Front. Mech. Eng. China 2007, 2(4): 483–488
DOI 10.1007/s11465-007-0084-9

RESEARCH ARTICLE

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Optimization model for rotor blades of horizontal axis wind turbines

Abstract This paper presents an optimization model for rotor blades of horizontal axis wind turbines. The model refers to the wind speed distribution function on the specific wind site, with an objective to satisfy the maximum annual energy output. To speed up the search process and guarantee a global optimal result, the extended compact genetic algorithm (ECGA) is used to carry out the search process. Compared with the simple genetic algorithm, ECGA runs much faster and can get more accurate results with a much smaller population size and fewer function evaluations. Using the developed optimization program, blades of a 1.3 MW stall-regulated wind turbine are designed. Compared with the existing blades, the designed blades have obviously better aerodynamic performance.

Keywords wind turbine, optimal design, genetic algorithm, strip theory

1 Introduction

The basic principle of a wind turbine converting wind energy into electricity comes from the lift produced by the air flowing through the rotor. The shape of the rotor blades plays a decisive role in determining the overall aerodynamic performance of a horizontal axis wind turbine [1–3]. Thus, aerodynamic optimization of the blade shape is a very important stage in the design and manufacturing of wind turbines [4].

In earlier days, the Glauert and Wilson methods were mostly used for blade design [5]. The objectives of these methods were to obtain the maximum power coefficient of each blade section at the design wind speed. Because the time variation characteristics of wind speed are not taken into account, blades designed by these methods cannot achieve the maximum annual energy output. Furthermore, design results from these methods must be substantially amended to get smooth chord length and twist distributions [5]. Because the amended results already deviate from the design points, effectively controlling design results pose problems.

The traditional methods are based on steady wind speed. In actuality, the wind speed changes stochastically. Two models are commonly used to define this variation: a power law model for variation in wind speed with elevation and a Weibull model for such a variation with time [5,6]. To avoid the disadvantages of traditional design methods, this paper presents an optimization model for rotor blade design that refers to the wind speed distribution function of the specific wind site, with an objective to satisfy the maximum annual energy output.

Conventional search algorithms, such as the feasible direction method and the complex method, are often prone to converging on the local optimal point [7]. For some complicated problems it is difficult to obtain a global optimal result and user interference must be carried out. For example, changing the design parameters or shifting the initial feasible domain to execute multiple search processes is needed to get the best local optimal result as the global optimal one. This prevents the designer from concentrating on the problem itself and guaranteeing that the designed result is globally optimal. To develop a generalized optimization program and obtain the desired result, genetic algorithms are used to carry out the optimization search procedure in this paper.

2 Statistical theory of wind speed

Variations in the wind speed with elevation above ground and with time have important influences on both the assessment of the wind energy resources and the design of wind turbines [5]. They are utilized to describe the characteristics of the wind site.

A variation in wind speed with elevation is referred to the vertical profile of the wind speed or the wind shear. A power

Translated from *Journal of Shantou University* (*Natural Science*), 2006, 21(1): 44–49 [译自: 汕头大学学报 (自然科学版)]

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law is commonly used for defining vertical wind profiles. The basic equation of the wind shear power law is

$$U(Z) = U_R \left(\frac{Z}{Z_R} \right)^{\alpha} \tag{1}$$

where Z is the altitude above the ground level; Z_R is the reference altitude; α is the empirical wind shear exponent; U_R is the wind speed at the reference altitude; and $U(Z)$ is the wind speed at the altitude Z. The exponent α is often changing from 0.1 to 0.4, varying with the roughness of the ground and the temperature gradient of the air.

The Weibull frequency distribution function of wind speed is widely used to characterize periodic variations in wind speed. It can be expressed as

$$f_w(U) = \frac{k}{C} \left(\frac{U}{C} \right)^{k-1} e^{-\left(\frac{U}{C}\right)^k} \tag{2}$$

where C is the empirical Weibull scale factor and k is the empirical Weibull shape factor. These two factors jointly reflect the wind speed time variation characteristics of a wind site. The range from 1.5 to 3.0 for k includes most site wind conditions.

