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RESEARCH ARTICLE

Entanglement measures of a new type pseudo-pure state in accelerated frames

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In this work we analyze the characteristics of quantum entanglement of the Dirac field in noninertial reference frames in the context of a new type pseudo-pure state, which is composed of the Bell states. This will help us to understand the relationship between the relativity and quantum information theory. Some states will be changed from entangled states into separable ones around the critical value $F = 1/4$, but there is no such a critical value for the variable y related to acceleration a. We find that the negativity $\mathcal{N}_{AB_I}(\rho_{AB_I}^{T_A})$ increases with F but decreases with the variable y, while the variation of the negativity $\mathcal{N}_{B_IB_{II}}(\rho_{B_IB_{II}}^{T_{B_I}})$ is opposite to that of the negativity $\mathcal{N}_{AB_I}(\rho_{AB_I}^{T_A})$. We also study the von Neumann entropies $S(\rho_{AB_I})$ and $S(\rho_{B_IB_{II}})$. We find that the $S(\rho_{AB_I})$ increases with variable y but $S(\rho_{B_IB_{II}})$ is independent of it. However, both $S(\rho_{AB_I})$ and $S(\rho_{B_IB_{II}})$ first decreases with F and then increases with it. The concurrences $C(\rho_{AB_I})$ and $C(\rho_{B_IB_{II}})$ are also discussed. We find that the former decreases with y while the latter increases with y but both of them first increase with F and then decrease with it.

Keywords negativity, pseudo-pure state, noninertial frame, entanglement, von Neumann entropy, concurrence

1 Introduction

The study of the entanglements within a noninertial frame is important in quantum information theory. Recently, a lot of interesting and significant works have been worked out in effects of both special and general relativity on quantum entanglement [1–17]. In the classical work, Alsing *et al.* analyzed the entanglement between two modes of a free Dirac field as regarded as two relatively accelerated parties [1]. Wang *et al.* generalized Alsing's study to three observers [9], which is revisited in our recent study [18]. They assume that Alice, Bob and Charlie initially share a GHZ state, and then let Alice stay stationary, while Bob and Charlie move with uniform acceleration. Considering the special characteristics for the GHZ state, i.e., the density matrix can be expressed as the X form. The corresponding eigenvalues can be obtained easily. Recently, Moradi [19] and Mehri-Dehnavi *et al.* [20] have studied the so-called Werner state [21] $\rho = F|\Psi^-\rangle\langle\Psi^-|+(1-F)/3(|\Phi^+\rangle\langle\Phi^+|+|\Phi^-\rangle\langle\Phi^-|+|\Psi^+\rangle\langle\Psi^+|)$ and pseudo-pure states combined by the Bell states such as the $|\psi\rangle = \alpha(|00\rangle+|11\rangle)+\beta|10\rangle$ [21], respectively. Moreover, we have restudied Werner state in accelerated frames [22]. Stimulated by these works we are going to propose an interesting new pseudo-pure state, which is composed of four Bell states

$$|\Psi\rangle = \sqrt{F}|\Psi^-\rangle + \sqrt{\frac{1-F}{3}}(|\Phi^+\rangle + |\Phi^-\rangle + |\Psi^+\rangle), \tag{1}$$

where F is a parameter such that $0 \le F \le 1$. The $|\Phi^\pm\rangle$ and $|\Psi^\pm\rangle$ are usual entangled Bell states

$$|\Phi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|0_A\rangle|0_B\rangle \pm |1_A\rangle|1_B\rangle), \tag{2}$$

$$|\Psi^\pm\rangle_{AB} = \frac{1}{\sqrt{2}}(|0_A\rangle|1_B\rangle \pm |1_A\rangle|0_B\rangle). \tag{3}$$

Our aim is to study its characteristics of the entanglement in noninertial frames such as the negativity, the von Neumann entropy, concurrence and also the effects of the variables F and y on them. We find that this state (1) has the simplest form $|\Psi^-\rangle$ for $F = 1$, while it is the combination of three Bell states $|\Phi^\pm\rangle$ and $|\Psi^+\rangle$ for $F = 0$.

