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RESEARCH ARTICLE

Oscillation sources and wave propagation paths in complex networks consisting of excitable nodes

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Self-sustained oscillations in complex networks consisting of nonoscillatory nodes have attracted long-standing interest in diverse natural and social systems. We study the self-sustained periodic oscillations in random networks consisting of excitable nodes. We reveal the underlying dynamic structure by applying a dominant phase-advanced driving method. The oscillation sources and wave propagation paths can be illustrated clearly via the dynamic structure revealed. Then we are able to control the oscillations with surprisingly high efficiency based on our understanding.

Keywords self-sustained oscillations, random networks, excitable nodes, oscillation sources, wave propagation

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1 Introduction

Self-sustained oscillations in complex networks consisting of nonoscillatory nodes are very common phenomena in nature [1], such as oscillations in genetic regulatory networks [2] and neural networks [3–5]. Revealing the mechanism of oscillations in complex networks will not only provide a better understanding of the physiological functions but also be beneficial for the diagnosis and treatment of unfavorable motions, such as epilepsy [6]. There have been many works focusing on oscillatory networks with simplified models [7–10], but some fundamental questions has not been answered. For instance, given an oscillatory complex network consisting of a large number of nonoscillatory nodes, one can hardly say anything about where the oscillation sources are, how oscillatory waves propagate in the whole network, not to mention how to efficiently control the oscillation.

The loop structure is ubiquitous in real networks [11] and the total number of loops can be analyzed [12]. The loops play an important role in the network dynamics. The relation between synchronization and the corresponding dominant loop was discussed [13]. Recurrent excitation has been proposed to be the reason supporting self-sustained oscillations in neural networks [7, 8, 14, 15]. However, efficiently identifying the functional structure in complex networks is remained a question.

In our previous papers [16, 17], we proposed a dominant phase-advanced driving (DPAD) method to reveal the underlying dynamic structure of self-sustained target waves in small-world networks of excitable nodes. Based on the information embedded in the DPAD structures, we successfully revealed the oscillation source. In small-world networks [18], numerous local regular connections coexist with some long-range links. The former play an important role in target wave propagation and the latter are crucial for maintaining the self-sustained oscillations. In the present paper we apply the above ideas to complicated random networks consisting of large numbers of excitable cells, where all connections are randomly distributed and no clear trace of wave propagation can be pursued. We are able to achieve the following results. (i) By applying the DPAD method, we simplify the original high-dimensional random networks to functional and one-dimensional (1D) DPAD structures. (ii) From the DPAD structures we can clearly identify the oscillation sources (unidirectional loops) and reveal the wave propagation paths (unidirectional tree branches); (iii) With these DPAD structures we are able to classify one or a few important nodes, by removing which we can control the oscillations of the whole networks with surprisingly

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high efficiency.

2 Results and analyses

2.1 DPAD method and universal structure

Here, we briefly recall the dynamical DPAD structure [16, 17]. Given a network consisting of N nodes with nonoscillatory local dynamics, there are M ($M > N$) interactions between different nodes. Dynamic variables of each node obey well defined coupled ordinary differential equations. In certain situations, the entire network displays self-sustained oscillations and all nodes are oscillating. Based on the network structure and oscillation data of each node, we try to find the mechanism supporting the oscillations.

It is clear that any individual nonoscillatory node can oscillate only if it is driven by one or more interactions with advanced phases. The definition of "advanced phase" differs in different systems. Among all phase-advanced interactions the interaction providing the most contribution to exciting the given node, is defined as the dominant phase-advanced driving (DPAD). Based on this idea, the corresponding DPAD for each node can be identified. With all DPAD interactions known, we draw the unidirectional DPAD paths between pairs of nodes from the nodes providing driving to the nodes being driven. Applying this complexity reduction method, the original oscillatory high-dimensional complex network of N nodes with M interactions can be reduced to a 1D network of size N with M' unidirectional dominant phase-advanced interactions. Here, the reduced functional network has the same number of nodes and interactions ($M' = N$), because each node has one and only one DPAD.

According to graph theory, if the reduced network with N links is connected, there is one and only one loop ($2^{M'-N+1} - 1 = 1$ [19]) in the network. When the reduced network consists of P mutually disconnected clusters with nodes in each cluster connected, there is one and only one loop in each cluster. Figure 1 gives an illustration of a DPAD structure consisting of two clusters, which agrees well with the conclusion of graph theory. There is one loop in each cluster, and the nodes not in loops are radiated from different nodes in loops. The DPAD structure reveals the dynamical relationship between different nodes. Based on this functional structure, we can identify the loops as the oscillation source, and illustrate the wave propagation along various branches.

All the above ideas are generally applicable to diverse fields for self-sustained oscillations of complex networks consisting of individual nonoscillatory nodes. The application to genetic regulatory networks are also studied [20]. The particular meanings of "advanced phase" and "dominant phase-advanced driving" should be properly defined to be physically meaningful and operable, based on the characteristics of different systems.

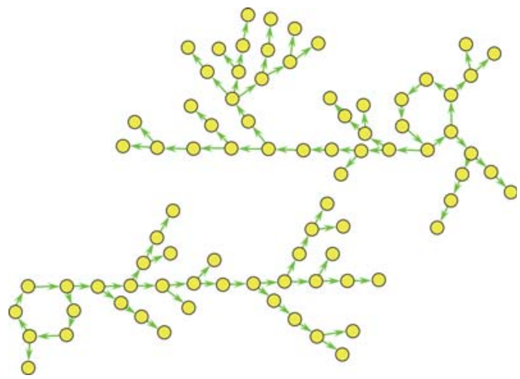

Fig. 1 A schematic diagram of the DPAD structure in oscillatory networks with nonoscillatory nodes. The arrows indicate unidirectional phase-advanced interactions.

