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RESEARCH ARTICLE

Electron–positron pair production in a strong asymmetric laser electric field

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By solving the quantum Vlasov equation, electron–positron pair production in a strong electric field with asymmetric laser pulses has been investigated. We consider three different situations of subcycle, cycle and supercycle laser pulses. It is found that in asymmetric laser pulse field, i.e., when the pulse length of one rising or falling side is fixed while the pulse length of the other side is changed, the pair production rate and number density can be significantly modified comparable to symmetric situation. For each case of these three different cycle pulses, when one side pulse length is constant and the other side pulse length becomes shorter, i.e., the whole pulse is compressed, the more pairs can be produced than that in the vice versa case, i.e., the whole pulse is elongated. In compressed pulse case there exists an optimum pulse length ratio of asymmetric pulse lengths which makes the pair number density maximum. Moreover, the created maximum pair number density by subcycle pulse is larger than that by cycle or/and supercycle pulse. In elongated pulse case, however, only for supercycle laser pulse the created pairs is enhanced and there exists also an optimum asymmetric pulse length ratio that maximizes the pair number density. On the other hand, surprisingly, in both cases of subcycle and cycle elongated laser pulses, the pair number density is monotonically decreasing as the asymmetry of pulse increases.

Keywords electron–positron pair production, quantum Vlasov equation (QVE), asymmetric laser electric field

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1 Introduction

Pair production in a strong electromagnetic field is one of the most interesting phenomena in relativistic quantum physics. Beginning with Sauter's pioneering paper [1] in 1931, the problem of spontaneous pair production in quantum electrodynamics (QED) vacuum in the presence of an external electric field has been investigated by many authors [2, 3]. Schwinger was the first to reformulate this problem by using proper-time technique in 1951 [3] in detail, and since then this process was often called the Schwinger effect. Using the Schwinger formula [3] one can find that electron–positron (e^-e^+) pairs are spontaneously produced in constant electric field if the field strength exceeds the critical value $E_{cr} = m_e^2c^3/(e\hbar) \sim 1.3\times10^{16}$ V/cm, which is Schwinger critical field strength corresponding to laser intensity $I \approx 4.3 \times 10^{29}$ W/cm^2 for 1 μm light. Unfortunately, it is extremely difficult to reach in the present experimental condition. However, with the advance of high-intensity laser technology such as the European extreme light infrastructure (ELI) [4] or the X-ray free electron laser (XFEL) facilities [5], the required field strength for investigation of QED vacuum decay to create e^-e^+ pair will be possible in the future. It is noted that the laser intensity $I = 2\times10^{22}$ W/cm^2 is already available [6] and projects to achieve $I = 10^{26-28}$ W/cm^2 [7, 8] are under way. The proposed XFEL facilities can provide a good spatial coherent beam, tunable energy and high peak power density [9, 10]. The XFEL facilities will generate field strength as [11] $E = 0.1E_{cr}$ so that to explore pair creation by a laser pulse is meaningful.

The e^-e^+ pair production in QED vacuum is in general non-perturbation, non-equilibrium and time-dependent processes. In recent years, various methods have been employed to cope with this challenging problem, in particular some semi-classical methods, for example, they are WKB approximation [13, 14], worldline instanton techniques [15, 16] and the quantum kinetic theory by solving the quantum Vlasov equation (QVE) [9, 10, 12, 17, 18, 22]. Among them the quantum kinetic method by solving the QVE has many advantages, which is not only getting the pair creation rate but also getting the e^-e^+ momentum information. By solving the QVE one can conveniently get the momentum distribution function and particles number density. Recently, special interest has been laid on the investigation of e^-e^+ pair production in alternating [9, 10, 12] and pulse-shaped electric fields [19–21] within the kinetic approach based on QVE. In a recent study [22], we have used this approach to study the dynamical assisted effect on pair production in the combined electric fields.

It should be noted that up to now the pair production in QED vacuum in an asymmetric laser electric field has still not been investigated in detail. Thus, in this paper, by solving the QVE we investigate e^-e^+ pair production in the time-depended asymmetric laser electric field. We use a fourth order Runge-Kutta method to solve the equivalent ordinary differential equations about the created pair distribution function. We have obtained numerically the momentum distribution function, the pair number density and its dependence on the pulse length ratio in three different situations of subcycle, cycle and supercycle laser pulses, respectively. These investigated results may be useful for possible e^-e^+ pair creation experiments in future.

