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RESEARCH ARTICLE

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Reliability-based robust design optimization of vehicle components, Part I: Theory

Abstract The reliability-based design optimization, the reliability sensitivity analysis and robust design method are employed to present a practical and effective approach for reliability-based robust design optimization of vehicle components. A procedure for reliability-based robust design optimization of vehicle components is proposed. Application of the method is illustrated by reliability-based robust design optimization of axle and spring. Numerical results have shown that the proposed method can be trusted to perform reliability-based robust design optimization of vehicle components.

Keywords vehicle components, reliability-based design optimization, reliability-based sensitivity analysis, multi-objective optimization, robust design

1 Introduction

The reliability and safe of vehicle components are main objectives of vehicle design. In case key components are damaged, the vehicle will break down, or even causes fatal accident. It is important to perform the reliability-based robust optimization design of vehicle components. Reliability of vehicle components can be verified by laboratory experiment. The experiment, however, is of limited applicable due to the demanding manpower, resource and time. Therefore, a numerical method is necessary to be introduced. During the last four decades, there have been considerable advances in the field. The reliability-based design, the reliability-based optimization design [1–22], and the reliability sensitivity analysis [23–30], the robust design and the reliability-based robust design have been widely employed for industrial product design with the uncertain information [31–42].

In engineering practice, the aforementioned methods are effective for vehicle component design under uncertainties, which include geometry parameters, material properties, loadings, etc. However, the method has been employed separately during different design stages. A robust design is, in general, considered to be one that the quality of a product is insensitive to variations of design variables. Based on the concept, the reliability-based robust optimization is defined as that reliability of the system should be insensitive to the variation of uncertain parameters, i.e., the reliability sensitivities with respect to design variables are low (or ideally zero). The minimum sensitivity formulation generally leads to a design that is more robust and hence of better quality. The Taguchi method has been successfully employed in the last few decades to develop robust products that will perform their intended functions with low sensitivity to variations of design variables. Current design practice tends to account for the variation by using of factors of safety and reliability. It is essential to combine the reliability-based optimization, the reliability sensitivity and the robust design and to develop a new design approach for reliability-based robust optimization of vehicle components.

Using the reliability-based optimization design, the reliability sensitivity analysis technique and the robust design method, this paper proposes a novel numerical approach for reliability-based robust design optimization of vehicle components. An effective method for the reliability-based robust optimization of vehicle components is developed. According to the numerical results, the proposed approach is a convenient and practical routine for reliability-based roust design optimization of vehicle components.

2 Perturbation method

A fundamental problem for reliability analysis is to

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compute the following multi-fold integral in terms of reliability R:

$$R = \int_{g(\boldsymbol{X})>0} f_{\boldsymbol{X}}(\boldsymbol{X})\mathrm{d}\boldsymbol{X}, \tag{1}$$

in which $f_{\boldsymbol{X}}(\boldsymbol{X})$ denotes joint probability density function of random variables $\boldsymbol{X}=(X_1, X_2, \ldots, X_n)^{\mathrm{T}}$, and $g(\boldsymbol{X})$ is the state function, representing safe state and failure state of the problem:

$$\left.\begin{array}{ll} g(\boldsymbol{X}) \leqslant 0, & \text{failure state} \\ g(\boldsymbol{X}) > 0, & \text{safe state} \end{array}\right\}, \tag{2}$$

where $g(\boldsymbol{X})= 0$ is the limit-state equation, representing an n-dimensional surface, which is termed as the "limit-state surface" or "failure surface".

The vector of random parameters $\boldsymbol{X}$ and the state function $g(\boldsymbol{X})$ are expanded as

$$\boldsymbol{X} = \boldsymbol{X}_d + \varepsilon \boldsymbol{X}_p, \tag{3}$$

$$g(\boldsymbol{X}) = g_d(\boldsymbol{X}) + \varepsilon g_p(\boldsymbol{X}), \tag{4}$$

where ε is a small parameter. A term with subscript d is the certain part, while the subscript p denotes the random part with zero mean value. Obviously, it is necessary to notice that the value of the random part should to be smaller than the value of deterministic part. Both sides of Eqs. (3) and (4) are evaluated about the mean value of random variables as follows:

$$E(\boldsymbol{X}) = E(\boldsymbol{X}_d) + \varepsilon E(\boldsymbol{X}_p) = \boldsymbol{X}_d, \tag{5}$$

$$E[g(\boldsymbol{X})] = E[g_d(\boldsymbol{X})] + \varepsilon E[g_p(\boldsymbol{X})] = g_d(\boldsymbol{X}). \tag{6}$$

