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Research article

Two-qubit entangled state teleportation via optimal POVM and partially entangled GHZ state

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Quantum teleportation is of significant meaning in quantum information. In this paper, we study the probabilistic teleportation of a two-qubit entangled state via a partially entangled Greenberger-Horne-Zeilinger (GHZ) state when the quantum channel information is only available to the sender. We formulate it as an unambiguous state discrimination problem and derive exact optimal positive-operator valued measure (POVM) operators for maximizing the probability of unambiguous discrimination. Only one three-qubit POVM for the sender, one two-qubit unitary operation for the receiver, and two cbits for outcome notification are required in this scheme. The unitary operation is given in the form of a concise formula, and the fidelity is calculated. The scheme is further extended to more general case for transmitting a two-qubit entangled state prepared in arbitrary form. We show this scheme is flexible and applicable in the hop-by-hop teleportation situation.

Keywords probabilistic teleportation, optimal POVM, state discrimination, average fidelity

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1 Introduction

Quantum teleportation [1] is the process of transmitting unknown quantum states via a pre-shared quantum channel between distant network nodes with the help of classical information. Since it was proposed by Bennett *et al.* [2], it has been at the heart of quantum information theory and also represents a fundamental step in the development of many quantum technologies, including quantum networks [3, 4], quantum cryptography [5], measurement-based quantum computing [6], quantum repeater [7], etc. Due to quantum teleportation's potential applications in the realm of quantum communication [8], a growing amount of theoretical and experimental progress [9, 10] has been made in this domain.

The transmission of quantum states relies on the quantum channel [11] that is composed of entangled states. Due to the fragility of entanglement [12, 13] itself and inevitable environmental noise, partially entangled states are utilized for teleporting quantum state probabilistically. Multi-qubit teleportation is not only useful for quantum communication but also for quantum computing and quantum search [14]. Probabilistic teleportation was introduced by Li *et al.* [15] and followed by additional studies [16–18] using different types of entanglement to teleport different unknown quantum states. In most schemes, the sender makes a standard measurement (e.g., Bell-state measurement), and then the receiver introduces an auxiliary particle and makes a corresponding unitary transformation to reconstruct the original state with the aid of full information of the shared partially entangled state. When only the sender has the coefficients of quantum channel, these probabilistic teleportation schemes are not applicable in this situation.

Upon analysis of its process, it is easy to discover that quantum teleportation takes advantage of the one-to-one correspondence between the quantum state measurement result and the state after measurement. The correct measurement result guides the receiver to reconstruct the original state exactly. As one of the key issues in quantum communication and quantum cryptography, considerable studies have been conducted on the discrimination of quantum states in various situations. In the quantum state discrimination problem, two settings have been commonly investigated. Minimum-error discrimination (MED) [19] aims to minimize the probability of making an error in distinguishing states and accepting erroneous results. However, unlike MED, unambiguous state discrimination (USD) identifies the states unambiguously without error at the expense of allowing an inconclusive result to indicate "I don't know". An unambiguous measurement that maximizes the average correct probability is called optimal, and a closed-form analytical expression has been obtained for some cases [20, 21]. Chefles *et al.* [22] showed that a necessary and sufficient condition for the existence of unambiguous measurements for distinguishing between m quantum states is that the states are linearly independent. They also proposed equal-probability measurement (EPM) for unambiguous discrimination. Eldar *et al.* [23] developped a general framework for unambiguous state discrimination that can be applied to any number of states with arbitrary prior probabilities and showed that the EPM is optimal when a certain constraint is satisfied in several state sets. Nakahira *et al.* [24] found optimal quantum measurement for the generalized quantum state discrimination problem. Some of the discriminators have been experimentally demonstrated [25, 26]. Most of the existing measurements are designed to identify one-qubit or two-qubit states; few measurements are conducted on the three-qubit state which is the main focus in this paper.