2 Kinetic formalism based on quantum Vlasov equation

A key quantity in the description of pair production process in the electric background field is the single-particle momentum distribution function $f(\boldsymbol{p},t)$. It can be computed from a QVE with a source term for e^-e^+ pair production. Once distribution function $f(\boldsymbol{p},t)$ is known we can easily calculate produced particles number density. To simplify our calculations, the laser field is treated as classical background field and the magnetic field is assumed to be zero. Because the laser wavelength is much larger than the pair creation scale as about electrons Compton wavelength [11] so the electric field is time dependent.

Usually the total electric field is $\boldsymbol{E} = \boldsymbol{E}_{ext} + \boldsymbol{E}_{int}$, where $\boldsymbol{E}_{int}$ is the internal field induced by the back-reaction mechanism. However, under the XFEL condition the inner field $\boldsymbol{E}_{int}$ can be neglected due to the small created pairs number density so that $\boldsymbol{E} = \boldsymbol{E}_{ext}$. The vector potential along the $\hat{z}$-direction is $\boldsymbol{A}(t) = (0, 0, A(t))$ which generates an electric field

$$\boldsymbol{E}(t) = -\frac{\mathrm{d}\boldsymbol{A}(t)}{\mathrm{d}t} = (0, 0, E(t)) \tag{1}$$

The quantum kinetic equations whose solution describes the evolution of the single-particle distribution function can be written as

$$\frac{\mathrm{d}f(\boldsymbol{p},t)}{\mathrm{d}t} = s(\boldsymbol{p},t) \tag{2}$$

which further can be written as QVE form

$$\frac{\mathrm{d}f(\boldsymbol{p},t)}{\mathrm{d}t} = \frac{eE(t)\varepsilon_\perp}{2\omega^2(\boldsymbol{p},t)}\int_{t_0}^{t}\mathrm{d}t'\frac{eE(t')\varepsilon_\perp}{\omega^2(\boldsymbol{p},t')} \times [1-2f(\boldsymbol{p},t')]\cos[2\int_{t'}^{t}\mathrm{d}\tau\omega(\boldsymbol{p},\tau)] \tag{3}$$

where the quantities appearing in this equation are the electrons/positron three-vector momentum $\boldsymbol{p} = (\boldsymbol{p}_\perp, p_{/\!/})$, the transverse energy squared $\varepsilon_\perp^2 = m^2 + p_\perp^2$, the total energy squared $\omega^2(\boldsymbol{p},t) = \varepsilon_\perp^2 + p_{/\!/}^2$, and the longitudinal momentum $p_{/\!/} = p_3 + eA(t)$. If we define $w(\boldsymbol{p},t) = \frac{eE(t)\varepsilon_\perp}{\omega^2(\boldsymbol{p},t)}$ and $\Theta(\boldsymbol{p};t,t') = \int_{t'}^{t}\omega(\boldsymbol{p},\tau)\mathrm{d}\tau$, then the Eq. (3) becomes

$$\frac{\mathrm{d}f(\boldsymbol{p},t)}{\mathrm{d}t} = \frac{1}{2}w(\boldsymbol{p},t)\int_{t_0}^{t}\mathrm{d}t'w(\boldsymbol{p},t')[1-2f(\boldsymbol{p},t')] \times \cos[2\Theta(\boldsymbol{p};t,t')] \tag{4}$$

In order to simplify numerical treatment of this equation, we reformulate it by introducing the relevant auxiliary quantities

$$u(\boldsymbol{p},t) = \int_{t_0}^{t}\mathrm{d}t'w(\boldsymbol{p},t')[1-2f(\boldsymbol{p},t')]\sin[2\int_{t'}^{t}\mathrm{d}\tau\omega(\boldsymbol{p},\tau)] \tag{5}$$

$$\nu(\boldsymbol{p},t) = \int_{t_0}^{t}\mathrm{d}t'w(\boldsymbol{p},t')[1-2f(\boldsymbol{p},t')]\cos[2\int_{t'}^{t}\mathrm{d}\tau\omega(\boldsymbol{p},\tau)] \tag{6}$$

so Eq. (4) can be expressed as a set of first order ordinary differential equations (ODE)

$$\frac{\mathrm{d}}{\mathrm{d}t}f(\boldsymbol{p},t) = \frac{1}{2}w(\boldsymbol{p},t)\nu(\boldsymbol{p},t) \tag{7}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}\nu(\boldsymbol{p},t) = w(\boldsymbol{p},t)[1-2f(\boldsymbol{p},t)] - 2\omega(\boldsymbol{p},t)u(\boldsymbol{p},t) \tag{8}$$

$$\frac{\mathrm{d}}{\mathrm{d}t}u(\boldsymbol{p},t) = 2\omega(\boldsymbol{p},t)\nu(\boldsymbol{p},t) \tag{9}$$