This work is organized as follows. In Section 2 we propose an interesting new pseudo-pure state and present the density operator. We are going to study the characteristics of entanglement of the Dirac field in noninertial reference frames subject to this state such as the negativity, von Neumann entropy and the concurrences. Finally, we will give our summary in Section 4.

2 New pseudo-pure state in noninertial reference frames

Based on the explicit expressions of the Bell states, we are able to write out this state as the following modified form:

$$|\Psi\rangle_m = \alpha|0_A 0_B\rangle + \beta_+|0_A 1_B\rangle + \beta_-|1_A 0_B\rangle, \tag{4}$$

where

$$\alpha = \sqrt{\frac{2(1-F)}{3}}, \quad \beta_\pm = \sqrt{\frac{1-F}{6}} \pm \sqrt{\frac{F}{2}}. \tag{5}$$

In order to describe the entanglement of the state as seen by Alice and Bob, let us expand the Minkowski states $|0\rangle_M$ and $|1\rangle_M$ into Rindler regions I and II particle and antiparticle states. As shown in Refs. [9, 13], the Minkowski vacuum and one particle states in terms of Rindler–Fock states are given by

$$|0\rangle_M = \cos r_b |0\rangle_I |0\rangle_{II} + \sin r_b |1\rangle_I |1\rangle_{II}, \tag{6}$$

$$|1\rangle_M = |1\rangle_I |0\rangle_{II}, \tag{7}$$

where $\tan r_b = \exp(-\pi\omega/a)$. The parameters a and ω represent the acceleration of the accelerated observer and the Minkowski frequency, respectively. In this case one has $r_b \in [0, \pi/4]$ for $0 \le a \le \infty$.

We first assume Alice is in an inertial frame and Bob is observing the system from accelerated frame. And then we let Alice stay stationary while Bob moves with uniform acceleration. Taking Eqs. (6) and (7) into account, this state can be expressed as the following form:

$$\begin{aligned}|\Psi\rangle_m = {}& \alpha[\cos(r_b)|0_A 0_{B_I} 0_{B_{II}}\rangle + \sin(r_b)|0_A 1_{B_I} 1_{B_{II}}\rangle] \\ & + \beta_+|0_A 1_{B_I} 0_{B_{II}}\rangle + \beta_-[\cos(r_b)|1_A 0_{B_I} 0_{B_{II}}\rangle \\ & + \sin(r_b)|1_A 1_{B_I} 1_{B_{II}}\rangle].\end{aligned} \tag{8}$$

The corresponding density matrix will take the form of

$$\rho_{AB_IB_{II}} = \begin{pmatrix} \alpha^2\cos^2 y & 0 & \alpha\beta_+\cos y & \alpha^2\cos y\sin y & \alpha\beta_-\cos^2 y & 0 & 0 & \alpha\beta_-\cos y\sin y \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha\beta_+\cos y & 0 & \beta_+^2 & \alpha\beta_+\sin y & \beta_+\beta_-\cos y & 0 & 0 & \beta_+\beta_-\sin y \\ \alpha^2\cos y\sin y & 0 & \alpha\beta_+\sin y & \alpha^2\sin^2 y & \alpha\beta_-\cos y\sin y & 0 & 0 & \alpha\beta_-\sin^2 y \\ \alpha\beta_-\cos^2 y & 0 & \beta_+\beta_-\cos y & \alpha\beta_-\cos y\sin y & \beta_-^2\cos^2 y & 0 & 0 & \beta_-^2\cos y\sin y \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \alpha\beta_-\cos y\sin y & 0 & \beta_+\beta_-\sin y & \alpha\beta_-\sin^2 y & \beta_-^2\cos y\sin y & 0 & 0 & \beta_-^2\sin^2 y \end{pmatrix}, \tag{9}$$

where $y = r_b$ for simplicity and we write the matrix on the basis of $|0\rangle = |000\rangle$, $|1\rangle = |001\rangle$, $|2\rangle = |010\rangle$, $|3\rangle = |011\rangle$, $|4\rangle = |100\rangle$, $|5\rangle = |101\rangle$, $|6\rangle = |110\rangle$, $|7\rangle = |111\rangle$.

