Front. Mech. Eng. 2014, 9(1): 81–94
DOI 10.1007/s11465-014-0287-9

RESEARCH ARTICLE

Sunil Dhingra, Gian Bhushan, Kashyap Kumar Dubey

Multi-objective optimization of combustion, performance and emission parameters in a jatropha biodiesel engine using non-dominated sorting genetic algorithm-II

Abstract The present work studies and identifies the different variables that affect the output parameters involved in a single cylinder direct injection compression ignition (CI) engine using jatropha biodiesel. Response surface methodology based on Central composite design (CCD) is used to design the experiments. Mathematical models are developed for combustion parameters (Brake specific fuel consumption (BSFC) and peak cylinder pressure (P_{max})), performance parameter brake thermal efficiency (BTE) and emission parameters (CO, NO_x, unburnt HC and smoke) using regression techniques. These regression equations are further utilized for simultaneous optimization of combustion (BSFC, P_{max}), performance (BTE) and emission (CO, NO_x, HC, smoke) parameters. As the objective is to maximize BTE and minimize BSFC, P_{max}, CO, NO_x, HC, smoke, a multi-objective optimization problem is formulated. Non-dominated sorting genetic algorithm-II is used in predicting the Pareto optimal sets of solution. Experiments are performed at suitable optimal solutions for predicting the combustion, performance and emission parameters to check the adequacy of the proposed model. The Pareto optimal sets of solution can be used as guidelines for the end users to select optimal combination of engine output and emission parameters depending upon their own requirements.

Keywords jatropha biodiesel, fuel properties, response surface methodology, multi-objective optimization, non-dominated sorting genetic algorithm-II

Received December 22, 2013; accepted December 31, 2013

Sunil Dhingra (✉)
Department of Mechanical Engineering, University Institute of Engineering and Technology, Kurukshetra University, Kurukshetra
E-mail: pecdhingra@gmail.com

Gian Bhushan
Department of Mechanical Engineering, National Institute of Technology, Kurukshetra

Kashyap Kumar Dubey
Department of Biotechnology, University Institute of Engineering and Technology, Maharshi Dayanand University, Rohtak

1 Introduction

The predicted shortage of petroleum and its products have increased the search for the substitute of petroleum derivatives. Energy is the most important input for the growth of every sector including industrial sector, transport services, agriculture etc. Around the world, the demand for energy is increasing exponentially, specifically based on petroleum. Fossil fuels are non-renewable source of energy and will be depleted one day due to its limited supply. The International Energy Agency (2007) suggested that the global energy demand will grow by more than 50% from the current consumption by 2030 with China and India alone making up 45% of the anticipating demand [1]. The search for petroleum derivative results an alternative fuel known as “Biodiesel”. It derives from animal fats, vegetable oils and using waste cooking oil including triglycerides. The serious global concern is on the availability of energy in future and climate change due to global warming. Greenhouse gases are the main cause of global warming which are produced or emitted by the combustion of petroleum and its products. Recent data confirmed that consumption of fossil fuels accounts for the majority of global anthropogenic Green House Gases emissions. Emissions continue to grow and CO_2 concentrations had increased to over 390 ppm (parts per million) or 39% above preindustrial levels by the end of 2010 [2]. India has maintained a high growth rate during the last decade resulting in increase in energy demand and consumption. Use of renewable and alternate sources of energy like bio-fuels especially based on non-edible oils is being promoted to overcome energy demand.

Jatropha Curcas plants are found in almost every part of developing countries like India, Brazil, Germany etc. Oil extracted from the seeds of these plants is used for the production of its biodiesel. Methods like trans-esterification, pyrolysis, ultrasonic cavitation etc [3–6]. have been used for the production of biodiesel. Among these trans-esterification is a significant method for biodiesel production. It is the process of reacting oil (Edible/Non-edible) with alcohol (Methanol/Ethanol) in the presence of catalyst (KOH/NaOH). Fatty acid alkyl ester (Biodiesel) and glycerol along with other by-products are produced. The production of biodiesels from various edible and non-edible oils is being enhanced by using different optimization techniques like taguchi, response surface methodology (RSM), artificial neural network (ANN), genetic algorithm (GA) etc. RSM and GA have been applied successfully for predicting optimum reaction conditions for maximum production of Jatropha and Karanja biodiesel by Dhingra et al. [3,4]. One of the reasons for using biodiesel is the emissions reduction. Bojan et al. [5] predicted the optimum value of biodiesel yield at a particular combination of reaction variables using RSM.

The performance tests of biodiesels from edible and non-edible oils were conducted by various researchers [6–20] for predicting the variation of BSFC, BTE, emission parameters (CO, NO_x, HC, smoke etc.) with load, speed and injection timing. The multi-objective optimization techniques like NSGA (Non-dominated sorting genetic algorithm), MOGA (Multi-objective genetic algorithm), Desirability approach etc.) have been applied for predicting the optimal sets of solution [21–23]. Deb et al. [24] proposed the modified version of NSGA in multi-objective technique (NSGA-II) by evaluating the best optimal solutions. Sharing parameter was then eliminated by the modification of NSGA II as developed by Agrawal et al. [25]. NSGA-II helps in predicting the different sets of optimal solution for the process parameters involved [26].

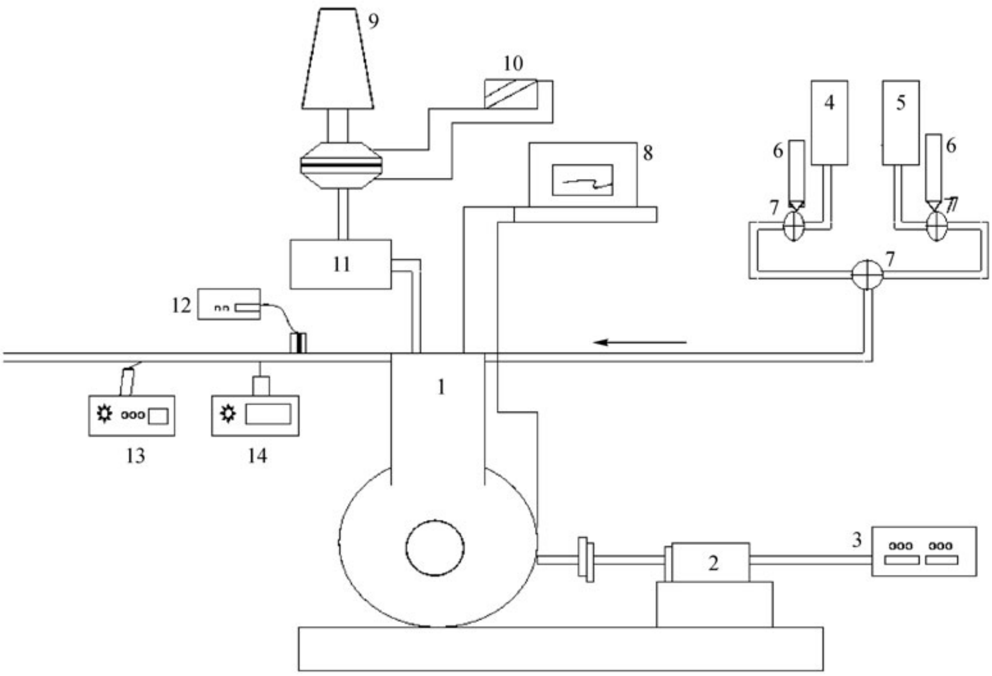
It is found that no work has been carried out for the evaluation of performance parameters in CI engine by using the NSGA-II approach. Earlier the authors have used RSM and GA techniques for optimizing the process parameters for the production of biodiesel from Jatropha and Karanja oils. This research paper focus on production of Jatropha biodiesel, study of combustion (BSFC, P_{max}), performance (BTE) and emission (CO, NO_x, HC and Smoke) parameters behavior in CI engine with variation of load, compression ratio and blending ratio using RSM. Regression equations were predicted from analysis of variance (ANOVA) for further use in NSGA-II developed computer program in MATLAB ver. 7.11.0.584 (R2010b) to predict optimal conditions of Jatropha biodiesel in a single cylinder direct injection diesel engine.