What we should emphasize here is that the original integral-differential equation becomes a set of ODEs makes not only the numerical treatment simpler but also the involved physical quantities or/and terms clearer. Obviously the term $\nu(\boldsymbol{p}, t)$ represents the integral part of Eq. (4) which constitutes the important contribution as a source to the pair production. In fact this term reveals also the quantum statistics character through the term $[1 - 2f(\boldsymbol{p}, t)]$ in Eq. (4) as well as in Eq. (8) due to the Pauli exclusive principle. On the other hand, $u(\boldsymbol{p}, t)$ denotes a countering term to pair production, which is associated to the pair annihilation in pair created process to some extent. Obviously Eq. (9) means that more pairs are created, more probability occurs for the pair annihilation in pair created process. Thus combining all factors mentioned above will conclude that the studied system exhibits a typical non-Markovian character as QVE, Eq. (4).

It is noted that the initial conditions can be given as $f(\boldsymbol{p}, t_0), u(\boldsymbol{p}, t_0), \nu(\boldsymbol{p}, t_0)$ in terms of concrete physical problems. By integrating the distribution function to momentum, we can get the time-dependent pair number density

$$n(t) = 2 \int \frac{\mathrm{d}^3\boldsymbol{p}}{(2\pi)^3} f(\boldsymbol{p}, t) \tag{10}$$

with the factor of 2 arising from spin degeneracy. This is a very useful quantity to help us to compare the effects of different electric field parameters on the pair creation. It is also noted that the quasi-particle has a definite physical meaning at $t \to \infty$, thus what we are really interested in are $f(\boldsymbol{p}, \infty)$ and the produced particles number density $n(\infty)$.

3 Numerical solutions and results

In order to study the e^-e^+ pair production in a time-dependent asymmetric laser electric field, we choose the background field as

$$E(t) = E_0[\mathrm{e}^{-(\frac{t}{\tau_1})^2}\theta(-t) + \mathrm{e}^{-(\frac{t}{\tau_2})^2}\theta(t)]\sin(\omega t + \varphi) \tag{11}$$

where τ_1 and τ_2 are the rising and falling pulse lengths, respectively, ω is the laser frequency, φ is the carrier phase, and $\theta(t)$ is the Heaviside step function.

In our studies, the numerical solutions are found by solving Eqs. (7)–(9) using the fourth order Runge–Kutta method. For convenience, the involving physical quantities are normalized as the electric field E_{cr}, wavelength λ_c and momentum $m_e c$, respectively, where E_{cr} is the critical electric field strength, $\lambda_c = 2.426 \times 10^{-12}$ m is the electron Compton wavelength and $m_e c = 0.51$ MeV/c is electron momentum. Therefore we use a set of normalized quantities in our numerical calculations as $E_0 = 0.1$, $\lambda = 0.618$ associated to normalized laser frequency $\omega = 1.62$. By the way the parameters we have chosen are only convenient for the numerical calculations. However it should be emphasized that this corresponds to an ultraintense hard x ray laser with photon energy about 800 keV. This laser and corresponding ultrashort pulse studied in this paper are still unavailable so far.

First, we investigate the $f(\boldsymbol{p}, \infty)$ and $n(\infty)$ of three different situations, i.e., subcycle, cycle and supercycle laser pulses, when one the falling pulse length τ_2 is fixed and the rising pulse length τ_1 becomes shorter. Concretely we set the parameters as $\tau_2 = a\frac{\pi}{\omega}$ and $\tau_1 = \frac{\tau_2}{N}$, where $N > 1$ is the ratio of falling to rising pulse length and a is the cycle parameters. Here we choose $a = 0.1$, $a = 1$ and $a = 10$ as typical representations for subcycle, cycle and supercycle laser pulses, respectively. We fix the carrier phase $\varphi = 0$ for all numerical calculations.