Similarly, according to the Kronecker algebra [43], variances of $\boldsymbol{X}$ and $g(\boldsymbol{X})$ are respectively determined as

$$Var(\boldsymbol{X}) = E\{[\boldsymbol{X} - E(\boldsymbol{X})]^{[2]}\} = \varepsilon^2 [\boldsymbol{X}_p^{[2]}], \tag{7}$$

$$\begin{aligned} Var[g(\boldsymbol{X})] &= E\{[g(\boldsymbol{X}) - E[g(\boldsymbol{X})]]^{[2]}\} \\ &= \varepsilon^2 E\{[g_p(\boldsymbol{X})]^{[2]}\}, \end{aligned} \tag{8}$$

in which, the Kronecker power $(\cdot)^{[2]}$ is represented as

$$\boldsymbol{A}^{[2]} = \boldsymbol{A} \otimes \boldsymbol{A} = \begin{bmatrix} a_{11}\boldsymbol{A} & a_{12}\boldsymbol{A} & \cdots & a_{1q}\boldsymbol{A} \\ a_{21}\boldsymbol{A} & a_{22}\boldsymbol{A} & \cdots & a_{2q}\boldsymbol{A} \\ \vdots & \vdots & \ddots & \vdots \\ a_{p1}\boldsymbol{A} & a_{11}\boldsymbol{A} & \cdots & a_{pq}\boldsymbol{A} \end{bmatrix}, \tag{9}$$

where $\boldsymbol{A}_{p\times q}$ is a $pp\times qq$ matrix, while $\otimes$ represents the Kronecker product.

By expanding the state function $g_p(\boldsymbol{X})$ at $E(\boldsymbol{X})=\boldsymbol{X}_d$, one obtains first-order approximation of the limit state function as

$$g_p(\boldsymbol{X}) = \frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^{\mathrm{T}}} \boldsymbol{X}_p. \tag{10}$$

Substituting Eq. (10) into Eq. (8), we obtain

$$\begin{aligned} Var[g(\boldsymbol{X})] &= \varepsilon^2 E\left[\left(\frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^{\mathrm{T}}}\right)^{[2]} \boldsymbol{X}_p^{[2]}\right] \\ &= \left(\frac{\partial g_d(\boldsymbol{X})}{\partial \boldsymbol{X}^{\mathrm{T}}}\right)^{[2]} Var(\boldsymbol{X}), \end{aligned} \tag{11}$$

in which, $Var(\boldsymbol{X})$ is computed with the variance and covariance of input random parameters.

The reliability index is defined as

$$\beta = \frac{\mu_g}{\sigma_g} = \frac{E[g(\boldsymbol{X})]}{\sqrt{Var[g(\boldsymbol{X})]}}. \tag{12}$$

It is to note that the first-order approximation of μ_g and σ_g should be evaluated at the mean values $(\mu_{X_1}, \mu_{X_2}, \ldots, \mu_{X_n})^{\mathrm{T}}$. The reliability index can be directly employed to measure safety margin of the state function. If random variables follow the normal distribution, a tangent plane may be used to approximate actual failure surface of the problem, which determines the following reliability index:

$$R = \Phi(\beta), \tag{13}$$

where $\Phi(\ )$ is the standard normal distribution function. In case of a limit state function containing non-normal random variables, the proposed perturbation method is convenient to evaluate mean-value, variance for reliability analysis of the vehicle component.

