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RESEARCH ARTICLE

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Reliability-based robust design optimization of vehicle components, Part II: Case studies

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Abstract The reliability-based optimization, the reliability-based sensitivity analysis and robust design method are employed to propose an effective approach for reliability-based robust design optimization of vehicle components in Part I. Applications of the method are further discussed for reliability-based robust optimization of vehicle components in this paper. Examples of axles, torsion bar, coil and composite springs are illustrated for numerical investigations. Results have shown the proposed method is an efficient method for reliability-based robust design optimization of vehicle components.

Keywords vehicle axles and springs, reliability-based design optimization, reliability-based sensitivity analysis, multi-objective optimization, robust design

1 Introduction

Reliability of vehicle component is a significant research field in engineering design. In order to achieve the desired level of reliability, an effective method for reliability-based robust design optimization has proposed in Part I [1]. Uncertainty is one of inherent characteristics of vehicle components. A probabilistic method is desired by both of engineering and academic communities to account for the uncertainty.

Tough and hard materials are usually employed to resist stress, wear and fatigue of structural component. Medium carbon alloys that contain elements such as nickel, chromium and molybdenum are a proper choice for such a circumstance. Material selected for springs must be capable of withstanding high level of stress and fatigue. High-carbon steel in early design stage of vehicle component has moved forward to low-percentage of carbon alloys such as silicon manganese.

This paper presents reliability-based robust design optimization of vehicle components. An effective approach for reliability-based robust design optimization of vehicle components, including of semi-axle, fore-axle, rear-axle housing, torsion bar, coil spring and leaf spring, is proposed. The method is easily to be integrated with commercial software for reliability-based robust design optimization of complex vehicle components.

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2 Reliability-based robust design optimization of semi-axle

Axle shafts are divided into three categories according to the external load (Fig. 1), i.e., fully floating, semi-floating, and three-quarter floating. Fully floating shaft is generally fitted on commercial vehicles where torque and axle loads are primary loads. The construction of fully floating consists of an independently mounted hub that rotates on two bearings widely spaced on the axle housing. This arrangement relieves all loads except torsion. The strength of the shaft is strong. Studs connecting the shaft to the hub transmit the drive. When the nuts on these studs are removed, the shaft may be withdrawn without jacking up the vehicle. The semi-floating shaft is suitable for light cars. A single bearing at the hub end is fitted between the shaft and the housing, so the shaft will have to resist in all stresses previously mentioned. To reduce the risk of fracture at the hub end which would allow the wheel to fall off, the shaft diameter is increased. However, any increase must be gradual, since a sudden change in cross-sectional areas would produce a stress-raiser and increase the risk of failure due to fatigue. Three-quarter floating shaft is defined by the fully floating and the semi-floating shaft between which any alternative may be regarded as a construction which has a single bearing mounted between hub and housing. The main shear stress on the shaft is relieved, but all other stresses still have to be resisted.

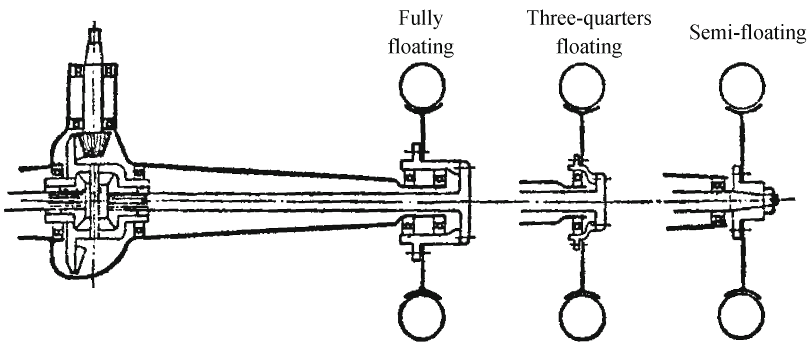

Fig. 1 Semi-axle structure

Shear and bending stresses of a semi-axle are separately defined as

$$\tau = \frac{16T}{\pi d^3}, \tag{1}$$

$$\sigma = \frac{32M}{\pi d^3}, \tag{2}$$

where T is torque, M is bending moment, d is diameter of the semi-axle section.

The state function for shearing fail of the semi-axle is defined as|

$$g(\boldsymbol{X}) = r - \tau, \tag{3}$$

where r is material strength, $\boldsymbol{X}$ is the vector of random variables.