For subcycle ($a = 0.1$) pulse, the distribution function $f(\boldsymbol{p}, \infty)$ is shown in Fig. 1. It is found that the peak value of $f(\boldsymbol{p}, \infty)$ does not locate at $p_\perp = 0$ for symmetric pulse case when $N = 1$. The dependence of e^-e^+ pair number density $n(\infty)$ on pulse length ratio N is shown in Fig. 2. It shows that the $n(\infty)$ increases rapidly with N until to $N = 3$, and when $N > 3$ it decreases with N until to

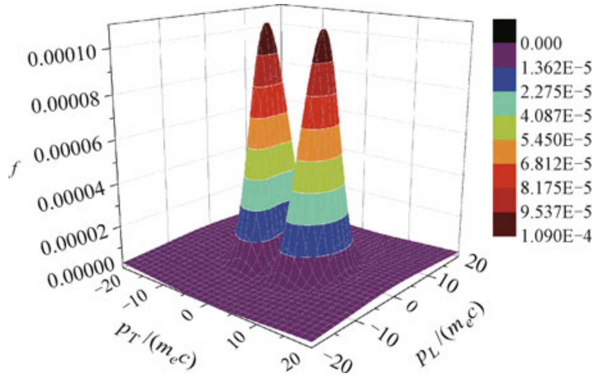

Fig. 1 Momentum dependence of the distribution function $f(\boldsymbol{p}, \infty)$ in symmetric laser electric field for subcycle ($a = 0.1$) pulse. Pulse length parameters are set by $\tau_2 = 0.1\frac{\pi}{\omega}$ and $\tau_1 = \frac{\tau_2}{N}$ with $N = 1$.

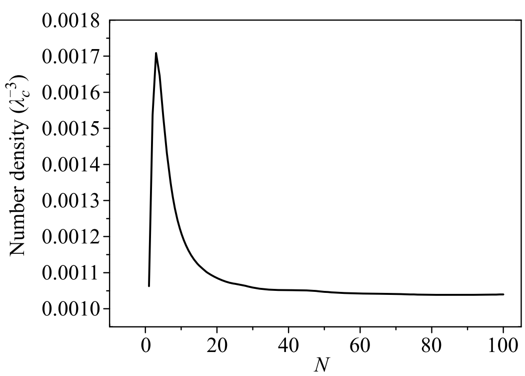

Fig. 2 Dependence of e^-e^+ pair number density $n(\infty)$ on pulse length ratio N in the asymmetric laser electric field for compressed subcycle ($a = 0.1$) pulse.

$N \approx 47$ and then the $n(\infty)$ is insensitive to N.

For cycle ($a = 1$) pulse. The distribution function $f(\boldsymbol{p}, \infty)$ is shown in Fig. 3. We can see that the $f(\boldsymbol{p}, \infty)$ get its maximum value $f(\boldsymbol{p}, \infty) = 5.61 \times 10^{-3}$ at $p_{/\!/} = p_{\perp} = 0$ when $N = 1$. The dependence of e^-e^+ pair number density $n(\infty)$ on pulse length ratio N is shown in Fig. 4. In this case $n(\infty)$ increases rapidly with N until to $N = 11$, at which the number density reaches its maximum value $n_{max}(\infty) = 4.771 \times 10^{-4}$, and when $N > 11$ the $n(\infty)$ decreases with N until to $N \approx 80$ and then $n(\infty)$ keeps almost unchangeable with a further increase of the N. By comparing the Fig. 4 with Fig. 2 we find that, in this case the optimum peak value of the $n(\infty)$ is one order smaller than subcycle pules. These results are also summarized in Table 1.

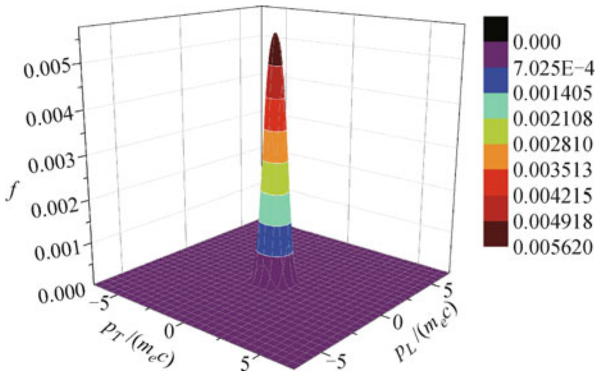

Fig. 3 Momentum dependence of the distribution function $f(\boldsymbol{p}, \infty)$ in symmetric laser electric field for cycle ($a = 1$) pulse. Pulse length parameters are set by $\tau_2 = \frac{\pi}{\omega}$ and $\tau_1 = \frac{\tau_2}{N}$ with $N = 1$.

