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Generalization of a theorem of Gonchar

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Abstract. Let X and Y be two complex manifolds, let $D\subset X$ and $G\subset Y$ be two nonempty open sets, let A (resp. B) be an open subset of ∂D (resp. ∂G), and let W be the 2-fold cross $((D\cup A)\times B)\cup(A\times(B\cup G))$. Under a geometric condition on the boundary sets A and B, we show that every function locally bounded, separately continuous on W, continuous on $A\times B$, and separately holomorphic on $(A\times G)\cup(D\times B)$ "extends" to a function continuous on a "domain of holomorphy" $\widehat{W}$ and holomorphic on the interior of $\widehat{W}$.

1. Introduction

In the works [6], [7] Gonchar has proved the following remarkable result.

Gonchar's theorem. *Let $D,G\subset\mathbf{C}$ be Jordan domains and A (resp. B) be a nonempty open subset of the boundary ∂D (resp. ∂G). Let*

$$f\colon W:=((D\cup A)\times B)\cup(A\times(B\cup G))\longrightarrow\mathbf{C}$$

be a continuous function such that $f(a,\cdot)|_G$ and $f(\cdot,b)|_D$ are holomorphic for all $a\in A$ and $b\in B$. Then there is a unique function $\hat{f}$ continuous on

$$\widehat{W}:=\{(z,w)\in(D\cup A)\times(G\cup B):\omega(z,A,D)+\omega(w,B,G)<1\},$$

and holomorphic on

$$\widehat{W}^{\circ}:=\{(z,w)\in D\times G:\omega(z,A,D)+\omega(w,B,G)<1\},$$

such that $\hat{f}=f$ on W, where $\omega(\cdot,A,D)$ and $\omega(\cdot,B,G)$ are the harmonic measures (see Subsection 2.2 below). Moreover, if $|f|_W<\infty$ then

$$|\hat{f}(z,w)|\le|f|_{A\times B}^{1-\omega(z,A,D)-\omega(w,B,G)}|f|_W^{\omega(z,A,D)+\omega(w,B,G)},\quad(z,w)\in\widehat{W},$$

where $|g|_M:=\sup_M|g|$ for a function g defined on a set M.

Gonchar's theorem generalizes the pioneering work of Malgrange–Zerner [15] on a boundary version of the cross theorem, and other results obtained by Komatsu [8] and Drużkowski [3]. At the same time as Gonchar's work in [6], Airapetyan and Henkin published a version of the edge-of-the-wedge theorem for CR manifolds (see [1] for a brief version and [2] for a complete proof). Gonchar's theorem could be deduced from the latter result.

Recently, the authors have been able to generalize Gonchar's result to the case where D and G are pseudoconvex domains in $\mathbf{C}^n$ (see [11]).

The main goal of the present work is to establish a generalization of Gonchar's theorem for the case where D and G are open subsets of arbitrary complex manifolds and $A\subset\partial D$ and $B\subset\partial G$ are open (boundary) subsets.

The proof of the result presented in this work is based on Gonchar's theorem, the techniques introduced in our previous work [11], the approach "Poletsky theory of holomorphic discs and Rosay's theorem" developed in a recent article of the second author [10], and a thorough geometric study of the plurisubharmonic measure.

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2. Statement of the main result and outline of the proof

In order to state the main result, we need to introduce some notation and terminology. In fact, we keep the main notation from the previous work [11]

2.1. Topological hypersurfaces in a complex manifold

For every open subset $U\subset\mathbf{R}^{2n-1}$ and every continuous function $h\colon U\to\mathbf{R}$, the graph

$$\{z=(z',z_n)=(z',x_n+iy_n)\in\mathbf{C}^n:(z',x_n)\in U \text{ and } y_n=h(z',x_n)\}$$

is called a *topological hypersurface in* $\mathbf{C}^n$.

Let X be a complex manifold of dimension n. A subset $A\subset X$ is a *topological hypersurface* if, for every point $a\in A$, there is a local chart$(U,\phi\colon U\to\mathbf{C}^n)$ around a such that $\phi(A\cap U)$ is a topological hypersurface in $\mathbf{C}^n$

Now let $D\subset X$ be an open subset and let $A\subset\partial D$ be an open subset (with respect to the topology induced on ∂D). Suppose in addition that A is a topological hypersurface. A point $a\in A$ is said to be *of type* 1 (*with respect to* D) if, for every neighborhood U of a there is an open neighborhood V of a such that $V\subset U$ and $V\cap D$ is a domain. Otherwise, a is said to be *of type* 2. We see easily that if a is of type 2, then for every neighborhood U of a, there are an open neighborhood V of a and two domains V_1 and V_2 such that $V\subset U$, $V\cap D=V_1\cup V_2$ and all points in $A\cap V$ are of type 1 with respect to V_1 and V_2.

We conclude this subsection with a simple example which may clarify the above definitions. Let G be the open square in $\mathbf{C}$ whose four vertices are $1+i$, $-1+i$, $-1-i$, and $1-i$. Define the domain

$$D:=G\setminus\left[-\tfrac{1}{2},\tfrac{1}{2}\right].$$

Then $A:=\partial G\cup\left(-\frac{1}{2},\frac{1}{2}\right)$ is not only an open subset of ∂D, but also a topological hypersurface. Every point of ∂G is of type 1 and every point of $\left(-\frac{1}{2},\frac{1}{2}\right)$ is of type 2 (with respect to D).

2.2. Plurisubharmonic measure

Let X be a complex manifold and let D be an open subset of X. For every function $u\colon D\to[-\infty,\infty)$, let

$$\hat{u}(z):=\begin{cases} u(z), & z\in D,\\ \limsup\limits_{\substack{w\in D\\ w\to z}} u(w), & z\in\partial D.\end{cases}$$

For a set $A\subset\overline{D}$ put

$$h_{A,D}:=\sup\{u : u\in\mathcal{PSH}(D),\ u\leq 1 \text{ on } D \text{ and } \hat{u}\leq 0 \text{ on } A\},$$

where $\mathcal{PSH}(D)$ denotes the set of all plurisubharmonic functions on D.