3 Reliability sensitivity design

It is important to evaluate the sensitivity of input random variables with respect to reliability of a system. The sensitivity of mean-value of a random variable is derived as follows:

$$\frac{\mathrm{D}R}{\mathrm{D}\overline{X}^{\mathrm{T}}} = \frac{\partial R}{\partial \beta} \frac{\partial \beta}{\partial \mu_g} \frac{\partial \mu_g}{\partial \overline{\boldsymbol{X}}^{\mathrm{T}}}, \tag{14}$$

where

$$\frac{\partial R}{\partial \beta} = \varphi(\beta), \tag{15}$$

$$\frac{\partial \beta}{\partial \mu_g} = \frac{1}{\sigma_g}, \tag{16}$$

$$\frac{\partial \mu_g}{\partial \overline{\boldsymbol{X}}^{\mathrm{T}}} = \left[\frac{\partial \overline{g}}{\partial X_1}, \frac{\partial \overline{g}}{\partial X_2}, \ldots, \frac{\partial \overline{g}}{\partial X_n}\right]^{\mathrm{T}}, \tag{17}$$

as well as the sensitivity of variance:

$$\frac{\mathrm{D}R}{\mathrm{D}Var(\boldsymbol{X})}=\frac{\partial R}{\partial \beta}\frac{\partial \beta}{\partial \sigma_g}\frac{\partial \sigma_g}{\partial Var(\boldsymbol{X})}, \tag{18}$$

where

$$\frac{\partial \beta}{\partial \sigma_g}=-\frac{\mu_g}{\sigma_g^2}, \tag{19}$$

$$\frac{\partial \sigma_g}{\partial Var(\boldsymbol{X})}=\frac{1}{2\sigma_g}\left[\frac{\partial \overline{g}}{\partial \boldsymbol{X}}\otimes\frac{\partial \overline{g}}{\partial \boldsymbol{X}}\right]. \tag{20}$$

Substituting for Eqs. (14) and (18), the reliability sensitivity indices $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{X}}^{\mathrm{T}}$ and $\mathrm{D}R/\mathrm{D}Var(\boldsymbol{X})$ can be obtained.

4 Reliability-based robust optimization

The reliability-based robust optimization design of vehicle components is proposed as a multi-objective optimization in terms of the minimum weight of vehicle component and the minimum mean-value sensitivity of design variables. The reliability-based robust optimization problem, hence, can be expressed as the following form:

$$\left.\begin{array}{l}\min f(\boldsymbol{x})=\sum_{k=1}^{n}w_k f_k(\overline{\boldsymbol{x}})\\ \text{s.t. } \overline{g}-\Phi^{-1}(R_0)\sigma_g\geqslant 0,\ q_i(\boldsymbol{x})\geqslant 0,\ (i=1,...,l)\end{array}\right\}, \tag{21}$$

where w_k is a kth weighting factor. It measures the significance of the kth sub-objective function:

$$\begin{aligned}w_1&=\frac{f_k(\boldsymbol{x}^{*1})-f_k(\boldsymbol{x}^{*k})}{[f_1(\boldsymbol{x}^{*k})-f_1(\boldsymbol{x}^{*1})]+[f_2(\boldsymbol{x}^{*(k-1)})-f_2(\boldsymbol{x}^{*2})]+...+[f_k(\boldsymbol{x}^{*1})-f_k(\boldsymbol{x}^{*k})]},\\ w_2&=\frac{f_{k-1}(\boldsymbol{x}^{*2})-f_{k-1}(\boldsymbol{x}^{*(k-1)})}{[f_1(\boldsymbol{x}^{*k})-f_1(\boldsymbol{x}^{*1})]+[f_2(\boldsymbol{x}^{*(k-1)})-f_2(\boldsymbol{x}^{*2})]+...+[f_k(\boldsymbol{x}^{*1})-f_k(\boldsymbol{x}^{*k})]},\\ &\vdots\qquad\qquad\vdots\\ w_k&=\frac{f_1(\boldsymbol{x}^{*k})-f_1(\boldsymbol{x}^{*1})}{[f_1(\boldsymbol{x}^{*k})-f_1(\boldsymbol{x}^{*1})]+[f_2(\boldsymbol{x}^{*(k-1)})-f_2(\boldsymbol{x}^{*2})]+...+[f_k(\boldsymbol{x}^{*1})-f_k(\boldsymbol{x}^{*k})]}.\end{aligned} \tag{22}$$

In the paper, two sub-objective functions are considered: $f_1(\overline{\boldsymbol{x}})$ is the area or volume of vehicle components and $f_2(\overline{\boldsymbol{x}})$ is the reliability sensitivity of mean-value of design variables. R_0 is the target reliability. $q_i(\boldsymbol{x})$ is an ith inequality constrain.