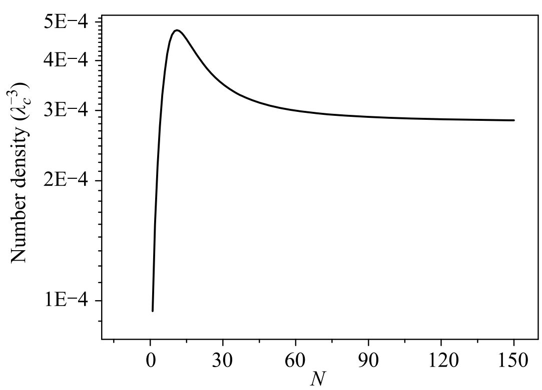

Fig. 4 Dependence of e^-e^+ pair number density $n(\infty)$ on pulse length ratio N in the asymmetric laser electric field for compressed cycle ($a = 1$) pulse.

For supercycle ($a = 10$) pulse, we plot the distribution function $f(\boldsymbol{p}, \infty)$ in Fig. 5 when $N = 1$. By comparing the Fig. 5 with Figs. 1 and 3 we find that the $f(\boldsymbol{p}, \infty)$ is very different from of the previous two cases. Beside that $f(\boldsymbol{p}, \infty)$ peak does not locate at $p_{\perp} = 0$ there exist multi-peak structure. The dependence of e^-e^+ pair

Table 1 The e^-e^+ pair number density (in unit of λ_c^{-3}) in an asymmetric laser electric field for subcycle ($a = 0.1$), cycle ($a = 1$) and supercycle ($a = 10$) laser pulses, respectively. When τ_2 is fixed and τ_1 becomes shorter, there exists an optimum pulse length ratio N_{opt} of asymmetric pulse lengths which makes the pair number density maximum $n_{max}(\infty)$, which is obtained by parameters set as $\tau_2 = a\frac{\pi}{\omega}$ and $\tau_1 = \frac{\tau_2}{N}$, where $N > 1$ is changed.

Cases	Maximum number density	Optimum pulse length ratio
Subcycle	$n_{max} = 1.709 \times 10^{-3}$	$N_{opt} = 3$
Cycle	$n_{max} = 4.771 \times 10^{-4}$	$N_{opt} = 11$
Supercycle	$n_{max} = 4.234 \times 10^{-4}$	$N_{opt} = 111$

number density $n(\infty)$ on pulse length ratio N is shown in Fig. 6. It is obvious that, in this case, the $n(\infty)$ is very sensitive to N which changes from 1 to 111. However, after $N = 111$ the value of $n(\infty)$ keeps almost unchangeable with the increase of the N. It is worthy to note that the $n(\infty)$ is enhanced five orders magnitude from 5.567×10^{-9} when $N = 1$ to 4.234×10^{-4} when $N = 111$. From the Table 1 we can also see that the maximum value of the $n(\infty)$ in this case is still smaller than previous two cases.

We also study the $f(\boldsymbol{p}, \infty)$ and $n(\infty)$ by the exchange

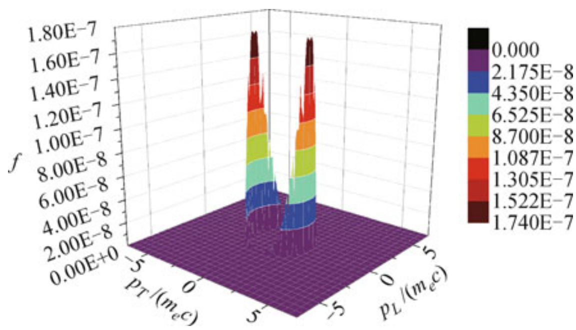

Fig. 5 Momentum dependence of the distribution function $f(\boldsymbol{p}, \infty)$ in symmetric laser electric field for supercycle ($a = 10$) pulse. Pulse length parameters are set by $\tau_2 = 10\frac{\pi}{\omega}$ and $\tau_1 = \frac{\tau_2}{N}$ with $N = 1$.

