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RESEARCH ARTICLE

Evolution of a two-mode squeezed vacuum for amplitude decay via continuous-variable entangled state approach

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Extending the recent work completed by Fan *et al.* [*Front. Phys.* 9(1), 74 (2014)] to a two-mode case, we investigate how a two-mode squeezed vacuum evolves when it undergoes a two-mode amplitude dissipative channel, with the same decay rate κ, using the continuous-variable entangled state approach. Our analytical results show that the initial pure-squeezed vacuum state evolves into a definite mixed state with entanglement and squeezing, decaying over time as a result of amplitude decay. We also investigate the time evolutions of the photon number distribution, the Wigner function, and the optical tomogram in this channel. Our results indicate that the evolved photon number distribution is related to Jacobi polynomials, the Wigner function has a standard Gaussian distribution (corresponding to the vacuum) at long periods, losing its nonclassicality due to amplitude decay, and a larger squeezing leads to a longer decay time.

Keywords two-mode squeezed vacuum, amplitude decay, continuous-variable entangled state representation, photon number distribution, Wigner function, optical tomogram

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1 Introduction

Quantum decoherence of a signal (a quantum state of a system) passing through a quantum channel usually happens since dissipation or dephasing in this process will deteriorate the degree of nonclassicality of this state. In this case, the reduced dynamics of the density matrix operator of the quantum system is described by a master equation. Thus in quantum physics, a master equation of a quantum system is established for a better understanding of how decoherence arises and influences unitary feature of the system in a dissipative or/and dephasing process. In order to solve master equations, various quasi-probability representations, for examples, P-representation, Q-representation, and Wigner functions, etc. (see Refs. [1–4]) to convert the master equations of density operators into the corresponding c-number equations are usually used. Lately, a new approach of treating master equations [5], namely the continuous-variable (CV) entangled state approach, is proposed via the thermal field dynamics theory [6], and its basic idea is to turn operator master equations into the evolution equations for the state vectors, which can directly obtain the corresponding Kraus operators (that is, the infinitive-sum representation of evolved density operators) in most cases. Also, abundant previous results have fully proved that the CV entangled state representation provides us with a new tool for insight about the nature of the dissipative process [7–12]. For instance, Fan *et al.* have analytically studied the evolution of the single-mode squeezed vacuum in an amplitude dissipative channel (ADC) and manifestly revealed how the initial squeezed vacuum evolves into a squeezed chaotic state with decreasing squeezing and with decoherence [10].

As an extension of the work in Ref. [10], in this paper we investigate how an initial two-mode squeezed vacuum state (TMSVS) evolves in a two-mode amplitude dissipative channel by deriving analytically some explicit expressions. We emphasize that this is not a simple extension of the single-mode case [10] since the presence of a two-mode squeezing operation causes the distribution of entanglement between two modes, which requires us to adopt completely different entangled state representations and derivation skills from the single-mode case to treat the complicated calculations involved, especially when the density operator of this state evolves undergoing decoherence in the ADC. Besides, the TMSVS as a Gaussian entangled resource is frequently used to carry out some certain CV quantum information tasks, such as quantum teleportation [13], metrology [14] and communications [15], and the dissipative noise from the environment always occurs in the process of completing these tasks. So, the investigation of the influence of amplitude dissipation on the TMSVS is essential for its practical applications in the quantum information technologies. Moreover, we will investigate what kind of mixed states does the TMSVS develops into and how the photon number distribution (PND), Wigner function and optical tomogram analytically evolve in a ADC. The reduced system dynamics in the two-mode ADC is described by the following master equation [1]:

$$\frac{\mathrm{d}\rho(t)}{\mathrm{d}t} = (L_a + L_b)\rho(t), \tag{1}$$

where $L_i\rho(t) = \kappa(2i\rho(t)i^\dagger - i^\dagger i\rho(t) - \rho(t)i^\dagger i)$, $(i = a, b)$, $\rho(t)$ is the evolved density operator of the system and κ is the decay rate. Actually, amplitude dissipation, different from pure phase dephasing, represents the transfer process of energy from the system to a zero-temperature environment. For simplicity but without loss of generality, the decay rates of the two decay modes are assumed to be the same. To achieve the above aims, we will make full use of the CV entangled state representation newly introduced in Ref. [5].

The work is arranged as follows. In Section 2 by virtue of the CV entangled state representation we clearly derive the infinitive-sum representation of density operator as a solution of the master equation (1). In Section 3, we discuss how an initial TMSVS evolves into a definite mixed state in the two-mode ADC, and its exact evolution law is presented. Based on this, in Section 4, the PND evolution of the mixed output state is obtained which turns out to be a Jacobi polynomial function related to the squeezing parameter r and the decay rate κ. In Sections 5 and 6, we respectively discuss the evolutions of Wigner function and optical tomogram of TMSVS in the ADC.

2 Deducing the infinitive-sum representation of $\rho(t)$

By noticing that modes a and b are independent of each other, the specific solutions of master equation (1) can be considered as the direct product of the solutions of two independent master equations for a- and b-modes. For this reason, we first use the CV entangled state representation to solve the single-mode (say, a-mode) master equation. Recalling that the two-mode CV entangled state is defined as [5, 16–18]

$$|\varsigma\rangle = \exp\left(-\frac{1}{2}|\varsigma|^2 + \varsigma a^\dagger - \varsigma^* \tilde{a}^\dagger + a^\dagger \tilde{a}^\dagger\right)|0\tilde{0}\rangle, \tag{2}$$

where $\tilde{a}^\dagger$ is a fictitious (non-physical) mode accompanying the real mode $a^\dagger$, $[\tilde{a}, a^\dagger] = [\tilde{a}^\dagger, a^\dagger] = 0$. So, the state $|\varsigma\rangle$ may describe the entanglement between the quantum system and the heat reservoir. For the entangled state $|\varsigma = 0\rangle \equiv |I\rangle$, it possesses the well-behaved properties

$$a|I\rangle = \tilde{a}^\dagger|I\rangle, \quad a^\dagger|I\rangle = \tilde{a}|I\rangle. \tag{3}$$