In this paper, we study the probabilistic teleportation of two-qubit entangled states via a partially entangled GHZ state and try to relate teleportation with quantum state discrimination. We show that to distinguishdistinguishing states unambiguously is crucial for successful teleportation. To realize the teleportation, the exact optimal POVM operator set is given and performed on three qubits on the sender side, the unitary operation for recoverying on the receiver side is provided in the form of a concise formula, and the average fidelity is calculated. The scheme is extended to the more general case where the quantum state that needs to be transmitted is prepared in arbitrary forms of two-qubit entangled states. Methods are provided for constructing POVM and conveniently obtaining correct unitary operation conveniently in that situation. Based on these manipulations, our scheme is proved demonstrated to be applicable to the situation where only the sender has full knowledge about the coefficients of the quantum channel.

The rest of paper is organized as follows: The scheme for teleporting two-qubit entangled states is proposed in Section 2. We construct the optimal POVM after formulating the problem as unambiguous state discrimination. Unitary operation is given in concise formula and average fidelity is calculated. In Section 3, we extend the scheme to more general case, convenient and efficient methods are provided avoiding repeated and tedious deduction. Finally, in Section 4, we make some discussion and conclude the full paper.

2 Scheme for two-qubit entangled state teleportation

Suppose two nodes, conveniently called Alice and Bob, are connected with shared entangled states as quantum channel. Through this channel, Alice wishes to send an unknown two-qubit entangled state to Bob, which is described as

$$|\chi\rangle = \alpha|00\rangle + \beta|11\rangle, \tag{1}$$

where α and β are complex numbers normalized so that $|\alpha|^2 + |\beta|^2 = 1$. The quantum channel shared between these two nodes is pure partially entangled GHZ state of the form say,

$$|GHZ\rangle = a|000\rangle + b|111\rangle, \tag{2}$$

where $|a|^2 + |b|^2 = 1$. Without loss of generality, we assume a and b to be real with $|a| \geq |b|$. Note that only Alice has the full knowledge of these two coefficients in our assumption. For the convenience of description, we numbered all the particles involved and located them either at Alice or Bob as shown in Fig. 1.

The combined five-qubit system state could be written as

$$|\Psi_{sys}\rangle = |\chi\rangle_{12} \otimes |GHZ\rangle_{345}. \tag{3}$$

Expanding the above equation, the system state could be expressed as

$$\begin{aligned}|\Psi_{sys}\rangle &= (\alpha|00\rangle + \beta|11\rangle)_{12} \otimes (a|000\rangle + b|111\rangle)_{345} \\ &= \frac{1}{2}(a|000\rangle + b|111\rangle)_{123} \otimes (\alpha|00\rangle + \beta|11\rangle)_{45} \\ &\quad + \frac{1}{2}(a|000\rangle - b|111\rangle)_{123} \otimes (\alpha|00\rangle - \beta|11\rangle)_{45} \\ &\quad + \frac{1}{2}(b|001\rangle + a|110\rangle)_{123} \otimes (\alpha|11\rangle + \beta|00\rangle)_{45} \\ &\quad + \frac{1}{2}(b|001\rangle - a|110\rangle)_{123} \otimes (\alpha|11\rangle - \beta|00\rangle)_{45}.\end{aligned} \tag{4}$$

In order to realize teleportation, in our scheme, Alice needs to make a measurement on particles $(1, 2, 3)$ jointly so that the state on particles $(4, 5)$ would collapse into one of the four possible states. We observe that in Eq. (4) there exists correspondence relationship between the states on Alice's and Bob's side. The correct discrimination of the quantum states on particles $(1, 2, 3)$ will indicate corresponding state on particles $(4, 5)$ precisely, and then lead to proper unitary operations at Bob

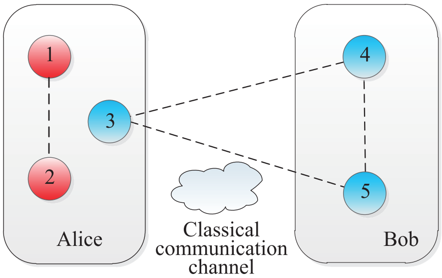

Fig. 1 System model for teleporting unknown two-qubit entangled state. The particle 1 and 2 are in the possession of Alice, particle 3 from partially entangled GHZ state is with Alice while Bob has the particle 4 and 5. The classical communication channel, wired or wireless, is also equipped between Alice and Bob.