While considering both of torque and bending moment, the total stress is represented as

$$s = \sqrt{\sigma^2 + 3\tau^2} = \frac{16}{\pi d^3}\sqrt{4M^2 + 3T^2}, \tag{4}$$

as well as the state equation:

$$g(\boldsymbol{X}) = r - s. \tag{5}$$

The random vector $\boldsymbol{X}$ is defined as $\boldsymbol{X}=(r, T, d)^{\mathrm{T}}$ or $\boldsymbol{X}=(r, T, M, d)^{\mathrm{T}}$, respectively.

1) Probabilistic characteristics of random variables are assumed as (μ_T,σ_T)=(11760, 980) N·m, (μ_r,σ_r)=(1050, 40) MPa, (μ_d,σ_d)=(42, 0.21) mm, in which $\mu_{(\ )}$ and $\sigma_{(\ )}$ represent mean-value and standard deviation, respectively.

Reliability index β, reliability R and the reliability-based sensitivity $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}$ are computed as

$$\beta = 3.047335,\ R = 0.998846,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}} = \frac{\partial R}{\partial \overline{d}} = 2.797 \times 10^{-3}.$$

Objective functions for robust optimization are

$$f_1(\boldsymbol{x}) = \frac{\pi}{4}x_1^2, \tag{6}$$

$$f_2(\boldsymbol{x}) = \left|\frac{\partial R}{\partial x_1}\right|, \tag{7}$$

where the design variables are $\boldsymbol{x} = x_1 = d$.

Assume target reliability is $R_0 = 0.999$. One has the following reliability constraint:

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0. \tag{8}$$

With initial value $d = 45$ mm, the optimum is determined as $d = 43.1242$ mm, as well as the reliability index and the reliability-based sensitivity coefficient:

$$\beta = 4.051936,\ R = 0.999974,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}} = \frac{\partial R}{\partial \overline{d}} = 7.539 \times 10^{-5}.$$

2) Torque T is (μ_T,σ_T)=(113500, 9200) N·mm, and the bending moment M is(μ_M,σ_M)=(14300, 1300) N·mm. The diameter d of the risk section (μ_d,σ_d)=(11.7, 0.0585) mm. The material strength r is (μ_r,σ_r)=(820, 32) MPa.

The reliability index β, the reliability R and the reliability sensitivity $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}$ of the semi-axle are computed as

$$\beta = 3.125381,\ R = 0.999112,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}} = \frac{\partial R}{\partial \overline{d}} = 8.116 \times 10^{-3}.$$

Define the objective functions:

$$f_1(\boldsymbol{x}) = \frac{\pi}{4}x_1^2, \tag{9}$$

$$f_2(\boldsymbol{x}) = \left|\frac{\partial R}{\partial x_1}\right|, \tag{10}$$

as well as the reliability constraint:

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0, \tag{11}$$

where R_0 is the target reliability $R_0 = 0.999$.

With initial value of $d = 15$ mm, one determines the optimum of design variable as $d = 12.1031$ mm. Reliability index β and the reliability-based sensitivity $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}$ are further verified as

$$\beta = 4.442173,\ R = 0.999995,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}} = \frac{\partial R}{\partial \overline{d}} = 5.217 \times 10^{-5}.$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ are, the more robust the reliability of the semi-axle is.

3 Reliability-based robust design optimization of fore-axle

An I section fore-axle in Fig. 2 is considered in this example for reliability-based robust design optimization of vehicle component.

The sectional coefficient of the axle is represented as

$$W_x = \frac{a(h-2t)^3}{6h} + \frac{b}{6h}[h^3 - (h-2t)^3], \tag{12}$$

$$W_\rho = 0.8bt^2 + \frac{0.4(h-2t)a^3}{t}. \tag{13}$$

Therefore, the maximal normal and shearing stresses are defined as

$$s = \frac{M}{W_x}, \tag{14}$$

$$\tau = \frac{T}{W_\rho}, \tag{15}$$

where M and T are torsion moment and bending moment, respectively. The complex stress of the fore-axle is represented as