The *plurisubharmonic measure* of A relative to D is given by

$$\omega(z,A,D):=\widehat{h^*_{A,D}}(z),\quad z\in D\cup A, \tag{2.1}$$

where u^* denotes the upper semicontinuous regularization of a function u.

Geometric properties of the plurisubharmonic measure will be discussed in Section 3 below.

2.3. Cross and separate holomorphicity

Let X and Y be two complex manifolds, let $D\subset X$ and $G\subset Y$ be two nonempty open sets, and let A (resp. B) be either an open subset of ∂D (resp. ∂G) or an open subset of D (resp. G). If, moreover, A (resp. B) is an open subset of ∂D (resp. ∂G), then we assume in addition that A (resp. B) is a topological hypersurface.

We define a 2-*fold cross* W and its *interior* W° as

$$\begin{aligned} W &= \mathbf{X}(A,B;D,G) := ((D\cup A)\times B)\cup(A\times(B\cup G)),\\ W^\circ &= \mathbf{X}^\circ(A,B;D,G) := (A\times G)\cup(D\times B). \end{aligned}$$

Moreover, put

$$\omega(z,w) := \omega(z,A,D)+\omega(w,B,G), \quad (z,w)\in(D\cup A)\times(G\cup B).$$

For a 2-fold cross $W:=\mathbf{X}(A,B;D,G)$ define *its wedge*

$$\widehat{W} := \widehat{\mathbf{X}}(A,B;D,G) := \{(z,w)\in(D\cup A)\times(G\cup B) : \omega(z,w)<1\}.$$

Then the set of all interior points of the wedge $\widehat{W}$ is given by

$$\widehat{W^\circ} := \widehat{\mathbf{X}}^\circ(A,B;D,G) := \{(z,w)\in D\times G : \omega(z,w)<1\}.$$

We say that a function $f\colon W\to\mathbf{C}$ is *separately holomorphic on* W° and write $f\in\mathcal{O}_s(W^\circ)$, if for any $a\in A$ (resp. $b\in B$) the function $f(a,\cdot)|_G$ (resp. $f(\cdot,b)|_D$) is holomorphic on G (resp. on D).

We say that a function $f\colon W\to\mathbf{C}$ is *separately continuous on* W and write $f\in\mathcal{C}_s(W)$, if for any $a\in A$ (resp. $b\in B$) the function $f(a,\cdot)|_{G\cup B}$ (resp. $f(\cdot,b)|_{D\cup A}$) is continuous on $G\cup B$ (resp. on $D\cup A$).

Throughout the paper, for a topological space M, $\mathcal{C}(M)$ denotes the space of all continuous functions $f\colon M\to\mathbf{C}$ equipped with the "sup-norm" $|f|_M:=\sup_M|f|$. Moreover, a function $f\colon M\to\mathbf{C}$ is said to be *locally bounded* on M if, for any point $z\in M$, there are an open neighborhood U of z and a positive number $K=K_z$ such that $|f|_U<K$.

2.4. Statement of the main result and an outline of its proof

We are now ready to state the main result.

Main theorem. *Let X and Y be two complex manifolds, let $D\subset X$ and $G\subset Y$ be two nonempty open sets, and let A (resp. B) be a nonempty open subset of ∂D (resp. ∂G). Suppose in addition that A and B are topological hypersurfaces. Let $f\colon W\to\mathbf{C}$ be such that:*

(i) *$f\in\mathcal{C}_s(W)\cap\mathcal{O}_s(W^\circ)$;*

(ii) *f is locally bounded on W;*

(iii) *$f|_{A\times B}$ is continuous.*

Then there exists a unique function $\hat{f}\in\mathcal{C}(\widehat{W})\cap\mathcal{O}(\widehat{W}^{\circ})$ *such that* $\hat{f}=f$ *on* W. *Moreover, if* $|f|_W<\infty$, *then*

$$|\hat{f}(z,w)|\le|f|_{A\times B}^{1-\omega(z,w)}|f|_W^{\omega(z,w)},\quad (z,w)\in\widehat{W}.$$

It is worth to remark that the formulation of the Main theorem bears a flavor of Drużkowski's theorem in [3]. In fact, when D and G are Jordan domains in $\mathbf{C}$, the Main theorem follows from Gonchar's theorem and the proof of Drużkowski's theorem. Now we give some ideas of how to prove the Main theorem.

In order to tackle "arbitrary" complex manifolds, the first key technique here is to apply the beautiful theorem of Rosay [14]. This "Poletsky theory of holomorphic discs" approach has been explored in the work [10], where the second author succeeded in removing the "pseudoconvex hypothesis" in the classical cross theorems. The second key technique is to apply a mixed cross type theorem (see also [11]). The third key technique is to use *level sets* of the plurisubharmonic measure (see [10] and [11]). More precisely, we exhaust D (resp. G) by the level sets of the plurisubharmonic measure $\omega(\,\cdot\,,A,D)$ (resp. $\omega(\,\cdot\,,B,G)$), i.e. by $D_\delta:=\{z\in D:\omega(z,A,D)<1-\delta\}$ (resp. $G_\delta:=\{w\in G:\omega(w,B,G)<1-\delta\}$) for $0<\delta<1$.

Our method consists of three steps. In the first step we suppose that G is a domain in $\mathbf{C}^m$ and A is an open subset of D. In the second step we treat the case where the pairs (D,A) and (G,B) are "good" enough in the sense of the slicing method. In the last one we consider the general case. For the first step we combine the mixed cross theorem with the technique of holomorphic discs. For the second step one applies the slicing method and Gonchar's theorem. The general philosophy is to prove the Main theorem with D (resp. G) replaced by D_δ (resp. G_δ). Then we construct the solution for the original open sets D and G by means of a gluing procedure (see also [10] and [11]). In the last step we transfer the holomorphicity from local situations to the global context.

Although our results have been stated only for the case of a 2-fold cross, they can be formulated for the general case of an N-fold cross with $N\ge2$ (see also [10] and [11]).