for successful original state recovery. It has been known that only when the state belongs to a known orthogonal set that it can be infallibly determined by a standard Von Neumann measurement. When confronted with the problem of trying to discriminate between non-orthogonal states, we must accept that no strategy will correctly reveal the state of the system with unit probability. The states to be discriminated in this paper is extracted from the system state from Eq. (4) in the form of vectors as follows,

$$|\phi_1\rangle = [a,0,0,0,0,0,0,b]^{\mathrm{T}}, \quad |\phi_2\rangle = [a,0,0,0,0,0,0,-b]^{\mathrm{T}},$$
$$|\phi_3\rangle = [0,b,0,0,0,0,a,0]^{\mathrm{T}}, \quad |\phi_4\rangle = [0,b,0,0,0,0,-a,0]^{\mathrm{T}}. \tag{5}$$

These four states are non-orthogonal and linearly independent pure states. Therefore, our task now is to specify the kind of POVM that Alice should perform on her three qubits to discriminate state probabilistically. Since the state discrimination here is crucial for the whole teleportation process, the measurement performed must distinguish the states unambiguously without error. We accept the failure of the teleportation with inclusive result other than recovering the state with erroneous information. Thus, we formulate this as a typical unambiguous state discrimination problem, and then describe the procedure of obtaining the exact optimal measurements.

2.1 Unambiguous state discrimination formulation

Assume that the possible state of particles $(1,2,3)$ would be drawn from a collection of non-orthogonal pure states $|\phi_i\rangle$ with priori probabilities η_i in r-dimensional complex Hilbert space $\mathcal{H}$. We have $1 \le i \le m$, $r \le m$ where m is the number of states in the collection, and the states span a subspace $\mathcal{U}$ of $\mathcal{H}$. The m states are linearly independent and the occurrence probabilities η_i are nonzero that satisfy $\sum_{i=1}^{m} \eta_i = 1$. A measurement is described by a positive operator-valued measure (POVM), and constructed comprising $m+1$ measurement operators $\{\Pi_i, 0 \le i \le m\}$ that

$$\sum_{i=0}^{m} \Pi_i = I_r. \tag{6}$$

Based on these measurement operators, either the state is correctly detected or the measurement returns an inconclusive result. Thus, each of the operators Π_i corresponds to the detection of the state $|\phi_i\rangle$, $1 \le i \le m$, and Π_0 corresponds to an inclusive result. Given that the actual quantum state is $|\phi_i\rangle$, the probability of obtaining outcome k is $\langle\phi_i|\Pi_k|\phi_i\rangle$. Therefore, to ensure the result obtained is either error-free or inclusive, we must have

$$\langle\phi_i|\Pi_k|\phi_i\rangle = p_i\delta_{ik}, \quad 1 \le i,k \le m \tag{7}$$

for some $0 \le p_i \le 1$. We could get $\Pi_0 = I_r - \sum_{i=1}^{m}\Pi_i$ from Eq. (6). Given that the state is $|\phi_i\rangle$, the state is correctly detected with probability p_i so that an inconclusive result is returned with probability $1-p_i$. Therefore, the total probability of correctly detecting the state and obtaining conclusive result is

$$P_{con} = \sum_{i=1}^{m} \eta_i\langle\phi_i|\Pi_i|\phi_i\rangle = \sum_{i=1}^{m} \eta_i p_i. \tag{8}$$