The relationship between the average wind speed $\bar{U}$ and the arbitrary wind speed U is

$$\bar{U} = \int_0^{\infty} U f_w(U) \mathrm{d}U \tag{3}$$

3 Genetic algorithms

3.1 Introduction

Genetic algorithms [7–9] are stochastic search techniques based on the mechanism of natural selection and natural genetics. These algorithms differ from conventional search techniques and start with an initial set of random solutions called population. Each individual in the population is called a chromosome and represents a solution to the problem at hand. A chromosome is a string of symbols; it is usually, but not necessarily, a binary bit string. The chromosomes evolve through successive iterations called generations. During each generation, the chromosomes are evaluated using some measures of fitness. To create the next generation, new chromosomes called offspring are formed by either merging two chromosomes from a current generation using a crossover operator or modifying a chromosome using a mutation operator. A new generation is formed by selecting, according to the fitness values, some of the parents and offspring and rejecting others to keep the population size constant. Fitter chromosomes have higher probabilities of being selected. After several generations, the algorithms converge to the best chromosome, which hopefully represents the optimum or suboptimal solution to the problem.

Assume $P(i)$ and $C(i)$ are parents and offspring in current generation i; the general structure of genetic algorithms is described as follows [7]

```
begin
    i←0
    initialize P(i)
    evaluate P(i)
    while (not termination condition) do
    begin
        recombine P(i) to yield C(i)
        evaluate C(i)
        select P(i+1) from P(i) and C(i)
        i←i+1
    end
end
```

Because genetic algorithms only use binary coding and fitness to represent problems and do not have many mathematical requirements for optimization, they are more generalized than traditional optimization methods. Due to their evolutionary nature, genetic algorithms will search for solutions without regard to the specific inner workings of the problem. Genetic algorithms can handle any kind of objective function and constraint defined on discrete, continuous or mixed search spaces. The ergodicity of evolution operators makes genetic algorithms very effective at performing global searches. As a self-instructive and self-adaptive search technique, genetic algorithms are successfully applied to resolve many complicated problems such as structural optimization, nonlinear optimization and parallel computing.

3.2 Extended compact genetic algorithm (ECGA)

The ECGA [10–12] is a kind of improved genetic algorithm in which linkage learning via probabilistic modeling is introduced. Compared with the traditional Simple GA, in terms of solving a problem with the same complexity, ECGA runs much faster and can get more accurate results with a much smaller population size and fewer function evaluations. Moreover, Simple GA demands users to deal with more parameters than ECGA. To obtain a satisfactory result, the users must sometimes modify parameters many times, which will prevent users from concentrating on the problem itself. It is very difficult for users who are not familiar with genetic algorithms to modify the parameters. The necessity of user interference makes it impractical to develop a generalized optimization program. Compared with Simple GA, ECGA only demands five parameters that can all be automatically set by a program.

These advanced characteristics make ECGA more suitable for solving some complicated problems. The process of rotor blades optimization involves a great deal of high-load computation. In particular, the complicated aerodynamic performance calculation for each chromosome will cost considerable computing time. Using ECGA to carry on the search process can save computing time remarkably, and the accuracy of the design result can be guaranteed. Furthermore, ECGA makes it feasible to build a generalized program for rotor blade design.

4 Optimization model

4.1 Decision variables

The shape of a blade is determined by the airfoil, chord length c and twist t of each blade section. When the airfoil family is defined, it needs to get the optimal chord length and twist distributions along the blade. To achieve smooth spanwise distributions of chord length and twist in the main power production part of the blade, both distributions are defined as a function of blade radius using the Bezier curve (Fig. 1).

The Bezier curves that define the distributions of chord length and twist both use four control points, thus there are 16 decision variables in total. They are coordinates of control points of chord length curve, respectively

$$(x_{cpi}, y_{cpi}) \qquad i = 1,2,3,4$$

and coordinates of control points of twist curve

$$(x_{tpi}, y_{tpi}) \qquad i = 1,2,3,4$$

4.2 Fitness function

The annual energy output equals the product of the average power of the wind turbine and the constant time of one year. Thus, annual average power can be used as the design objective and the fitness function $f(x)$ can be defined as

$$f(x) = \overline{P} = \int_{U_{in}}^{U_{out}} f_w(U)P(U)\mathrm{d}U \tag{4}$$

where $\overline{P}$ is the annual average power; U_{in} is the cut in wind speed; U_{out} is the cut out wind speed; and $P(U)$ is the shaft power produced by the rotor at the wind speed U.