$$\sigma = \sqrt{s^2 + 3\tau^2}, \tag{16}$$

which determines state function of the fore-axle as

$$g(\boldsymbol{X}) = r - \sigma. \tag{17}$$

Random variables are $\boldsymbol{X} = (r, M, T, a, t, h, b)^{\mathrm{T}}$, in which mean-value and standard deviation of random variables are assumed as (μ_T, σ_T)=(3026710, 245160) N·mm, (μ_M, σ_M)=(3517220, 319715) N·mm, (μ_a, σ_a)=(6.3164, 0.031582) mm, (μ_t, σ_t)=(13.7916, 0.068958) mm, (μ_h, σ_h)=(81.4675, 0.4073375) mm, (μ_b, σ_b)=(64.6425, 0.3232125) mm, and (μ_r, σ_r)=(667, 25.3) MPa.

Reliability index β and the reliability-based sensitivity index are computed as

$$\beta = 3.09033,\ R = 0.9990004,$$

$$\left[\frac{\partial R}{\partial \overline{a}}\ \frac{\partial R}{\partial \overline{t}}\ \frac{\partial R}{\partial \overline{h}}\ \frac{\partial R}{\partial \overline{b}}\right] = \begin{bmatrix} 6.469 \times 10^{-4} \\ 4.780 \times 10^{-3} \\ 3.576 \times 10^{-5} \\ 5.321 \times 10^{-4} \end{bmatrix}^{\mathrm{T}}.$$

Define the objective functions:

$$f_1(\boldsymbol{x}) = x_1(x_3 - 2x_2) + 2x_4x_2, \tag{18}$$

$$f_2(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{4}\left(\frac{\partial R}{\partial x_i}\right)^2}, \tag{19}$$

and the reliability-based constraint:

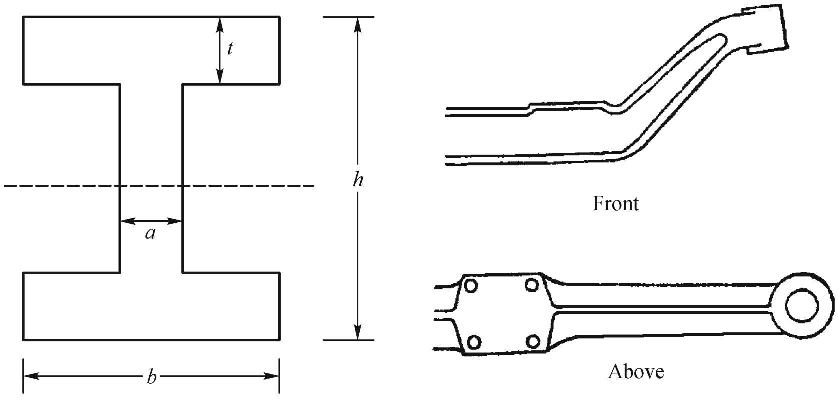

Fig. 2 Fore-axle structure

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0, \tag{20}$$

$$x_2 - x_1 \geqslant 2, \tag{21}$$

where $\boldsymbol{x} = (x_1, x_2, x_3, x_4)^{\mathrm{T}} = (a, t, h, b)^{\mathrm{T}}$ are design variables, and $R_0 = 0.999$.

With initial values $a = 12$ mm, $t = 14$ mm, $h = 85$ mm and $b = 65$ mm, optimum solution of design variables is determined as

$$a = 11.1434 \text{ mm},\ t = 13.1437 \text{ mm},\ h = 83.6931 \text{ mm},$$

$$b = 63.1607 \text{ mm}.$$

One can further verify the reliability and the reliability-based sensitivity index of example as

$$\beta = 4.214802,\ R = 0.999987,$$

$$\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}} = \begin{bmatrix} \frac{\partial R}{\partial \bar{a}} & \frac{\partial R}{\partial \bar{t}} & \frac{\partial R}{\partial \bar{h}} & \frac{\partial R}{\partial \bar{b}} \end{bmatrix} = \begin{bmatrix} 3.311 \times 10^{-5} \\ 5.422 \times 10^{-5} \\ 2.331 \times 10^{-6} \\ 7.173 \times 10^{-6} \end{bmatrix}^{\mathrm{T}}.$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}}$ are, the more robust the reliability of the fore-axle is.