3. Preparatory results

We present here the auxiliary results needed for the proof of the Main theorem.

3.1. Poletsky theory of discs and Rosay's theorem on holomorphic discs

Let E denote as usual the unit disc in $\mathbf{C}$. For a complex manifold $\mathcal{M}$, let $\mathcal{O}(\overline{E},\mathcal{M})$ denote the set of all holomorphic mappings $\phi\colon E\to\mathcal{M}$ which extend holomorphically to a neighborhood of $\overline{E}$. Such a mapping ϕ is called a *holomorphic disc* on $\mathcal{M}$. Moreover, for a subset A of $\mathcal{M}$, let

$$1_{A,\mathcal{M}}(z) := \begin{cases} 1, & z \in A, \\ 0, & z \in \mathcal{M} \setminus A. \end{cases}$$

In the work [14] Rosay proved the following remarkable result.

Theorem 3.1. *Let u be an upper semicontinuous function on a complex manifold $\mathcal{M}$. Then the Poisson functional of u defined by*

$$\mathcal{P}[u](z) := \inf\left\{ \frac{1}{2\pi} \int_0^{2\pi} u(\phi(e^{i\theta}))\, d\theta : \phi \in \mathcal{O}(\overline{E}, \mathcal{M}) \text{ and } \phi(0) = z \right\},$$

is plurisubharmonic on $\mathcal{M}$.

Special cases of Theorem 3.1 have been considered, for the first times, by Poletsky (in [12] and [13]), and then by Lárusson–Sigurdsson (see [9]) and Edigarian (see [4]).

The following result is an immediate consequence of Rosay's theorem.

Proposition 3.2. *Let $\mathcal{M}$ be a complex manifold and A a nonempty open subset of $\mathcal{M}$. Then $\omega(z, A, \mathcal{M}) = \mathcal{P}[1_{\mathcal{M}\setminus A,\mathcal{M}}](z)$, $z \in \mathcal{M}$.*

Proof. See, for example, the proof of Proposition 3.4 in [10]. □

The following result is simple but very useful.

Lemma 3.3. *Let T be an open subset of $\overline{E}$. Then*

$$\omega(0, T \cap E, E) \le \frac{1}{2\pi} \int_0^{2\pi} 1_{\partial E \setminus T, \partial E}(e^{i\theta})\, d\theta.$$

Proof. See Lemma 3.3 in [10]. □

3.2. Slicing method

Let X be a complex manifold of dimension n, let D be an open subset of X and let $A \subset \partial D$ be an open boundary subset which is also a topological hypersurface. We like to study the "slicing" property of D near an arbitrary point $a \in A$. Since our study is local, we look at sufficiently small open neighborhoods V of a such that V is contained in a chart. Therefore, $V \cap D$ may be identified with an open neighborhood of 0 in $\mathbf{C}^n$. In addition, we may choose V so that there are an open subset $U\subset\mathbf{R}^{2n-1}$ and a continuous function $h\colon U\to\mathbf{R}$ such that

$$V\cap\partial D=V\cap A=\{z=(z',z_n)=(z',x_n+iy_n)\in\mathbf{C}^n:(z',x_n)\in U \text{ and } y_n=h(z',x_n)\}.$$

Assume without loss of generality that $[-1,1]^{2n-1}\subset U$ and that $a:=0\in\mathbf{C}^n$. By shrinking U and V (if necessary), and using the continuity of h, we may find an $\varepsilon>0$ such that there are only the following two cases:

Case 1. All points of $A\cap V$ are of type 1 and

$$E_{z'}:=\{z=(z',z_n)=(z',x_n+iy_n)\in\mathbf{C}^n:-\varepsilon<y_n-h(z',x_n)<0\}\subset D\cap V$$

for all $z'\in(-1,1)^{2n-2}$.

Case 2. All points of $A\cap V$ are of type 2 and

$$E_{z'}:=\{z=(z',z_n)=(z',x_n+iy_n)\in\mathbf{C}^n:-\varepsilon<y_n-h(z',x_n)<0 \text{ or } 0<y_n-h(z',x_n)<\varepsilon\}\subset D\cap V$$

for all $z'\in(-1,1)^{2n-2}$.

For a subset $S\subset\mathbf{C}^n$, let $(S)_n$ denote the image of S under the canonical projection of $\mathbf{C}^n$ onto the nth coordinate. Observe that in Case 1, $(E_{z'})_n$ is a Jordan domain, but in Case 2, $(E_{z'})_n$ is a disjoint union of two Jordan domains. For all $z'\in(-1,1)^{2n-2}$, let

$$(A\cap V)_{z'}:=\{z=(z',z_n)=(z',x_n+iy_n)\in\mathbf{C}^n:y_n=h(z',x_n)\}.$$

Definition 3.4. Under the above hypothesis and notation, $(D\cap V,A\cap V)$ is said to be a *good pair*.

In summary, we have shown the following result.

Proposition 3.5. *Let X be a complex manifold, let D be an open subset of X and let $A\subset\partial D$ be an open boundary subset which is also a topological hypersurface. Then for all points $a\in A$, there is an open neighborhood V of a such that the pair $(D\cap V,A\cap V)$ is good.*

Using the construction above and the continuity of h, we may apply Radó's theorem (see Theorem 2 in [5, p. 59]). Consequently, the family of harmonic measures $\omega(\,\cdot\,,((A\cap V)_{z'})_n,(E_{z'})_n)$ depends "continuously" on the parameter $z'\in(-1,1)^{2n-2}$. Therefore, we obtain the following result.

Proposition 3.6. *We keep the above hypothesis and notation. Consider the set*

$$(A\cap V)_\delta := \operatorname{int}\Big(\bigcup_{z'\in(-1,1)^{2n-2}} \{z=(z',z_n):\ z_n\in(E_{z'})_n,\ \omega(z_n,((A\cap V)_{z'})_n,(E_{z'})_n)<\delta\}\Big),$$

where $\operatorname{int} S$ *denotes the set of all interior points of* $S\subset\mathbf{C}^n$. *Then* $(A\cap V)_\delta\cup(A\cap V)$ *is a neighborhood of* $A\cap V$ *in* $D\cup(A\cap V)$.