2 Experimental set-up design and methodology

The jatropha ethyl ester is produced from trans-esterification process by Authors already predicted optimum conditions by RSM approach [3]. The different fuel properties of *jatropha curcas*, its biodiesel blends, diesel fuel are shown in Table 1 along with American standard for testing and materials (ASTM) standards of biodiesel. It has been found that most of the important properties of jatropha and its blends are closer to the standard values. A diesel engine setup has been designed for predicting the behavior of different combustion, performance and emis-

Table 1 Fuel properties of Jatropha oil, its biodiesel blends, diesel fuel and standard values of biodiesel

Fuel property	Jatropha Oil	Diesel	B11.25	B15.81	B22.5	B29.18	B33.75	B100	Biodiesel standards ASTM D6751-02	Biodiesel standards EN 14214
Density at 15°C /($kg\cdot m^{-3}$)	865	866	867	868	869	871	872	873	–	860–900
Viscosity at 40°C /($mm^2\cdot s^{-1}$)	54	3.4	3.6	3.8	4	4.3	4.35	4.23	1.9 – 6.0	3.5 – 5.0
Calorific value /($kJ\cdot kg^{-1}$)	42275	43500	42305	42330	42450	42500	42525	42673	–	–
Acid value / (mg $KOH\cdot g^{-1}$)	2.5	0.07	0.08	0.17	0.19	0.21	0.225	0.25	<0.8	<0.50
Flash point/°C	220	71	95	125	146	156	153	148	<130	>120
Pour point/°C	3.1	1	–3	0	1	2	2.5	4.2	–	–
Water content/%	0.07	0.06	0.04	0.02	0.02	0.03	0.025	0.02	<0.03	<0.05
Ash content/%	0.75	0.01	0.012	0.014	0.014	0.014	0.013	0.015	<0.02	<0.02
Carbon residue	3.54	0.17	0.19	0.20	0.21	0.20	0.19	0.15	–	<0.3
Sulphur content/%	0.03	0.025	0.02	0.015	0.01	0.05	0.02	NIL	15 ppm max	–



1-Diesel engine; 2-Alternator indicator; 3-Loading device; 4-Biodiesel tank; 5-Diesel tank; 6-Fuel Metering system; 7-Control valve; 8-Data acquisition system; 9-Air filter; 10-Inclined manometer; 11-Surge tank; 12-Temperature; 13-Gas analyzer; 14-Smoke meter

Fig. 1 Schematic diagram of the engine setup

Table 2 Engine Specification

Component	Specification
Make type	Kirloskar
Engine type	Single Cylinder 4-Stroke, Water Cooled
Compression ratio	Variable ranging from 12 to 18
Rated power	3.5 kW@1500 R.P.M
Stroke	110 mm
Bore	87.5 mm
Connecting rod length	234 mm
Loading device	Eddy current dynamometer
Load indicator	Digital, Range 0–50 Kg, Supply 230V AC
Load sensor	Load cell, type strain gauge, range 0–50 Kg
Speed indicator	Digital with non contact type speed sensor
Temperature sensor	Thermocouple, Type K
Rotameter	Engine cooling 40-400 LPH; Calorimeter
Injection pressure	220 bar
Injection timing	23° bTDC
Fuel spray angle	120°

sion parameters using jatropha biodiesel as fuel. The specification of the setup is as shown in Table 2. The schematic diagram of the testing engine with their different components is as shown in Fig. 1. The variables affecting the engine parameters are found from various research articles [9–17]. Three significant variables blending ratio, load torque and compression ratio with their limits to perform the experiments are analyzed as shown in Table 3. Provisions of all the factors (Three) variation were provided in the engine setup at a constant injection timing, fuel spray angle and speed. Experiments were designed on the basis of input variables in the engine and corresponding response was measured at a particular combination of variables obtained from design expert version 6.0 stat ease inc. USA as shown in Table 3. Following are the steps for the measurement of different responses:

(1) The fuel consumption rate and brake thermal efficiency is measured by the data acquisition system using software (Engine soft) installed in it.

(2) Cylinder pressure measured using AVL GM12D miniature pressure sensor.

(3) Concentration of exhaust gases (CO, NO_x and HC) are measured by AVL Di gas 4000 light analyzer.

(4) The smoke concentration is measured using flue gas analyzer.

The individual response values were then entered in the experimental design for the prediction of regression models of all the responses and its behavior with regards to input variables.

Table 3 Variable constraints in response surface methodology

	Variables	Symbol	Unit	Low	High
1	Blending ratio	X_1	% V/V	11.25	33.75
2	Load torque	X_2	N-m	7.5	12.5
5	Compression ratio	X_5	V/V	13.5	16.5

Table 4 Experimental design for combination of input variables in the engine and their responses

Exp. No.	Type	X_1	X_2	X_3	BSFC[a)]	BTE[b)]	[c)]P_{max}	Sqrt(CO)[d)]	[e)] Sqrt(NO_x)	Log_{10} (HC)[f)]	Sqrt (Smoke)[g)]
1	Factorial	15.81	8.51	14.10	0.608	13.95	54	0.196	185	14.55	13.56
2	Factorial	29.18	8.51	14.10	0.572	17.63	53	0.234	185	14.86	13.97
3	Factorial	15.81	11.48	14.10	0.418	23.97	56	0.278	213.7	43.94	34.97
4	Factorial	29.18	11.48	14.10	0.437	25.68	53	0.272	210	45.61	38.74
5	Factorial	15.81	8.51	15.89	0.684	7.638	58	0.149	170.5	15.06	14.58
6	Factorial	29.18	8.51	15.89	0.583	16.86	61	0.219	185	15.75	14.94
7	Factorial	15.81	11.48	15.89	0.339	29.98	54	0.581	252.5	46.84	40.73
8	Factorial	29.18	11.48	15.89	0.369	25.95	56	0.351	208.6	48.92	41.72
9	Axial	11.25	10	15	0.495	22.54	51	0.265	185	11.74	24.58
10	Axial	33.75	10	15	0.478	22.76	53	0.26	185	12.73	24.95
11	Axial	22.5	7.5	15	0.785	7.5	53	0.5	279.6	15	7
12	Axial	22.5	12.5	15	0.295	32.94	52	0.65	308.5	65.54	46.53
13	Axial	22.5	10	13.5	0.503	24.57	52	0.294	214.5	13.06	20.74
14	Axial	22.5	10	16.5	0.522	25.98	66	0.317	213.8	13.75	23.85
15	Center	22.5	10	15	0.9	45	90	2.1	450	750	13.56
16	Center	22.5	10	15	0.9	45	90	2.1	450	750	13.56
17	Center	22.5	10	15	0.9	45	90	2.1	450	750	13.56

a) kg/kwh; b) N-m; c) bar; d) vol. %; e) parts per million (ppm); f) parts per million (ppm); g) vol.