2.2 Oscillatory excitable cell networks

We have known that the excitability of cell dynamics and the complexity of the interaction structure are two major features of neural networks [21, 22]. To achieve a better understanding of the basic mutual interaction between the excitable local dynamics and the complicated topological structure, we make a simplification to consider the following complex excitable cell networks (CECNs) of the Bär model [23]

$$\frac{\mathrm{d}u_i}{\mathrm{d}t} = -\frac{1}{\varepsilon}u_i(u_i-1)\left(u_i-\frac{v_i+b}{a}\right)+D_u\sum_{j=1}^{N}A_{ij}(u_j-u_i)$$

$$\frac{\mathrm{d}v_i}{\mathrm{d}t} = f(u_i)-v_i, \quad i=1,2,\ldots,N \tag{1}$$

$$f(u_i)=\begin{cases} 0, & u_i<\frac{1}{3} \\ 1-6.75u_i(u_i-1)^2, & \frac{1}{3}\leqslant u_i\leqslant 1 \\ 1, & u_i>1 \end{cases}$$

where A_{ij} means the node–node adjacency matrix element. $A_{ij} = 1$ if there is an interaction between node i and node j and $A_{ij} = 0$ otherwise. The Bär model keeps the basic properties of the excitable system with simple dynamics, with which we can analyze the oscillation mechanism conveniently. Without couplings, all cells of CECNs are not oscillatory individually for given a, b. They evolve asymptotically to the rest state $u = v = 0$ and will stay there forever unless some external force drives them away from this state. Therefore, all the analyses in the former section are applicable to this type of systems. Whenever a cell is kicked from the rest state by a stimulus large enough, the cell can excite by its own internal dynamics (so called excitable dynamics). For simplicity, we assume symmetric couplings, and adopt random complex networks with identical coupling degree

k (i.e., each cell couples to equal number k of other cells, and the bidirectional couplings are chosen randomly). With proper coupling strength given, the excitation of any given node can cause firings of the neighbor nodes at rest. We also take identical parameters for all cells. One advantage of this homogeneous assumption is to make sure that all self-sustained oscillatory behavior here is not due to any heterogeneity in the topological structure, but due to the self-organization of dynamical mutual excitation.

Figure 2(a) and (c) show the same CECN with each node having interaction degree $k = 3$, and the topological structure looks rather complicated. With the given interaction structure and parameters, we simulate the system by taking different sets of random initial conditions. The system evolves asymptotically to the homogeneous rest state in most cases. However, about 3% of tests provide oscillations. The spatio-temporal patterns in Fig. 2(b) and (d) are two of these oscillation states. Both oscillations are periodic and self-sustained, but the excitation distribution in different nodes looks irregular. It is thus difficult to unveil the mechanism supporting the oscillations and the excitation propagation paths in the network.

We further study how sensitive the oscillations are to control. Here the control is to remove one or mutiple nodes from the network and to examine the system response to this operation. By removing a node we mean to discard all interactions from and toward the given node. After exhaustive tests of the single node removal we find that the oscillation in Fig. 2(b) can be terminated (i.e., turned to the homogeneous rest state $u_i = v_i = 0$, $i = 1, 2, ..., N$) by removing a single node 60 [red node shown in Fig. 2(a)] while the oscillation persists safely if we remove as many as 60 nodes [empty square nodes shown also in Fig. 2(a)]. The sharp difference between these operations is surprising. It clearly demonstrates that though the topological structure of Fig. 2(a) is random and homogeneous, the dynamic organization supporting the oscillation is strongly heterogeneous, and the roles played by different nodes in this organization are considerably different. The central task of the present work is to reveal these self-organized patterns under the condition of full knowledge of the coupling structure and the oscillation data, and then to find effective methods for controlling the oscillatory networks.

