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RESEARCH ARTICLE

# Modeling mobile ad hoc communication networks on two-dimensional square lattice

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In this paper, we model the mobile ad hoc communication network on a two-dimensional square lattice. Both structure and function of it depend on transmission range and site-occupancy of nodes. Critical occupancies $\sigma_c$ for different transmission ranges $r$ to maintain global connection are found. Universal scaling function behaves as $\eta \sim f(R^{\beta}\sigma)$, where $R = (r - r_0)/r_0$, and the scaling exponent $\beta = -0.61$, which distinguishes itself from percolation in previous lattice or network models. When the occupancy $\sigma$ is near the threshold $\sigma_c$, individual nodes self-organize into a dynamic small world network relative to geometric distance. The network has a cut-off degree below which clustering coefficient keeps constant, which distinguish itself from other systems and has its potential application in technical designs.

**Keywords** mobile ad hoc communication network, complex network, global connectivity

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## 1 Introduction

Since the models of small-world [1] and scale-free networks [2] were proposed, people in the physical community have paid much attention on the study of complex networks. A complex network can be developed by nodes representing individuals in a real system and by links representing interactions between them. Topological effects on dynamic behaviors in a network such as synchronization [3–5] and epidemic spreading [6] have been investigated widely. Most previous study on the complex networks focuses on topological properties of them ignoring the geometric distance between nodes, which is reasonable for some networks such as food web [7], citation networks [8], metabolic networks [9] etc. However, in some real problems involving transportation systems [10, 11], the internet [12, 13], WWW [14], power grids [15, 16] and mobile ad hoc communication networks [17, 18], the geometric distance is very important for the functions of them. With more in depth research on complex networks, physicists gradually take it into account. Rozenfeld [19] proposed a method for embedding scale-free networks in regular Euclidean lattices accounting for geographical properties. Xie *et al.* [20] proposed a growing network model based on an optimal policy involving both topological and geographical measures. Xulvi-Brunet *et al.* [21] introduced several network-generating mechanisms relative to the constraints that geography imposes on the evolution of large-scale network systems in physical space. A review of geographical networks by Yang *et al.* has appeared in the present journal [22].

On the other hand, mobile ad hoc networks have become an attractive topic in the field of technology, because of their potential applications such as in the battle field, disaster relief providing, outdoor assemblies and other settings in temporal use. In such a network, there is neither infrastructure nor central authority as seen in the existing systems. Each of mobile nodes serves not only as a host but also as a router forwarding packets for other ones [23]. The topology of the ad hoc network changes with time since nodes in it keep moving randomly. Therefore, to take collective duties effectively, the network should self-organize into a dynamically stationary network as needed. A key ingredient for the operation of the ad hoc network is transmission range of mobile nodes. It is a circle with radius $r$ from the center where a node is located. The energy consumption of a node is proportional to the square of $r$ [24]. Any other nodes inside this circle can communicate with it directly, while nodes outside it need to be connected by successive indirect transmissions (multi-hop links). It seems easier to realize the global connection with large transmission range $r$ than with smaller ones. However, it would consume more energy of nodes which is difficult to be recharged when they are in motion. Moreover, large range may cause strong interference in wireless communication. Inversely, smaller range $r$ would reduce the possibility of direct connection and demand more multi-hop steps of linking, which increases the probability of network breaking. Therefore, a balance between both sides is anticipated. The mobile ad hoc communication network is a sort of dynamic complex networks relative to geometric distance, which has not got in depth investigation from the view of statistical physics [25, 26].

In this paper, we model the two-dimensional (2-D) plane on which nodes move randomly with a square lattice, and determine distance-dependent connection by the transmission range. Demanding global connection of the mobile ad hoc network, we investigate critical phenomena relying on site occupancy for various transmission ranges, and find out a scaling relation for the percolation in self-organized dynamic complex networks and particular topological properties of them.