4 Reliability-based robust design optimization of rear-axle

The example considers reliability-based robust design optimization of a rear horsing axle as shown in Fig. 3. A tubular section of the rear-axle is considered. Shear stress and bending stress of the component are defined as

$$s = \frac{32DM}{\pi(D^4 - d^4)}, \tag{22}$$

$$\tau = \frac{16DT}{\pi(D^4 - d^4)}, \tag{23}$$

where T and M are external moments, while D and d are outside and inside diameters of the housing rear-axle.

Stress of the structure is represented as

$$\sigma = \sqrt{s^2 + 3\tau^2} = \frac{32D}{\pi(D^4 - d^4)}\sqrt{M^2 + 0.75T^2}. \tag{24}$$

In case of the quadrate section containing a hole, stress of the housing rear-axle is defined as

$$\sigma = \frac{M}{W_n}, \tag{25}$$

where W_n is the sectional coefficient:

$$W_n = \frac{bh^2}{6}\left(1 - 0.59\frac{d^4}{bh^3}\right), \tag{26}$$

where d is the inside diameter, b and h are side lengths of the quadrate section.

State equation for reliability analysis of the housing rear-axle is defined as

$$g(\boldsymbol{X}) = r - \sigma. \tag{27}$$

1) Circular section

Mean-value and standard deviation of $\boldsymbol{X}$ are assumed as $(\mu_T, \sigma_T) = (4472475, 362270)$ N·mm, $(\mu_M, \sigma_M) = (6432658, 584729)$ N·mm, $(\mu_d, \sigma_d) = (90.97, 0.45485)$ mm, $(\mu_D, \sigma_D) = (98.22, 0.4911)$ mm and $(\mu_r, \sigma_r) = (443, 27.5)$ MPa.

Reliability index and reliability-based sensitivity coefficients are determined as

$$\beta = 3.094426,\ R = 0.999014,$$

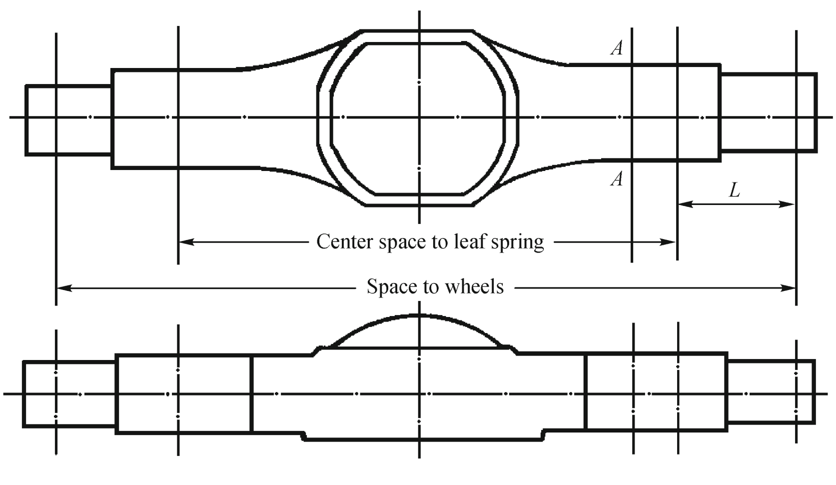

Fig. 3 Structure of rear-axle housing

$$DR/D\bar{\boldsymbol{x}}^{\mathrm{T}} = \left[\frac{\partial R}{\partial \overline{D}}\ \frac{\partial R}{\partial \overline{d}}\right] = \begin{bmatrix} 3.293 \times 10^{-3} \\ -2.802 \times 10^{-3} \end{bmatrix}^{\mathrm{T}}.$$

Define the objective function for reliability-based robust design optimization:

$$f_1(\boldsymbol{x}) = \frac{\pi}{4}(x_1^2 - x_2^2), \tag{28}$$

$$f_2(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{2}\left(\frac{\partial R}{\partial x_i}\right)^2}, \tag{29}$$

and constraint functions:

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0, \tag{30}$$

$$0 \leqslant x_1 \leqslant 100, \tag{31}$$

$$0 \leqslant x_2 \leqslant 100, \tag{32}$$

$$x_1 - x_2 \geqslant 0, \tag{33}$$

where the target reliability R_0= 0.999.