4.3 Constraints

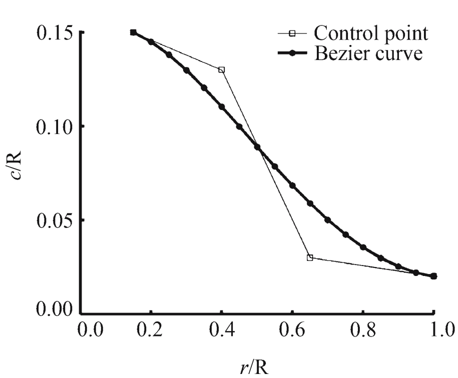
When applying binary coding for genes, the length of the chromosome is determined by the constraints of the decision variables [7]. To develop a generalized optimization program, the length of the chromosome is automatically calculated by the program after the constraint values are given by the user. The constraint equations of the decision variables are defined as

$$\begin{cases} r_{\min} \leqslant x_{cp1} < x_{cp2} < x_{cp3} < x_{cp4} \leqslant r_{\max} \\ c_{\max} \geqslant y_{cp1} > y_{cp2} > y_{cp3} > y_{cp4} \geqslant c_{\min} \\ r_{\min} \leqslant x_{tp1} < x_{tp2} < x_{tp3} < x_{tp4} \leqslant r_{\max} \\ t_{\max} \geqslant y_{tp1} > y_{tp2} > y_{tp3} > y_{tp4} \geqslant t_{\min} \end{cases} \tag{5}$$

where $r_{\min}$ and $r_{\max}$ are the minimum and the maximum radius of the optimization segment of the blade, respectively; $c_{\min}$ and $c_{\max}$ are the permitted minimum and maximum chord length along the blade respectively; and $t_{\min}$, $t_{\max}$ are the permitted minimum and maximum twist along the blade defined by the user, respectively.

4.4 Design program

For resolving the fitness function, the aerodynamic performance of each individual must be calculated. To guarantee the accuracy of fitness evaluation, the amended strip theory [13–15] is applied to calculate wind turbine aerodynamics. This theory is based on the momentum theory and blade element theory, taking into account the tip loss, hub loss, cascade effects, the invalidation of momentum theory when the wind turbine stalls, wind shear, yawing, structural parameters and installation parameters [15].



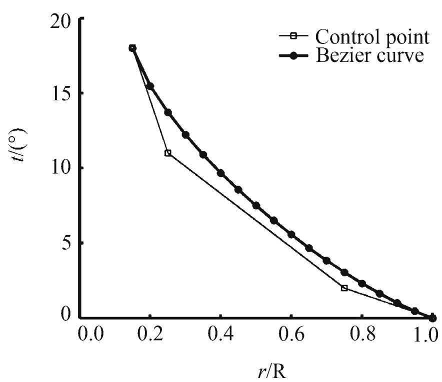


Fig. 1 Chord length and twist distributions described by Bezier curves

Applying ECGA and the amended strip theory, an optimization program is developed. Figure 2 shows the flowchart.

5 Design examples

For the convenience of evaluating the optimization model, referring to the parameters provided by a power cooperation company of an existing 1.3 MW stall-regulated wind turbine and the Weibull distribution function of the Nan'ao wind site, blades of a 1.3 MW stall-regulated wind turbine are designed by applying the developed optimization program. The comparison object for the designed blade is the one manufactured by the cooperation company for the existing wind turbine.

5.1 Design parameters

Table 1 shows the design parameters for blades of the 1.3 MW wind turbine.

Table 1 Parameters of rotor, wind conditions and ECGA

Turbine diameter /m	60
Blade number	3
Rated wind speed /($m \cdot s^{-1}$)	15
Rated power /MW	1.3
Rotational speed /($r \cdot min^{-1}$)	19
Hub altitude /m	60
Blade set angle /(°)	0
Rotor cone angle /(°)	0
Tilt angle/(°)	0
Rotor overhang/m	4.2
Hub diameter/m	2
Airfoil family	NACA63-4
Air density/($kg \cdot m^{-3}$)	1.225
Empirical wind shear exponent	0.166 67
Reference altitude /m	60
Empirical Weibull shape factor	1.85
Empirical Weibull scale factor /($m \cdot s^{-1}$)	11.5
Cut in wind speed /($m \cdot s^{-1}$)	5
Cut out wind speed /($m \cdot s^{-1}$)	25
Length of the chromosome	95
Population size	1,000
Tournament size	16