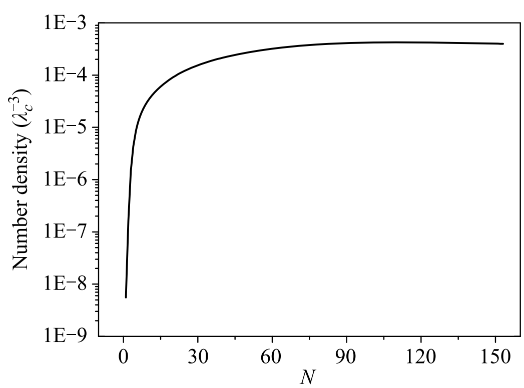

Fig. 6 Dependence of e^-e^+ pair number density $n(\infty)$ on pulse length ratio N in the asymmetric laser electric field for compressed supercycle ($a = 10$) pulse.

of τ_2 and τ_1, i.e., with pulse length parameters set by $\tau_1 = a\frac{\pi}{\omega}$ and $\tau_2 = \frac{\tau_1}{N}$ for subcycle, cycle and supercycle laser pulses, respectively. Surprisingly the obtained results for $f(\boldsymbol{p}, \infty)$ and $n(\infty)$ are exactly the same as what have been done and mentioned above by $\tau_2 = a\frac{\pi}{\omega}$ and $\tau_1 = \frac{\tau_1}{N}$. This indicates that there exists an exchange symmetry for pulse rising and falling in studied system.

We now investigate the $f(\boldsymbol{p}, \infty)$ and $n(\infty)$ when the falling pulse length τ_2 is fixed and the rising pulse length τ_1 becomes longer. Now the pulse length parameters are set by $\tau_2 = a\frac{\pi}{\omega}$ and $\tau_1 = N\tau_2$. Obviously when $N = 1$ the $f(\boldsymbol{p}, \infty)$ for three cycle pulses are the same as in the previous cases as shown in Figs. 1, 3 and 5, respectively. So our attention here will be focused on the behavior of $n(\infty)$, especially its dependence of N.

For subcycle ($a = 0.1$) pulse this dependence is shown in Fig. 7. Surprisingly, there exists not an optimum N for maximum pair number density in this case of elongated pulses, which is contrast to the compressed pulses studied above. It is found that the $n(\infty)$ is only rapidly decreasing with N until to $N \approx 45$ and then exhibits insensitivity of N. Similarly for cycle ($a = 1$) pulse the created $n(\infty)$ decreases with N also but in a more rapid way, see Fig. 8, because after $N \approx 10$ the $n(\infty)$ has been almost unchangeable with the further increase of the N.

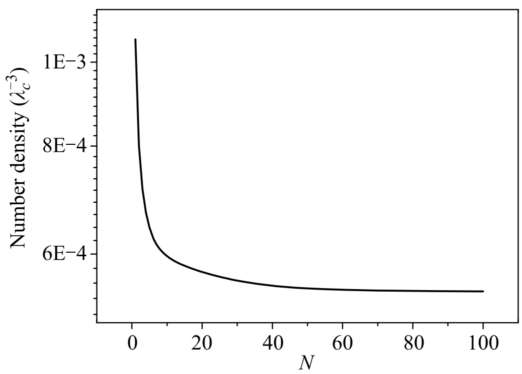

Fig. 7 Dependence of e^-e^+ pair number density $n(\infty)$ on pulse length ratio N in the asymmetric laser electric field for elongated subcycle ($a = 0.1$) pulse.

On the other hand for supercycle ($a = 10$) pulse we plot the dependence of e^-e^+ pair number density $n(\infty)$ on pulse length ratio N in Fig. 9. In this case we choose N ranging from 1 to 600. By comparing the Fig. 9 with Figs. 7 and 8 we find that the $n(\infty)$ increases with the N until to $N = 461$. When $N = 461$ the $n(\infty)$ reaches its maximum value $n_{max}(\infty) = 5.778 \times 10^{-6}$ and then $n(\infty)$ decreases very slowly with the increase of N. However, in this case, the optimum peak value of the $n(\infty)$ is still smaller two orders comparing with the case of supercycle pulse when pulse length becomes compressed.