3 Behavior of engine parameters with input variables using RSM

RSM is a group of statistical tools for predicting optimal responses based on different input conditions. It is useful for analyzing the problems in which a response is influenced by several variables. The experimental design is developed from the levels of variables affecting the various responses. A Central composite rotatable design (CCRD) is selected for predicting the behavior of engine parameters with load, blending ratio and compression ratio. Five levels of three factors are proposed in design expert version 6.0 stat ease inc. USA along with their limits to perform the experiments as shown in Table 3. A total of 20 experiments are performed by using full factorial design. Engine parameters are measured from the standard instruments mentioned above. The predicted model response must be quadratic as represented by Eq. (1).

$$\mathrm{Y} = \beta_0 + \sum \beta_i x_i + \sum \beta_{ii} x_i^2 + \sum \beta_{ij} x_i x_j \qquad (1)$$

3.1 Significance test from analysis of variance (ANOVA) for different response terms and its precision

The various precision values of predicted models and probability value of all the terms for response models are shown in Table 5 and Table 6. The model terms are significant for p-value less than 0.05 and insignificant for value greater than 0.1. R^2, Adjusted R^2 and predicted R^2 are closer for all the response models which indicate that error between actual and predicted responses are less. Adequate precision measures the signal to noise ratio. A ratio greater than 4 is desirable. It has been shown in Table 5 that all the response models have adequate precision greater than 4. So these models can be used to navigate the design space. Moreover PRESS values of predicted models are smaller that shows the model predictions closer to the experimental values.

3.2 BSFC model

The most significant terms of BSFC model are X_2, X_1^2,

Table 5 Precision index values of different response models

Model	Precision index values			
	Adjusted-[a)]R^2	Predicted R^2	PRESS[b)]	Adeq- precision
BSFC	0.9872	0.9477	0.046	32.891
BTE	0.9802	0.9127	272.56	28.845
Sqrt(P_{max})	0.9794	0.9180	436.21	23.227
Sqrt(CO)	0.9901	0.9580	0.15	33.556
Sqrt(NO_x)	0.9789	0.9125	18.15	23.830
Log_{10}HC	0.9533	0.8144	2.02	15.519
Sqrt(Smoke)	0.9951	0.9800	0.56	69.124

a) Coefficient of determination, b) Predicted residual sum of square

Table 6 Probability values (*p*-values) of different performance and emission parameter models by Analysis of variance (ANOVA)

Model terms	P [a)] - values						
	BSFC	BTE	P_{max}	Sqrt(CO)	Sqrt(NO_x)	Log_{10} (HC)	Sqrt (Smoke)
X_1	**0.2250**	**0.1316**	**0.6334**	**0.7095**	**0.5736**	**0.8430**	0.1097
X_2	< 0.0001	< 0.0001	**0.3508**	0.0005	0.0028	0.0005	< 0.0001
X_3	**0.7621**	**0.8210**	0.0021	**0.1210**	**0.7258**	**0.8264**	0.0007
X_1^2	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
X_2^2	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
X_3^2	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001	< 0.0001
X_1X_2	0.0225	0.0138	**0.6680**	0.0335	**0.1502**	**0.9891**	**0.2631**
X_1X_3	**0.4520**	**0.9695**	**0.2145**	**0.3338**	**0.5807**	**0.9782**	**0.3388**
X_2X_3	0.0069	0.0257	**0.1363**	0.0108	**0.2225**	**0.9698**	0.0977

a) Significant for value < 0.05

X_2^2, X_3^2, X_1X_2 and X_2X_3 as indicated from Table 6. Even though X_1 and X_2 are not significant terms but their square terms are significant since probability values are < 0.05. Adequate precision of $32.89 > 4$ indicates that the model prediction values are closer to the experimental values. A lesser value of PRESS shows that error between experimental and predicted values of the BSFC model is smaller which further concludes the significance. Its regression Eq. (2) of quadratic nature shows that there must be one optimum point occurs at particular combination of input variables. The three-dimensional surface behavior of BSFC with blending ratio, load torque and compression ratio has been shown in Figs. 2(a) and 2(b), respectively. The constant contour lines in corresponding figures show the lower BSFC has been attained by decreasing the blending ratio, increasing the load torque and decreasing the compression ratio. It can also be observed that BSFC decreases when the load toque is increased at any given blending ratio. However, BSFC increases when compression ratio is increased at any given blending ratio in engine setup.

$$\begin{aligned}\mathrm{BSFC} =& -46.68493 + 0.13685^* X_1 + 1.32378^* X_2 \\ & + 5.31712^* X_3 - 3.21186^* 10^{-3} X_1^2 \\ & - 0.056480^* X_2^2 - 0.16911^* X_3^2 \\ & + 2.33817^* 10^{-3} X_1 X_2 - 1.13137^* 10^{-3} X_1 X_3 \\ & - 0.022062^* X_2 X_3 \end{aligned} \tag{2}$$

3.3 BTE model

It is seen from Table 6 that the significant model terms of BTE model are load torque, square and interaction terms of (blending ratio, load torque and compression ratio). The response surface variations of BTE model at 1500 $r \cdot min^{-1}$ and 350 CAD are shown in Figs. 3(a) and 3(b), respectively. It shows that as the load torque advances BTE increases at a given blending ratio. This may be due to the higher cetane number at a particular blending ratio which leads to better combustible mixture of air and fuel. Figure 3(b) indicates that when the compression ratio is increased BTE initially increases and later it starts to

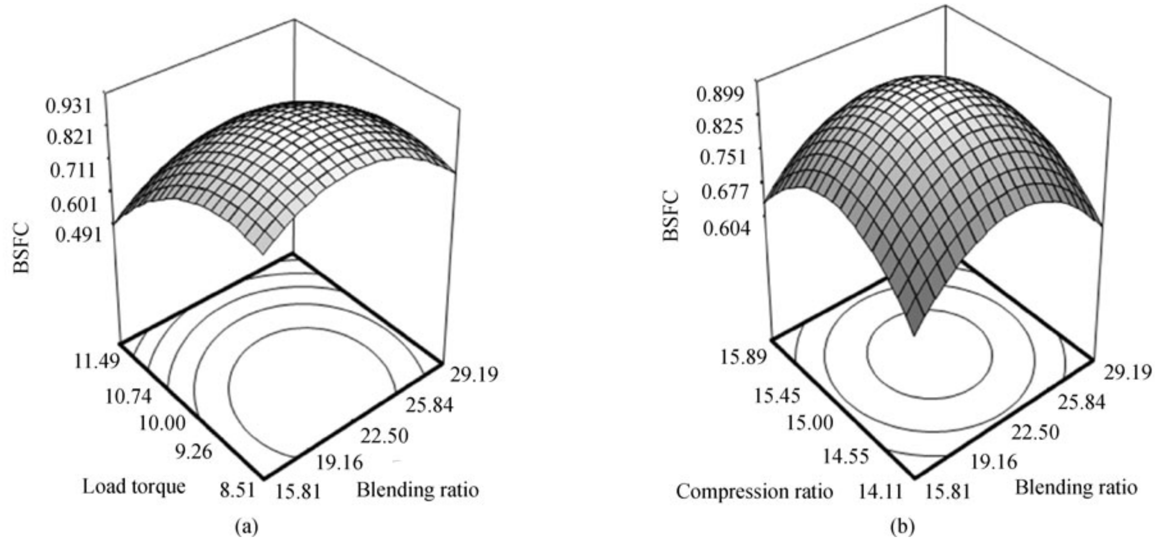


Fig. 2 Surface variation of BSFC response model across blending ratio, load torque and compression ratio