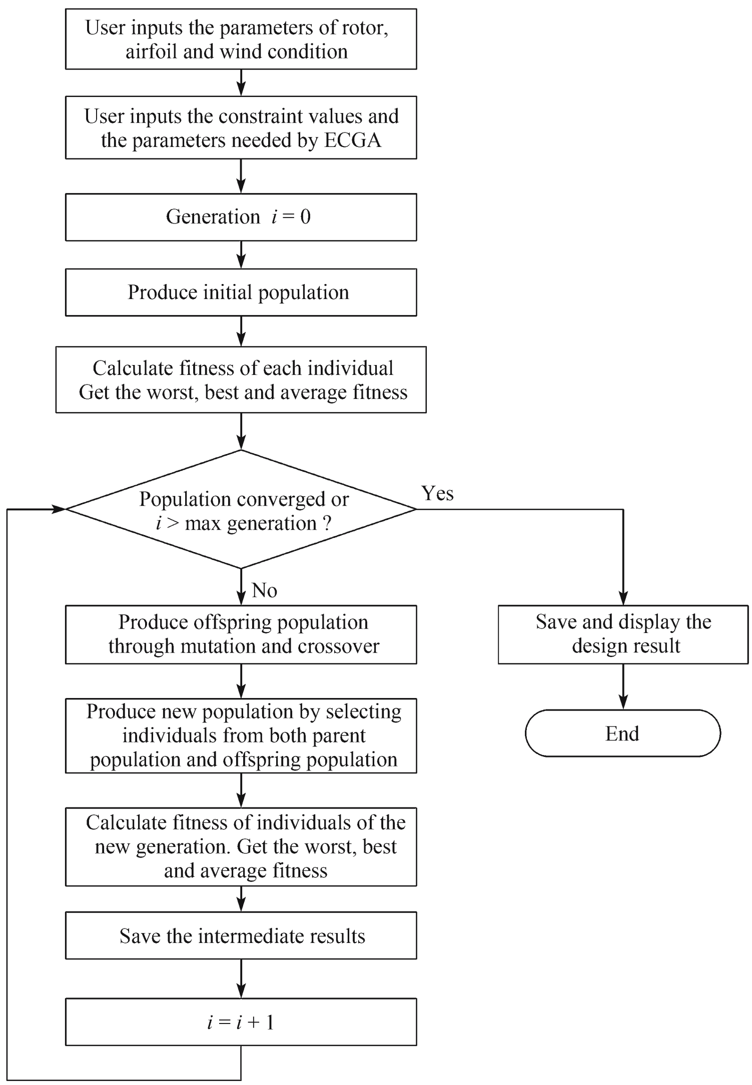


Fig. 2 Flowchart of the optimization program

5.2 Design results

Figure 3 shows the design results of chord length, twist and relative thickness distributions of the blade, and a comparison has been made with the existing blade.

Figure 4 shows the power and power coefficient curves of the 1.3 MW stall-regulated wind turbine applying the designed blades.

The aerodynamic performances of the 1.3 MW stall-regulated wind turbine are calculated applying the existing blades and the designed blades, respectively. The calculation results of power and power coefficient are listed in Table 2.

Table 2 shows that in the whole range of operating wind speed, the designed blades have better performance than their existing counterparts. At a lower wind speed, the power yielded by the designed blades is nearly twice that yielded by the existing blades. Therefore, applying the designed blades would make the wind turbine start up at a lower wind speed. Assuming that the wind turbine never stops at the operating wind speed when applying the designed blades, the annual energy it produced is 5.16×10^6 kWh. Compared with the 4.8×10^6 kWh produced by applying the existing ones, the annual energy output is increased by 7.5%.

6 Conclusions

1) An optimization model for rotor blades of horizontal axis wind turbines that accounts for the wind speed distribution function of the specific wind site and targets maximum annual energy output is presented.

2) To develop a generalized optimization program, ECGA is used as the search algorithm. The ECGA can speed up the search process and guarantee a global optimal result.