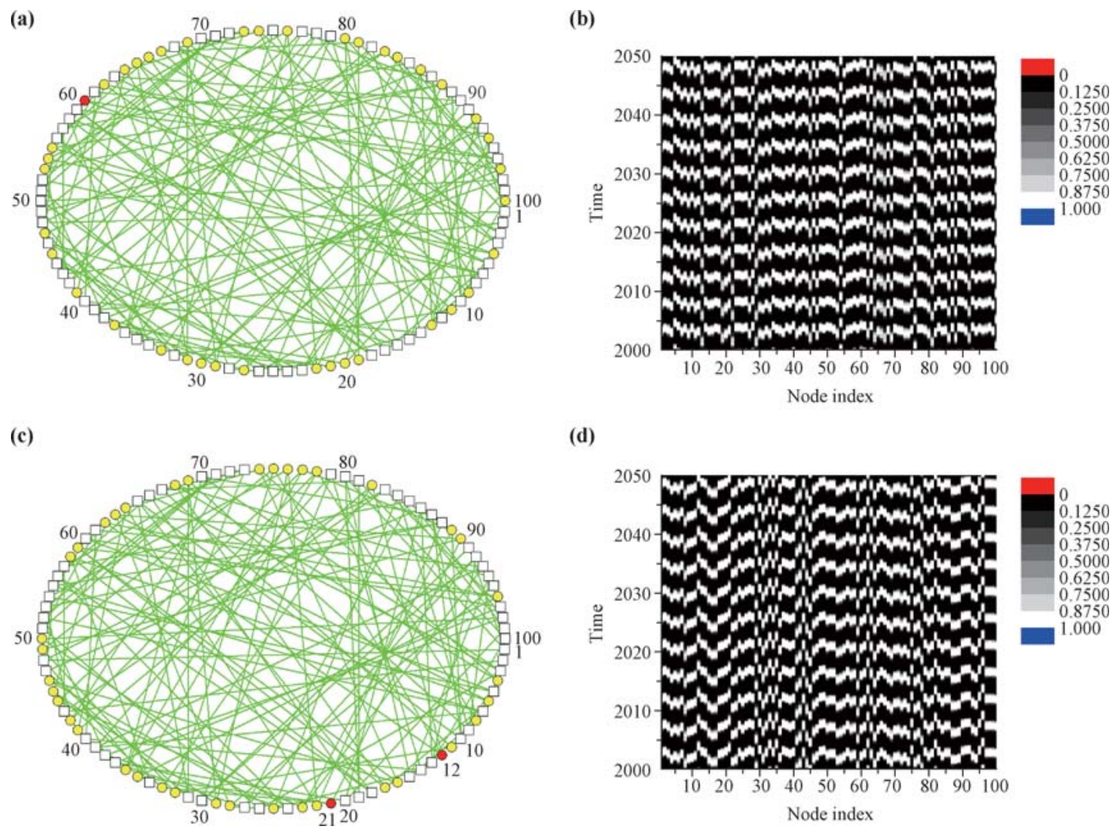

Fig. 2 Different oscillation states in the same random network consisting of $N = 100$ nodes. All couplings with strength $D_u = 1.0$ are randomly chosen between N nodes, each having coupling degree $k = 3$. The nodes have the same parameters set as follows: $\alpha = 0.84$, $\beta = 0.07$, $\varepsilon = 0.04$. All these parameters will be used throughout Figs. 2–5. **(a)**, **(c)** Random networks and their modulations corresponding to oscillation states (b) and (d), respectively. Green lines between nodes display the mutual interactions generated randomly. When we remove the red nodes, the corresponding oscillation is suppressed. In contrast, both oscillations persist when the indicated 60 empty square nodes are removed. **(b)**, **(d)** Spatio-temporal patterns of two oscillation states in the same network. Both the patterns display the evolution of local variable u. The nodes are spatially arranged according to their indexes.

On the other hand, we can never terminate the oscillation in Fig. 2(d) by discarding any single node. Instead, this oscillation can be terminated by removing a pair of nodes [red nodes 12 and 21 shown in Fig. 2(c)] in contrast with Fig. 2(a) where the oscillation can be terminated by removing only a single red node. The difference between Fig. 2(a) and (c) is again a mystery of oscillatory networks. Similar to Fig. 2(a) the oscillation persists when we simultaneously remove all 60 empty square nodes shown in Fig. 2(c).

In order to apply the DPAD method to the oscillation states in Fig. 2, the first task is to define phase-advanced interactions of excitable nodes. For a given node that enters the region of the rest state ($u < u_{th} = b/a$) at time t_s and departs from this region at time t_e, we define the "phase-advanced driving" as the interaction from the neighbor node that leaves the rest state earlier than the given node [i.e., in the period(t_s, t_e)], which thereby provides a favorable contribution to kicking the given cell from the rest state around excitation time t_e. Among all phase-advanced interactions the dominant phase-advanced driving is defined as the interaction from the node first leaving the resting state in the period (t_s, t_e). There is no doubt that the dominant driving makes the most important contribution to exciting the given node, i.e., to driving the given node to oscillate.

2.3 DPAD structure and oscillation control

In Fig. 3(a), we display the dominant phase-advanced driving of node 55 in state Fig. 2(b) as an example. With the definition of DPAD shown in Fig. 3(a) we can draw the DPAD structures corresponding to various oscillation states of Fig. 2. In Fig. 3(b), we show the DPAD structure of state Fig. 2(b). We find a DPAD pattern, which is just the type shown in Fig. 1. The only difference is that there is only one cluster in Fig. 3(b). The single dynamical loop plays the role of oscillation source, with cells in the loop exciting sequentially to maintain the self-sustained oscillation. We can observe waves propagating downstream along the tree branches rooted at various cells in the loop. Wave propagation in two different paths is illustrated in Fig. 3(c) and (d), respectively. In Fig. 3(c) and (d) nodes are arranged in order according to the sequence in the loop and the purple branch, respectively. We find regular and perfect wave propagation patterns in both figures. These figures demonstrate that the DPAD structure well illustrates the wave propagation paths.

The DPAD structure in Fig. 3(b) clearly shows distinctive significance of different cells in the oscillation which cannot be observed in Fig. 2(a). In Fig. 2(a) all cells stand equivalently in the homogeneous and randomly coupled network, and no cell takes any priority over