5 Numerical example

5.1 Reliability-based robust design optimization of tension bar

A bar subjected to axial tension is considered as shown in Fig. 1.

Tensile stress of the bar is defined as

$$\sigma=\frac{4Q}{\pi(d_1^2-d_0^2)}, \tag{23}$$

where Q is the tensile load, d_0 is inside diameter, while d_1 is outside diameter of the tubular section.

The state equation for reliability analysis of the tension bar is defined as

$$g(\boldsymbol{X})=r-\frac{4Q}{\pi(d_1^2-d_0^2)}, \tag{24}$$

where r is material strength. The random vector $\boldsymbol{X}$ is defined as $\boldsymbol{X}=(r,\ Q,\ d_1,\ d_0)^{\mathrm{T}}$, where mean-value and variance random variables (μ_{d_0},σ_{d_0})=(25, 0.125) mm, (μ_{d_1},σ_{d_1})=(35, 0.175) mm, (μ_Q,σ_Q)=(170, 2.6) kN and (μ_r,σ_r)=(400, 11) MPa. Specially, the d_0 and d_1 are correlative random variables with correlation coefficient $\rho=0.70$.

The reliability index β, the reliability R and the reliability sensitivity $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ of the tension bar, are determined as

$$\beta=2.916838,\ R=0.998232,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}=\begin{bmatrix}\frac{\partial R}{\partial \overline{d}_1} & \frac{\partial R}{\partial \overline{d}_0}\end{bmatrix}=\begin{bmatrix}1.773\times 10^{-2}\\ -1.266\times 10^{-2}\end{bmatrix}^{\mathrm{T}}.$$

Given the target reliability $R_0=0.999$, objective functions for reliability-based robust design optimization of the tensile bar are formulated as

$$f_1(\boldsymbol{x})=\frac{\pi}{4}(x_1^2-x_2^2), \tag{25}$$

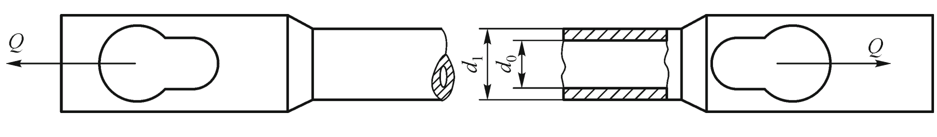

Fig. 1 Structure of tension bar

$$f_2(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{2}\left(\frac{\partial R}{\partial x_i}\right)^2}, \tag{26}$$

where the design variables are $\boldsymbol{x}=(x_1,x_2)^{\mathrm{T}}=(d_1,d_0)^{\mathrm{T}}$.

The design constraints are

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0, \tag{27}$$

$$x_1 - x_2 \geqslant 0. \tag{28}$$

Given initial values d_0= 30 mm and d_1= 40 mm, the optimum result is determined as d_0= 20.0011 mm and d_1= 31.989 mm.

The proposed method determines the reliability index β, the reliability R and the reliability sensitivity $\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}}$ as

$$\beta = 4.055630,\ R = 0.999975,$$

$$\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}} = \begin{bmatrix}\dfrac{\partial R}{\partial \bar{d}_1} & \dfrac{\partial R}{\partial \bar{d}_0}\end{bmatrix} = \begin{bmatrix} 2.934\times10^{-4} \\ -1.835\times10^{-4}\end{bmatrix}^{\mathrm{T}}.$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}}$ are, the more robust the reliability of the tension bar is.

5.2 Reliability-based robust design optimization of screw bolt

Screw bolt subjected to shearing load has shown in Fig. 2.

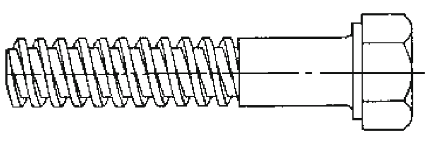

Fig. 2 Structure of screw bolt

Stress of the screw bolt is defined as

$$\sigma = \frac{4p}{N\pi d^2}, \tag{29}$$

where p is the shearing load, d is the diameter of screw bolt section, N is the number of shearing sections.