Given the initial values of d = 85 mm and D = 95 mm, one can harvest optimum results as

$$d = 92.8014 \text{ mm}, D = 99.9996 \text{ mm},$$

as well as reliability of the structure:

$$\beta = 3.338399, R = 0.999579,$$

$$\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}} = \left[\frac{\partial R}{\partial \overline{D}}\ \frac{\partial R}{\partial \overline{d}}\right] = \begin{bmatrix} 1.478 \times 10^{-3} \\ -1.263 \times 10^{-3} \end{bmatrix}^{\mathrm{T}}.$$

2) Quadrate section

Probability information of the component is assumed as (μ_M,σ_M)=(106742122.26, 9702858.912) N·mm, (μ_d,σ_d) =(102.92, 0.5146) mm, (μ_b,σ_b)=(122.92, 0.6146) mm, (μ_r,σ_r)=(433, 27.5) MPa and (μ_h,σ_h)=(143.33, 0.71665) mm, respectively. Reliability of the housing rear-axle with quadrate section can be determined as

$$\beta = 3.341242, R = 0.999583,$$

$$\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}} = \left[\frac{\partial R}{\partial \bar{b}}\ \frac{\partial R}{\partial \bar{h}}\ \frac{\partial R}{\partial \bar{d}}\right] = \begin{bmatrix} 1.170 \times 10^{-4} \\ 2.190 \times 10^{-4} \\ -1.022 \times 10^{-4} \end{bmatrix}^{\mathrm{T}}.$$

Objective functions for reliability-based robust design optimization are defined as

$$f_1(\boldsymbol{x}) = x_1 x_2 - \frac{\pi}{4}x_3^2, \tag{34}$$

$$f_2(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{3}\left(\frac{\partial R}{\partial x_i}\right)^2}, \tag{35}$$

where the design variables are $\boldsymbol{x} = (x_1, x_2, x_3)^{\mathrm{T}} = (b, h, d)^{\mathrm{T}}$.

Optimization constraints are

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0, \tag{36}$$

$$x_1 - x_3 \geqslant 10, \tag{37}$$

$$x_2 - x_3 \geqslant 10, \tag{38}$$

$$x_2 - x_1 \geqslant 0. \tag{39}$$

With initial values b = 160 mm, h = 164 mm and d = 120 mm, one determines the design result as b =123.5748 mm, h = 152.4063 mm, d = 113.5747 mm, and the reliability-based sensitivity coefficients of the component:

$$\beta = 4.060122, R = 0.999975,$$

$$\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}} = \left[\frac{\partial R}{\partial \bar{b}}\ \frac{\partial R}{\partial \bar{h}}\ \frac{\partial R}{\partial \bar{d}}\right] = \begin{bmatrix} 8.242 \times 10^{-6} \\ 1.486 \times 10^{-5} \\ -8.049 \times 10^{-6} \end{bmatrix}^{\mathrm{T}}.$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}}$ are, the more robust the reliability of the rear-axle housing is.

5 Reliability-based robust design optimization of torsion bar

The example considers reliability-based robust design optimization of a bar subjected to torque moment (Fig. 4).

Shear stress of the bar is defined as

$$\tau = \frac{16DT}{\pi(D^4 - d^4)}, \tag{40}$$

where T is external moment, D is outside diameter and d is inside diameter of the tubular section (for circular section d = 0).

Limit state function is defined as

$$g(\boldsymbol{X}) = r - \frac{16DT}{\pi(D^4 - d^4)}, \tag{41}$$

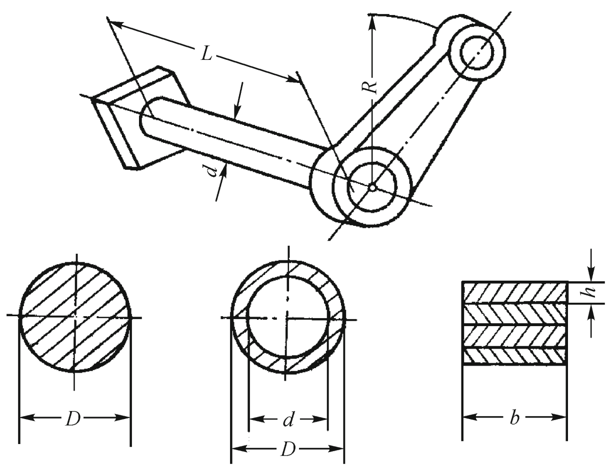

Fig. 4 Torsion bar structure

where r is material strength. Random variable $\boldsymbol{X}$ is defined as $\boldsymbol{X}=(r, T, D, d)^{\mathrm{T}}$ where mean-value and standard deviation are (μ_T,σ_T)=(677400, 8891.28) N·mm, (μ_r,σ_r) =(686.9, 35.8) MPa, (μ_d,σ_d)=(27.87, 0.13935) mm and (μ_D,σ_D)=(30, 0.15) mm, respectively.