Equation (3) indicates that there exists a non-trivial interchange relation between the real and fictitious modes, that is $a \Leftrightarrow \tilde{a}^\dagger, a^\dagger \Leftrightarrow \tilde{a}$ and $a^\dagger a \Leftrightarrow \tilde{a}^\dagger\tilde{a}$, which plays a key role in obtaining the formal solutions of master equation (1), with details as follows. Acting the both sides of the master equation for a-mode on the state $|I\rangle$, and denoting $|\rho_a(t)\rangle = \rho_a(t)|I\rangle$, we have

$$\frac{\mathrm{d}|\rho_a(t)\rangle}{\mathrm{d}t} = \kappa(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a})|\rho_a(t)\rangle. \tag{4}$$

Letting $\rho_a(0)$ denote the initial density operator, so the formal solution of Eq. (4) is

$$|\rho_a(t)\rangle = \exp[\kappa t(2a\tilde{a} - a^\dagger a - \tilde{a}^\dagger\tilde{a})]|\rho_a(0)\rangle. \tag{5}$$

Noticing that the operators in Eq. (5) follow the commutative relation $[a\tilde{a}, a^\dagger a] = [a\tilde{a}, \tilde{a}^\dagger\tilde{a}] = \tilde{a}a$ and $[a^\dagger a + \tilde{a}^\dagger\tilde{a}, a\tilde{a}] = -2\tilde{a}a$, and using the operator identity $\mathrm{e}^{\lambda(A+\sigma B)} = \mathrm{e}^{\lambda A}\mathrm{e}^{\sigma(1-\mathrm{e}^{-\lambda\tau})B/\tau}$ which holds for $[A, B] = \tau B$ [5], we have

$$\mathrm{e}^{-2\kappa t\left(\frac{a^\dagger a+\tilde{a}^\dagger\tilde{a}}{2} - a\tilde{a}\right)} = \mathrm{e}^{-\kappa t(a^\dagger a+\tilde{a}^\dagger\tilde{a})}\mathrm{e}^{\mathcal{T}a\tilde{a}}, \tag{6}$$

where $\mathcal{T} = 1 - \mathrm{e}^{-2\kappa t}$. Then substituting Eq. (6) into Eq. (5) yields

$$|\rho_a(t)\rangle = \sum_{n=0}^{\infty} \frac{\mathcal{T}^n}{n!}\mathrm{e}^{-\kappa t a^\dagger a}a^n\rho_a(0)a^{\dagger n}\mathrm{e}^{-\kappa t a^\dagger a}|I\rangle. \tag{7}$$

Getting rid of $|I\rangle$ simultaneously on both sides of Eq. (7), we obtain the infinitive operator-sum representation of $\rho_a(t)$, i.e.,

$$\rho_a(t) = \sum_{n=0}^{\infty} \frac{\mathcal{T}^n}{n!}\mathrm{e}^{-\kappa t a^\dagger a}a^n\rho_a(0)a^{\dagger n}\mathrm{e}^{-\kappa t a^\dagger a}. \tag{8}$$

Noting the independence between two modes, thus in terms of Eq. (8), the infinitive-sum representation of $\rho(t)$

as the solutions of master equation (1) can be obtained as

$$\rho(t) = \sum_{m,n=0}^{\infty} \mathcal{M}_{m,n}\rho(0)\mathcal{M}_{m,n}^{\dagger}, \tag{9}$$

where $\mathcal{M}_{m,n}$ is a kind of Kraus operator corresponding to the density operator $\rho(t)$,

$$\mathcal{M}_{m,n} \equiv \sqrt{\frac{\mathcal{T}^{m+n}}{m!n!}} \mathrm{e}^{-\kappa t(a^{\dagger}a+b^{\dagger}b)} a^n b^m. \tag{10}$$

Therefore, the evolution of any given initial state $\rho(0)$ in the ADC can be readily calculated from Eq. (9). The CV entangled state representation provides an elegant approach to solve the master equation (1) and obtain the infinitive-sum representation of density operator $\rho(t)$.

Further, using the i-mode operator formula

$$\mathrm{e}^{\lambda i^{\dagger} i} i \mathrm{e}^{-\lambda i^{\dagger} i} = \mathrm{e}^{-\lambda} i, \tag{11}$$

we can prove the completeness relation of the operator $\mathcal{M}_{m,n}$, i.e.,

$$\begin{aligned}&\sum_{m,n} \mathcal{M}_{m,n}^{\dagger}\mathcal{M}_{m,n} \\ &= \sum_{m,n} \frac{\mathcal{T}^{m+n}\mathrm{e}^{2(m+n)\kappa t}}{m!n!} : a^{\dagger n} a^n b^{\dagger m} b^m : \mathrm{e}^{-2\kappa t(a^{\dagger}a+b^{\dagger}b)} \\ &= I,\end{aligned} \tag{12}$$

where the symbol $: \ :$ denotes normal ordering and I is the unit identity operator. Equation (12) leads to the operator identity tr $\rho(t) = \mathrm{tr}(\sum_{m,n=0}^{\infty} \mathcal{M}_{m,n}\rho(0)\mathcal{M}_{m,n}^{\dagger}) = \mathrm{tr}\rho(0)$, so $\rho(t)$ in Eq. (9) is qualified to be a density operator, and $\mathcal{M}_{m,n}$ is a trace-preserving quantum operation.