The above result has several useful consequences.

Proposition 3.7. *Let* X *be a complex manifold, let* D *be an open subset of* X *and let* $A\subset\partial D$ *be an open boundary subset which is also a topological hypersurface. For all* $0\le\varepsilon<1$, *let*

$$D_\varepsilon := \{z\in D : \omega(z,A,D)<1-\varepsilon\}.$$

Then:

(1) A *is also an open set of* ∂D_ε *and* $\lim_{z\to\zeta}\omega(z,A,D)=0$ *for all* $\zeta\in A$.

(2) *Moreover,*

$$\omega(z,A,D_\varepsilon)=\frac{\omega(z,A,D)}{1-\varepsilon},\quad z\in D_\varepsilon.$$

(3) (*The Uniqueness theorem*) *If* $f\in\mathcal{O}(D_\varepsilon)$ *is such that* $\lim_{z\to\zeta}f(z)=0$ *for all* $\zeta\in A$, *then* $f\equiv 0$.

Proof. Applying Proposition 3.6 locally, the first assertion follows. Using the first assertion, the second and third ones follow from standard arguments. □

3.3. A mixed cross theorem

Theorem 3.8. *Let* D *be the unit disc in* $\mathbf{C}$, *let* G *be an open subset in* $\mathbf{C}^m$, *let* A *be an open subset of* D, *and let* B *be an open subset of* ∂G. *Suppose in addition that* B *is a topological hypersurface. Put* $W:=\mathbf{X}(A,B;D,G)$ *and* $\widehat{W}:=\widehat{\mathbf{X}}(A,B;D,G)$. *Let* $f: W\to\mathbf{C}$ *be such that*:

(i) $f\in\mathcal{C}_s(W)\cap\mathcal{O}_s(W^\circ)$;

(ii) f *is locally bounded on* W;

(iii) $f|_{A\times B}$ *is continuous.*

Then there exists a unique function $\hat f\in\mathcal{C}(\widehat{W})\cap\mathcal{O}(\widehat{W}^\circ)$ *such that* $\hat f=f$ *on* W. *Moreover, if* $|f|_W<\infty$, *then*

$$|\hat f(z,w)|\le |f|_{A\times B}^{1-\omega(z,w)}|f|_W^{\omega(z,w)},\quad (z,w)\in\widehat{W}.$$

Proof. Using Proposition 3.7, the proof of Theorem 4.1 and Theorem 4.2 in [11] also works in the present context after making the obviously necessary changes. In fact, the hypothesis on D (i.e. $D=E$) implies that D is pseudoconvex. Therefore, we are able to apply the classical method of doubly orthogonal bases of Bergman type. □

4. Part 1 of the proof of the Main theorem

The main purpose of this section is to prove the following mixed cross theorem.

Theorem 4.1. *Let D be a complex manifold, let G be an open subset in $\mathbf{C}^m$, let A be an open subset of D, and let B be an open subset of ∂G. Suppose in addition that B is a topological hypersurface. Put $W:=\mathbf{X}(A,B;D,G)$ and $\widehat{W}:=\widehat{\mathbf{X}}(A,B;D,G)$. Let $f\colon W\to\mathbf{C}$ be such that:*

(i) $f\in\mathcal{C}_s(W)\cap\mathcal{O}_s(W^\circ)$;

(ii) *f is locally bounded on W;*

(iii) *$f|_{A\times B}$ is continuous.*

Then there exists a unique function $\hat{f}\in\mathcal{C}(\widehat{W})\cap\mathcal{O}(\widehat{W}^\circ)$ such that $\hat{f}=f$ on W. Moreover, if $|f|_W<\infty$, then

$$|\hat{f}(z,w)|\le|f|_{A\times B}^{1-\omega(z,w)}|f|_W^{\omega(z,w)},\quad (z,w)\in\widehat{W}.$$

It is worth to remark that Theorem 4.1 removes the hypothesis "pseudoconvex" in Theorem 3.8.

Proof. It follows essentially the proof of Theorem 4.1 in [10]. We begin the proof with the following lemma.

Lemma 4.2. *We keep the hypothesis of Theorem 4.1. For $j\in\{1,2\}$, let $\phi_j\in\mathcal{O}(\overline{E},D)$ be a holomorphic disc, and let $t_j\in E$ such that $\phi_1(t_1)=\phi_2(t_2)$ and*

$$\frac{1}{2\pi}\int_0^{2\pi}1_{D\setminus A,D}(\phi_j(e^{i\theta}))\,d\theta<1,\quad j=1,2.$$

Then:

(1) *For $j\in\{1,2\}$, the function $(t,w)\mapsto f(\phi_j(t),w)$ belongs to*

$$\mathcal{C}_s(\mathbf{X}(\phi_j^{-1}(A)\cap E,B;E,G))\cap\mathcal{O}_s(\mathbf{X}^\circ(\phi_j^{-1}(A)\cap E,B;E,G)),$$

and is continuous on $(\phi_j^{-1}(A)\cap E)\times B$, where $\phi_j^{-1}(A):=\{t\in\overline{E}:\phi_j(t)\in A\}$.

(2) *For $j\in\{1,2\}$, in virtue of Part (1) and applying Theorem 3.8, let $\hat{f}_j$ be the unique function in*

$$\mathcal{C}(\widehat{\mathbf{X}}(\phi_j^{-1}(A)\cap E,B;E,G))\cap\mathcal{O}(\widehat{\mathbf{X}}^\circ(\phi_j^{-1}(A)\cap E,B;E,G))$$

such that $\hat{f}_j(t,w)=f(\phi_j(t),w)$, $(t,w)\in\mathbf{X}(\phi_j^{-1}(A)\cap E, B; E, G)$. *Then*

$$\hat{f}_1(t_1,w)=\hat{f}_2(t_2,w)$$

for all $w\in G$ *such that* $(t_j,w)\in\widehat{\mathbf{X}}(\phi_j^{-1}(A)\cap E, B; E, G)$, $j\in\{1,2\}$.