The reliability index β, the reliability R and the reliability sensitivity $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ of the torsion bar, therefore, are computed as

$$\beta = 3.132776,\ R = 0.999134,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}} = \begin{bmatrix} \dfrac{\partial R}{\partial \overline{D}} & \dfrac{\partial R}{\partial \overline{d}} \end{bmatrix} = \begin{bmatrix} 1.216 \times 10^{-2} \\ -1.042 \times 10^{-2} \end{bmatrix}^{\mathrm{T}}.$$

Define the objective functions:

$$f_1(\boldsymbol{x}) = \frac{\pi}{4}(x_1^2 - x_2^2), \tag{42}$$

$$f_2(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{2}\left(\frac{\partial R}{\partial x_i}\right)^2}, \tag{43}$$

and the constraint functions:

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0, \tag{44}$$

$$0 \leqslant x_1 \leqslant 30, \tag{45}$$

$$0 \leqslant x_2 \leqslant 30, \tag{46}$$

$$x_1 - x_2 \geqslant 0. \tag{47}$$

With initial values $d=20$ mm and $D=30$ mm, one harvests the optimum design variables as

$$d = 16.8542\ \text{mm},\ D = 21.8537\ \text{mm},$$

as well as the reliability and the reliability-based sensitivity coefficients:

$$\beta = 4.477526,\ R = 0.999996,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}} = \begin{bmatrix} \dfrac{\partial R}{\partial \overline{D}} & \dfrac{\partial R}{\partial \overline{d}} \end{bmatrix} = \begin{bmatrix} 5.483 \times 10^{-5} \\ -3.000 \times 10^{-5} \end{bmatrix}^{\mathrm{T}}.$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ are, the more robust the reliability of the torsion bar is.

6 Reliability-based robust design optimization of coil spring

Helical spring (Fig. 5) is normally used in conjunction with independent suspensions. The maximum stress of a coil spring occurs at the inner wall of the spring, namely,

$$\tau = \frac{(1 + d/2D)dGy}{\pi D^2 n}, \tag{48}$$

where d is the diameter, D is the median diameter, G is the elastic modulus in shear, n is the number of active coils, y is deformation of the spring.

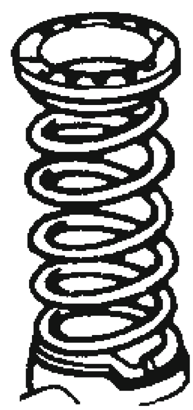

Fig. 5 Coil spring structure

Limit state function of the coil spring is defined as

$$g(\boldsymbol{X}) = r - \tau, \tag{49}$$

where r is material strength. Random variables are $\boldsymbol{X} = (r, d, D, G, n, y)^{\mathrm{T}}$, where mean-value and standard deviation of $\boldsymbol{X}$ are listed in Table 1.

Reliability index β, reliability R and reliability sensitivity $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ are computed as

$$\beta = 3.344368,\ R = 0.999588,$$

Table 1 Random variable for robust design optimization of a coil spring

r/MPa		d/mm		D/mm		G/MPa		n		y/mm	
μ_r	σ_r	μ_d	σ_d	μ_D	σ_D	μ_G	σ_G	μ_n	σ_n	μ_y	σ_y
1714.02	83.202	14	0.07	90	0.45	79250	1585	7	0.0833	208	4.16

$$\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}} = \left[\frac{\partial R}{\partial \bar{d}}\ \frac{\partial R}{\partial \bar{D}}\ \frac{\partial R}{\partial \bar{n}}\right] = \begin{bmatrix} -1.673\times10^{-3} \\ 5.031\times10^{-4} \\ 3.121\times10^{-3} \end{bmatrix}^{\mathrm{T}}.$$

Define objective functions:

$$f_1(\boldsymbol{x}) = \frac{\pi^2}{4}x_1^2x_2x_3, \tag{50}$$

$$f_2(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{3}\left(\frac{\partial R}{\partial x_i}\right)^2}, \tag{51}$$

where the design variables are $\boldsymbol{x}=(x_1, x_2, x_3)^{\mathrm{T}}=(d, D, n)^{\mathrm{T}}$.