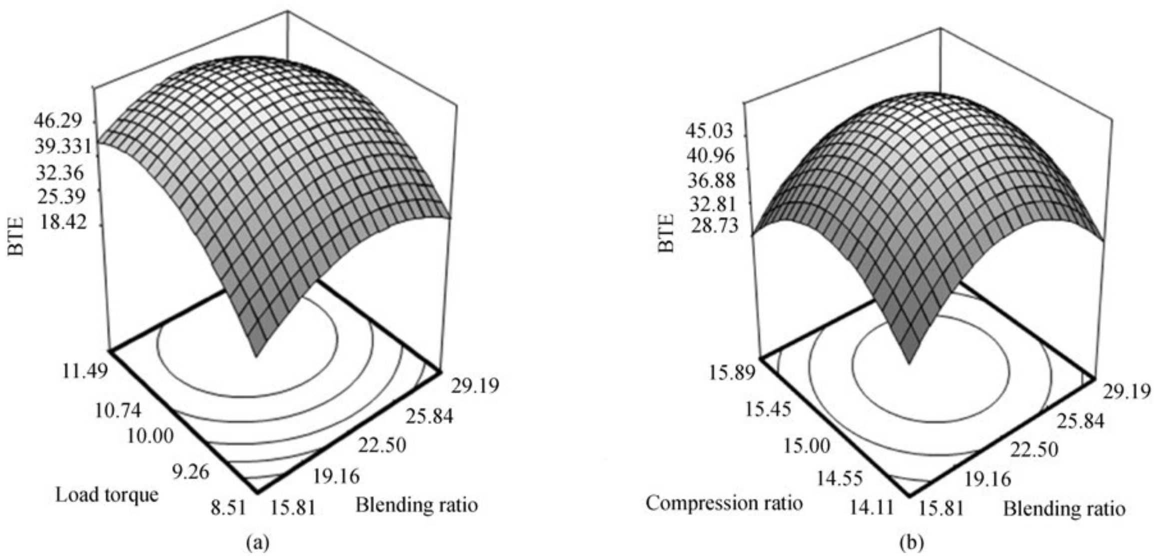


Fig. 3 Surface variation of BTE response model across blending ratio, load torque and compression ratio

decrease at a constant blending ratio. This is due to poor mixture formation at high compression ratio because of unbalance mixture of air and fuel enters into the combustion chamber. Also similar trends occur with the advancement of blending ratio at a constant compression ratio and load torque. The regression Eq. (3) of BTE model has been predicted by the use of analysis of variance (ANOVA).

$$\begin{aligned}\mathrm{BTE} = &-2400.88522 + 10.28829^{*}\mathrm{X}_1 + 71.43483^{*}\mathrm{X}_2\\ &+ 259.72937^{*}\mathrm{X}_3 - 0.18203^{*}\mathrm{X}_1^2 - 4.07476^{*}\mathrm{X}_2^2\\ &-9.07056^{*}\mathrm{X}_3^2 - 0.19143^{*}\mathrm{X}_1\mathrm{X}_2\\ &-4.19026^{*}10^{-3}\mathrm{X}_1\mathrm{X}_3 + 1.26091^{*}\mathrm{X}_2\mathrm{X}_3 \qquad (3)\end{aligned}$$

3.4 P_{max} model

The *p*-values of square of peak cylinder pressure are shown in Table 6. For simplicity take square root of P_{max} due to its very low values. It is clear from the table that compression ratio and square of individual terms are significant except load torque, blending ratio and interaction terms. Also precision index values of the P_{max} model measure adeq. Precision of 23.227 along with adj. R^2 and pred. R^2 values which are found to be closer as shown in Table 5. Hence predicted model can be used to measure the responses as well. Also the mathematical Eq. (4) shows that P_{max} is very well predictor of optimum conditions at a given variables. The surface plots in three dimensions are indicated in

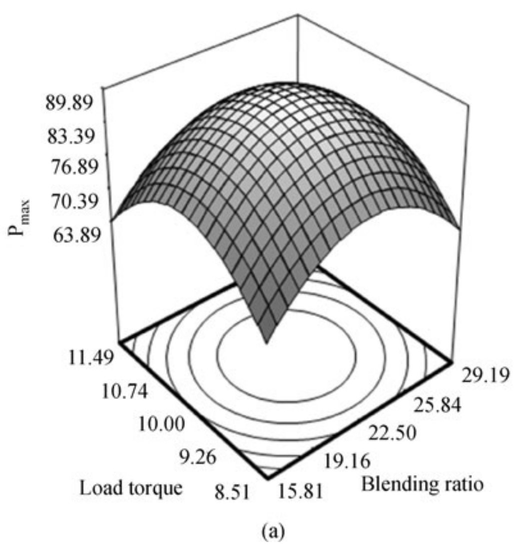


Fig. 4 Surface variation of P_{max} response model across blending ratio, load torque and compression ratio

Figs. 4(a) and 4(b). One can visualize from the variations that as the load toque is advanced the P_{max} starts increasing and maximum at a particular load toque then starts decreasing. Similar behaviors occur for the individual variable of either blending ratio or compression ratio on P_{max} model. This is because of improved combustion characteristics at higher compression ratio and load torque.

$$\begin{aligned} P_{max} = & -3669.50268 + 10.41183^{*}X_1 + 129.71550^{*}X_2 \\ & + 396.37136^{*}X_3 - 0.28479^{*}X_1^2 - 5.68691^{*}X_2^2 \\ & - 12.90809^{*}X_3^2 - 0.037712^{*}X_1X_2 + 0.18856^{*}X_1X_3 \\ & - 1.03709^{*}X_2X_3 \end{aligned} \tag{4}$$

3.5 CO emission model

The precision values for CO model are found to be significant as listed in Table 5. These values indicate that the model can be employed to validate the response. Also the probability values listed in Table 6 shows that the most significant variable to influence CO model is load torque along with the interaction terms. Equation (5) predicts the variation of responses in relation to blending ratio, load torque and compression ratio. The corresponding three dimensional surface plots are shown in Figs. 5(a) and 5(b), respectively. It can be observed from these figures that CO emission levels are increased with the advancement of both load torque and blending ratio. Similar trend occurs for the enhancement of blending ratio and compression ratio. This may be due to better combustion characteristics at higher loads and compression ratio.

$$\begin{aligned} \mathrm{Sqrt(CO)} = & -102.47076 + 0.41566^{*}X_1 + 1.86746^{*}X_2 \\ & + 11.94069^{*}X_3 - 7.55818^{*}10^{-3}X_1^2 \\ & - 0.11396^{*}X_2^2 - 0.40724^{*}X_3^2 \\ & - 3.75222^{*}10^{-3}X_1X_2 - 2.57927^{*}10^{-3}X_1X_3 \\ & + 0.035673^{*}X_2X_3 \end{aligned} \tag{5}$$