We need to choose the measurement operators Π_i and the probabilities $p_i \ge 0$ to maximize P_{con} subject to the constraint in Eq. (6). It can be formulated as a convex optimization problem that several solution algorithms have been proposed [27]. We may choose

$$\Pi_i = p_i|\widetilde{\phi}_i\rangle\langle\widetilde{\phi}_i| \tag{9}$$

as the measurement operator, where $|\widetilde{\phi}_i\rangle \in \mathcal{U}$ are the reciprocal state associated with the states $|\phi_i\rangle$. To obtain the optimal measurement operators Π_i, the exact form of $|\widetilde{\phi}_i\rangle$ needs to be worked out. We consider the structure described in [23] and construct the matrices Φ and $\widetilde{\Phi}$ with $|\phi_i\rangle$ and $|\widetilde{\phi}_i\rangle$ as columns, respectively, so that we have

$$\widetilde{\Phi} = \Phi(\Phi^*\Phi)^{-1}. \tag{10}$$

Then the exact $|\widetilde{\phi}_i\rangle$ could be obtained through

$$|\widetilde{\phi}_i\rangle = |\phi_i\rangle(\Phi^*\Phi)^{-1}. \tag{11}$$

Afterwards, we can construct the measurement operators by which Alice could measure particles $(1,2,3)$ conclusively and lay good foundation for the following original state recovery at Bob.

2.2 Optimal POVM construction

For the case in our scheme, the states to be discriminated $|\phi_i\rangle$ have been exacted as Eq. (5) with equal priori probability $\eta_i = 1/4$ for $1 \le i \le 4$. We construct matrix Φ and calculate the reciprocal states accordingly. Through Eq. (10), we give $|\widetilde{\phi_i}\rangle$ directly as follows:

$$|\widetilde{\phi_1}\rangle = \frac{1}{2a}|000\rangle + \frac{1}{2b}|111\rangle, \quad |\widetilde{\phi_2}\rangle = \frac{1}{2a}|000\rangle - \frac{1}{2b}|111\rangle,$$
$$|\widetilde{\phi_3}\rangle = \frac{1}{2a}|001\rangle + \frac{1}{2b}|110\rangle, \quad |\widetilde{\phi_4}\rangle = \frac{1}{2a}|001\rangle - \frac{1}{2b}|110\rangle. \tag{12}$$

In this case, the state $|\phi_i\rangle$ and the prior probabilities η_i satisfy the sufficient conditions for equal-probability measurement [22] to maximize the total probability of correct detection P_{con} so that the EPM is optimal [23]. For optimal EPM, equal measurement probabilities $p_i = p$ for all $1 \le i \le 4$ such that p is equal to the inverse of maximum eigenvalue of the corresponding frame operator S defined as

$$S = \sum_{i=i}^{4} |\widetilde{\phi_i}\rangle\langle\widetilde{\phi_i}|. \tag{13}$$

Thus, we get $p = 2|b|^2$ and specify the optimal POVM for detecting the four states unambiguously as

$$\Pi_i = 2|b|^2|\widetilde{\phi_i}\rangle\langle\widetilde{\phi_i}|, \quad \text{for } 1 \le i \le 4,$$
$$\Pi_0 = I_r - \sum_{i=1}^{4} \Pi_i. \tag{14}$$

The POVM operators are related with the coefficients of quantum channel which is available when preparing entangled states. The total probability of correctly detecting the state is $P_{con} = \sum_{i=1}^{4} \eta_i p_i = 4\times 1/4 \times 2|b|^2 = 2|b|^2$. Ignoring the error may arise with following unitary operation, it is also the probability of successful teleportation with fidelity one. Note that for $|b| = 1/\sqrt{2}$, which corresponds to maximally entangled state, the teleportation is always successful with certainty.

2.3 Original state reconstruction

After optimal POVM on particles $(1, 2, 3)$, Alice obtains corresponding measurement outcome. For convenience, the outcomes of Π_i $(1 \le i \le 4)$ are expressed as classical bit strings m_1m_2 and the correspondence relationship is defined as

$$\Pi_1 \to 00, \quad \Pi_2 \to 01, \quad \Pi_3 \to 10, \quad \Pi_4 \to 11. \tag{15}$$