3 Negativity, von Neumann entropy and concurrence

As we know, when Bob moves with uniform acceleration a in Rindler region I, he is causally disconnected from region II. This means that he has no access to field modes in this region II. Thus, the observer has to trace over this inaccessible region. By taking the trace over the modes in II, we are able to obtain the following mixed density between Alice and Bob:

$$\begin{aligned}\rho_{AB_I} = {}& \alpha^2\cos^2 y\,|0_A 0_{B_I}\rangle\cdot\langle 0_A 0_{B_I}| + \alpha\beta_+\cos y\,|0_A 1_{B_I}\rangle\cdot\langle 0_A 0_{B_I}| + \alpha\beta_-\cos^2 y\,|1_A 0_{B_I}\rangle\cdot\langle 0_A 0_{B_I}| \\ & + \alpha\beta_+\cos y\,|0_A 0_{B_I}\rangle\cdot\langle 0_A 1_{B_I}| + \beta_+^2\,|0_A 1_{B_I}\rangle\cdot\langle 0_A 1_{B_I}| + \alpha^2\sin^2 y\,|0_A 1_{B_I}\rangle\cdot\langle 0_A 1_{B_I}| \\ & + \beta_+\beta_-\cos y\,|1_A 0_{B_I}\rangle\cdot\langle 0_A 1_{B_I}| + \alpha\beta_-\sin^2 y\,|1_A 1_{B_I}\rangle\cdot\langle 0_A 1_{B_I}| + \alpha\beta_-\cos^2 y\,|0_A 0_{B_I}\rangle\cdot\langle 1_A 0_{B_I}| \\ & + \beta_+\beta_-\cos y\,|0_A 1_{B_I}\rangle\cdot\langle 1_A 0_{B_I}| + \beta_-^2\cos^2 y\,|1_A 0_{B_I}\rangle\cdot\langle 1_A 0_{B_I}| + \alpha\beta_-\sin^2 y\,|0_A 1_{B_I}\rangle\cdot\langle 1_A 1_{B_I}| \\ & + \beta_-^2\sin^2 y\,|1_A 1_{B_I}\rangle\cdot\langle 1_A 1_{B_I}|.\end{aligned} \tag{10}$$

Its corresponding matrix form is given explicitly by

$$\rho_{AB_I} = \begin{pmatrix} \alpha^2\cos^2 y & \alpha\beta_+\cos y & \alpha\beta_-\cos^2 y & 0 \\ \alpha\beta_+\cos y & \beta_+^2+\alpha^2\sin^2 y & \beta_+\beta_-\cos y & \alpha\beta_-\sin^2 y \\ \alpha\beta_-\cos^2 y & \beta_+\beta_-\cos y & \beta_-^2\cos^2 y & 0 \\ 0 & \alpha\beta_-\sin^2 y & 0 & \beta_-^2\sin^2 y \end{pmatrix} \tag{11}$$