3.6 NO_x emission model

The significant terms obtained from *p*-values for nitrogen oxide model are load torque (X_2), Interaction between all the variables i.e. X_1X_2, X_1X_3 and X_2X_3 as listed in Table 6. Its surface variation at $1500\,\mathrm{r \cdot min^{-1}}$ and 350 CAD injection timing are shown in Figs. 6(a) and 6(b), respectively. It can be inferred from the figures that as the load torque advances NO emission increases at a constant blending ratio. Similarly as the compression ratio increases NO emissions increases. The reason for this is that as the load torque and compression ratio advances premixed combustion phase occurs. The response predicted values of NO model are represented in Eq. (6) in terms of blending ratio, load torque and compression ratio.

$$\begin{aligned} \mathrm{Sqrt}(NO_x) = & -765.06345 + 3.33115^{*}X_1 + 12.59961^{*}X_2 \\ & + 91.17741^{*}X_3 - 0.062945^{*}X_1^2 \\ & - 0.70774^{*}X_2^2 - 3.08084^{*}X_3^2 \\ & - 0.026616^{*}X_1X_2 - 0.016250^{*}X_1X_3 \\ & + 0.16660^{*}X_2X_3 \end{aligned} \tag{6}$$

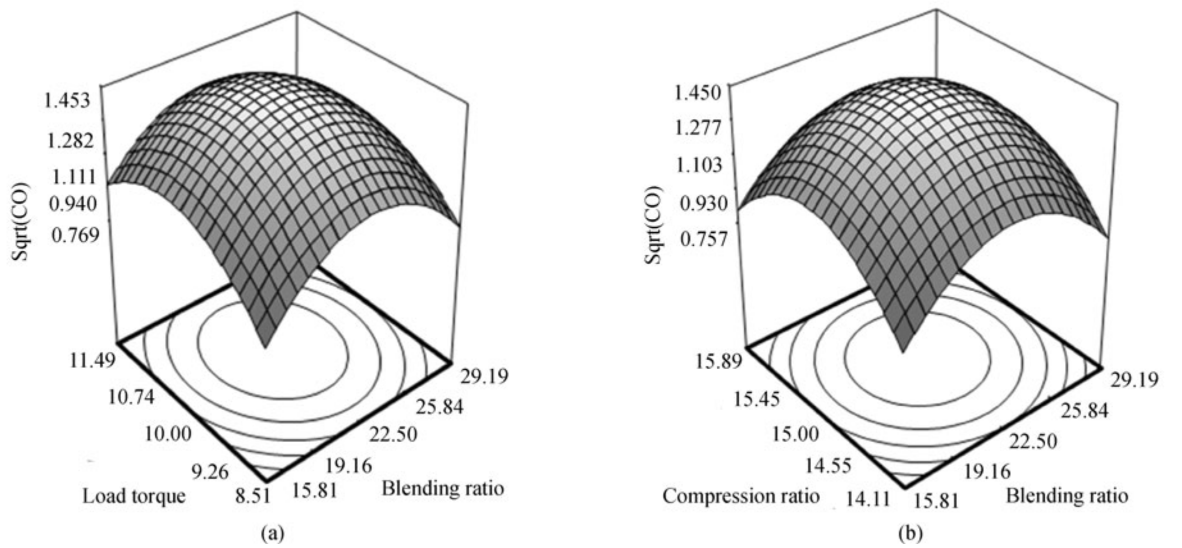


Fig. 5 Surface variation of Sqrt(CO) response model across blending ratio, load torque and compression ratio

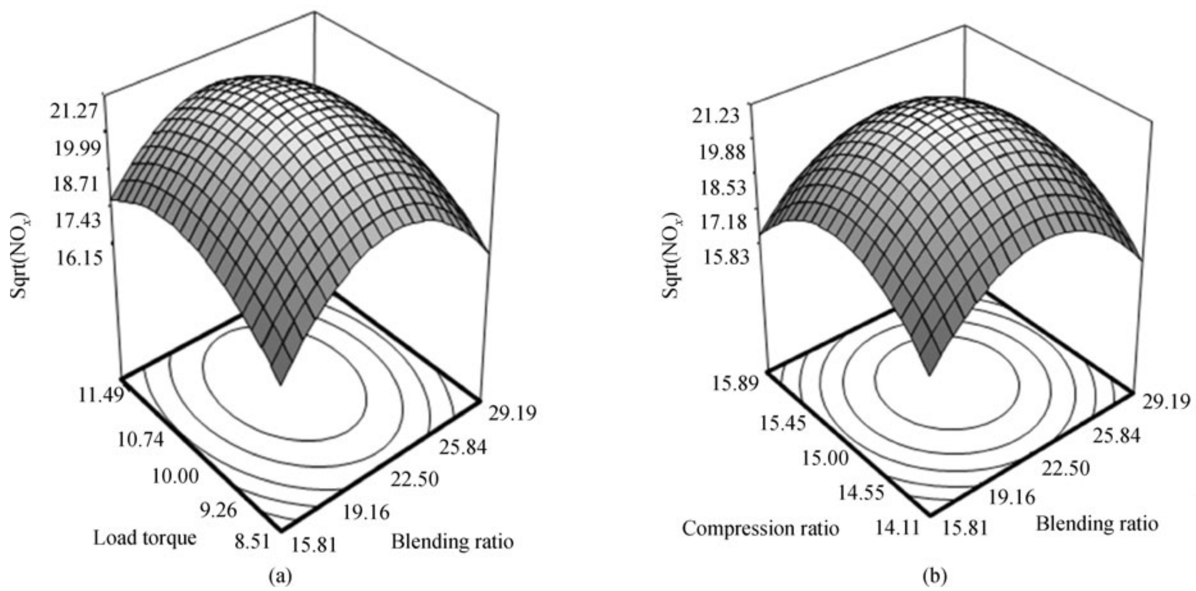


Fig. 6 Surface variation of Sqrt(NO_x) response model across blending ratio, load torque and compression ratio

3.7 Un-burnt hydrocarbon (HC) emissions model

The reasons for un-burnt hydrocarbon emissions in engine running on jatropha biodiesels are incomplete combustion in the cylinder, improper mixing of fuel with air and quenching of the oxidation process. The precision and *p*-values of HC emissions model are represented in Tables 5 and 6. It has been observed that the model is very well fitted to the actual values since all the precision parameters lie in the range of significant values. The response predictions of HC model can be represented by the Eq. (7) and total of four terms (X_2, X_1X_2, X_1X_3 and X_2X_3) are significant in producing HC emissions as indicated in Table 6. The three dimensional surface plots of HC model at 1500 $r \cdot min^{-1}$ and 350 CAD are shown in Figs. 7(a) and 7(b). As the load torque increases with the advancement of blending ratio HC emission levels are increased due to the inefficient combustion. Similarly as the compression ratio advances with the increase in blending ratio un-burnt hydrocarbon levels initially increases and at certain point it starts decreasing. This might be due to under mixing in the cylinder.