3 Evolution of two-mode squeezed vacuum for amplitude decay

In this section, we investigate how the TMSVS as an initial state evolves when it passes through the ADC. Supposing that the initial state in Eq. (9) is a realistic two-mode squeezed vacuum [19, 20], which may be generated by performing the unitary squeezing operator $S(r) = \mathrm{e}^{r(a^{\dagger}b^{\dagger}-ab)}$ with the squeezing parameter r on the vacuum, i.e.,

$$\rho(0) = \mathrm{sech}^2 r \mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle\langle 00|\mathrm{e}^{ab\tanh r}, \tag{13}$$

where the vacuum state $|00\rangle$ is annihilated by either a or b, $[a, a^{\dagger}] = [b, b^{\dagger}] = 1$. Substituting (13) into (9) yields

$$\begin{aligned}\rho(t) = \mathrm{sech}^2 r \sum_{m,n=0}^{\infty} \frac{\mathcal{T}^{n+m}}{n!m!} &\mathrm{e}^{-\kappa t(a^{\dagger}a+b^{\dagger}b)} a^n b^m \mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r} \\ &|00\rangle\langle 00|\mathrm{e}^{ab\tanh r} a^{\dagger n} b^{\dagger m} \mathrm{e}^{-\kappa t(a^{\dagger}a+b^{\dagger}b)}.\end{aligned} \tag{14}$$

To simplify (14) we firstly use the operator identity

$$[i, f(i, i^{\dagger})] = \frac{\partial}{\partial i^{\dagger}} f(i, i^{\dagger}) \tag{15}$$

to analyze the part

$$a^n b^m \mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle = a^n (a^{\dagger}\tanh r)^m \mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle. \tag{16}$$

Equation (16) is the multiple-photon-subtracted TMSVS, serving as a useful non-Gaussian entangled resource in quantum information [21, 22]. Using the operator identity [23]

$$a^n a^{\dagger m} = (-\mathrm{i})^{m+n} : H_{m,n}(\mathrm{i}a^{\dagger}, \mathrm{i}a) : , \tag{17}$$

which can be deduced by virtue of the technique of integration within an ordered product of operators [24], and where $H_{m,n}(\cdot,\cdot)$ is the two-variable Hermite polynomial [25], thus we can rewrite Eq. (16) as

$$\begin{aligned}&a^n b^m \mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle \\ &= \tanh^m r \sum_{l=0}^{\min(m,n)} \frac{m!n!a^{\dagger m-l}a^{n-l}}{l!(m-l)!(n-l)!} \mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle.\end{aligned} \tag{18}$$

Further, using the operator relation (15), we finally obtain

$$\begin{aligned}&a^n b^m \mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle \\ &= (-\mathrm{i})^{m+n} \tanh^m r H_{m,n}(\mathrm{i}a^{\dagger}, \mathrm{i}b^{\dagger}\tanh r)\mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle.\end{aligned} \tag{19}$$

Comparing the first and last terms in (19) shows that the multiple-photon-subtracted state $a^n b^m \mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle$ can be regarded as the two-variable Hermite polynomial excitation on squeezed vacuum (THPESV) $H_{m,n}(\mathrm{i}a^{\dagger}, \mathrm{i}b^{\dagger}\tanh r)\mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle$ since $H_{m,n}(\mathrm{i}a^{\dagger}, \mathrm{i}b^{\dagger}\tanh r)$ is the two-variable Hermite polynomial operator of orders (m, n) as a function of the variables $(a^{\dagger}, b^{\dagger})$, a remarkable result. This provides us with a new sight for studying some nonclassical properties and specific applications involving the state $a^n b^m \mathrm{e}^{a^{\dagger}b^{\dagger}\tanh r}|00\rangle$. Some unique and valuable research results of the THPESV have been published recently [25–27].

Substituting Eq. (19) into Eq. (14) and using the normally ordered form of the vacuum projector $|00\rangle\langle 00| =: \mathrm{e}^{-a^{\dagger}a-b^{\dagger}b}:$ and the operator identity (11), we have

$$\begin{aligned}\rho(t) = \mathrm{sech}^2 r \sum_{m,n=0}^{\infty} &\frac{\mathcal{T}^{n+m}\tanh^{2m} r}{n!m!} \\ &\times : H_{m,n}(\mathrm{i}a^{\dagger}\mathrm{e}^{-\kappa t}, \mathrm{i}b^{\dagger}\mathrm{e}^{-\kappa t}\tanh r)\mathrm{e}^{(a^{\dagger}b^{\dagger}+ab)\mathrm{e}^{-2\kappa t}\tanh r} \\ &\times H_{m,n}(-\mathrm{i}a\mathrm{e}^{-\kappa t}, -\mathrm{i}b\mathrm{e}^{-\kappa t}\tanh r)\mathrm{e}^{-a^{\dagger}a-b^{\dagger}b} : .\end{aligned} \tag{20}$$

Further, using the generating function of the product of two two-variable Hermite polynomials

$$\sum_{m,n=0}^{\infty}\frac{t^n s^m}{m!n!}H_{m,n}(\xi,\eta)H_{m,n}(\sigma,\kappa) = \frac{1}{1-st}\exp\left(\frac{s\sigma\xi+t\eta\kappa-st\sigma\kappa-st\xi\eta}{1-st}\right), \tag{21}$$

we present the compact expression of density operator $\rho(t)$ as follows

$$\rho(t)=\text{ß}_1 \colon \exp[\text{ß}_2(a^\dagger a+b^\dagger b)+\text{ß}_3(ab+a^\dagger b^\dagger)]\colon, \tag{22}$$

where we have set

$$\text{ß}_1=\frac{\mathcal{A}}{\cosh^2 r},\ \text{ß}_2=\mathcal{A}(\mathcal{T}\tanh^2 r-1),\ \text{ß}_3=\mathcal{A}\mathrm{e}^{-2\kappa t}\tanh r.$$

with $\mathcal{A}=(1-\mathcal{T}^2\tanh^2 r)^{-1}$. It is worth noting that, using the normal ordering product of $\rho(t)$ in (22), one can readily calculate the analytical evolutions of the PND, Wigner function and optical tomogram as an indicator of quantifying nonclassicality of $\rho(t)$.