from which we can obtain partially transpose subsystems A and B_I of Eq. (10) as

$$\begin{aligned} \rho_{AB_I}^{T_A} = \rho_{AB_I}^{T_{B_I}} &= \alpha^2\cos^2 y\,|0_A0_{B_I}\rangle\cdot\langle 0_A0_{B_I}| + \alpha\beta_+\cos y\,|0_A1_{B_I}\rangle\cdot\langle 0_A0_{B_I}| + \alpha\beta_-\cos^2 y\,|1_A0_{B_I}\rangle\cdot\langle 0_A0_{B_I}| \\ &+\beta_+\beta_-\cos y\,|1_A1_{B_I}\rangle\cdot\langle 0_A0_{B_I}| + \alpha\beta_+\cos y\,|0_A0_{B_I}\rangle\cdot\langle 0_A1_{B_I}| + [\beta_+^2+\alpha^2\sin^2 y]\,|0_A1_{B_I}\rangle\cdot\langle 0_A1_{B_I}| \\ &+\alpha\beta_-\sin^2 y\,|1_A1_{B_I}\rangle\cdot\langle 0_A1_{B_I}| + \alpha\beta_-\cos^2 y\,|0_A0_{B_I}\rangle\cdot\langle 1_A0_{B_I}| + \beta_-^2\cos^2 y\,|1_A0_{B_I}\rangle\cdot\langle 1_A0_{B_I}| \\ &+\beta_+\beta_-\cos y\,|0_A0_{B_I}\rangle\cdot\langle 1_A1_{B_I}| + \alpha\beta_-\sin^2 y\,|0_A1_{B_I}\rangle\cdot\langle 1_A1_{B_I}| + \beta_-^2\sin^2 y\,|1_A1_{B_I}\rangle\cdot\langle 1_A1_{B_I}|. \end{aligned} \tag{12}$$

The reason why we take the partial transposed density matrix of a system is to make the entangled quantum system have at least one negative eigenvalue.

Their matrix forms are given by

$$\rho_{AB_I}^{T_A} = \rho_{AB_I}^{T_{B_I}} = \begin{pmatrix} \alpha^2\cos^2 y & \alpha\beta_+\cos y & \alpha\beta_-\cos^2 y & \beta_+\beta_-\cos y \\ \alpha\beta_+\cos y & \beta_+^2+\alpha^2\sin^2 y & 0 & \alpha\beta_-\sin^2 y \\ \alpha\beta_-\cos^2 y & 0 & \beta_-^2\cos^2 y & 0 \\ \beta_+\beta_-\cos y & \alpha\beta_-\sin^2 y & 0 & \beta_-^2\sin^2 y \end{pmatrix}. \tag{13}$$

Let us study the so-called two-tangle $\mathcal{N}_{AB} = \|\rho_{AB}^{T_A}\| - 1$, which represents the negativity of the mixed state ρ_{AB}, while the "one-tangle" is given by $\mathcal{N}_{A(BC)} = \|\rho_{ABC}^{T_A}\| - 1$, and $\|A\| = \mathrm{tr}(\sqrt{AA^\dagger})$ is the trace norm of matrix A, i.e., the sum of the singular values of A [23]. Alternatively, $\|A\| - 1$ is equal to two times of the sum of absolute values of negative eigenvalues of A. It should be addressed that the eigenvalues of the above equation (13) can be obtained analytically, but their expressions are too complicated to write down here. In Fig. 1, we show the variation of the negativity $\mathcal{N}_{AB_I}(\rho_{AB_I}^{T_A})$ on the acceleration parameter y and F. We find that the negativity increases with F but decreases with the acceleration parameter y. In the special case $F = 1$, the eigenvalues of above matrix (13) are given by

$$\lambda_{AB_I}^1 = \lambda_{AB_I}^2 = \frac{1}{2}, \lambda_{AB_I}^3 = -\lambda_{AB_I}^4 = \frac{1}{2}\cos^2 y, \tag{14}$$

from which one has $\mathcal{N}_{AB_I}(\rho_{AB_I}^{T_A}) = \cos^2 y$. It is not difficult to see that $\lambda_{AB_I}^4$ is always negative. Also, we find that there is only one negative eigenvalue for the case $F = 0$.