Alice sends this 2cbit information m_1m_2 to Bob through classical communication channel. If the inclusive result is obtained, Alice sends nothing to Bob and starts another teleportation. For Bob, if getting the classical information, he performs the unitary operation on particles $(4, 5)$ to reconstruct the original state according to the classical information received as

$$T = (X^{m_1})_4 \otimes (Z^{m_2}X^{m_1})_5, \tag{16}$$

where $X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ are Pauli matrices. Through this operation, Bob yields the original state successfully. Otherwise, if Bob do not receive any information concerning the teleportation after a fixed time period, he would know that Alice obtained an inclusive result that the teleportation failed. With the help of unambiguous state discrimination and optimal POVM, the total success probability of teleportation is $2|b|^2$ and when $|b| = 1/\sqrt{2}$ it reaches up to 1.

We summarize the whole teleportation process simply and quantum circuit is presented in Fig. 2. When Alice wishes to send an unknown two-qubit entangled state to Bob through the shared partially entangled GHZ state, she should construct the optimal POVM operators firstly. By formulating unambiguous state discrimination problem, the measurement operators could be specified accordingly. Alice makes this three-qubit optimal POVM on particles $(1, 2, 3)$ and sends the two cbit classical information of measurement outcome obtained to Bob. Upon receiving the information, Bob makes proper unitary operation on particles $(4, 5)$, in the light of Eq. (16), to reconstruct the unknown original state exactly. This scheme is applicable in the situation where only Alice has the full knowledge of the coefficients of the quantum channel since Bob needs to perform operation only consists of standard Pauli matrices.

2.4 Average fidelity

Although the original state to be teleported is unknown, it is necessary to study how much information is transferred from the sender to the receiver efficiently. The density operator ρ_k of the teleported pure state relies on the outcome k, and the average fidelity is defined as

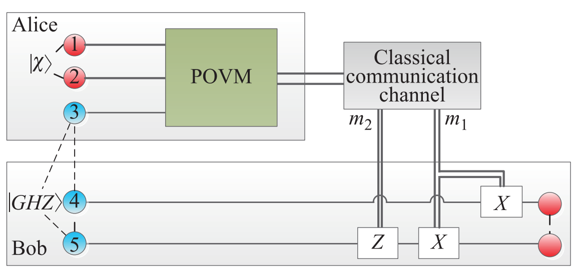

Fig. 2 Quantum circuit for the whole scheme. The solid line represents the quantum path at each node while the double solid line denotes the classical information path. The dashed line connects the entangled particles.

$$\mathcal{F} \equiv \frac{1}{V}\int \mathrm{d}\boldsymbol{\Omega}\sum_k p_k(\boldsymbol{\Omega})f_k(\boldsymbol{\Omega}), \tag{17}$$

where k denotes the outcome at the sender, $f_k(\boldsymbol{\Omega}) = \langle\psi(\boldsymbol{\Omega})|\rho_k|\psi(\boldsymbol{\Omega})\rangle$ and an unknown pure state $\psi(\boldsymbol{\Omega})$ is parameterized by a real vector $\boldsymbol{\Omega}$ in the parameter space of volume V [28]. We separate the average fidelity into two parts, conclusive events and inclusive event, for easy calculation as follows:

$$\begin{aligned}\mathcal{F} &= \mathcal{F}_{con} + \mathcal{F}_{inc}\\ &= \frac{1}{V}\int \mathrm{d}\boldsymbol{\Omega}\left[\sum_{k=1}^{4} p_k^{con}(\boldsymbol{\Omega})f_k^{con}(\boldsymbol{\Omega}) + p_0^{inc}(\boldsymbol{\Omega})f_0^{inc}(\boldsymbol{\Omega})\right],\end{aligned} \tag{18}$$

where $\mathcal{F}_{con}$ indicates the average fidelity conclusive events with $f_k^{con} = 1$ while $\mathcal{F}_{inc}$ denotes the average fidelity of inconclusive events with $f_k^{inc} = 2/3$. We then have the average fidelity

$$\mathcal{F} = \sum_{k=1}^{4}\left(\frac{1}{4}\times 2|b|^2\times 1\right) + (1-2|b|^2)\times\frac{2}{3} = \frac{2}{3}(1+|b|^2). \tag{19}$$

For $|b| = 1/\sqrt{2}$, $\mathcal{F}$ will equal to one which would change to a faithful teleportation. If $|b| = 0$, it implies that Bob cannot reconstruct the original state.