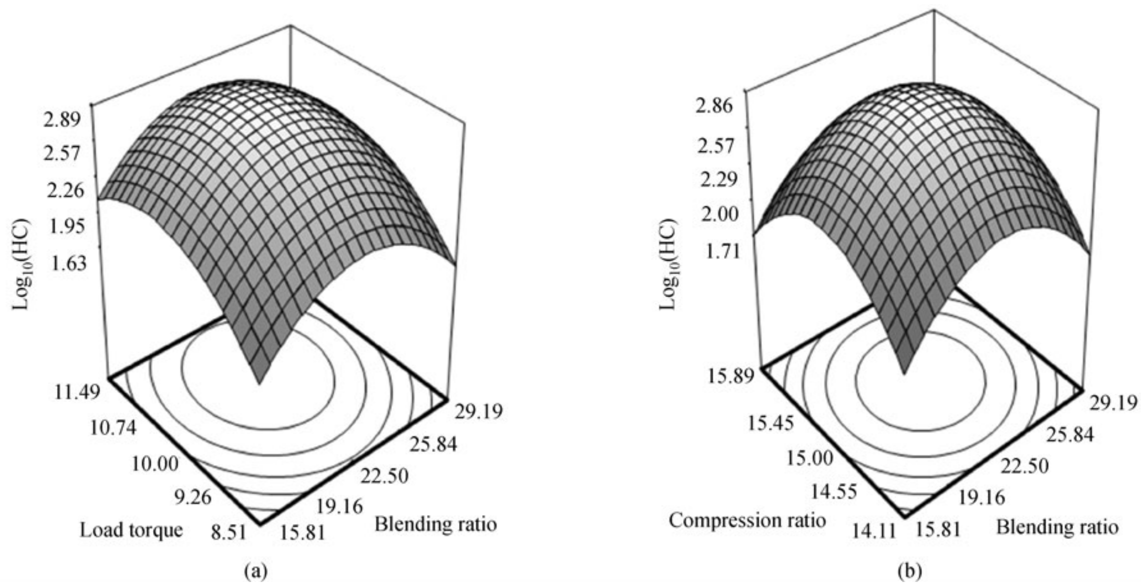


Fig. 7 Surface variation of Log_{10}(HC) response model across blending ratio, load torque and compression ratio

$$\begin{aligned}\mathrm{Log_{10}(HC)} = & -182.12527 + 0.57186^* X_1 + 3.99162^* X_2 \\ & + 21.03397^* X_3 - 0.012787^* X_1^2 \\ & - 0.19348^* X_2^2 - 0.70153^* X_3^2 \\ & + 8.11844^* 10^{-5} X_1 X_2 + 2.71724^* 10^{-4*} X_1 X_3 \\ & + 1.69324^* 10^{-3} X_2 X_3 \end{aligned} \tag{7}$$

3.8 Smoke emissions model

The ANOVA and precision results indicate that except interaction terms X_1X_2, X_1X_3 remaining terms are significant in the regression Eq. (8). This shows current model is very well predictor of responses in relation to input variables. The parameters involved in the precision index are in range of significant values (Table 5). The three dimensional surface plots of smoke model are shown in Figs. 8(a) and 8(b), respectively. One can observed from these figures that smoke density increases with the increase in load torque and blending ratio. This may be due to the lean mixture of fuel and air at higher load condition. Also smoke emissions increases with the advancement of compression ratio with corresponding increase in blending ratio at a speed of 1500 $r \cdot min^{-1}$ and 350 CAD injection timing. This is due to improper mixing of air fuel ratio at higher compression ratio.

$$\begin{aligned}\mathrm{Sqrt(Smoke)} = & 133.78384 - 0.22401^* X_1 - 3.48682^* X_2 \\ & - 15.04873^* X_3 + 0.010848^* X_1^2 \\ & + 0.18082^* X_2^2 + 0.49577^* X_3^2 \\ & + 3.59009^* 10^{-3} X_1 X_2 - 5.06832^* 10^{-3} X_1 X_3 \\ & + 0.041469^* X_2 X_3 \end{aligned} \tag{8}$$

It has been concluded from the above discussion that response surface methodology is a competent approach in predicted the behavior of multi-objective responses in relation to significant input variables. Moreover the mathematical equation developed from ANOVA result helps for further evaluating the optimum solutions using multi-objective optimization methods. So, there is need to develop the multi-objective optimization code for further selecting the solution that will fit to use in the diesel engines.

4 Multi-objective optimization

The objective is to maximize BTE and minimize BSFC, P_{max}, CO, NO_x, HC, smoke in a single cylinder direct injection CI engine using jatropha biodiesel. Therefore a multi-objective optimization problem is formulated. The simultaneous optimization of BTE, BSFC, P_{max}, CO, NO_x, HC, smoke has been carried out.

Non dominated sorting genetic algorithm (NSGA) is criticized for its computational complexity and lack of elitism. To get the equivalent solutions of various varieties, NSGA is relied on the sharing concept. There must be a requirement of sharing parameter σ_{share} specifications which is the major problem for using NSGA proposed by Raghuvanshi et al. [22] To overcome the elitism and sharing parameter chosen as priority, NSGA II is used which is better sorting algorithm proposed by Deb et al. [24] and a modified version is developed by Agarwal et al. [25] NSGA-II code is used in MATLAB 7.11.0.584 (R2010b) for selecting the proposed optimum solutions. The important steps/functions to run the computer program are [21–26].

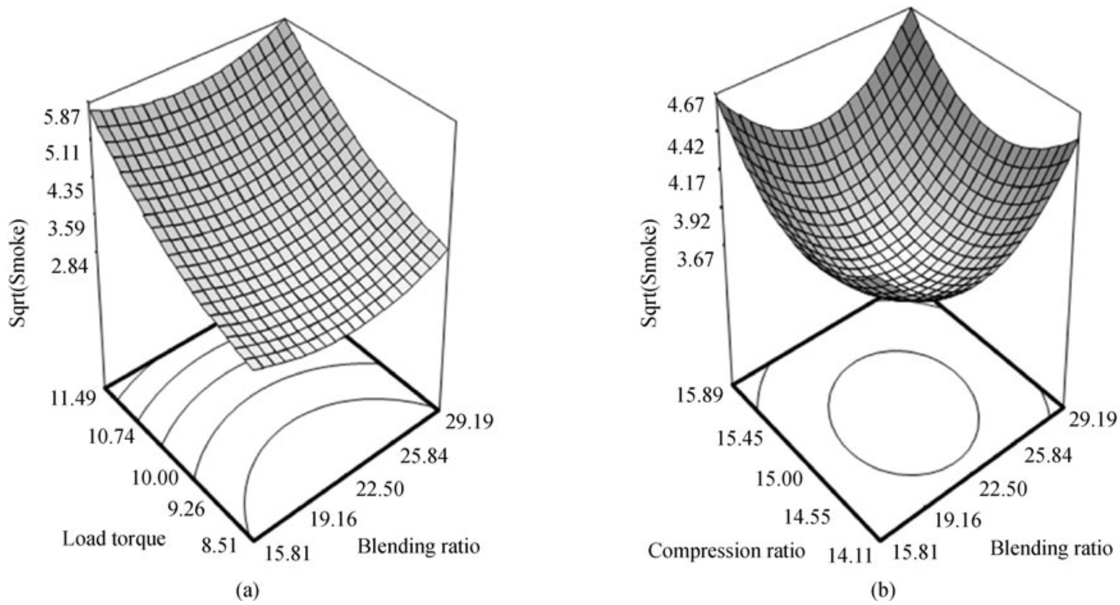


Fig. 8 Surface variation of Sqrt(Smoke) response model across blending ratio, load torque and compression ratio

Step I. Population Initialization

A random population P is created initially which depends upon the constraints and problem range, if any.

Step II. Non-dominated sorting

Based on the domination the initialized population is sorted. In a non-dominated set with respect to all the objectives, no member cannot be said to better than any other member. For every solution two entities are evaluated:

(a) Number of solutions (n_p) that dominate the solution p and

(b) Solution sets (S_p) that dominate p.