We are now in the position to calculate the von Neumann entropy $S(\rho_{AB_I}) = -\sum_{i=1}^4 \lambda_i \log_2 \lambda_i$ of density ρ_{AB_I}. To this end, let us calculate the eigenvalues of the density ρ_{AB_I} (11), which are given by

$$\begin{aligned} &\lambda_{1,2} = 0, \\ &\lambda_{3,4} = \frac{1}{4}\{2\alpha^2 + 2\left(\beta_+^2+\beta_-^2\right) \mp \sqrt{2}[\alpha^4 + 2\alpha^2\left(2\beta_+^2+\beta_-^2\right) + 2\beta_+^4 + \beta_-^4 + \left(\alpha^2+\beta_-^2\right)^2\cos(4y) + 4\beta_+^2\beta_-^2\cos(2y)]^{1/2}\}, \end{aligned} \tag{15}$$

from which we have

$$S(\rho_{AB_I}) = -\frac{6[\ln(\sqrt{2}\sigma+12) + \ln(12-\sqrt{2}\sigma) - 2\ln 24] + \sqrt{2}\sigma\,\mathrm{arccoth}\left(\frac{6\sqrt{2}}{\sigma}\right)}{12\ln 2}, \tag{16}$$

where the parameter σ is defined as

$$\begin{aligned} &\sigma = [(8\nu F - 8F^2 - 8F - 20\nu + 25)\cos(4y) + 4(1-4F)^2\cos(2y) - 8F(7F+\nu-5) + 20\nu + 43]^{1/2}, \\ &\nu = \sqrt{3F(1-F)}. \end{aligned} \tag{17}$$

Let us study the concurrence $C(\rho_{AB_I})$ of this system. Here we choose the approach $\rho(\sigma_y\otimes\sigma_y)\rho^\dagger(\sigma_y\otimes\sigma_y)$ to calculate the eigenvalues of the matrix. This approach is different from our recent study [24], in which the density matrix can

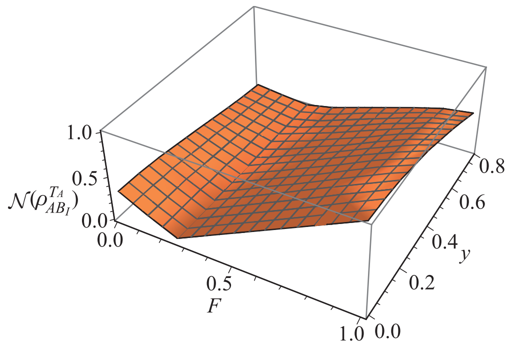

Fig. 1 A plot of $\mathcal{N}_{AB_I}(\rho_{AB_I}^{T_A})$ as function of acceleration parameter y and F.

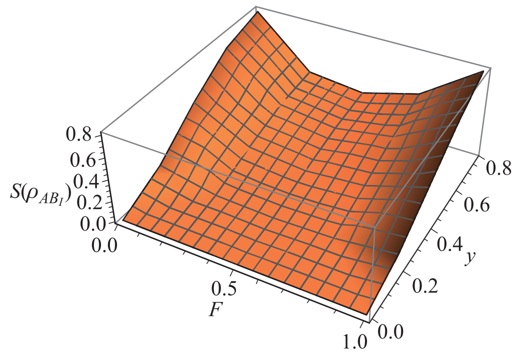

Fig. 2 A plot of $S(\rho_{AB_I})$ as function of acceleration parameters y and F.

be written as an X form. The $(\sigma_y \otimes \sigma_y)$ is given by

$$(\sigma_y \otimes \sigma_y) = \begin{pmatrix} 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \end{pmatrix}. \tag{18}$$

The matrix $\rho_{AB_I}(\sigma_y \otimes \sigma_y)\rho_{AB_I}^{\dagger}(\sigma_y \otimes \sigma_y)$ is calculated as

$$\begin{pmatrix} 0 & 2\alpha\beta_+\beta_-^2\cos^3 y & 2\alpha\beta_+^2\beta_-\cos^2 y & -2\alpha^2\beta_+\beta_-\cos^3 y \\ 0 & 2\beta_+^2\beta_-^2\cos^2 y & 2\beta_+^3\beta_-\cos y & -2\alpha\beta_+^2\beta_-\cos^2 y \\ 0 & 2\beta_+\beta_-^3\cos^3 y & 2\beta_+^2\beta_-^2\cos^2 y & -2\alpha\beta_+\beta_-^2\cos^3 y \\ 0 & 0 & 0 & 0 \end{pmatrix}, \tag{19}$$