Proof. Part (1) follows immediately from the hypothesis. Therefore, it remains to prove Part (2). To do this fix $w_0\in G\cup B$ such that $(t_j,w_0)\in\widehat{\mathbf{X}}(\phi_j^{-1}(A)\cap E, B; E, G)$ for $j\in\{1,2\}$. We need to show that $\hat{f}_1(t_1,w_0)=\hat{f}_2(t_2,w_0)$. Observe that both functions $w\in\mathcal{G}\mapsto\hat{f}_1(t_1,w)$ and $w\in\mathcal{G}\mapsto\hat{f}_2(t_2,w)$ belong to $\mathcal{O}(\mathcal{G})$, where $\mathcal{G}$ is the connected component which contains w_0 of the following open set

$$\Big\{w\in G:\omega(w,B,G)<1-\max_{j\in\{1,2\}}\omega(t_j,\phi_j^{-1}(A)\cap E,E)\Big\}.$$

On the other hand, for any $j\in\{1,2\}$ and $w\in B$, $(t_j,w)\in\widehat{\mathbf{X}}(\phi_j^{-1}(A)\cap E, B; E, G)$. This, combined with the equality $\phi_1(t_1)=\phi_2(t_2)$, implies that

$$\hat{f}_1(t_1,w)=f(\phi_1(t_1),w)=f(\phi_2(t_2),w)=\hat{f}_2(t_2,w),\quad w\in B.$$

Therefore, by the Uniqueness theorem (see Part (3) of Proposition 3.7), $\hat{f}_1(t_1,w)=\hat{f}_2(t_2,w)$, $w\in\mathcal{G}$. Hence, $\hat{f}_1(t_1,w_0)=\hat{f}_2(t_2,w_0)$, which completes the proof of the lemma. □

Now we return to the proof of the theorem. We define $\hat{f}$ as follows: Let $\mathcal{W}$ be the set of all pairs $(z,w)\in D\times(G\cup B)$ with the property that there are a holomorphic disc $\phi\in\mathcal{O}(\overline{E},D)$ and $t\in E$ such that $\phi(t)=z$ and $(t,w)\in\widehat{\mathbf{X}}(\phi^{-1}(A)\cap E, B; E, G)$. In virtue of Theorem 3.8, let $\hat{f}_\phi$ be the unique function in

$$\mathcal{C}(\widehat{\mathbf{X}}(\phi^{-1}(A)\cap E, B; E, G))\cap\mathcal{O}(\widehat{\mathbf{X}}^\circ(\phi^{-1}(A)\cap E, B; E, G))$$

such that

$$\hat{f}_\phi(t,w)=f(\phi(t),w),\quad (t,w)\in\mathbf{X}(\phi^{-1}(A)\cap E, B; E, G). \tag{4.1}$$

Then the desired extension function $\hat{f}$ is given by

$$\hat{f}(z,w):=\hat{f}_\phi(t,w). \tag{4.2}$$

In virtue of Part (2) of Lemma 4.2, $\hat{f}$ is well-defined on $\mathcal{W}$. We next prove that

$$\mathcal{W}=\widehat{W}. \tag{4.3}$$

Taking (4.3) for granted, $\hat{f}$ is well-defined on $\widehat{W}$.

Now we return to (4.3). To prove the inclusion $\mathcal{W}\subset\widehat{W}$, let $(z,w)\in\mathcal{W}$. By the above definition of $\mathcal{W}$, one may find a holomorphic disc $\phi\in\mathcal{O}(\overline{E},D)$, a point $t\in E$ such that $\phi(t)=z$ and $(t,w)\in\widehat{\mathbf{X}}(\phi^{-1}(A)\cap E,B;E,G)$. Since $\omega(\phi(t),A,D)\le \omega(t,\phi^{-1}(A)\cap E,E)$, it follows that

$$\omega(z,A,D)+\omega(w,B,G)\le\omega(t,\phi^{-1}(A)\cap E,E)+\omega(w,B,G)<1.$$

Hence $(z,w)\in\widehat{W}$. This proves the above mentioned inclusion.

To finish the proof of (4.3), it suffices to show that $\widehat{W}\subset\mathcal{W}$. To do this, let $(z,w)\in\widehat{W}$ and fix an $\varepsilon>0$ such that

$$\varepsilon<1-\omega(z,A,D)-\omega(w,B,G). \tag{4.4}$$

Applying Theorem 3.1 and Proposition 3.2, there is a holomorphic disc $\phi\in\mathcal{O}(\overline{E},D)$ such that $\phi(0)=z$ and

$$\frac{1}{2\pi}\int_0^{2\pi}1_{D\setminus A,D}(\phi(e^{i\theta}))\,d\theta<\omega(z,A,D)+\varepsilon. \tag{4.5}$$

Observe that

$$\begin{aligned}\omega(0,\phi^{-1}(A)\cap E,E)+\omega(w,B,G)&\le\frac{1}{2\pi}\int_0^{2\pi}1_{D\setminus A,D}(\phi(e^{i\theta}))\,d\theta+\omega(w,B,G)\\&<\omega(z,A,D)+\omega(w,B,G)+\varepsilon<1,\end{aligned}$$

where the first inequality follows from Lemma 3.3, the second one from (4.5), and the last one from (4.4). Hence, $(0,w)\in\widehat{\mathbf{X}}(\phi^{-1}(A)\cap E,B;E,G)$, which implies that $(z,w)\in\mathcal{W}$. This complete the proof of (4.3). Hence, the construction of the extension function $\hat{f}$ on $\widehat{W}$ has been completed.

Using (4.1)–(4.3), the proof given in Steps 2 and 3 of Section 4 in [10] still works in the present context after making the obviously necessary changes. This gives that $\hat{f}=f$ on W and $\hat{f}\in\mathcal{C}(\widehat{W})\cap\mathcal{O}(\widehat{W}^{\circ})$. Consequently, arguing as in the proof of Theorem 4.2 in [11], the desired estimate of the theorem follows.