Constraint functions of the problem are

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0, \tag{52}$$

$$0 \leqslant x_1 \leqslant 25, \tag{53}$$

$$100 \leqslant x_2 \leqslant 150, \tag{54}$$

$$4 \leqslant \frac{x_2}{x_1} \leqslant 10, \tag{55}$$

$$4 \leqslant x_3 \leqslant 15, \tag{56}$$

$$H_0 - \delta_{\max} \geqslant H_b, \tag{57}$$

where, R_0= 0.999, H_0 is free height of the spring, $H_0= nt + 1.5d$, $\delta_{\max} = \frac{8F_{\max}D^3n}{Gd^4}$. Solid height of the coil spring $H_\mathrm{b} \approx (n+1.5)d$.

$$b = \frac{H_0}{D} = \frac{nt+1.5d}{D} = 0.5x_3 + 1.5\left(\frac{x_1}{x_2}\right) \leqslant b_c, \tag{58}$$

where b_c is critical proportion of free height and median diameter. If double ends are fixed, b_c= 5.3.

$$f \leqslant 0.5 f_r, \tag{59}$$

where f is natural frequency of coil spring. If double ends are fixed, $f = \frac{d}{2\pi D^2 n}\sqrt{\frac{Gg}{2\gamma}}$. f_r is excitation frequency, which is assume as f_r= 127.8 Hz. γ is the density of wire material, $\gamma = 7.4872\times10^{-5}$ N/mm^3.

With initial values, d = 13.5 mm, D = 109.5 mm and n = 6.5, the optimized solutions of d, D and n are

$$d= 10.823 \text{ mm}, D= 108.232 \text{ mm}, n= 5.241.$$

And the reliability, the reliability index β, and the reliability-based sensitivity are determined as

$$\beta= 7.696796, R= 1.000000,$$

$$\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}} = \left[\frac{\partial R}{\partial \bar{d}}\ \frac{\partial R}{\partial \bar{D}}\ \frac{\partial R}{\partial \bar{n}}\right] = \begin{bmatrix} -5.947\times10^{-14} \\ 1.162\times10^{-14} \\ 1.228\times10^{-14} \end{bmatrix}^{\mathrm{T}}.$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}}$ are, the more robust the reliability of the coil spring is.

7 Reliability-based robust design optimization of composite springs

The example considers reliability-based robust design optimization of a composite spring as depicted in Fig. 6.

Stress of the composite springs is computed as

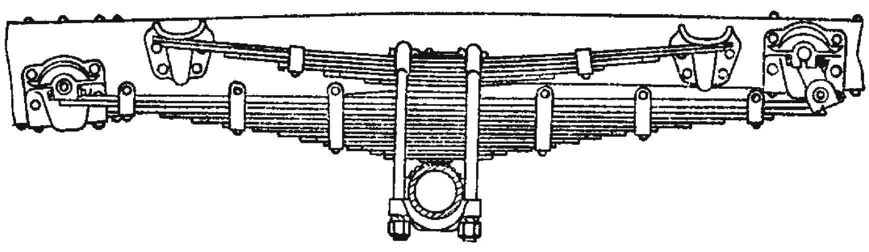

Fig. 6 Structure of composite springs

$$\sigma = \frac{3Pl}{2b} \cdot \frac{h_i}{n_1 h_1^3 + n_2 h_2^3 + \ldots + n_m h_m^3}, \tag{60}$$

which determines the maximal stress as

$$\sigma_{\max} = \frac{3Pl}{2b} \cdot \frac{h_{\max}}{n_1 h_1^3 + n_2 h_2^3 + \ldots + n_m h_m^3}, \tag{61}$$

in which, P is external load, h_i is the height of an ith panel, and n_i is the number of panels of with thickness h_i.