## 2 Model

We study the mobile ad hoc communication network on a 2-D plane with the following model. For the convenience of discrete simulation, we model the plane with a 2-D square lattice of size $L$, and with periodic boundary condition. The total number of the sites on it is $N = L \times L$. The length of the edge between two nearest geometric adjacent sites is defined as $r_0$. We assign $n_0 (n_0 < N)$ nodes to the lattice randomly and define the occupancy $\sigma$ of sites by mobile nodes as $n_0/N$. Each site of the square lattice can be occupied by just one node at most. Every node has its transmission range $r (r > r_0)$. It can gain links to other nodes within its transmission range. To simplify our study, we assume that every node in the network has the same transmission range. And it is noted that a node cannot connect with itself, and two neighbor nodes just can have one link between them, since it makes no sense to communicate with itself or to have multiple channels between any two direct communication nodes. Take $r = 2r_0$ shown in Fig. 1 as an example. If node 1 occupies a site of the lattice, its transmission range is denoted by the circle around it. Every node inside the circle can directly link to node 1. Considering the mobility of the nodes in the mobile ad hoc communication network, we suppose that every node can randomly move to one of its four closest neighboring sites if it is not occupied by any other node at that moment. Different configurations formed by the motion of the nodes at the same occupancy can be regarded as the ensemble for averaged quantities.

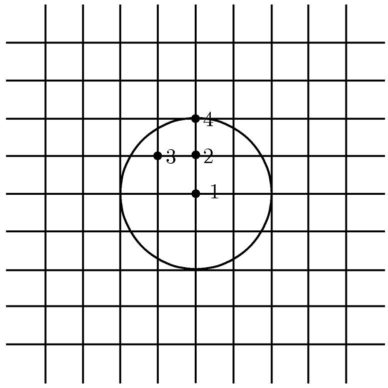


**Fig. 1** Square lattice model, with transmission range $r = 2r_0$.

The first problem for a communication network is the connectivity of it. Moreover, nodes in an ad hoc network may take duties of rescue, search, tracing or so, which demands that the integrated network contains nodes as much as possible. Here, we define the probability of global connection for an ad hoc network as averaged quantity $\eta = n/n_0$, where $n$ is the number of nodes included in the largest linked network. In comparison, previous researchers are interested in the probability for a site to be included in the giant component of percolation problem [25, 27–30] which extends from a border to its opposite one of a lattice with finite size, leaving many isolated nodes outside it. Therefore, we have a stricter demand than that in traditional percolation problems. And the probability of global connection $\eta$ is taken as an order parameter.

## 3 Results

The probability of global connection $\eta$ (global connectivity) is calculated by the burning algorithm [31]. It is naturally expected by the argument above that the global connectivity should be related to the transmission range $r$ of nodes and the occupancy $\sigma$ of the sites by them. We numerically calculate the relation between the global connectivity and the occupancy of sites for different transmission ranges. Figure 2 shows the behavior of $\eta$ versus the occupancy $\sigma$, where the simulation area is a square lattice of size $L = 200$ and transmission ranges of nodes are $2r_0, 3r_0, 4r_0, 5r_0$, and $6r_0$, respectively. All simulation results are averaged over 300 realizations of configurations formed by the moving nodes on the square lattice. One can observe the drastic transition phenomena of the global connectivity versus $\sigma$ for different transmission ranges. When $\sigma$ goes larger and passes the critical value $\sigma_c$, the mobile nodes in the ad hoc network self-organize from a disconnected state to a surely globally connected one. For our model, $\sigma_c$=0.4, 0.25, 0.14, 0.1 and 0.08, for $r = 2r_0, 3r_0, 4r_0, 5r_0$, and

$6r_0$, respectively. It is meaningful to find the critical values of the occupancy for different transmission ranges to the design of the ad hoc network. For a certain density of nodes of the ad hoc network, it can help us to choose proper transmission range to balance the maintenance of the global connection of the ad hoc network and minimum energy consumption of nodes in it.