from which we have the square roots of the eigenvalues as $\lambda_1 = 2|\beta_+\beta_-|\cos y = |(1-4F)|\cos y/3, \lambda_{2,3,4} = 0$. The concurrence is calculated by

$$C(\rho) = \max\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\}, \quad \lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4. \tag{20}$$

We plot the concurrence of $C(\rho_{AB_I})$ in Fig. 3. We find that $C(\rho_{AB_I})$ first decreases with F and then increases with it, but it decreases with the increasing y. In particular, we find that $C(\rho_{AB_I})$ becomes zero for $F = 1/4$. This implies that the entanglement is broken. This means that $C(\rho_{AB_I})$ weakens when the acceleration increases.

Finally, by tracing over the Alice qubit we obtain the density matrix of Bob in region I and the modes in region II

$$\rho_{B_I B_{II}} = \begin{pmatrix} (\alpha^2+\beta_-^2)\cos^2 y & 0 & \alpha\beta_+\cos y & (\alpha^2+\beta_-^2)\cos y\sin y \\ 0 & 0 & 0 & 0 \\ \alpha\beta_+\cos y & 0 & \beta_+^2 & \alpha\beta_+\sin y \\ (\alpha^2+\beta_-^2)\cos y\sin y & 0 & \alpha\beta_+\sin y & (\alpha^2+\beta_-^2)\sin^2 y \end{pmatrix}. \tag{21}$$

The partial transposes of $\rho_{B_I B_{II}}$ is given by

$$\rho_{B_I B_{II}}^{TB_I} = \rho_{B_I B_{II}}^{TB_{II}} = \begin{pmatrix} (\alpha^2+\beta_-^2)\cos^2 y & 0 & \alpha\beta_+\cos y & 0 \\ 0 & 0 & (\alpha^2+\beta_-^2)\cos y\sin y & 0 \\ \alpha\beta_+\cos y & (\alpha^2+\beta_-^2)\cos y\sin y & \beta_+^2 & \alpha\beta_+\sin y \\ 0 & 0 & \alpha\beta_+\sin y & (\alpha^2+\beta_-^2)\sin^2 y \end{pmatrix}. \tag{22}$$

Likewise, we find that the eigenvalues for above matrix (22) are too complicated to write them out explicitly. In Fig. 4, we illustrate the variation of the negativity $\mathcal{N}_{B_I B_{II}}(\rho_{B_I B_{II}}^{TB_I})$ on the acceleration parameter y and the F. In comparison with the negativity $\mathcal{N}_{AB_I}(\rho_{AB_I}^{T_A})$, we find that the negativity decreases with both the acceleration

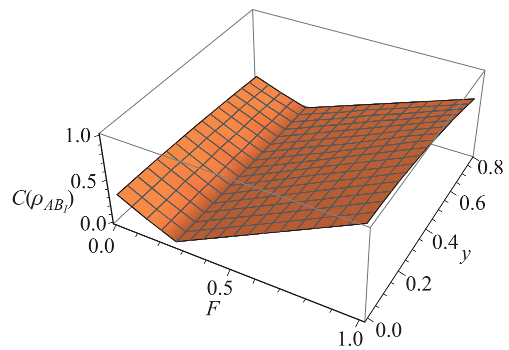

Fig. 3 A plot of $C(\rho_{AB_I})$ as function of acceleration parameters y and F.