This completes the proof of Theorem 4.1. □

5. Part 2 of the proof: local result

The main purpose of this section is to prove the following "local" result.

Theorem 5.1. *Let (D,A) and (G,B) be two good pairs. Let $f\colon W\to\mathbf{C}$ be such that*:

(i) $f\in\mathcal{C}_s(W)\cap\mathcal{O}_s(W^{\circ})$;

(ii) *f is locally bounded on W*;

(iii) *$f|_{A\times B}$ is continuous.*

Then there exists a unique function $\hat{f}\in\mathcal{C}(\widehat{W})\cap\mathcal{O}(\widehat{W}^{\circ})$ *such that* $\hat{f}=f$ *on* W. *Moreover, if* $|f|_W<\infty$, *then*

$$|\hat{f}(z,w)|\le |f|_{A\times B}^{1-\omega(z,w)}|f|_W^{\omega(z,w)},\quad (z,w)\in\widehat{W}.$$

Proof. We assume without loss of generality that $D\subset\mathbf{C}^n$ and $G\subset\mathbf{C}^m$.

For every $0<\delta<\frac{1}{2}$, $z'\in(-1,1)^{2n-2}$ and $w'\in(-1,1)^{2m-2}$ define

$$\begin{aligned}
E_{z',\delta} &:= \{z=(z',z_n): z_n\in(E_{z'})_n \text{ and } \omega(z_n,(A_{z'})_n,(E_{z'})_n)<\delta\},\\
E_{w',\delta} &:= \{w=(w',w_m): w_m\in(E_{w'})_m \text{ and } \omega(w_m,(B_{w'})_m,(E_{w'})_m)<\delta\},\\
D_\delta &:= \{z\in D:\omega(z,A,D)<1-\delta\},\\
G_\delta &:= \{w\in G:\omega(w,B,G)<1-\delta\},\\
A_\delta &:= \operatorname{int}\Big(\bigcup_{z'\in(-1,1)^{2n-2}} E_{z',\delta}\Big),\\
B_\delta &:= \operatorname{int}\Big(\bigcup_{w'\in(-1,1)^{2m-2}} E_{w',\delta}\Big).
\end{aligned}\tag{5.1}$$

The proof is divided into two steps.

Step 1. *G is a Jordan domain.* Firstly, we apply the slicing method: For all $z'\in(-1,1)^{2n-2}$, consider the function

$$f_{z'}(z_n,w):=f(z,w),\quad (z_n,w)\in\mathbf{X}((A_{z'}\cap\partial E_{z'})_n,B;(E_{z'})_n,G).\tag{5.2}$$

Applying Gonchar's theorem, we obtain an extension function

$$\hat{f}_{z'}\in\mathcal{C}(\widehat{\mathbf{X}}((A_{z'}\cap\partial E_{z'})_n,B;(E_{z'})_n,G))\cap\mathcal{O}(\widehat{\mathbf{X}}^{\circ}((A_{z'}\cap\partial E_{z'})_n,B;(E_{z'})_n,G))$$

such that

$$\hat{f}_{z'}(z_n,w)=f_{z'}(z_n,w),\quad (z_n,w)\in\mathbf{X}((A_{z'}\cap\partial E_{z'})_n,B;(E_{z'})_n,G).\tag{5.3}$$

Using (5.1)–(5.3), we are able to define a new function $\tilde{f}_\delta$ on $\mathbf{X}(A_\delta,B;D,G_\delta)$ as follows

$$\tilde{f}_\delta(z,w):=\begin{cases}\hat{f}_{z'}(z_n,w), & (z,w)\in A_\delta\times G_\delta,\\ f(z,w), & (z,w)\in D\times B.\end{cases}\tag{5.4}$$

Applying Theorem 4.1, we obtain an extension function

$$\hat{f}_\delta\in\mathcal{C}(\widehat{\mathbf{X}}(A_\delta,B;D,G_\delta))\cap\mathcal{O}(\widehat{\mathbf{X}}^{\circ}(A_\delta,B;D,G_\delta))$$

such that

$$\hat{f}_\delta(z,w)=\tilde{f}_\delta(z,w),\quad (z,w)\in\mathbf{X}(A_\delta,B;D,G_\delta). \tag{5.5}$$

On the other hand, using Proposition 3.7 and (5.1), we see that

$$\lim_{\delta\to 0^+}\omega(z,A_\delta,D)=\omega(z,A,D)\quad\text{and}\quad \lim_{\delta\to 0^+}\omega(w,B,G_\delta)=\omega(w,B,G). \tag{5.6}$$

We are now in a position to define the desired extension function $\hat{f}$. Indeed, one glues $(\hat{f}_\delta)_{0<\delta<1/2}$ together to obtain $\hat{f}$ in the following way

$$\hat{f}:=\begin{cases}\lim_{\delta\to 0}\hat{f}_\delta & \text{on } \widehat{W}^\circ\cup(D\times B),\\ f & \text{on } A\times G.\end{cases} \tag{5.7}$$

Using (5.2)–(5.6) and a gluing argument as in Lemma 6.5 in [10], it can be checked that the limit (5.7) exists and possesses all the required properties.

