from the initial point to the final solution, which is constant much less than the number of variables, the complexity of NNK is polynomial. On the other hand, max-min iteratively solves MILP. Although the number of iterations in max-min is constant, MILP itself is categorized into NP-hard problem; that is, there is no polynomially solvable algorithm for MILP. This implies that max-min can not be solved in polynomial time. Thus, the gap in computation time between NNK and max-min increases significantly as the problem size increases.

5. Conclusion

A recently proposed heuristic method, the max-min approach, is analyzed and compared with a classical heuristic method, the Nakagawa and Nakashima method, for reliability allocation problems. The considered problems have a series-parallel structure with several constraints and several choices of components in each stage. The max-min approach maximizes the minimum system reliability based on the fact that the system reliability of a series system is greater than, or equal to, the minimum subsystem reliability.

Brief descriptions are presented for each algorithm and two examples are used to illustrate their strengths and weaknesses. The experimental results show that the max-min approach is superior to the Nakagawa and Nakashima method on small-scale test sets with regard to the defined overall performance measures. However, the Nakagawa and Nakashima method is slightly better than max-min approach on problems with more variables and sub-systems. From what has been discussed above, we can conclude that limits on the resources and the number of variables are critical to the quality of the solutions, and the max-min approach runs on exponential time while the Nakagawa and Nakashima method runs on polynomial time.

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