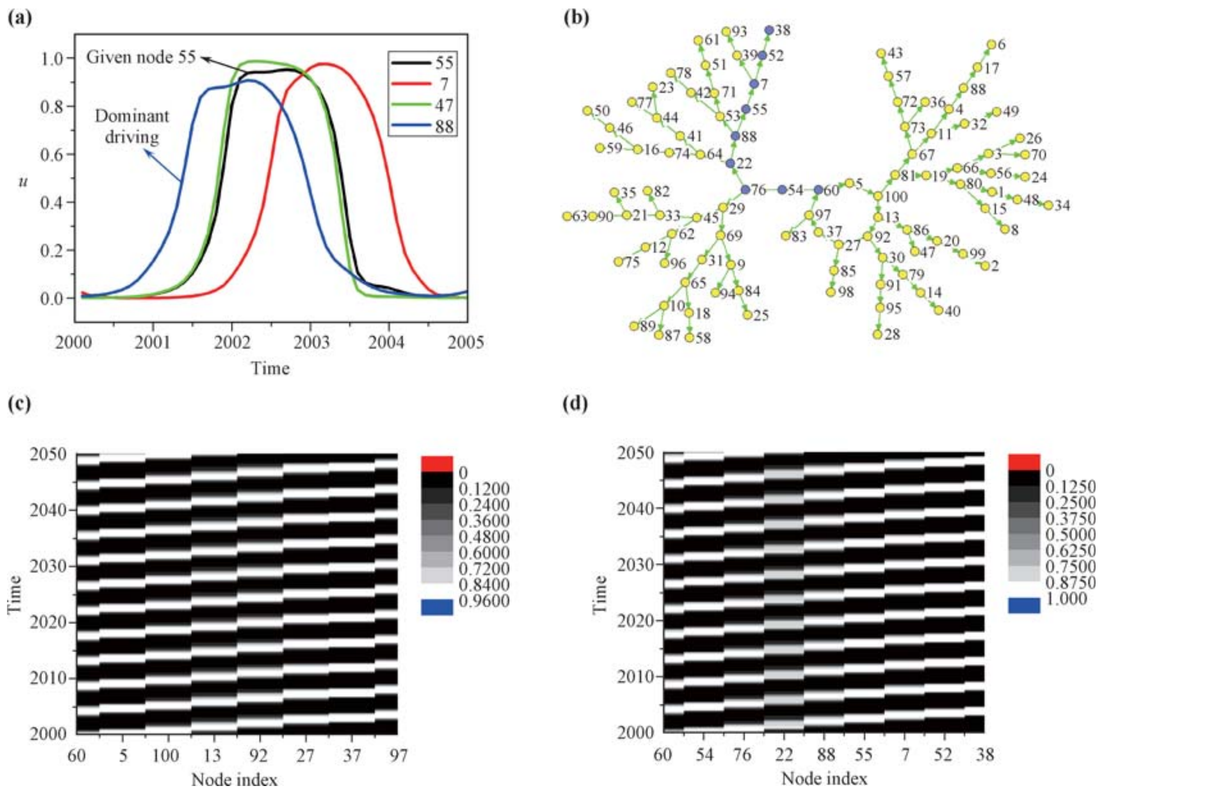

Fig. 3 **(a)** Illustration of the dominant phase-advanced driving (DPAD). The black curve denotes the given node 55. Three nodes, nodes 7, 47, and 88, interact with node 55. The blue and green lines provide driving for exciting the given node, so they are called phase-advanced interactions. The blue one, making the most significant contribution, is the dominant phase-advanced driving (DPAD). **(b)** DPAD structure corresponding to the oscillation state in Fig. 2(b). **(c)** Evolution of variable u of nodes in the loop. These nodes are ordered according to the sequence in the loop shown in (b). **(d)** Evolution of variable u of different nodes in the purple branch in (b). Both figures (c) and (d) show perfect wave propagation.

others in topology. The situation in Fig. 3(b) is different. Because the unidirectional loop works as the oscillation source, cells in the loop must be important to the oscillation. Furthermore, the cells at various turning points of large branches, which control large numbers of downstream cells, are also likely to be important. In particular, node 60 in Fig. 3(b) is likely to be the most important cell because it locates in the source loop on one hand and controls the largest number of downstream nodes on the other hand. It is interesting to observe that node 60 is the very single red node in Fig. 2(a) that we found important for controlling the oscillation but did not known the reason then. Now, the significance of node 60 to the oscillation is unveiled clearly. In Fig. 4(a) we show how the oscillation collapses quickly to the homogeneous rest state after removing only a single node 60. On the other hand, cells near the branch ends must be less important to the oscillation. We remove simultaneously a large number of cells (60 nodes) in branch tails [See Fig. 4(b)] which are exactly the empty square nodes in Fig. 2(a). The network not only continues its periodic oscillation [Fig. 4(b)], but also keeps the DPAD structure similar with Fig. 3(b) for the remaining cells [Fig. 4(c)]. Therefore, the puzzling question that why so many empty square nodes together in Fig. 2(a) are much less important than a single red cell, is clearly answered in Fig. 3(b). Because all these empty square nodes are far from the source loop, they have little influence on the self-sustained oscillation. On the other hand the single red cell 60 controls both the oscillation source and a large number of downstream nodes, so it is of crucial importance for the given oscillation. Removing one or few other cells in the source loop may not stop the oscillation but can considerably change the oscillation frequency as well as the DPAD structure of the oscillation.

Period is an important quantity describing the properties of oscillatory networks. We further study the influence of the DPAD loops on the periods of network oscillations. We simulated Eq. (1) in different homogeneous random networks ($N = 100$, $k = 3$) with random initial conditions. We found 217 oscillatory realizations (all are periodic). Then we measured the period T of each oscillatory network, and plotted T against n in Fig. 4(d) with n being the size of the loop in the corresponding DPAD structure. In Fig. 4(d) a monotonous and approximately linear increase of $\langle T \rangle$ with n is clearly demonstrated. These observations convincingly support the conclusion that DPAD loops indeed play the role of oscillation sources in complex networks. For a given node in the source loop, while the neighbor with the most advanced phase dominates the driving, how quick the node will be excited is influenced by the input from other