If $n_p = 0$; it means out of all solution, no solution dominates p and p lie in the first front which further given 1st rank to individual p i.e., $i_{rank} = 1$. This has been done for all individuals' population P. For every solution p with $n_p = 0$, visit each member q of its set S_p and reduced the domination count to 1. If for any member of q, n_p becomes zero, it must be put in a separate list Q. Members listed in Q denotes second non-dominated front. This is continued until identification of all the fronts has been done.

Step III. Crowding distance

It is defined as the sum of individual solution distance values with respect to each objective. Also it is the average distance of two solutions on the consecutive side of current solution corresponding to each objective function. The main requirement for crowding distance is population sorting of each objective function value in increasing order of magnitude for each front. Assign the infinite distance values for the solutions with a minimum and maximum function values for every objective function. Remaining solutions are given a normalized difference in the function values of two consecutive solutions.

Step IV. Selection

Two main criteria on the basis of which the solution is selected: Non domination rank (i_{rank}) and crowding distance ($i_{distance}$). A solution is selected if (i) $i_{rank} < j_{rank}$ or $i_{rank} = j_{rank}$ AND ($i_{distance} > j_{distance}$). Hence between two solutions of different ranks, better rank (Front) solution is selected. And if the solutions are of same rank then solution lie in less crowded distance is selected.

Step V. Genetic operators

Simulate Binary Crossover is used for reproduction. It is worked on two parent solutions and creating an offspring. It is simulated by the working principle of single point crossover on binary strings. Two properties of SBX operator are: Offspring difference is proportional to parent solutions and near parent solutions are significantly chosen as offspring than distant solutions from parents.

To keep the diversity of population, mutation is needed and a random number is generated based on small mutation probability p_m. It is evaluated by $1/n$, where n is the number of input variables ($n = 3$ for the present work). Multiply the generated number by difference of minimum and maximum limits of parents for introducing the changes in the offspring.

Step VI. Recombination and selection

Combine the offspring population with the current generation population and the selection is done to assign the individuals of next generation. Due to the inclusion of previous and current populations in the new population, elitism is ensured. Unless the population size exceeds the current population size the newly developed generation is filled by each front.

Simultaneous optimization of seven objectives are done by considering regression Eqs. (2)–(8) with input variables blending ratio (X_1), load torque (X_2) and compression ratio (X_3) along with their minimum and maximum values from Table 4. The square root response of CO, NO_x and smoke is taken because their range of variation is small while the logarithmic HC is taken due to the larger variation in range. These predicted models are utilized in NSGA-II code as fitness function. Figure 9 shows the flowchart of NSGA-II. Table 7 shows the genetic algorithm parameters used in the MATLAB program and pare to optimal front of 100 solutions are obtained after 500 generations. Out of these, selection of solutions is done on the basis of rank and crowding distance. Few optimal solutions are as shown in Table 8. It is seen from the optimal solutions that no solutions in the non-dominated sets are absolutely better than any other. Any of these solutions are better in terms of individual objective requirements.

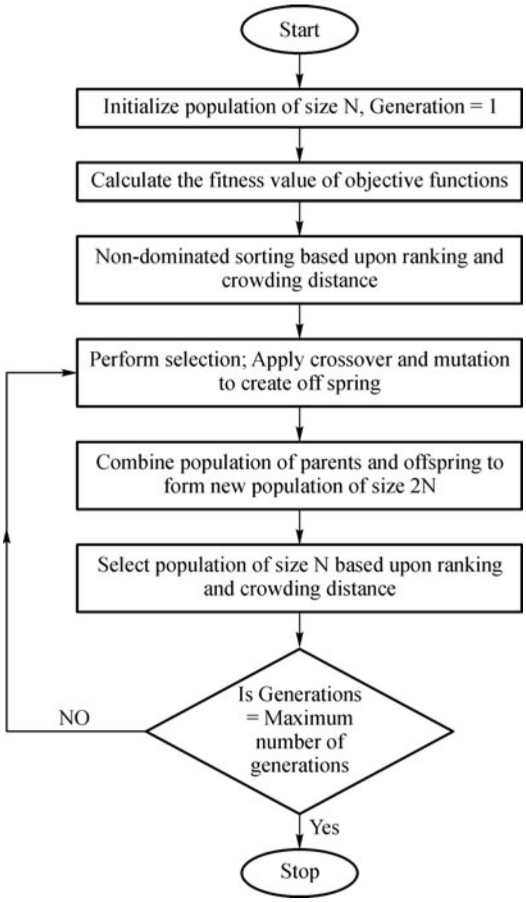


Fig. 9 NSGA-II flowchart

Biodiesel has low value of brake thermal efficiency as compared to commercial diesel when used in compression ignition engine due to high viscosity and low density. The combustion and emission parameters optimization has been predicted with primary objective is to minimize them except performance parameter (BTE) which is to be maximized. So from the 100 solutions predicted the variation of BTE with input variables (Blending ratio, load and compression ratio) has been drawn as shown in Figs. 10(a), 10(b) and 10(c). It has been found that significant changes in the values of BTE are predicted as compared to response surface methodology approach due to the accuracy in the simultaneous optimization using genetic algorithm code by varying the population and generation.

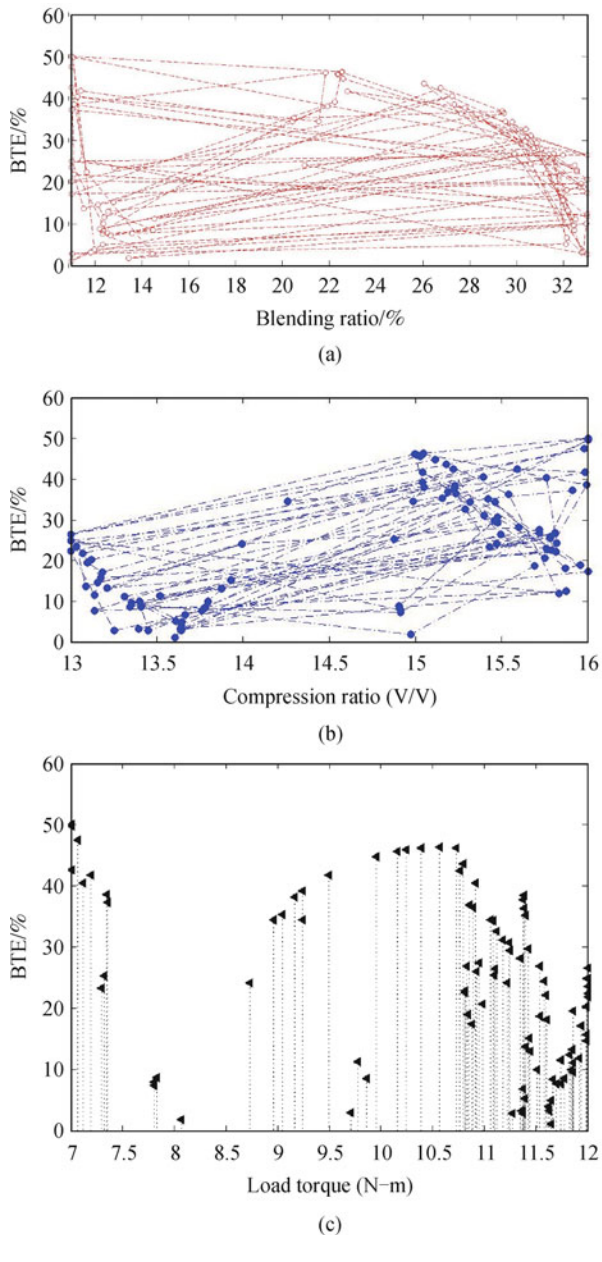


Fig. 10 (a) Variation of brake thermal efficiency against the blending ratio. (b) Variation of brake thermal efficiency against the compression ratio. (c) Variation of brake thermal efficiency against load torque