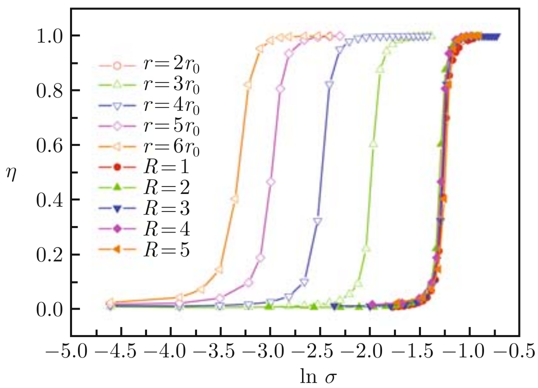


**Fig. 2** The probabilities of global connection versus the occupancy for various transmission ranges $r$.

From the direct observation on Fig. 2, we rescale $\eta(\sigma, r)$ into a universal scaling function:

$$\eta \sim f(R^{\beta}\sigma) \tag{1}$$

where $R = (r - r_0)/r_0$, and $\beta$ is the scaling exponent. As shown in Fig. 2, the curves with $r = 3r_0, 4r_0, 5r_0$ and $6r_0$ collapse into the one with $r = 2r_0$ when we choose the scaling exponent $\beta = -0.61$. It is well known that percolation in various 2-D lattices have exact analytical solutions [27]. The standard exponent for square lattice is $\beta = -0.59$. With a slight deviation from it, our model demonstrates a novel type of percolation–geometric distance-dependent percolation in a dynamic complex network based on the background of 2-D square lattice. Actually, the node at the center of its transmission range links to any other node inside it directly, irrelevant to the distance between them, which is different from traditional percolation in a 2-D square lattice [27]; while it is also different from percolation in a static small world network as investigated by Moore and Newman [29] since it is $r$-dependent and on the square lattice. Here we have not derived out the exponent analytically due to time-varying topology formed by moving nodes. A parallel scheme to model mobile ad hoc network with 2-D triangular lattice has appeared on another journal [32], with its scaling exponent $\beta = -0.49$ as the comparison.

The self-organized ad hoc network has a dynamically stationary structure when $\sigma$ exceeds $\sigma_c$. The structural properties of it can be characterized by the distribution of the degree and the clustering coefficient. Degree of a node in the network is defined as the total number of its topological links connecting others. The distribution of the degree $p(k)$ is the probability that a randomly selected node has $k$ edges. We draw the degree distributions of the ad hoc networks in Fig. 3 when the occupancy of sites gets to the threshold. They all display Poisson distribution which is one of the characteristics of random networks or small world networks. As shown in Fig. 3, one can find that there is a cut-off degree $k_c$ which means that the probability for any node in the ad hoc network with degree $k > k_c$ is very small. For instances, $k_c = 11, 15, 16, 17$ and 18 for $r = 2r_0, 3r_0, 4r_0, 5r_0$, and $6r_0$, respectively. Because the appearance of large degree is nearly impossible, the ad hoc network can not be a scale-free network from our model, which is different from previous results [26].