3 Other forms of two-qubit entangled state transmission

In the above section, we present a scheme for teleportation of one specific form of unknown two-qubit entangled state. However, the two-qubit entngled state to be transmitted may be prepared in other forms. In this section, we extend our scheme to more general cases. Suppose the unknown two-qubit entangled state is given in another form of

$$|\chi'\rangle = \alpha|01\rangle + \beta|10\rangle, \tag{20}$$

which is different from $|\chi\rangle$ we have discussed above. Accordingly, the system state should be expanded as

$$\begin{aligned}|\Psi'_{sys}\rangle &= (\alpha|01\rangle + \beta|10\rangle)_{12}\otimes(a|000\rangle + b|111\rangle)_{345}\\ &= \frac{1}{2}(a|010\rangle + b|101\rangle)_{123}\otimes(\alpha|00\rangle + \beta|11\rangle)_{45}\\ &\quad+\frac{1}{2}(a|010\rangle - b|101\rangle)_{123}\otimes(\alpha|00\rangle - \beta|11\rangle)_{45}\\ &\quad+\frac{1}{2}(b|011\rangle + a|100\rangle)_{123}\otimes(\alpha|11\rangle + \beta|00\rangle)_{45}\\ &\quad+\frac{1}{2}(b|011\rangle - a|100\rangle)_{123}\otimes(\alpha|11\rangle - \beta|00\rangle)_{45}.\end{aligned} \tag{21}$$

Obviously, in the language of unambiguous state discrimination, the non-orthogonal state set that needs to be distinguished changes, turning into

$$\begin{aligned}&|\phi'_1\rangle = [0,0,a,0,0,b,0,0]^{\mathrm{T}}, \quad |\phi'_2\rangle = [0,0,a,0,0,-b,0,0]^{\mathrm{T}},\\ &|\phi'_3\rangle = [0,0,0,b,a,0,0,0]^{\mathrm{T}}, \quad |\phi'_4\rangle = [0,0,0,b,-a,0,0,0]^{\mathrm{T}}.\end{aligned} \tag{22}$$

The existing POVM operators Π_i in Eq. (14) are unable to distinguish these four states $|\phi'_i\rangle$ $(1 \le i \le 4)$ unambiguously. That's because the constrains imposed on unambiguous state discrimination in Eq. (7) cannot be satisfied as

$$\langle\phi'_i|\Pi_k|\phi'_i\rangle \neq p_i\delta_{ik}, \quad 1 \le i,k \le m. \tag{23}$$

It is not a difficult task to construct new optimal POVM Π'_i accordingly but the derivation and calculation are tedious and redundant for all the other forms of two-qubit entangled state. We notice that there exists transformation relationship among these two two-qubit entangled states that

$$|\chi'\rangle = (I\otimes X)\cdot(\alpha|00\rangle + \beta|11\rangle) = (I\otimes X)|\chi\rangle. \tag{24}$$

Besides we have mentioned, the two-qubit entangled state has other two basic forms that we generalize them into one expression as

$$\begin{aligned}|\chi_{st}\rangle &= \alpha|0,t\rangle + (-1)^s\beta|1,1-t\rangle\\ &= (I\otimes Z^sX^t)|\chi_{00}\rangle,\end{aligned} \tag{25}$$

where $s,t = 0,1$ and $|\chi_{00}\rangle = \alpha|00\rangle + \beta|11\rangle$. We could express arbitrary two-qubit entangled state in the form that transformed from state $|\chi_{00}\rangle$ as $|\chi_{st}\rangle = U^{st}|\chi_{00}\rangle$, which is similar with Bell state even though they are non-maximal. The general operation $U^{st} = I\otimes Z^sX^t$ is a 4×4 unitary matrix which may change according to the form of quantum state transmitted. One intuitive method is to perform corresponding unitary operation U^{st} on these two qubits to transform them into the state $|\chi_{00}\rangle$, then follow our scheme to construct the POVM. But extra unitary operation may increase the probability of error occurrence. Thus, we try to find out whether this relationship will benefit further POVM construction. Eq. (21) is rewritten as