For checking the validity of the above derivation, we calculate $\mathrm{tr}\rho(t)$ to see if it equals one. In fact, using the completeness relation of the coherent states $|\alpha,\beta\rangle$ [28, 29], we have

$$\begin{aligned}\mathrm{tr}\rho(t) &= \text{ß}_1\int\frac{\mathrm{d}^2\alpha d^2\beta}{\pi^2}\langle\alpha\beta|\colon\exp\{\text{ß}_2(a^\dagger a\\ &\quad +b^\dagger b)+\text{ß}_3(ab+a^\dagger b^\dagger)\}\colon|\alpha\beta\rangle\\ &=\frac{\mathrm{sech}^2 r}{1-\mathcal{T}\tanh^2 r}\int\frac{\mathrm{d}^2\beta}{\pi}\exp\left(\frac{\tanh^2 r-1}{1-\mathcal{T}\tanh^2 r}|\beta|^2\right)\\ &=1,\end{aligned} \tag{23}$$

as expected, where we have used the identity in the last step

$$\int\frac{\mathrm{d}^2 z}{\pi}\mathrm{e}^{\zeta|z|^2+\xi z+\eta z^*}=-\frac{1}{\zeta}\mathrm{e}^{-\frac{\xi\eta}{\zeta}},\ \mathrm{Re}\zeta<0. \tag{24}$$

Further, using the operator identity $\mathrm{e}^{\lambda a^\dagger a}=\colon\exp[(\mathrm{e}^\lambda-1)a^\dagger a]\colon$ [30, 31], equation (22) can be rewritten as

$$\begin{aligned}\rho(t) &= \text{ß}_1\exp(\text{ß}_3 a^\dagger b^\dagger)\exp[(a^\dagger a+b^\dagger b)\\ &\quad\times\ln(\text{ß}_2+1)]\exp(\text{ß}_3 ab).\end{aligned} \tag{25}$$

Comparing (25) with the initial state expression (13) we clearly see that, after undergoing the ADC, the squeezing amount $\tanh r\to\frac{\mathrm{e}^{-2\kappa t}\tanh r}{1-\mathcal{T}^2\tanh^2 r}$, and the vacuum state $|00\rangle\langle 00|\longrightarrow\exp\left[(a^\dagger a+b^\dagger b)\ln\left(\frac{\mathcal{T}\mathrm{e}^{-2\kappa t}\tanh^2 r}{1-\mathcal{T}^2\tanh^2 r}\right)\right]$, so $\rho(t)$ becomes an mixed state as a result of amplitude decay, but is still entangled and squeezed. Due to $\mathcal{T}=1-\mathrm{e}^{-2\kappa t}$, and $\mathcal{T}>\mathcal{T}^2\tanh^2 r$, so

$$\frac{\mathrm{e}^{-2\kappa t}}{1-\mathcal{T}^2\tanh^2 r}<1, \tag{26}$$

which implies the squeezing amount decreases in the process of dissipation. Thus we know that, as time evolves, the squeezing effect decreases whereas decoherence increases.

Particularly, when $\kappa t=0$, owing to $\mathcal{T}=0$, $\text{ß}_1=\mathrm{sech}^2 r$, $\text{ß}_2=-1$ and $\text{ß}_3=\tanh r$, equation (22) just reduces to the TMSVS shown in Eq. (13). At long times $\kappa t\to\infty$, noting that $\mathcal{T}\longrightarrow 1$, $\text{ß}_1=-\text{ß}_2\longrightarrow 1$, $\text{ß}_3\longrightarrow 0$, thus the state $\rho(t)$ entirely loses its entanglement and squeezing, and eventually decays to vacuum, as expected.

4 Photon number distribution

The PND is usually used to acquire quantum statistical properties of light fields since its oscillations explained in terms of the phase-space interference effects are regarded as a qualitative signature of nonclassicality [32]. In this section we shall study the PND evolution of the TMSVS and its oscillating behavior.

For the TMSVS, the photon number is given by $p(m,n,t)=\mathrm{tr}[\rho(t)|mn\rangle\langle mn|]$. So, using the normal ordering of $\rho(t)$ in (22), and the relation between the unnormalized coherent state $|\alpha\beta\rangle$ and the number state $|mn\rangle$, i.e., $|mn\rangle=\frac{1}{\sqrt{m!n!}}\frac{\partial^{m+n}}{\partial\alpha^m\partial\beta^n}|\alpha\beta\rangle|_{m=n=0}$, we obtain

$$\begin{aligned}&p(m,n,t)\\ &=\frac{\text{ß}_1}{m!n!}\frac{\partial^{2(m+n)}}{\partial\alpha^m\partial\beta^n\partial\alpha'^{*m}\partial\beta'^{*n}}\exp\{(\text{ß}_2+1)\\ &\quad\times(\alpha\alpha'^*+\beta\beta'^*)+\text{ß}_3(\alpha\beta+\alpha'^*\beta'^*)\}|_{\alpha,\beta,\alpha',\beta'=0}.\end{aligned} \tag{27}$$