Limit state equation for reliability analysis of the composite spring is defined as

$$g(\boldsymbol{X}) = r - \sigma_{\max} , \tag{62}$$

where r is material strength. Random vector is defined as $\boldsymbol{X}= (r, P, l, b, h_{\max})^{\mathrm{T}}$, where mean-value and standard deviation of $\boldsymbol{X}$ are (μ_b,σ_b)=(89, 0.445) mm, (μ_l,σ_l)=(1475, 7.375) mm, (μ_{h_1},σ_{h_1})=(10, 0.05) mm, (μ_{h_2},σ_{h_2})=(9.3, 0.0465) mm, (μ_{h_3},σ_{h3})=(8.3, 0.0415) mm, (μ_P,σ_P) =(16503.2, 825.16) N and (μ_r,σ_r)=(614, 45.8) MPa. The numbers of panels are n_1= 2, n_2= 6 and n_3= 4.

Define the objection function as

$$f_1(\boldsymbol{x}) = x_1(n_1 x_2 + n_2 x_3 + n_3 x_4), \tag{63}$$

$$f_2(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{4} \left(\frac{\partial R}{\partial x_i}\right)^2}, \tag{64}$$

as well as the constraint functions:

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0, \tag{65}$$

$$x_2 - x_3 \geqslant 1.0, \tag{66}$$

$$x_3 - x_4 \geqslant 1.0, \tag{67}$$

$$x_1 - 80 \geqslant 0, \tag{68}$$

where design variables are $\boldsymbol{x}=(x_1, x_2, x_3, x_4)^{\mathrm{T}}=(b, h_1, h_2, h_3)^{\mathrm{T}}$, and target reliability R_0= 0.9999.

With initial values b = 90 mm, h_1= 11 mm, h_2= 10 mm and h_3= 9 mm, the optimum solution for b, h_1, h_2, h_3 are

$$b = 80.0009 \text{ mm}, h_1 = 11.8283 \text{ mm},$$

$$h_2 = 10.8277 \text{ mm}, h_3 = 8.4690 \text{ mm},$$

and the reliability R, the reliability-based sensitivity $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ of the composite springs:

$$\beta = 4.15909, R = 0.9999839,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}} = \left[\frac{\partial R}{\partial \overline{b}} \ \frac{\partial R}{\partial \overline{h}_1} \ \frac{\partial R}{\partial \overline{h}_2} \ \frac{\partial R}{\partial \overline{h}_3}\right] = \begin{bmatrix} 7.006 \times 10^{-6} \\ 9.477 \times 10^{-5} \\ 8.856 \times 10^{-5} \\ 3.612 \times 10^{-5} \end{bmatrix}^{\mathrm{T}}.$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ are, the more robust the reliability of the composite springs is.

8 Conclusions

An effective method for reliability-based robust design optimization of vehicle components has been proposed in Ref. [1]. Application of the method is further illustrated by several examples for robust design optimization of vehicle components. Numerical results have further confirmed the high efficiency and accuracy of the proposed method.

Nomenclature

τ	Torsional stress
T	Torsional moment
d	Diameter of circular section, inside diameter
σ	Stress
M	Bending moment
$\boldsymbol{X}=(X_1, X_2, \ldots, X_n)^{\mathrm{T}}$	Vector of original random parameters
$g(\boldsymbol{X})$	State function
r	Material strength
s	Complex stress
$E(\ )$	Mean value
$Var(\ )$	Variance
β	Reliability index
$\mu_{(\ )}$	Mean value of ()
$\sigma_{(\ )}$	Standard deviation of ()
R	Reliability
$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$	Reliability-based sensitivity index
b	Width
t	Pitch
h	Thickness of a connecting rod
$W_{(\ )}$	Sectional coefficient
D	Outside diameter
n	Active number of a coil spring
G	Elastic modulus
y	Deformation
H_0	Free height of a coil spring

δ_{max}	Maximal deformation of a spring
F_{max}	Maximal load
b_c	Critical proportion of free height, the medium diameter
f	Natural frequency of coil spring
f_r	Excitation frequency
h_i	Thickness of plate
n_i	The numbers of panels with thickness h_i
H_b	Solid height of a coil spring
P	External load of composite spring
γ	Density of steel
l	Span of composite spring

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References

1. Zhang Y. Reliability-based robust design optimization of vehicle components, Part I: Theory. Frontiers of Mechanical Engineering, 2015, 10(2): 138–144