$$\begin{aligned}|\Psi'_{sys}\rangle &= |\chi_{st}\rangle_{12}\otimes|GHZ\rangle_{345} = U|\chi_{00}\rangle_{12}\otimes|GHZ\rangle_{345}\\ &= \frac{1}{2}(U^{st}\otimes I)|\phi_1\rangle_{123}\otimes(\alpha|00\rangle + \beta|11\rangle)_{45}\\ &\quad+\frac{1}{2}(U^{st}\otimes I)|\phi_2\rangle_{123}\otimes(\alpha|00\rangle - \beta|11\rangle)_{45}\\ &\quad+\frac{1}{2}(U^{st}\otimes I)|\phi_3\rangle_{123}\otimes(\alpha|11\rangle + \beta|00\rangle)_{45}\\ &\quad+\frac{1}{2}(U^{st}\otimes I)|\phi_4\rangle_{123}\otimes(\alpha|11\rangle - \beta|00\rangle)_{45}.\end{aligned} \tag{26}$$

Using U' to represent $U^{st} \otimes I$, we have the relationship between the new states set $|\phi_i'\rangle$ to be discriminated and the set we have discussed $|\phi_i\rangle$ as $|\phi_i'\rangle = U'|\phi_i\rangle$. The matrix Φ' can be constructed with $|\phi_i'\rangle$ as columns and we also have $\Phi' = U'\Phi$. The matrix $\widetilde{\Phi'}$ with new POVM operators $|\widetilde{\phi_i'}\rangle$ as columns can be obtained through Eq. (10) that

$$\begin{aligned}\widetilde{\Phi'} &= \Phi'(\Phi'^*\Phi')^{-1} \\ &= U'\Phi((U'\Phi)^*(U'\Phi))^{-1} \\ &= U'\Phi(\Phi^*U'^*U'\Phi)^{-1} \\ &= U'\Phi(\Phi^*\Phi)^{-1}.\end{aligned} \tag{27}$$

Obviously, we have the relationship between new POVM operators and the existing POVM operators that

$$|\widetilde{\phi_i'}\rangle = U'|\widetilde{\phi_i}\rangle. \tag{28}$$

After derivation and calculation case by case, we have the conclusion that equal-probability measurement is also optimal in unambiguous state discrimination for sending two-qubit entangled state of arbitrary form. The equal probability is $p = 2|b|^2$ and the specific new optimal POVM Π_i' can be constructed with Eq. (14) accordingly. Thus, when other forms of two-qubit entangled state are prepared to be transmitted, the existing POVM Π_i can be used as starting point for constructing the new optimal POVM.

On the receiver side, the expression of unitary operation performed by Bob should change accordingly as

$$T' = U^{st}T = (X^{m_1})_4 \otimes (Z^{m_2+s}X^{m_1+t})_5, \tag{29}$$

where s, t indicate the form of two-qubit entangled state transmitted. However, Bob does not know the specific form of two-qubit entangled state transmitted so that Alice would have to send additional 2cbit type code st to Bob together with the 2cbit measurement outcome, that is 4cbit classical information in total. We still want to reduce the classical communication cost as much as possible.

Suppose that if we let Bob believe that the two-qubit entangled state transmitted is always in the form of $|\chi_{00}\rangle$ so that the default unitary operation formula should be Eq. (16). In that case, the classical information that Alice sends to Bob should indicate the correct unitary operation required when being substituted into the formula even if the information is not the original measurement outcome but processed result. Based on this consideration, Alice could map the information to proper form by defining the mapped result as $m_1m_zm_x$ where $m_1 = m_1$, $m_z = m_2 \oplus s$ and $m_x = m_1 \oplus t$. The Eq. (16) changes to

$$T = (X^{m_1})_4 \otimes (Z^{m_z}X^{m_x})_5. \tag{30}$$

Alice applies the mapping and sends the mapped result to Bob, Bob substitutes the mapped result into Eq. (30) directly to get the proper unitary operation to perform. Obviously, the classical communication cost required is only 3cbits other than 4cbits by using this method.