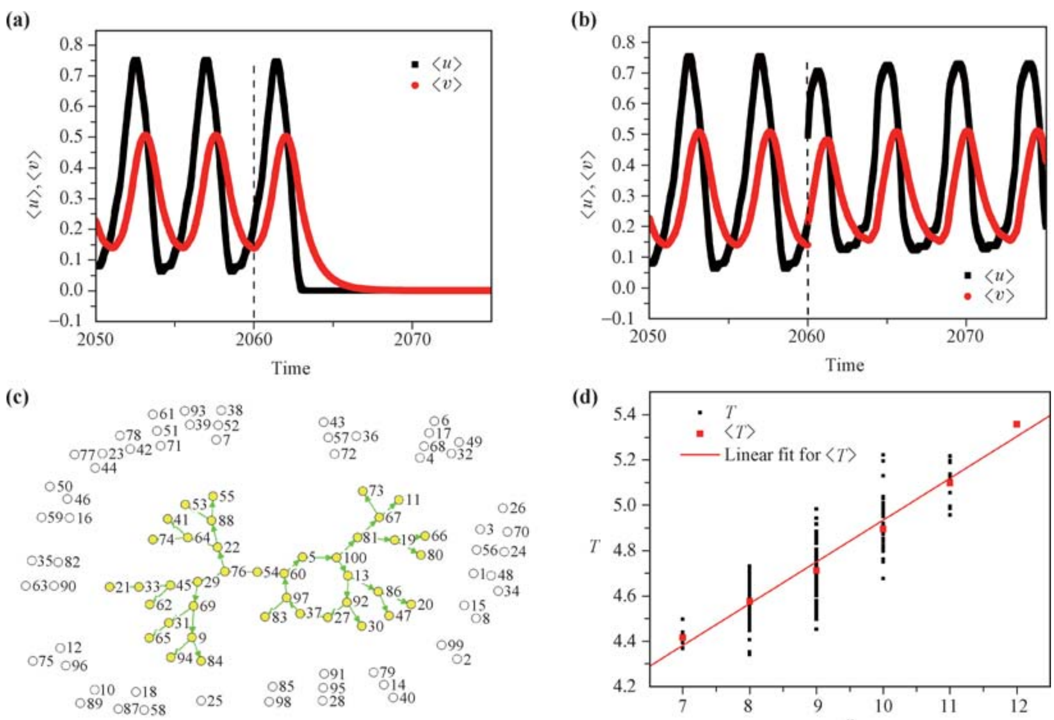

Fig. 4 **(a)**, **(b)** Evolution of average variables $\langle u \rangle$ and $\langle v \rangle$. $\langle u \rangle = 1/N \sum_{i=1}^{N} u_i$, $\langle v \rangle = 1/N \sum_{i=1}^{N} v_i$. **(a)** The oscillation of Fig. 2(b) is suppressed when we remove the single node 60 at $t = 2060$ denoted by the dashed line. **(b)** The oscillation of Fig. 2(b) are modulated slightly by removing 60 side nodes in total at $t = 2060$ denoted by the dashed line. **(c)** The DPAD structure corresponding to modulated oscillation in (b) with 40 nodes remaining. The resulting DPAD structure remains similar after the control. **(d)** Periods T's of 217 oscillation states plotted against the sizes of corresponding DPAD loops. The oscillations are generated in different networks with random initial conditions. The larger red square represents the average period $\langle T \rangle$ at certain loop size and the red solid line denotes the linear fitting of the tendency. $\langle T \rangle$ increases with the loop size n almost linearly.

connecting nodes. Therefore the periods can be different for the same loop size n.

In Fig. 5(a) we show the DPAD structure corresponding to the oscillation in Fig. 2(d). The interesting difference between Fig. 5(a) and Fig. 3(b) is that Fig. 5(a) contains two DPAD clusters (similar to Fig. 1) instead of the single one in Fig. 3(b). Comparing the DPAD structure in Fig. 5(a) with that in Fig. 3(b), we can understand the difference of Fig. 2(a) and (c) immediately. Since there are two oscillation source loops in Fig. 5(a), we have to destroy both loops for terminating the oscillation by removing the two red nodes in Fig. 5(a) simultaneously, each from a loop. The two red nodes shown here (node 12 and 21) are exactly identical to those in Fig. 2(c) with the same color. We show that the oscillation is suppressed after removing these two red nodes in Fig. 5(b). In Fig. 5(c) and (d) we derive the DPAD structure with only one of the two red nodes (node 12 or node 21) removed, respectively. It is interesting to see that in both cases when one DPAD loop of Fig. 5(a) is destroyed, the other remaining loop serves as the unique oscillation source of the whole system. All nodes in the original destroyed DPAD cluster will move to the survival loop cluster to continue their periodic oscillations. Based on the former experience in Fig. 4(c) we can easily identify that the empty square nodes removed in Fig. 2(c) are all the side nodes in the DPAD structure in Fig. 5(a). That is the reason why the oscillation persists safely when this large number of nodes are removed. The removal of these nodes does not effectively affect the oscillation source.

On the basis of the DPAD structure, we are able to identify the oscillation source and distinguish the roles played by different nodes. We can make a better understanding of the wave propagation in complex networks consisting of excitable nodes. Furthermore, we are able to suppress the oscillations in networks with high efficiency.

2.4 Extensions to other systems

We have sampled a number of CECNs with different initial conditions and different structures (including different system sizes and interaction degrees). Some highly heterogeneous scale-free networks [24] are also considered. We even studied CECNs with different local dynamics, such as Fitzhugh–Nagumo (FHN) [21] neural cell networks. For each oscillatory network we are able to derive the DPAD structure, which has the universal structure of tree branches from loops like Fig. 1. The general conclusions about DPAD structures are well confirmed. Some of the results in different networks are shown here.