Using multiple differential operations and noting the range of exponentials lead to the form:

$$\begin{aligned}&p(m,n,t)\\ &=\frac{\text{ß}_1}{n!m!}\frac{\partial^{2(m+n)}}{\partial\alpha^m\partial\beta^n\partial\alpha'^{*m}\partial\beta'^{*n}}\sum_{l,j,k,p=0}^{\infty}\text{ß}_3^{j+p}\\ &\quad\times\frac{(\text{ß}_2+1)^{l+k}}{l!k!j!p!}\alpha^{l+j}\beta^{j+k}(\alpha'^*)^{l+p}(\beta'^*)^{k+p}\Big|_{\alpha,\beta,\alpha',\beta'=0}\\ &=\text{ß}_1\sum_{p=0}^{\min(m,n)}\frac{m!n!(\text{ß}_2+1)^{m+n-2p}\text{ß}_3^{2p}}{(p!)^2(m-p)!(n-p)!}.\end{aligned} \tag{28}$$

Without loss of generality, supposing $m\leqslant n$ and comparing Eq. (28) with the standard expression of Jacobi polynomials $P_m^{(\alpha,\beta)}(\cdot)$, one can put Eq. (28) into the following compact form:

$$\begin{aligned}p(m,n,t)&=\text{ß}_3^{n+1}\mathcal{T}^{m+n}\mathrm{e}^{-2(m-1)\kappa t}\mathrm{sech}^2 r\\ &\quad\times\tanh^{2m+n-1}r P_m^{(0,n-m)}\left(\frac{1+\mathcal{T}^2\tanh^2 r}{1-\mathcal{T}^2\tanh^2 r}\right),\end{aligned} \tag{29}$$

where we have used the identity $P_m^{(\alpha,\beta)}(-x)=(-1)^m P_m^{(\beta,\alpha)}(x)$. Equation (29) shows that the PND evolution of the TMSVS in the ADC is just related to the Jacobi polynomials, a new expression. Noting that the m-dimensional power-series expansion in Eq. (28) is very convenient to examine the variation of $p(m,n,t)$, so let us use it to analyze several special cases below. At the initial time $\kappa t = 0(\mathcal{T}=0)$, Eq. (29) becomes the PND of the TMSVS, i.e.,

$$
\begin{aligned}
&p(m,n,0)\\
&=\operatorname{sech}^2 r \lim_{\mathcal{T}\to 0} \sum_{p=0}^{\min(m,n)} \frac{m!n!(\tanh r)^{2m+2n-2p}\mathcal{T}^{m+n-2p}}{(p!)^2(m-p)!(n-p)!}\\
&=\begin{cases} \operatorname{sech}^2 r \tanh^{2n} r, & m=n\\ 0, & m\neq n \end{cases}, \qquad (30)
\end{aligned}
$$

which shows that the PND is not zero only when $m=n$, and the PND decreases with increasing the squeezing parameter r. However, we also have $p(m,n,\infty)=0$ when $\kappa t\to\infty$ $(\mathcal{T}\to 1)$, which indicates that there is no photon after a long time interaction with the amplitude dissipative environment, a reasonable result.

In Fig. 1, the PND of the TMSVS is shown in the Fock space (m,n) for different values r and κt, from which we see that the probabilities of finding the smaller (m,n) numbers of photons in this mixed state are larger for small squeezing r, but the probabilities gradually become small and more photons arise in larger number states as r increases. This result seems understandable because the coefficient $\tanh^l r$ of the initial TMSVS as shown in the equation $e^{a^\dagger b^\dagger \tanh r}|00\rangle = \sum_{l=0}^{\infty} \tanh^l r|ll\rangle$ shows a higher weight to the larger-number photon component. Also, the probabilities of finding any photon number (m,n) except for $(0,0)$ decrease as κt increases. At long times $(\kappa t\to\infty)$, the probability of finding $(0,0)$ is one and the others turn to zero resulting from amplitude decay, coinciding with the above analytical result.

5 Wigner function

In quantum optics, the Wigner function is a useful tool with which to study the nonclassical light fields. In this section, we investigate the Wigner function evolution of the TMSVS in the ADC. Noting that the normal ordering form of $\rho(t)$ is shown in Eq. (22), and using the coherent state representation of two-mode Wigner operator $\Delta(\alpha,\beta)$ [33], thus the Wigner function evolution of the TMSVS for amplitude decay can be calculated as

$$
\begin{aligned}
&W(\alpha,\beta;t)\\
&=\text{ß}_1 e^{2(|\alpha|^2+|\beta|^2)}\int \frac{d^2\gamma d^2\zeta}{\pi^4}\exp[(\text{ß}_3\zeta-2\alpha^*)\gamma-\mathcal{B}|\gamma|^2\\
&\quad+(\text{ß}_3\zeta^*+2\alpha)\gamma^*-\mathcal{B}|\zeta|^2-2(\zeta\beta^*-\zeta^*\beta)], \qquad (31)
\end{aligned}
$$

where γ,ζ are the complex amplitudes of two-mode coherent states [28, 29], and $\mathcal{B}=[1+(1-2\mathcal{T})\mathcal{T}\tanh^2 r]/(1-\mathcal{T}^2\tanh^2 r)$. Using the integral formula (24) again, we directly obtain

$$
\begin{aligned}
W(\alpha,\beta,t)=\frac{\text{ß}_1}{\pi^2(\mathcal{B}^2-\text{ß}_3^2)}\exp\bigg[&-\frac{4|\text{ß}_3\alpha-\mathcal{B}\beta^*|^2}{\mathcal{B}(\mathcal{B}^2-\text{ß}_3^2)}\\
&-\frac{2(2-\mathcal{B})|\alpha|^2-2\mathcal{B}|\beta|^2}{\mathcal{B}}\bigg]. \qquad (32)
\end{aligned}
$$

Obviously, the evolution of Wigner function for the TMSVS always keeps Gaussian in amplitude decay environment. In particular, when $\kappa t = 0$ and $\kappa t\to\infty$, Eq. (32) becomes $W(\alpha,\beta,0)=\pi^{-2}e^{-2(|\alpha|^2+|\beta|^2)\cosh 2r+2(\beta\alpha+\beta^*\alpha^*)\sinh 2r}$ and $W(\alpha,\beta,\infty)\to\pi^{-2}e^{-2(|\alpha|^2+|\beta|^2)}$, which just correspond to Wigner functions for the TMSVS and the vacuum, respectively.