The state equation for reliability analysis of the screw bolt is defined as

$$g(\boldsymbol{X}) = r - \frac{4p}{N\pi d^2}, \tag{30}$$

where r is material strength. The random vector $\boldsymbol{X}$ is defined as $\boldsymbol{X}=(r,\ p,\ d)^{\mathrm{T}}$ where mean-value and standard deviation of the normal random variables are given as (μ_p,σ_p)=(24, 1.44) kN, (μ_d,σ_d)=(12.5, 0.062) mm, (μ_r,σ_r)=(143.3, 11.5) MPa, and N= 2.

The reliability index β, the reliability R and the reliability sensitivity $\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}$ are determined as

$$\beta = 3.515459,\ R = 0.999780,$$

$$\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}} = \frac{\partial R}{\partial \bar{d}} = 9.990\times10^{-4}.$$

If target reliability R_0= 0.999 is given, the minimum diameter d of the screw bolt can be optimized with the proposed robust design procedure.

Define the objective function in terms of minimizing sectional area and reliability-based sensitivity index, respectively:

$$f_1(\boldsymbol{x}) = \frac{\pi}{4}x_1^2, \tag{31}$$

$$f_2(\boldsymbol{x}) = \left|\frac{\partial R}{\partial x_1}\right|, \tag{32}$$

where the design variables are $\boldsymbol{x}=x_1=d$.

And the reliability constraint is

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0. \tag{33}$$

Given the initial value d= 15 mm, one can harvest the robust result as d= 13.083 mm, as well as the reliability index β, the reliability R and the reliability sensitivity $\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}$ of the screw bolt:

$$\beta = 4.248981,\ R = 0.999989,$$

$$\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}} = \frac{\partial R}{\partial \bar{d}} = 5.142\times10^{-5}.$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $\mathrm{D}R/\mathrm{D}\bar{\boldsymbol{x}}^{\mathrm{T}}$ are, the more robust the reliability of the screw bolt is.

5.3 Reliability-based robust design optimization of connecting rod

The connecting rod is a component subjected to tension and pressure. Its section falls into I bar, circularity, square, etc. (shown in Fig. 3).

Tensile stress of the connecting rod with I section is defined as

$$\sigma = \frac{F}{a(h-2t)+2bt}, \tag{34}$$

where F is the tension load, and a, b, h, and t are dimensional parameters.

The state equation is defined as

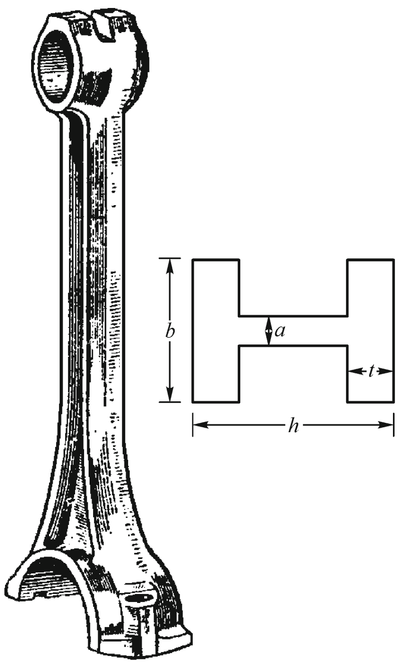

Fig. 3 Structure of connecting rod

$$g(\boldsymbol{X}) = r - \sigma, \tag{35}$$

where r is material strength. The original variable vector $\boldsymbol{X}$ is defined as $\boldsymbol{X}=(r, F, a, t, h, b)^{\mathrm{T}}$.

The probabilistic characteristics of $\boldsymbol{X}$ are assumed as $(\mu_r,\sigma_r)=(691, 38)$ MPa, $(\mu_a,\sigma_a)=(8, 0.04)$ mm, $(\mu_t,\sigma_t)=(11, 0.055)$ mm, $(\mu_h,\sigma_h)=(130.63, 0.65315)$ mm and $(\mu_b,\sigma_b)=(80.47, 0.40235)$ mm. The load F is $(\mu_F,\sigma_F)=(60000, 4000)$ N.