In summary, when the two-qubit entangled state to be transmitted is not prepared in the form $|\chi_{00}\rangle$, Alice can still construct and perform the optimal POVM through Eq. (28), and then send Bob the mapped 3cibt classical information $m_1m_zm_x$. Bob substitutes the received classical information into Eq. (30) and performs the unitary operation obtained to recover the original state. By our method, we can get the corresponding POVM operator and proper unitary operation accurately and conveniently, avoiding repeated and cumbersome derivation.

4 Discussion and conclusions

Our scheme utilizes unambiguous state discrimination to obtain conclusive measurement result, that is similar with the conclusive teleportation presented in Refs. [29–31]. However, compared with these schemes, we use different type of quantum channel for teleportation and introduce no auxiliary particle which brings ours higher efficiency. Novel optimal POVM operator set is given for maximizing probability of correct discrimination in our scheme. In the meanwhile, we utilize three-qubit partially entangled GHZ state for teleporting two-qbuit entangled state via optimal POVM, that is different from existing schemes to the best of our knowledge.

In the future quantum network, quantum states would be sent between nodes that not directly connected by entangled state. They could utilize the intermediate nodes which was connected with both of them forming a chain of nodes as shown in Fig. 3. Information could be transmitted in a simple way by hop-by-hop teleportation, from sender Alice to next hop Bob, then Bob transmits to Candy, finally to the receiver David. Not all nodes are aware of the coefficients of entangled state shared be-

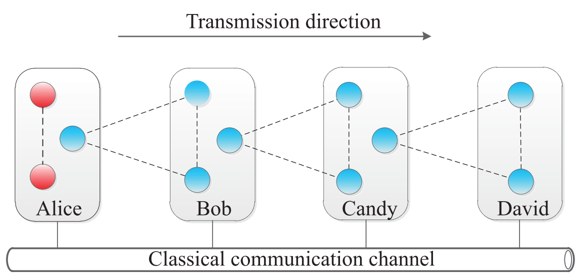

Fig. 3 Nodes chain for teleportation hop-by-hop. Nodes share non-maxinally entangled GHZ state with adjacent nodes to form a chain (channel) from the sender to the receiver. All nodes are connected by classical communication channel for information transmission.

tween so that the node which initiates the transmission to next hop should choose scheme between our scheme and typical scheme. Assume that at least one of the two nodes connected by entangled state has the knowledge of the quantum channel. The node which initiates the transmission to next hop would make a decision based on the fact whether he gets the information or not. If yes, utilize our scheme and make an optimal POVM, otherwise, make a standard Bell-state measurement and follow the typical scheme. In principle, any POVM can be implemented by adding an ancilla in a known state, and performing a standard measurement in the enlarged Hilbert space [32] and is experimentally demonstrated in Ref. [33]. Our scheme is flexible and implementable.

In this paper, one novel scheme is proposed for probabilistic teleportation of two-qubit entangled state via partially entangled GHZ state. We relate the quantum teleportation with quantum state discrimination, and thereby, derive optimal POVM operators for maximizing the total probability of correct detection. To realize the teleportation, Alice makes the derived optimal POVM and Bob performs unitary operation according to the classical information sent from Alice to yield original state. No auxiliary particle is required and the unitary operation for recovering is provided in the form of concise formula. A simple and generalized method is presented for constructing the optimal POVM and obtaining correct unitary operation when the two-qubit state is prepared in arbitrary form, avoiding cumbersome derivation and extra classical communication cost. Our scheme is especially applicable to the situation where only Alice knows the coefficients of quantum channel. We also show that it makes important part of hop-by-hop teleportation and compose a complete solution together with typical schemes.

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