In Fig. 6, we plot various DPAD structures for oscillations in networks with even larger size $N = 2500$. In Fig. 6(a)–(c) we display three DPAD structures, respectively, corresponding to different self-sustained periodic oscillations in different networks. DPAD structures

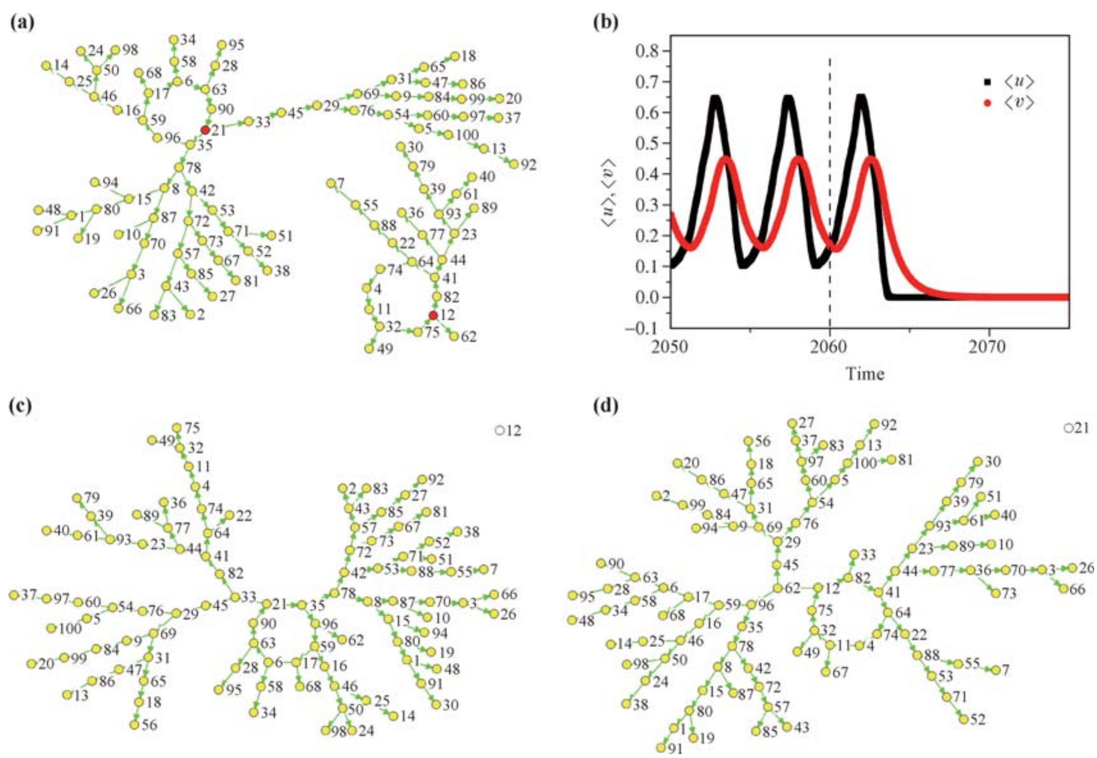

Fig. 5 DPAD structure and corresponding modulations of the oscillation state in Fig. 2(d). **(a)** DPAD structure with two loops working as the source of oscillation in Fig. 2(d). **(b)** Evolution of average variables $\langle u \rangle$, $\langle v \rangle$ of the state (a). The oscillation is suppressed when one removes nodes 12 and 21 simultaneously at $t = 2060$. The time of the removal is denoted by the dashed line. **(c)**, **(d)** DPAD structures of modulated oscillation states after removing node 12 and node 21, respectively.

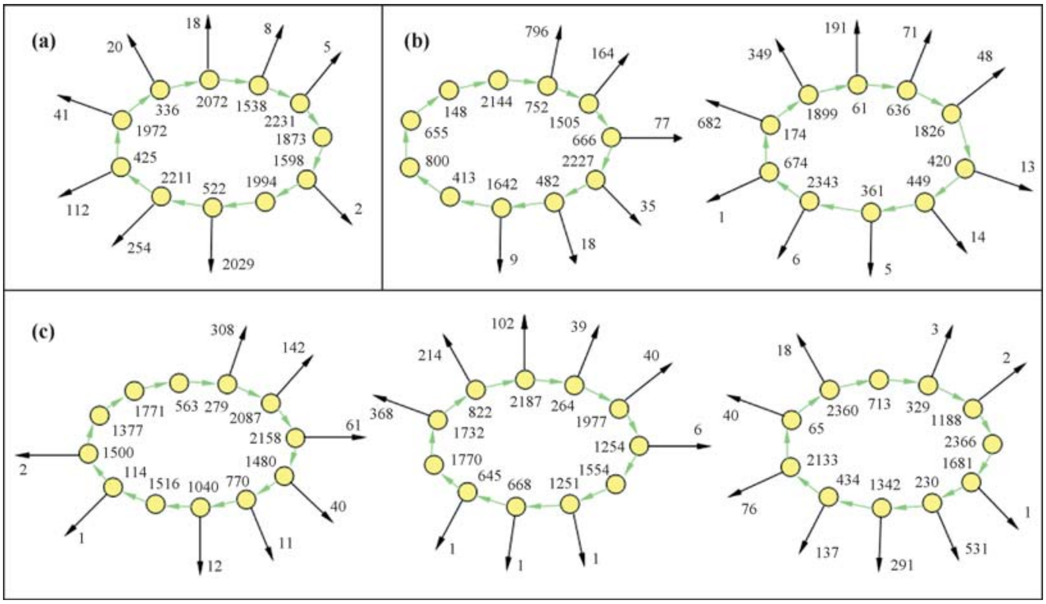

Fig. 6 The schematic diagrams of different DPAD structures in larger random networks with $N = 2500$. The other parameters are the same as Fig. 2. The numbers inside the loops mean the indexes of corresponding nodes in the loop. The sizes of branches radiated from different nodes in loops are denoted by the numbers associated with the the corresponding arrows. **(a)** Single loop DPAD structure. **(b)** Double-loop DPAD structure. **(c)** Triple-loop DPAD structure.

with one loop [Fig. 6(a)], double loops [Fig. 6(b)], and triple loops [Fig. 6(c)], are illustrated. The numbers inside the loops indicate the indexes of the loop nodes while those outside the loops associated with arrows denote the size of branches rooted at the given loop nodes. Currently, the network size is much larger than that in Fig. 2(a); however, the prediction of the universal DPAD structure like that in Fig. 1 is still verified. An interesting phenomenon is that we find triple DPAD clusters in Fig. 6(c).