In Fig. 2 we show the Wigner function evolution of the TMSVS in the dissipative channel as a function of Reα and Reβ for different values of r and κt. Apparently, it is easy to see that, as a signal of the nonclassicality of this state, squeezing in one of the quadratures is clear. For larger squeezing r, the peak is further compressed along the diagonal direction, which means that the larger squeezing r, the more slowly the nonclassicality is lost. In other word, the larger r leads to much longer decay time than that for the smaller one. However, as κt increases, squeezing properties can deteriorate quickly. For $\kappa t\to$

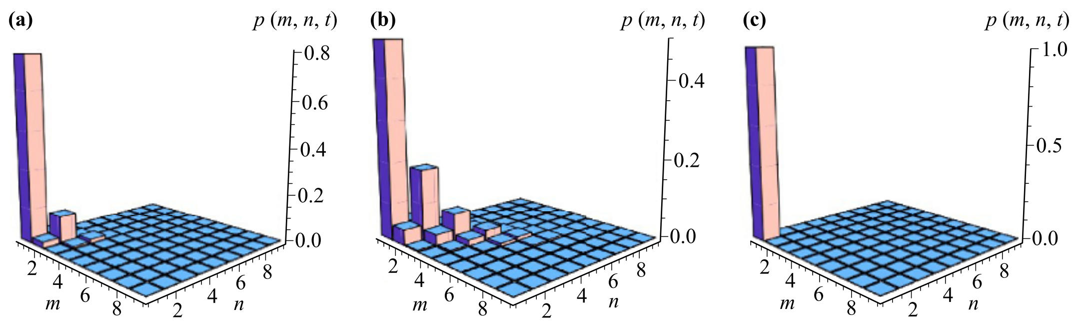

Fig. 1 Evolution law of photon number distribution of the TMSVS for amplitude decay when **(a)** $r=0.5$, $\kappa t=0.1$; **(b)** $r=0.9$, $\kappa t=0.1$; **(c)** $r=0.9$, $\kappa t=3$.

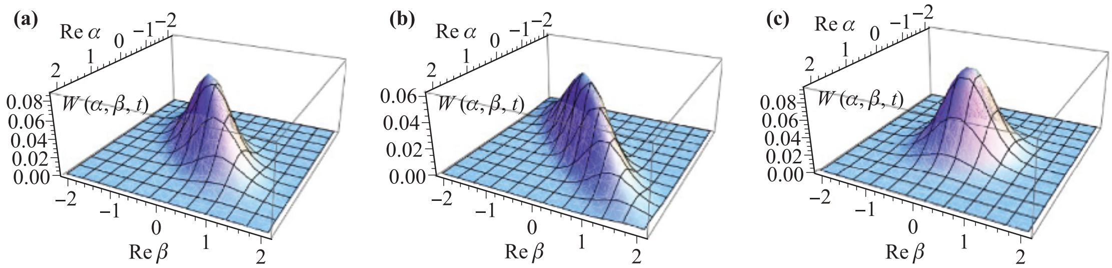

Fig. 2 Wigner function evolution for the TMSVS for amplitude decay when **(a)** $r = 0.5$, $\kappa t = 0.1$; **(b)** $r = 0.9$, $\kappa t = 0.1$; **(c)** $r = 0.9$, $\kappa t = 3$.

∞, the Wigner function may finally become the standard Gaussian distribution, corresponding to vacuum, which is perfectly accordant with the analytical result outlined in Section 3.

Next, we want to figure out how the marginal distributions of Wigner function $W(\alpha,\beta,t)$ evolves in the ADC. So, introducing the two-mode Wigner operator in the CV entangled state $|\eta\rangle$ representation [34], i.e.,

$$\Delta(\sigma,\gamma) = \int \frac{\mathrm{d}^2\eta}{\pi^3}|\gamma-\eta\rangle\langle\gamma+\eta|\mathrm{e}^{\eta\sigma^*-\eta^*\sigma} \tag{33}$$

and integrating $\Delta(\sigma,\gamma)$ over all complex values of γ and σ, thus we have the evolved marginal distributions of the Wigner function $W(\sigma,\gamma,t)$ for the density operator $\rho(t)$ in σ- and γ-space, i.e.,

$$\begin{aligned}&\int \mathrm{d}^2\gamma W(\sigma,\gamma,t) = \pi^{-1}\mathrm{tr}[\rho(t)|\eta\rangle\langle\eta|]_{\eta=\sigma},\\ &\int \mathrm{d}^2\sigma W(\sigma,\gamma,t) = \pi^{-1}\mathrm{tr}[\rho(t)|\tau\rangle\langle\tau|]_{\tau=\gamma},\end{aligned} \tag{34}$$

where $W(\sigma,\gamma,t)$ equals to the form $W(\alpha,\beta,t)$ since $\Delta(\sigma,\gamma)$ is rewritten as $\Delta(\alpha,\beta)$ only when $\sigma=\alpha+\beta^*$ and $\gamma=\alpha-\beta^*$. Here, it is worth noting that this representation $\Delta(\sigma,\gamma)$ can help one derive the Wigner functions for general squeezed states more conveniently since there exists the squeezing transform for the Wigner operator $\Delta(\sigma,\gamma)$, i.e., $S^{-1}(r)\Delta(\sigma,\gamma)S(r) = \Delta(\sigma/\mu,\mu\gamma)$, which manifestly exhibits the "squeezing" behavior [35, 36]. Also, $|\eta\rangle$ and $|\tau\rangle$ are two mutually conjugate entangled states [37], which are physically used in the description of the bipartite entangled system with the positions (X_1,X_2) and the momenta (P_1,P_2).