Step 2. *The general case.* Firstly, we apply the slicing method: For all $z'\in(-1,1)^{2n-2}$ and $w'\in(-1,1)^{2m-2}$, consider the functions

$$\begin{aligned} f_{z'}(z_n,w)&:=f(z,w),\quad (z_n,w)\in\mathbf{X}((A_{z'}\cap\partial E_{z'})_n,B;(E_{z'})_n,G),\\ f_{w'}(z,w_m)&:=f(z,w),\quad (z,w_m)\in\mathbf{X}(A,(B_{w'}\cap\partial E_{w'})_m;D,(E_{w'})_m).\end{aligned} \tag{5.8}$$

Applying the result of Step 1, we obtain extension functions

$$\begin{aligned}\hat{f}_{z'}&\in\mathcal{C}(\widehat{\mathbf{X}}((A_{z'}\cap\partial E_{z'})_n,B;(E_{z'})_n,G))\cap\mathcal{O}(\widehat{\mathbf{X}}^\circ((A_{z'}\cap\partial E_{z'})_n,B;(E_{z'})_n,G)),\\ \hat{f}_{w'}&\in\mathcal{C}(\widehat{\mathbf{X}}(A,(B_{w'}\cap\partial E_{w'})_m;D,(E_{w'})_m))\cap\mathcal{O}(\widehat{\mathbf{X}}^\circ(A,(B_{w'}\cap\partial E_{w'})_m;D,(E_{w'})_m))\end{aligned}$$

such that

$$\begin{aligned}\hat{f}_{z'}(z_n,w)&=f_{z'}(z_n,w),\quad (z_n,w)\in\mathbf{X}((A_{z'}\cap\partial E_{z'})_n,B;(E_{z'})_n,G),\\ \hat{f}_{w'}(z,w_m)&=f_{w'}(z,w_m),\quad (z,w_m)\in\mathbf{X}(A,(B_{w'}\cap\partial E_{w'})_m;D,(E_{w'})_m).\end{aligned} \tag{5.9}$$

Using (5.1)–(5.3) and (5.8)–(5.9), it can be checked that

$$\hat{f}_{z'}(z_n,w)=f_{w'}(z,w_m),\quad (z,w)\in A_\delta\times B_\delta.$$

Therefore, we are able to define a new function $\tilde{f}_\delta$ on $\mathbf{X}(A_\delta,B_\delta;D_\delta,G_\delta)$ as follows

$$\tilde{f}_\delta(z,w):=\begin{cases}\hat{f}_{z'}(z_n,w), & (z,w)\in A_\delta\times G_\delta,\\ \hat{f}_{w'}(z,w_n), & (z,w)\in D_\delta\times B_\delta.\end{cases} \tag{5.10}$$

Applying Theorem A or Theorem 5.1 in [10], we obtain an extension function $\hat{f}_\delta \in \mathcal{O}(\widehat{\mathbf{X}}^\circ(A_\delta, B_\delta; D_\delta, G_\delta))$ such that

$$\hat{f}_\delta(z,w) = \tilde{f}_\delta(z,w), \quad (z,w) \in \mathbf{X}(A_\delta, B_\delta; D_\delta, G_\delta). \tag{5.11}$$

We are now in a position to define the desired extension function $\hat{f}$. Indeed, one glues $(\hat{f}_\delta)_{0<\delta<1/2}$ together to obtain $\hat{f}$ in the following way

$$\hat{f} := \begin{cases} \lim_{\delta\to 0} \hat{f}_\delta & \text{on } \widehat{W}^\circ, \\ f & \text{on } W. \end{cases} \tag{5.12}$$

Using the first identity in (5.6) and (5.8)–(5.11) and applying Lemma 6.5 in [10], we can prove that the function given by the limit (5.12) exists. Moreover, $\hat{f}=f$ on W and $\hat{f} \in \mathcal{C}(\widehat{W}) \cap \mathcal{O}(\widehat{W}^\circ)$. Consequently, arguing as in the proof of Theorem 4.2 in [11], the desired estimate of the theorem follows. Hence, the proof is complete. □

6. Proof of the Main theorem

By Proposition 3.5, for all a (resp. $b \in B$) we may fix an open neighborhood U_a of a (resp. V_b of b) such that $(D\cap U_a, A\cap U_a)$ (resp. $(G\cap V_b, B\cap V_b)$) is a good pair. For any $0<\delta<\frac{1}{2}$, define

$$\begin{aligned} U_{a,\delta} &:= \{z \in U_a \cap D : \omega(z, A\cap U_a, U_a \cap D) < \delta\}, \quad a \in A, \\ V_{b,\delta} &:= \{w \in V_b \cap G : \omega(w, B\cap V_b, V_b \cap G) < \delta\}, \quad b \in B, \\ A_\delta &:= \bigcup_{a\in A} U_{a,\delta}, \\ B_\delta &:= \bigcup_{b\in B} V_{b,\delta}, \\ D_\delta &:= \{z \in D : \omega(z, A, D) < 1-\delta\}, \\ G_\delta &:= \{w \in G : \omega(w, B, G) < 1-\delta\}. \end{aligned} \tag{6.1}$$

We divide the proof into two steps.

Step 1. *(G,B) is a good pair.* Suppose without loss of generality that $G \subset \mathbf{C}^m$. For each $a \in A$, let $f_a := f|_{\mathbf{X}(A\cap U_a, B; D\cap U_a, G)}$. Using the hypothesis on f we deduce that f_a is locally bounded,

$$f_a \in \mathcal{C}_s(\mathbf{X}(A\cap U_a, B; D\cap U_a, G)) \cap \mathcal{O}_s(\mathbf{X}(A\cap U_a, B; D\cap U_a, G))$$

and that $f_a|_{(A\cap U_a)\times B}\in\mathcal{C}((A\cap U_a)\times B)$. Recall that $(D\cap U_a, A\cap U_a)$ and (G,B) are good pairs. Consequently, applying Theorem 5.1 to f_a yields that there is a unique function

$$\hat{f}_a\in\mathcal{C}(\widehat{\mathbf{X}}(A\cap U_a, B; D\cap U_a, G))\cap\mathcal{O}(\widehat{\mathbf{X}}^\circ(A\cap U_a, B; D\cap U_a, G))$$

such that

$$\hat{f}_a(z,w)=f_a(z,w)=f(z,w),\quad (z,w)\in\mathbf{X}(A\cap U_a, B; D\cap U_a, G). \tag{6.2}$$

Arguing as in Lemma 6.4 in [10] and using Definition 6.3 therein, we can show that the family $(\hat{f}_a|_{U_{a,\delta}\times G_\delta})_{a\in A}$ is collective for all $0<\delta<\frac{1}{2}$. In virtue of (6.1), let

$$\tilde{\tilde{f}}_\delta\in\mathcal{O}(A_\delta\times G_\delta) \tag{6.3}$$

denote the collected function of this family. In virtue of (6.2)–(6.3), we are able to define a new function $\tilde{f}_\delta$ on $\mathbf{X}(A_\delta, B; D, G_\delta)$ as follows

$$\tilde{f}_\delta:=\begin{cases}\tilde{\tilde{f}}_\delta & \text{on } A_\delta\times G_\delta,\\ f & \text{on } D\times B.\end{cases}$$