In Fig. 7, we analyze the self-sustained periodic oscillations in a scale-free network with degree distribution obeying a power-law rule [24]. The network consists of $N = 200$ nodes with average degree $\langle k \rangle = 4$. Because of the topological heterogeneity, there are some hub nodes in the network, which have interactions with many other nodes. The dynamic equations (1) are modified as follows:

$$\begin{aligned} \frac{\mathrm{d}u_i}{\mathrm{d}t} &= -\frac{1}{\varepsilon}u_i(u_i-1)(u_i-\frac{v_i+b}{a}) + D_u w_i \\ \frac{\mathrm{d}v_i}{\mathrm{d}t} &= f(u_i) - v_i, \quad i = 1, 2, \ldots, N \end{aligned} \tag{2}$$

$$w_i = \frac{\sum_{j=1}^{N} A_{ij}u_j}{\sum_{j=1}^{N} A_{ij}u_j + K} - u_i$$

$$f(u_i) = \begin{cases} 0, & u_i < \frac{1}{3} \\ 1 - 6.75u_i(u_i-1)^2, & \frac{1}{3} \leqslant u_i \leqslant 1 \\ 1, & u_i > 1 \end{cases}$$

Being different from the homogeneous random network, a specific type of interplay w_i between interacting nodes are considered here, to ensure that any rest node can be successfully exited by its any exciting neighbor node. This type of interplay has been widely used in neural networks [7, 8, 25] and other excitable networks [26–28]. Given oscillation states generated by random initial conditions, similar DPAD structures are unveiled, one of which is shown in Fig. 7(a). With the information embedded in the DPAD structure, we can suppress the oscillation effectively. The control on the evolution is demonstrated in Fig. 7(b).

The DPAD structure analysis can also be extended to complex networks consisting of nodes with more complicated local dynamics. Let us consider the network of FHN model [21] with diffusive coupling

$$\begin{aligned} \frac{\mathrm{d}u_i}{\mathrm{d}t} &= \frac{1}{\varepsilon}(u_i - \frac{u_i^3}{3} - v_i) + D_u \sum_{j=1}^{N}(A_{ij}(u_j - u_i)) \\ \frac{\mathrm{d}v_i}{\mathrm{d}t} &= \varepsilon(u_i + \beta - \gamma v_i), \quad i = 1, 2, ..., N \end{aligned} \tag{3}$$

where A_{ij} is also the element of node-node adjacency matrix. The FHN model has been extensively used to describe neural dynamics. For the given parameters, the individual neural cell is excitable. With certain initial conditions the network of coupled cells can self-organize into sustained oscillations. In Fig. 8(a) and (b), we display two different DPAD structures, respectively, corresponding to two periodic states in the same homogeneous network with different initial conditions. The single loop [Fig. 8(a)] and double-loop [Fig. 8(b)] DPAD structures are identified by applying the DPAD method. We can also suppress the oscillations efficiently by removing the corresponding red nodes in Fig. 8(a) and (b).

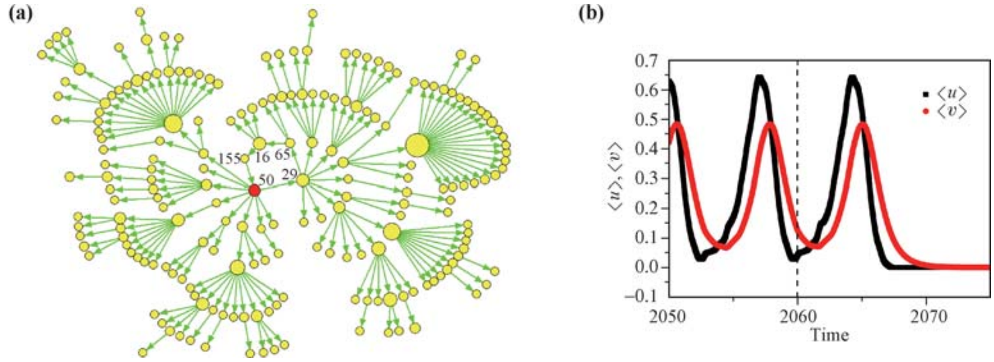

Fig. 7 Oscillation in a scale-free network consisting of $N = 200$ nodes. The size of each node is proportional to its degree k. The degree distribution of the network obeys a power-law distribution with an exponent $\gamma = -3$. The average degree is $\langle k \rangle = 4$. The bidirectional links between nodes are chosen randomly with coupling strength $D_u = 0.4$ and $K = 1.8$ in Eq. (2). The local dynamics parameters are the same as Fig. 2. **(a)** DPAD structure corresponding to a certain periodic oscillation generated randomly. Only the indexes of the nodes in the loop are indicated. **(b)** Time evolution of average variables $\langle u \rangle$, $\langle v \rangle$. One can suppress the oscillation with the red node 50 removed. The time of the removal is indicated by the dashed line.