By the way the exchange symmetry of the pulse of

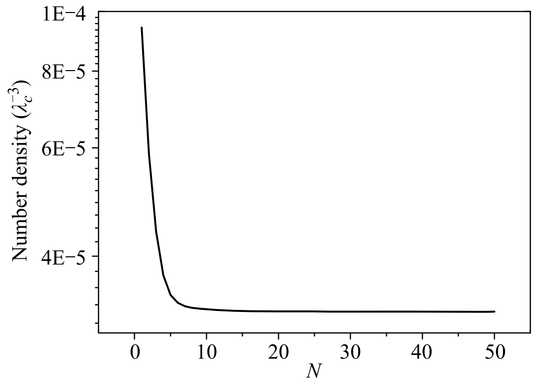

Fig. 8 Dependence of e^-e^+ pair number density $n(\infty)$ on pulse length ratio N in the asymmetric laser electric field for elongated cycle ($a = 1$) pulse.

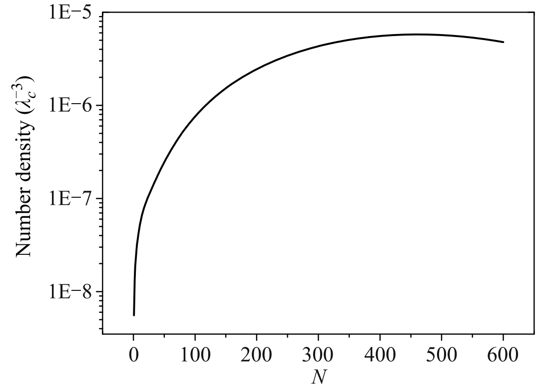

Fig. 9 Dependence of e^-e^+ pair number density $n(\infty)$ on pulse length ratio N in the asymmetric laser electric field for elongated supercycle ($a = 10$) pulse.

rising and falling lengthes holds again for elongation of pulse as that for compression of pulse.

4 Discussion and conclusion

Before giving a summary of this paper it is worthy to discuss briefly some more practical problems, for example, the pair production in plasmas with available laser pulses. To our knowledge as early as the beginning of this century some authors have studied involved problems theoretically. For example, Shen *et al.* [23] have suggested a very interesting method to produce pairs with an ultrathin foil illuminated by two intense circularly polarized laser pulses of zero phase difference. They estimated that the γ-photon intensity is of 7×10^{27} (sr s)$^{-1}$ and the positron density is of $5 \times 10^{22}/\mathrm{cm}^3$ when two 330 fs and 7×10^{21} W/cm^2 laser pulses are used. We noticed that in a recent publication Ridgers *et al.* [24] reported that 10 PW laser interacting with solid will generate dense e^-e^+ plasmas and ultraintense bursts of γ rays. The positron density of $10^{26}/\mathrm{m}^3$ can be achieved. This is relevant strongly to the laboratory study of pair production in high-energy astrophysical environments. However, more theoretical and experimental research are still needed in the future to have a deeper understanding in plasmas by an ultrastrong laser field to create e^-e^+ pairs, which has some important potential applications including the higher positrons and neutrons source production and possible new nuclear fusion schemes and so on.

In a summary, in this paper, we have numerically investigated e^-e^+ pair production in a strong asymmetric laser electric field by using QVE. We consider three different situations of subcycle, cycle and supercycle laser pulses. We have obtained numerically the momentum distribution function, the pair number density and its dependence on the pulse length ratio of the asymmetric laser pulses. It is found that in asymmetric laser electric field, i.e., when the pulse length of one rising or falling side is fixed while the pulse length of the other side is changed, the pair production rate and number density can be significantly modified comparable to symmetric situation. For each case of three different cycle pulses, when the pulse is compressed, the more pairs can be produced than that the pulse is elongated. In former case there always exists an optimum pulse length ratio of fixed to shorter pulse lengths which makes the pair number density maximum. Moreover, the created maximum pair number density by subcycle pulse is larger than that by cycle or/and supercycle pulses. In later case, however, on one hand, only for supercycle laser pulse the created pairs is enhanced and there exists also an optimum pulse length ratio of longer to fixed pulse segments that maximizes the pair number density. On the other hand, surprisingly, in both cases of subcycle and cycle laser pulses, the pair number density is monotonically decreasing as the ratio of longer to fixed pulse lengths increases. The present work could be useful for understanding and theoretical considerations on Schwinger mechanism and these investigated results may be useful for possible e^-e^+ pair production experiments in future.

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