Using this and (6.2)–(6.3), we see that $\tilde{f}_\delta\in\mathcal{O}_s(\mathbf{X}(A_\delta, B; D, G_\delta))$, and

$$\tilde{f}_\delta=f\quad\text{on } D\times B. \tag{6.4}$$

Since A_δ is open in D, and B is not only an open subset of ∂G_δ, but also a topological hypersurface (by Proposition 3.7), we are able to apply Theorem 4.1 to $\tilde{f}_\delta$ in order to obtain a function

$$\hat{f}_\delta\in\mathcal{C}(\widehat{\mathbf{X}}(A_\delta, B; D, G_\delta))\cap\mathcal{O}(\widehat{\mathbf{X}}^\circ(A_\delta, B; D, G_\delta))$$

such that

$$\hat{f}_\delta=\tilde{f}_\delta\quad\text{on } \mathbf{X}(A_\delta, B; D, G_\delta). \tag{6.5}$$

We are now in a position to define the desired extension function $\hat{f}$. Indeed, one glues $(\hat{f}_\delta)_{0<\delta<1/2}$ together to obtain $\hat{f}$ in the following way

$$\hat{f}:=\begin{cases}\lim_{\delta\to 0}\hat{f}_\delta & \text{on } \widehat{W}^\circ,\\ f & \text{on } W.\end{cases} \tag{6.6}$$

Using (6.2)–(6.6) and arguing as in (6.12)–(6.14) in [10], we see that $\hat{f}$ is well-defined and possesses all the required properties.

Step 2. *The general case.* For each $a\in A$, let $f_a := f|_{\mathbf{X}(A\cap U_a,B;D\cap U_a,G)}$. Using the hypothesis on f and the fact that $(D\cap U_a, A\cap U_a)$ is a good pair, we are able to apply the result of Step 1 to f_a. Consequently, there is a unique function

$$\hat{f}_a \in \mathcal{C}(\widehat{\mathbf{X}}(A\cap U_a, B; D\cap U_a, G))\cap \mathcal{O}(\widehat{\mathbf{X}}^{\circ}(A\cap U_a, B; D\cap U_a, G))$$

such that

$$\hat{f}_a(z,w) = f(z,w), \quad (z,w)\in \mathbf{X}(A\cap U_a, B; D\cap U_a, G). \tag{6.7}$$

Let $0<\delta<\frac{1}{2}$. In virtue of (6.1) and (6.7), we may apply Lemma 6.4 in [10]. Consequently, we can collect the family $(\hat{f}_a|_{U_{a,\delta}\times G_\delta})_{a\in A}$ in order to obtain the collected function $\tilde{f}_\delta^A \in \mathcal{O}(A_\delta\times G_\delta)$.

Similarly, for each $b\in B$, one obtains a unique function

$$\hat{f}_b \in \mathcal{C}(\widehat{\mathbf{X}}(A, B\cap V_b; D, G\cap V_b))\cap \mathcal{O}(\widehat{\mathbf{X}}^{\circ}(A, B\cap V_b; D, G\cap V_b))$$

such that

$$\hat{f}_b(z,w) = f(z,w), \quad (z,w)\in \mathbf{X}(A, B\cap V_b; D, V_b). \tag{6.8}$$

Moreover, one can collect the family $(\hat{f}_b|_{D_\delta\times V_{b,\delta}})_{b\in B}$ in order to obtain the collected function $\tilde{f}_\delta^B \in \mathcal{O}(D_\delta\times B_\delta)$.

Arguing as in the proof of (6.17)–(6.18) in [10], we can show that

$$\tilde{f}_\delta^A = \tilde{f}_\delta^B \quad \text{on } A_\delta\times B_\delta.$$

Consequently, we are able to define a new function $\tilde{f}_\delta$ on $\mathbf{X}(A_\delta, B_\delta; D_\delta, G_\delta)$ as follows

$$\tilde{f}_\delta := \begin{cases} \tilde{f}_\delta^A & \text{on } A_\delta\times G_\delta, \\ \tilde{f}_\delta^B & \text{on } D_\delta\times B_\delta. \end{cases} \tag{6.9}$$

Using formula (6.9) it can be readily checked that $\tilde{f}_\delta \in \mathcal{O}_s(\mathbf{X}(A_\delta, B_\delta; D_\delta, G_\delta))$. Since we know that A_δ (resp. B_δ) is an open subset of D_δ (resp. G_δ), we are able to apply Theorem A or Theorem 5.1 in [10] to $\tilde{f}_\delta$ for every $0<\delta<\frac{1}{2}$. Consequently, one obtains a unique function $\hat{f}_\delta \in \mathcal{O}(\widehat{\mathbf{X}}(A_\delta, B_\delta; D_\delta, G_\delta))$ such that

$$\hat{f}_\delta = \tilde{f}_\delta \quad \text{on } \mathbf{X}(A_\delta, B_\delta; D_\delta, G_\delta). \tag{6.10}$$

We are now in a position to define the desired extension function $\hat{f}$.

$$\hat{f} := \begin{cases} \lim_{\delta\to 0} \hat{f}_\delta & \text{on } \widehat{W}^\circ, \\ f & \text{on } W. \end{cases} \tag{6.11}$$

To prove that $\hat{f}$ is well-defined, $\hat{f}=f$ on W and $\hat{f}\in\mathcal{C}(\widehat{W})\cap\mathcal{O}(\widehat{W}^\circ)$, one proceeds as in the end of the proof of Theorem 6.1 in [10] using (6.7)–(6.11). Consequently, arguing as in the proof of Theorem 4.2 in [11], the desired estimate of the theorem follows. Hence, the proof of the Main theorem is complete. □

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