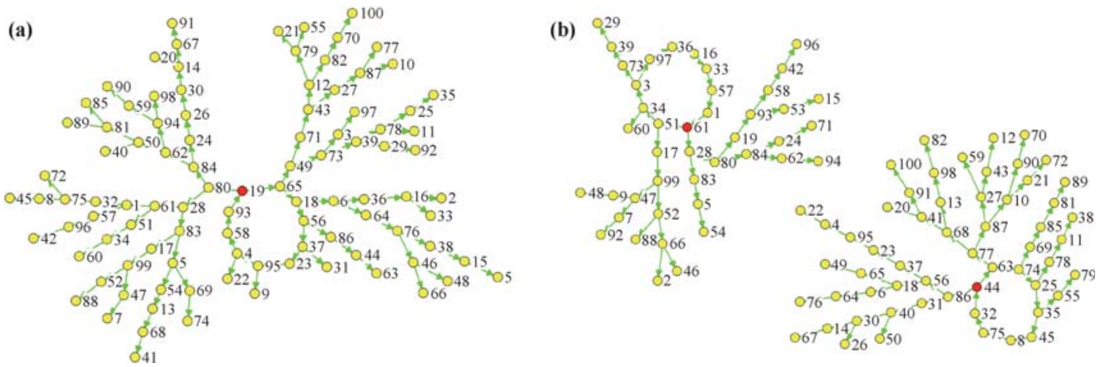

Fig. 8 DPAD structures corresponding to different oscillation states in an FHN network. The network consists of $N = 100$ nodes with homogeneous degree $k = 3$. The interactions between different nodes are generated randomly with coupling strength $D_u = 0.1$. FHN model is applied as the local dynamics. The local parameters in Eq. (3) are set as follows: $\beta = 0.7$, $\gamma = 0.5$, $\varepsilon = 0.2$. Two oscillation states are generated by different initial conditions. **(a)** DPAD structure with a single loop. **(b)** DPAD structure with double loops. The important nodes in both oscillation states are marked by red nodes.

3 Conclusions

We studied the problem of periodic oscillations in complex networks consisting of excitable nodes. With the dominant phase-advanced driving method we revealed DPAD structures in complicated high-dimensional networks. On the basis of the DPAD structures we can easily identify the oscillation sources and wave propagation paths in oscillatory complex networks. All these messages are deeply hidden in the original complex structures and seemingly random phase distribution. These DPAD structures are extremely important for understanding and efficiently controlling self-sustained oscillations in complex systems. We successfully used these ideas and methods to analyze the model of excitable cell networks, and these ideas and methods are expected to be applicable to self-sustained oscillations of complex networks in a broad range of fields.

In the present paper we have considered only cases of periodic oscillations where DPAD structures are stationary. If an oscillation is quasiperiodic or even chaotic, the DPAD structure may vary during the evolution, and this opens a new field for further study. Moreover, throughout this paper we have studied how to reveal DPAD sturctures with full knowledge of the interaction structures and oscillation data. These conditions are not fulfilled in many experiments. Thus, it is another crucial task to extend the investigation to the cases with partial or even without any oscillation data.

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References

1. L. Glass, Nature, 2001, 410: 277
2. M. B. Elowitz and S. Leibler, Nature, 2000, 403: 335
3. A. I. Selverston and M. Moulins, Ann. Rev. Physiol., 1985, 47: 29
4. G. Buzsáki and A. Draguhn, Nature, 2004, 304: 1926
5. T. P. Vogels, K. Rajan, and L. F. Abbott, Ann. Rev. Neurosci., 2005, 28: 357
6. G. A. Worrell, L. Parish, S. D. Cranstoun, R. Jonas, G.

Baltuch, and B. Litt, Brain, 2004, 127: 1496
7. T. J. Lewis and J. Rinzel, Network: Comput. Neural Syst., 2000, 11: 299
8. A. Roxin, H. Riecke, and S. A. Solla, Phys. Rev. Lett., 2004, 92: 198101
9. S. Sinha, J. Saramäki, and K. Kaski, Phys. Rev. E, 2007, 76: 015101(R)
10. A. J. Steele, M. Tinsley, and K. Showalter, Chaos, 2006, 16: 015110
11. A. Zeng, Y. Q. Hu, and Z. R. Di, Phys. Rev. E, 2010, 81: 046121
12. G. Bianconi, N. Gulbahce, and A. E. Motter, Phys. Rev. Lett., 2008, 100: 118701
13. X. J. Ma, L. Huang, Y.-C. Lai, and Z. G. Zheng, Phys. Rev. E, 2009, 79: 056106
14. M. J. O'Donovan, Curr. Opin. Neurobiol., 1999, 9: 94
15. M. V. Sanchez-Vives and D. A. McCormick, Nature Neurosci., 2000, 3: 1027
16. X. H. Liao, W. M. Ye, X. D. Huang, Q. Z. Xia, X. H. Huang, P. F. Li, Y. Qian, X. Q. Huang, and G. Hu, arXiv: 0906.2356, 2009
17. Y. Qian, X. D. Huang, G. Hu, and X. H. Liao, Phys. Rev. E, 2010, 81: 036101
18. D. J. Watts and S. H. Strogatz, Nature, 1998, 393: 440
19. B. Bollobás, Modern Graph Theory: Springer-Verlag, 1998
20. W. M. Ye, X. D. Huang, X. H. Huang, P. F. Li, Q. Z. Xia, and G. Hu, Phys. Lett. A, 2010, 374: 2521
21. R. Fitzhugh, Biophys. J., 1961, 1: 445
22. E. M. Izhikevich, IEEE Trans. on Neural Networks, 2004, 15: 1063
23. M. Bär and M. Eiswirth, Phys. Rev. E, 1993, 48: R1635
24. A.-L. Barabási and R. Albert, Science, 1999, 286: 509
25. M. Müller-Linow, C. C. Hilgetag, and M. T. Hütt, PLoS Comput. Biol., 2008, 4: e1000190
26. M. Kuperman and G. Abramson, Phys. Rev. Lett. 2001, 86: 2909
27. J. M. Greenberg and S. P. Hastings, SIAM J. Appl. Math., 1978, 34: 515
28. P. Bak, K. Chen, and C. Tang, Phys. Lett. A, 1990, 147: 297