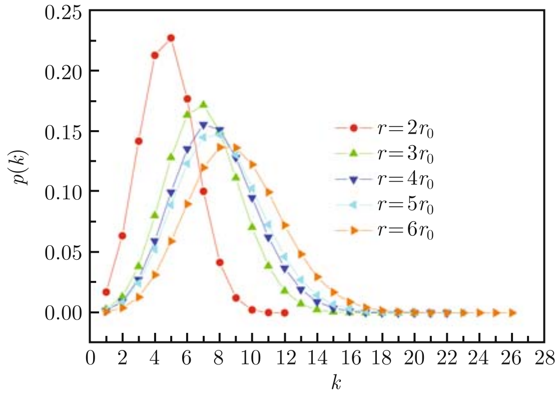


**Fig. 3** The degree distributions $p(k)$ of ad hoc networks for different transmission ranges $r$.

Clustering coefficient of a node $i$, $C_i$ in the network is defined as the averaged fraction of pairs of node i's neighbors that are also neighboring with each other in the topological sense. Taking node 1 in Fig. 1 as an example, we assume that transmission range of all the nodes in the network is $2r_0$. For convenience, first we assume that all the sites are occupied by the nodes. From Fig. 1, we can see that node 1 has 12 neighbors. The most possible number of closed topological triangles as the denominator of the clustering coefficient is $C_{12}^2$. The number of nodes with the distance $r \leqslant 2r_0$ to both nodes 1 and 2 should be 6, so there is 6 topological triangles with both nodes 1 and 2 as two vertices in the topological sense. When one of the vertices of the topological triangle is node 1, and the other is node 3 or 4, it can make 6 or 3 triangles, respectively. Due to the symmetry of the square lattice, we can get that the actual number of the triangles should be $(6 + 6 + 3) \times 4/2 = 30$. Therefore, when the sites of the square lattice is fully occupied by the nodes, the clustering coefficient is $30/C_{12}^2$=0.454. Under the assumption of homogeneous mixing, this result is also valid for any occupancy, because we only need to multiply both numerator and denominator by the occupancy $\sigma$ simultaneously. It is true for the whole network. We also get the averaged clustering coefficient $C = 0.54, 0.56, 0.57$ and $0.57$ for $r = 3r_0, 4r_0, 5r_0$, and $6r_0$, respectively. $C(k)$ of the network is defined as $C(k) = \sum_{k_i \in V} C_i / \sum_{k_i \in V} 1$, where $V$ is the subset of the nodes with the same value of degree $k$. We calculate $C(k)$ at various transmission ranges at corresponding critical threshold $\sigma_c$, and the results are shown in Fig. 4. From the simulation results in it, we can conclude that the clustering coefficient of the network is independent of the degree when $k \leqslant k_c$. And $C(k) = 0.45, 0.54, 0.56, 0.57$ and $0.57$, for $r = 2r_0, 3r_0, 4r_0, 5r_0$, and $6r_0$, respectively. As an example, the averaged clustering coefficient is verified by the value of 0.454. The simulation results fit well into the analytical ones. The appearance of tails (Fig. 4) in the range of $k > k_c$ is due to very small probabilistic occurrence in the simulation on 300 realizations. With large clustering coefficients, we can conclude that such mobile ad hoc networks are dynamical small world networks [33], and they have large redundance in communication if only the strategy of occupancy-increasing is taken into account.

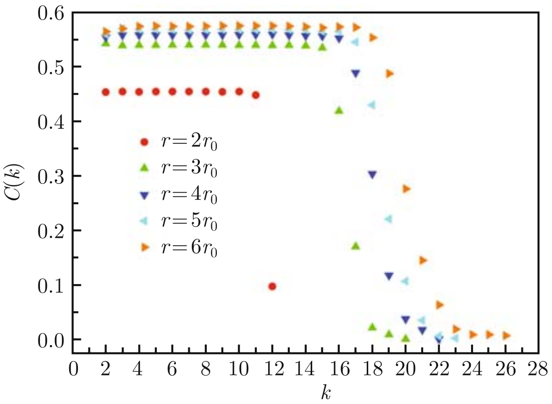


**Fig. 4** The clustering coefficient $C(k)$ for different transmission ranges $r$ in the 2-D square lattice.

## 4 Discussion and conclusion

Our study on connectivity and the topological properties of the mobile ad hoc communication network is based on observables of the system, such as the occupancy of sites and the transmission ranges. And the critical phenomena of global connectivity reveal the property of percolation on complex networks which is different from traditional problems. However, a square lattice applied to modeling the communication network with circular transmission range of nodes may be not a perfect one, although it is the simplest one. A better model with triangular lattice which resembles the circle of transmission range naturally appeared [32]. Besides, in the process of communication, the transmission ranges of nodes should decrease as messages are sent out for themselves and others. Actually, the energy consumption of every node is different. Therefore, the model with random and changeable transmission ranges is also an interesting one to be investigated.

To conclude, in this paper, we have presented a two-dimensional square lattice as the background for mobile ad hoc communication networks with uniform transmission range and the homogenous distribution of nodes on it. We found the critical phenomenon of global connectivity of the networks and scaling behavior of them by numerical simulations. The model presents a kind of percolation on the dynamic complex network relative to geometric distance. Moreover, we revealed the Poisson distribution of degree, cut-off degree, invariant clustering coefficient as the function of degree for different transmission ranges at the critical occupancy. These properties indicate that mobile ad hoc communication network is a kind of dynamical small-world network instead of a scale-free one.

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