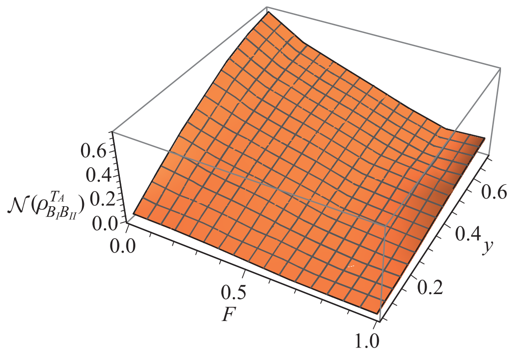

Fig. 4 A plot of $\mathcal{N}_{B_IB_{II}}(\rho^{T_{B_I}}_{B_IB_{II}})$ as function of acceleration parameters y and F.

parameter y and the F. In the special case $F=1$, we find that the eigenvalues are given by

$$\begin{aligned}\lambda^1_{B_IB_{II}} &= \frac{1}{2}\cos^2 y,\\ \lambda^2_{B_IB_{II}} &= \frac{1}{8}\left[2-\sqrt{6-2\cos(4y)}\right],\\ \lambda^3_{B_IB_{II}} &= \frac{1}{8}\left[2+\sqrt{6-2\cos(4y)}\right],\\ \lambda^4_{B_IB_{II}} &= \frac{1}{2}\sin^2 y.\end{aligned}\tag{23}$$

from which we have $\mathcal{N}_{B_IB_{II}}(\rho^{T_{B_I}}_{B_IB_{II}}) = -2\lambda^2_{B_IB_{II}}$. We see that $\lambda^2_{B_IB_{II}} \le 0$. In the special case $y=0$, i.e. $a=0$, there is no entanglement for this case. Similarly, when $F=0$ the analytical eigenvalues are also too complicated to write out them explicitly, but we find there is no entanglement for $y=0$.

In a similar way, let us calculate the von Neumann entropy $S(\rho_{B_IB_{II}})$ of density $\rho_{B_IB_{II}}$. The eigenvalues of Eq. (21) are given by

$$\begin{aligned}\lambda_{1,2} &= 0,\\ \lambda_{3,4} &= \frac{1}{2}\{\alpha^2+\beta_+^2+\beta_-^2\\ &\quad \mp[\alpha^4+2\alpha^2(\beta_+^2+\beta_-^2)+(\beta_+^2-\beta_-^2)^2]^{1/2}\},\end{aligned}\tag{24}$$

where the signs "$\mp$" correspond the eigenvalues λ_3 and λ_4, respectively. Based on them we get analytical expression of the von Neumann entropy $S(\rho_{B_IB_{II}})$

$$\begin{aligned}S(\rho_{B_IB_{II}}) = \frac{1}{\ln 64}\Bigg[&-3\ln(2\tau+3)-3\ln(3-2\tau)\\ &-4\tau\operatorname{arctanh}\left(\frac{2}{3}\tau\right)+\ln 46656\Bigg],\end{aligned}\tag{25}$$

where $\tau=\sqrt{-4F^2+2F+2}$. It is interesting to see that $S(\rho_{B_IB_{II}})$ is independent of the variable y as shown above. his can be explained well since Bob is moving in Regions I and II with same acceleration.

Similarly, we study the concurrence $C(\rho_{B_IB_{II}})$. We obtain the matrix expression of the $\rho_{B_IB_{II}}(\sigma_y\otimes\sigma_y)\rho^\dagger_{B_IB_{II}}(\sigma_y\otimes\sigma_y)$ as follows:

$$\begin{pmatrix} 2(\alpha^2+\beta_-^2)^2\cos^2 y\sin^2 y & -2\alpha\beta_+(\alpha^2+\beta_-^2)\cos^2 y\sin y & 0 & 2(\alpha^2+\beta_-^2)^2\cos^3 y\sin y\\ 0 & 0 & 0 & 0\\ \alpha\beta_+(\alpha^2+\beta_-^2)\sin y\sin 2y & -2\alpha^2\beta_+^2\cos y\sin y & 0 & 2\alpha\beta_+(\alpha^2+\beta_-^2)\cos^2 y\sin y\\ 2(\alpha^2+\beta_-^2)^2\cos y\sin^3 y & -2\alpha\beta_+(\alpha^2+\beta_-^2)\cos y\sin^2 y & 0 & 2(\alpha^2+\beta_-^2)^2\cos^2 y\sin^2 y\end{pmatrix},\tag{26}$$

from which we have the square roots of the eigenvalues of this matrix as

$$\lambda_1=\sqrt{(\alpha^2+\beta_-^2)^2\sin y[\sin y+\sin(3y)]},\qquad \lambda_{2,3,4}=0.$$