In order to obtain the distributions, we first derive the anti-normal ordering product of the evolved density operator $\rho(t)$. Substituting Eq. (22) into the integration formula converting an operator into its anti-normal ordering product [38], and using the inner product $\langle-\alpha,-\beta|\alpha,\beta\rangle = \mathrm{e}^{-2(|\alpha|^2+|\beta|^2)}$, we arrive at

$$\rho(t) = h_1 \vdots \exp[h_2(a^\dagger a+b^\dagger b)+h_3(ab+a^\dagger b^\dagger)]\vdots, \tag{35}$$

where $\vdots\ \vdots$ represents anti-normal ordering whose ordering rules are adverse to those of normal ordering, and we have set

$$h_1=\mathcal{C}\text{ß}_1,\ h_2=\mathcal{C}[(\text{ß}_2+1)\text{ß}_2-\text{ß}_3^2],\ h_3=\mathcal{C}\text{ß}_3$$

with $\mathcal{C}=[(\text{ß}_2+1)^2-\text{ß}_3^2]^{-1}$. Substituting Eq. (24) into Eq. (35) and using the completeness relation of two-mode coherent states $|\alpha\beta\rangle$, we directly obtain the inner product as follows:

$$\begin{aligned}&\langle\eta|\rho(t)|\eta\rangle\\ &=h_1\mathrm{e}^{-|\eta|^2}\int\frac{\mathrm{d}^2\alpha\mathrm{d}^2\beta}{\pi^2}\exp(\eta^*\alpha-\eta\beta+\eta\alpha^*-\eta^*\beta^*)\\ &\quad\times\exp[(h_2-1)(|\alpha|^2+|\beta|^2)+(h_3+1)(\alpha\beta+\alpha^*\beta^*)]\\ &=\varkappa(h_1,h_2,h_3)\mathrm{e}^{\varrho(h_1,h_2,h_3)|\eta|^2},\end{aligned} \tag{36}$$

where $\varkappa(h_1,h_2,h_3)=h_1/[(h_2+h_3)(h_2-h_3-2)]$ and $\varrho(h_1,h_2,h_3)=(h_2-h_3)/(h_3-h_2+2)$. Therefore, the compact form of the evolved marginal distribution of $W(\sigma,\gamma,t)$ in the "σ-direction" is directly obtained as

$$\int\mathrm{d}^2\gamma W(\sigma,\gamma,t)=\pi^{-1}\varkappa(h_1,h_2,h_3)\mathrm{e}^{\varrho(h_1,h_2,h_3)|\sigma|^2}. \tag{37}$$

In a similar way to deriving Eq. (37), we obtain another evolved marginal distribution of $W(\sigma,\gamma,t)$ in the "γ-direction", i.e.,

$$\int\mathrm{d}^2\sigma W(\sigma,\gamma,t)=\pi^{-1}\varkappa(h_1,h_2,-h_3)\mathrm{e}^{\varrho(h_1,h_2,-h_3)|\gamma|^2}. \tag{38}$$

Equations (37) and (38) represent the measurement probability evolutions of two entangled particles with total momentum $\sqrt{2}\mathrm{Im}\sigma$ (or relative momentum $\sqrt{2}\mathrm{Im}\gamma$) and simultaneously relative position $\sqrt{2}\mathrm{Re}\sigma$ (or center-of-mass position $\sqrt{2}\mathrm{Re}\gamma$) in the TMSVS for amplitude decay [24].

6 Optical tomogram

Optical tomography is connected with the property to be a standard positive probability distribution function describing quantum state in the field of quantum statistics and quantum optics, so it also provides the possibility of obtaining photon-number statistics distributions and reconstructing the Wigner functions of photon states. Here, with the help of the CV entangled state $|\eta,\tau_1,\tau_2\rangle$ representation, we aim to address the optical tomogram evolution of the TMSVS in the ADC.

For any two-mode quantum system, for tomographic approach there exists the entangled state $|\eta,\tau_1,\tau_2\rangle$, and the Radon transform of the Wigner operator $\Delta(\sigma,\gamma)$ is just the entangled-state density matrices $|\eta,\tau_1,\tau_2\rangle\langle\eta,\tau_1,\tau_2|$, i.e.,

$$|\eta,\tau_1,\tau_2\rangle\langle\eta,\tau_1,\tau_2| = \pi\iint \mathrm{d}^2\sigma\mathrm{d}^2\gamma\delta(\eta_1-\mu_1\gamma_1-\nu_1\sigma_2)\times\delta(\eta_2-\nu_2\gamma_2-\mu_2\sigma_1)\Delta(\sigma,\gamma), \quad (39)$$

where $\eta=\eta_1+\mathrm{i}\eta_2$ and $\tau_j=|\tau_j|\mathrm{e}^{\mathrm{i}\theta_j}=\mu_j+\mathrm{i}\nu_j$ $(j=1,2)$, and the entangled state $|\eta,\tau_1,\tau_2\rangle$ in two-mode Fock space reads [39]

$$|\eta,\tau_1,\tau_2\rangle = g\exp[g_1+g_2a^\dagger+g_3b^\dagger+g_4a^\dagger b^\dagger-g_5(a^{\dagger 2}+b^{\dagger 2})]|00\rangle, \quad (40)$$