Confirmatory experiments are done to check the accuracy of the optimization results. Five solutions are

Table 7 NSGA-II operators used in MATLAB code

S. No	Parameter	Type	Value/probability
1	Population size	———————————	100
2	Number of Generations	———————————	500
3	Crossover	Simulated binary	0.9
4	Mutation	Polynomial	0.166
5	Selection	Non-dominated sorting and crowding distance	—————

Table 8 Pareto optimum solution sets predicted by NSGA-II package

S. No	X_1	X_2	X_3	BSFC	BTE	P_{max}	CO	NO_x	HC	Smoke
1	11.25	12.5	13.5	0.43	13.46	8.7	1.27	7.12	15.8	126.11
2	11.89	12.28	13.76	0.197	0.44	9.76	0.33	43.42	2.13	112.14
3	12.51	7.8	14.91	0.55	7.9	33.39	10	106.2	2.04	42.77
4	**17.35**	**8.88**	**14.96**	**0.85**	**28.34**	**75.13**	**1.09**	**331.13**	**128.96**	**49.44**
5	**18.51**	**7.98**	**14.92**	**0.8**	**14.55**	**62.23**	**0.58**	**272.08**	**36.89**	**48.27**
6	**19.08**	**11.62**	**15.8**	**0.43**	**36.11**	**63.17**	**0.91**	**313.08**	**100.32**	**104.82**
7	19.15	11.62	15.92	0.39	34.56	60.69	0.78	293.37	73.11	107.76
8	19.16	11.88	15.9	0.31	32.71	55.54	0.66	279.27	55.64	115.64
9	21.37	10.88	14.76	0.77	44.87	83.49	1.83	429.03	609.49	84.78
10	21.57	9.15	15.02	0.93	37.78	85.91	1.76	414.56	386.23	61.064
11	21.62	10.7	15.63	0.73	43.03	82.6	1.65	398.49	384.56	87.87
12	21.64	11.18	15.72	0.6	41.07	75.57	1.87	369.14	252.03	100.54
13	**21.93**	**9.23**	**15.04**	**0.93**	**38.95**	**86.92**	**1.82**	**421.34**	**430.7**	**63.02**
14	21.95	11.09	15.89	0.58	39.24	73.8	1.18	343.32	175.85	102.83
15	21.97	10.95	15.72	0.66	41.83	78.83	1.47	378.58	293.42	96.17
16	22.02	10.87	15.85	0.64	40.25	77.34	1.31	357.42	218.41	96.96
17	22.28	10.26	15.02	0.87	45.94	89.4	2.11	452.66	777.53	77.06
18	22.49	10.38	14.99	0.85	46.17	88.85	2.1	451.96	783.83	79.71
19	22.5	10.16	15.04	0.88	45.68	89.75	2.11	452.36	766.72	76.36
20	22.52	10.56	15.04	0.83	46.32	87.89	2.06	449.4	770.67	83.35
21	**22.63**	**10.98**	**15.64**	**0.67**	**42.64**	**79.88**	**1.55**	**389.39**	**347.2**	**98.44**
22	22.64	9.95	15.11	0.9	44.71	90.08	2.09	448.34	707.6	73.99
23	28.63	10.58	15.5	0.65	37.63	75.68	1.08	320.41	175.38	123.43
24	30.66	11.45	16.13	0.192	20.04	44.89	0.06	139.94	8.70	184.41
25	30.69	12.25	16.34	0.15	7.17	19.93	0.06	70.04	1.288	229.82
26	31.47	12.21	16.35	0.18	4.994	17.39	0.12	56.7	1.07	237.16
27	32.662	12.47	16.5	0.423	6.68	1.42	0.67	15.84	6.41	276.22
28	33.75	7.5	13.5	0.16	23.9	19.64	0.96	6.2	81.28	158.5
29	33.75	7.5	13.6	0.112	21.37	14.73	0.75	11.56	48.97	154.5
30	33.75	12.5	16.5	0.5	11.7	8.53	1.04	5.38	13.8	294.46

randomly chosen from Table 8 and they are at serial number 4, 5, 6, 13, 21. Experiments have been performed at these five optimal input variable combinations and the responses have been measured. It is seen from Table 9 that error between the actual and predicted value of responses are less than 5% which shows the excellent reproduction of results.

Table 9 Confirmatory experiments by randomly selecting the five solution sets

No	X_1	X_2	X_3	BSFC	BTE	P_{max}	CO	NO_x	HC	Smoke	Error/%						
											BSFC	BTE	P_{max}	CO	NO_x	HC	Smoke
1	21.93	9.23	15.04	0.90	40.68	87.85	1.85	423.45	433.76	66.21	−3.3	4.2	1.05	1.6	0.4	0.7	4.8
2	19.08	11.62	15.80	0.45	37.75	64.65	0.89	315.65	98.75	102.56	4.4	4.3	2.28	−2.2	0.8	−1.5	−2.2
3	18.51	7.98	14.92	0.81	15.67	60.35	0.61	269.56	38.78	50.65	2.4	1.2	−3.1	4.9	−0.9	4.8	4.6
4	17.35	8.88	14.96	0.87	29.42	77.63	1.04	333.25	130.75	50.74	2.2	3.6	3.2	−4.8	0.64	1.3	2.5
5	22.63	10.98	15.64	0.69	43.54	82.76	1.59	382.46	349.3	99.29	2.8	2.06	3.4	2.5	−1.8	0.6	0.8

5 Conclusions

This research has applied multi-objective optimization of performance and emission parameters in a single cylinder four stroke direct injection diesel engine using RSM and NSGA-II. Mathematical relations are predicted for combustion parameters (BSFC and P_{max}), performance parameters (BTE) and emission parameters (CO, NO_x, unburnt HC and smoke) using regression analysis. NSGA-II algorithm is then used for multi-objective optimization of performance and emission parameters. Based on the findings of the work, following specific conclusions are drawn:

(a) Empirical relation of BSFC, BTE, P_{max}, CO, NO_x, unburnt HC and smoke are predicted by using response surface methodology (RSM) approach.

(b) Three dimensional surface and contour plots of performance and emission parameters are studied with significant variation in relation to blending ratio, load torque and compression ratio.

(c) Significant change in values of performance parameter (BTE) by NSGA-II is obtained in comparison to response surface methodology approach.

(d) A non-dominated solution sets are obtained. Confirmatory experiments are then conducted to verify the predicted optimal sets of solution, to be useful for end users in selecting the optimal combination of engine combustion, performance and emission parameters. Any one solution can be chosen depending upon the engineer's requirement.

Acknowledgements The authors gratefully acknowledge Mr. Mohinder Pal Garg, Assistant professor, Mechanical Engineering Department, M. M. University, Ambala, Haryana and Mr. Deepak Chhabra, Assistant professor, Mechanical Engineering Department, UIET, M.D.U, Rohtak, Haryana for helping them in carrying out their research.

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