The reliability index β, the reliability R and the reliability sensitivity $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ of the connecting rod, therefore, are computed as

$$\beta = 3.316011,\ R = 0.999543,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}} = \left[\frac{\partial R}{\partial \overline{a}}\ \frac{\partial R}{\partial \overline{t}}\ \frac{\partial R}{\partial \overline{h}}\ \frac{\partial R}{\partial \overline{b}}\right] = \begin{bmatrix} 6.794\times 10^{-4} \\ 9.065\times 10^{-4} \\ 5.004\times 10^{-5} \\ 1.376\times 10^{-4} \end{bmatrix}^{\mathrm{T}}.$$

Assume the target reliability $R_0 = 0.999$ and the design parameters are, a, t, h, and b of the connecting rod. Objective functions are to minimize the sectional area $f_1(\boldsymbol{x})$ and the reliability-based sensitivity coefficients $f_2(\boldsymbol{x})$:

$$f_1(\boldsymbol{x}) = x_1(x_3 - 2x_2) + 2x_4x_2, \tag{36}$$

$$f_2(\boldsymbol{x}) = \sqrt{\sum_{i=1}^{4}\left(\frac{\partial R}{\partial x_i}\right)^2}, \tag{37}$$

where the design variables are $\boldsymbol{x}=(x_1, x_2, x_3, x_4)^{\mathrm{T}}=(a, t, h, b)^{\mathrm{T}}$.

Design constraints are

$$\mu_g - \Phi^{-1}(R_0)\sigma_g \geqslant 0, \tag{38}$$

$$x_1 \geqslant 8.0, \tag{39}$$

$$x_2 - x_1 \geqslant 3.0, \tag{40}$$

$$x_3 - 2x_2 \geqslant 0, \tag{41}$$

$$1.4 \leqslant \frac{x_3}{x_4} \leqslant 1.8. \tag{42}$$

Given initial values $a = 14$ mm, $t = 27.5$ mm, $h = 140$ mm and $b = 96$ mm, the dimensional variables, a, t, h, b, are computed as $a = 9.2751$ mm, $t = 13.5427$ mm, $h = 129.0557$ mm and $b = 71.6976$ mm.

One can further verify the reliability index β, the reliability R and the reliability sensitivity $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ of the connecting rod as

$$\beta = 4.351443,\ R = 0.999993,$$

$$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}} = \left[\frac{\partial R}{\partial \overline{a}}\ \frac{\partial R}{\partial \overline{t}}\ \frac{\partial R}{\partial \overline{h}}\ \frac{\partial R}{\partial \overline{b}}\right] = \begin{bmatrix} 1.046\times 10^{-5} \\ 1.281\times 10^{-5} \\ 9.514\times 10^{-7} \\ 2.778\times 10^{-6} \end{bmatrix}^{\mathrm{T}}.$$

On the basis of the above results, the bigger the reliability index β and the reliability R are, the less the values of the reliability sensitivities $\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{x}}^{\mathrm{T}}$ are, the more robust the reliability of the connecting rod is.

6 Conclusions

This paper presents an effective method for reliability-based robust design optimization of vehicle components. Optimum results for reliability-based robust design optimization of vehicle components are obtained accurately and quickly. The method provides an effective approach to conduct reliability-based robust optimization of vehicle components.

Nomenclature

R	Reliability
$f_X(\boldsymbol{X})$	Probability density function
$g(\boldsymbol{X})$	State function
$\boldsymbol{X}=(X_1, X_2, \ldots, X_n)^{\mathrm{T}}$	Vector of random variables
ε	Small parameter
$E(\ )$	Mean value
$Var(\ )$	Variance
$\mu_{(\ )}$	Mean value
$\sigma_{(\ )}$	Standard variance
β	Reliability index
Φ	Standard normal distribution function
$\mathrm{D}R/\mathrm{D}\overline{\boldsymbol{X}}^{\mathrm{T}}$	Reliability sensitivity with respect to mean value of random variables
$\mathrm{D}R/\mathrm{D}Var(\boldsymbol{X})$	Reliability sensitivity with respect to standard variance of random variables
w_k	kth weighting factor
$\boldsymbol{x}=(x_1, x_2,\ldots, x_n)^{\mathrm{T}}$	Vector of design variables
$q_i(\boldsymbol{X})\geq 0$	Inequality constrain
σ	Stress
Q	External load of tension bar
d_0	Inside diameter of the tubular section
d_1	Outside diameter of the tubular section
r	Material strength
ρ	Correlation coefficient
p	External load
d	Diameter of the circular section, inside diameter
N	The number of shearing sections
F	External load
b	Width
t	Pitch
h	Thickness of connecting rod

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