where we have set

$$g=\frac{1}{\sqrt{|\tau_1\tau_2|}},\quad g_1=-\frac{\eta_1^2|\tau_2|^2+\eta_2^2|\tau_1|^2}{2|\tau_1|^2|\tau_2|^2},$$
$$g_2=\frac{\eta_1\tau_2^*+\eta_2\tau_1^*}{\tau_1^*\tau_2^*},\quad g_3=\frac{\eta_2\tau_1^*-\eta_1\tau_2^*}{\tau_1^*\tau_2^*},$$
$$g_4=\frac{\mathrm{e}^{\mathrm{i}2\theta_1}-\mathrm{e}^{\mathrm{i}2\theta_2}}{2},\quad g_5=\frac{\mathrm{e}^{\mathrm{i}2\theta_1}+\mathrm{e}^{\mathrm{i}2\theta_2}}{4}.$$

Therefore, the Radon transform of Wigner function $W(\sigma,\gamma)$ for any two-mode quantum state ρ, namely, the optical tomogram denoted as $T(\eta,\tau_1,\tau_2)$ can be calculated as

$$\langle\eta,\tau_1,\tau_2|\rho|\eta,\tau_1,\tau_2\rangle = \pi\iint \mathrm{d}^2\sigma\mathrm{d}^2\gamma\delta(\eta_1-\mu_1\gamma_1-\nu_1\sigma_2)\times\delta(\eta_2-\nu_2\gamma_2-\mu_2\sigma_1)W(\sigma,\gamma)\equiv T(\eta,\tau_1,\tau_2), \quad (41)$$

which indicates that the tomogram $T(\eta,\tau_1,\tau_2)$ of the state ρ can be considered as the matrix element $\langle\eta,\tau_1,\tau_2|\rho|\eta,\tau_1,\tau_2\rangle$, a concise expression.

Thus, using Eqs. (35) and (41), the optical tomogram evolution of the TMSVS in the ADC is expressed as

$$T(\eta,\tau_1,\tau_2;t) = h_1\langle\eta,\tau_1,\tau_2|\vdots\exp[h_2(a^\dagger a+b^\dagger b)+h_3(ab+a^\dagger b^\dagger)]\vdots|\eta,\tau_1,\tau_2\rangle. \quad (42)$$

Inserting the completeness relation of the coherent states $|\alpha,\beta\rangle$ into Eq. (42) and using the following integration formula twice

$$\int\frac{\mathrm{d}^2z}{\pi}\mathrm{e}^{\zeta|z|^2+\xi z+\eta z^*+fz^2+hz^{*2}}=\frac{1}{\sqrt{\zeta^2-4fh}}\mathrm{e}^{\frac{-\zeta\xi\eta+\xi^2h+\eta^2f}{\zeta^2-4fh}}, \quad (43)$$

which holds for the conditions $\mathrm{Re}(\zeta\pm f\pm h)<0$ and $\mathrm{Re}\left(\frac{\zeta^2-4fh}{\zeta\pm f\pm h}\right)<0$, to carry out the integral over α and β respectively, we obtain

$$T(\eta,\tau_1,\tau_2;t) = \frac{h_1g^2}{\sqrt{c_1(c_2^2-4|c_4|^2)}}\exp\left[-\frac{(h_2-1)|g_2|^2}{c_1}+2\mathrm{Re}\left(\frac{c_1g_1-g_2^2g_5^*}{c_1}+\frac{c_3^2c_4^*}{c_2^2-4|c_4|^2}\right)-\frac{c_2|c_3|^2}{c_2^2-4|c_4|^2}\right], \quad (44)$$

where the parameters c_1,c_2,c_3 and c_4 are, respectively,

$$c_1=(h_2-1)^2-4|g_5|^2,$$
$$c_2=\frac{(h_2-1)(c_1-|g_4+h_3|^2)}{c_1},$$
$$c_3=\frac{c_1g_3^*-(g_4^*+h_3)[g_2(h_2-1)+2c_1g_2^*g_5]}{c_1},$$
$$c_4=-\frac{c_1g_5^*+g_5(g_4^*+h_3)^2}{c_1}.$$

In particular, for the case of $\kappa t=0$, Eq. (44) becomes the optical tomogram of the TMSVS, whose analytical expression can be obtained by making use of the substitutions $h_1\longrightarrow -\mathrm{csch}^2 r$, $h_2\longrightarrow 1$, $h_3\longrightarrow -\coth r$ in Eq. (44). While for the limited time $\kappa t\to\infty$, equation (44) reduces to the optical tomogram of vacuum, i.e., $T(\eta,\tau_1,\tau_2;\infty)=g^2e^{2g_1}$, a highlighted Gaussian distribution.

7 Conclusions

In summary, we have obtained the evolution law of the TMSVS in a two-mode amplitude dissipative channel via the CV entangled state approach, which shows manifestly that the initial pure state evolves into a definite mixed state in this process, but it still maintains entangled and squeezed. Moreover, we have investigated the evolutions of PND, Wigner function and optical tomogram for amplitude decay. It is analytically found that in this process, the PND evolution turns out to be a Jocobi polynomial function, the marginal distributions in Eqs. (37) and (38) give the measurement probability evolutions of two entangled particles in the TMSVS in an entangled way, and the optical tomogram evolution is just the matrix element $\langle \eta, \tau_1, \tau_2 | \rho(t) | \eta, \tau_1, \tau_2 \rangle$. The numerical results show that the larger squeezing can lead to a longer decay time, and the TMSVS completely loses its entanglement and squeezing and ultimately decays to vacuum at long times.

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