We plot it in Fig. 6. We find that the $C(\rho_{B_IB_{II}})$ increases with y and first decreases with the variable F and then increases with it.

4 Conclusions

In this work we have analyzed the characteristics of the quantum entanglement of this system by taking a new pseudo-pure state into account. It is found that this state is composed by four Bell states and essentially by the combination of the two qubits $\alpha|00\rangle+\beta_+|01\rangle+\beta|10\rangle$. What is the advantage of this state is that the Bell states are orthogonal. The parameter F plays an important role in deciding whether this system is entangled or not. That is, some states will be changed from entangled states into separable ones around the critical value $F=1/4$. We have studied the negativities, von Neumann entropies and also the concurrences. We find that the negativity $\mathcal{N}_{AB_I}(\rho^{T_A}_{AB_I})$ increases with F but decreases with the variable y, while $\mathcal{N}_{B_IB_{II}}(\rho^{T_{B_I}}_{B_IB_{II}})$ decreases with F but increases the acceleration variable y. We also study the von Neumann en-

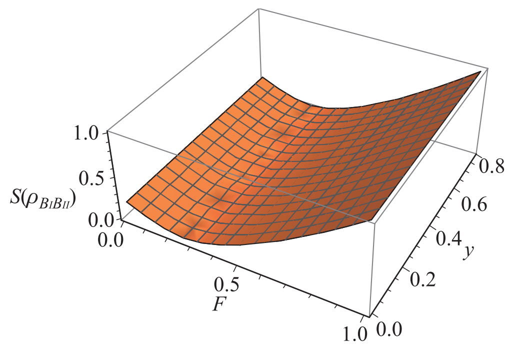

Fig. 5 A plot of $S(\rho_{B_I B_{II}})$ as function of acceleration parameters y and F.

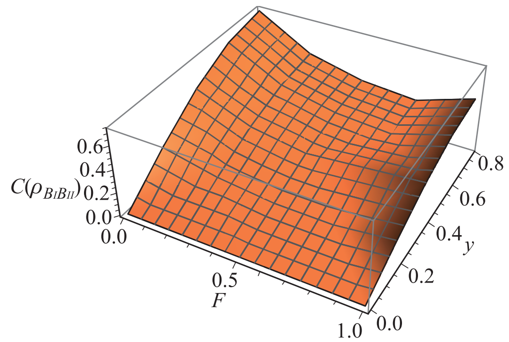

Fig. 6 A plot of $C(\rho_{B_I B_{II}})$ as function of acceleration parameters y and F.

tropies $S(\rho_{AB_I})$ and $S(\rho_{B_I B_{II}})$. We find that $S(\rho_{AB_I})$ increases with variable y but $S(\rho_{B_I B_{II}})$ is independent of y. However, both $S(\rho_{AB_I})$ and $S(\rho_{B_I B_{II}})$ first decreases with F and then increases with it. The concurrences $C(\rho_{AB_I})$ and $C(\rho_{B_I B_{II}})$ are also discussed. We find that the former decreases with y while the latter increases with y but both of them first increase with F and then decrease with it. Before ending this work, we give a useful remark on the von Neumann entropy, which is different from the Shannon entropy in quantum mechanics problem as shown in Refs. [25, 26]. On the other hand, we are going to see whether the concurrence vectors proposed by employing the fundamental representation of Lie algebra [27] can be used to study the present case.

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