3) Utilizing the developed program, blades of a 1.3 MW stall-regulated wind turbine are designed. Compared with the

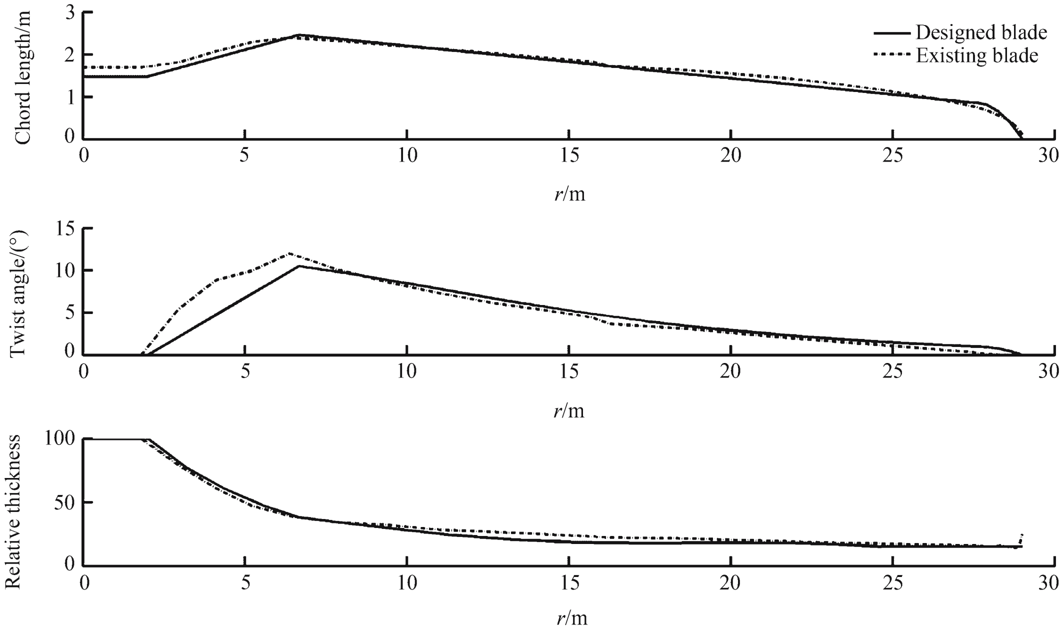


Fig. 3 Comparisons of chord length, twist and relative thickness distributions between the designed blade and the existing blade

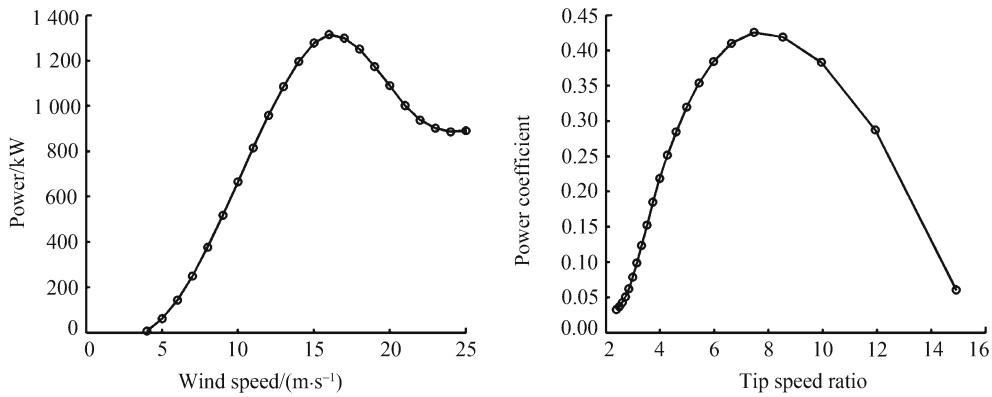


Fig. 4 Power and power coefficient curves of the designed wind turbine

Table 2 Comparison of power and power coefficient between the designed blade and the existing blade

Wind speed /(m·s^{-1})	Tip speed ratio	Power P_1/kW (existing blade)	Power coefficient (existing blade)	Power P_2/kW (designed bladed)	Power coefficient (designed blade)	$(P_2-P_1)/P_1$
5	11.94	31.280	0.144 5	62.208	0.287 4	98.87%
6	9.95	112.120	0.299 7	143.451	0.383 5	27.94%
7	8.53	221.079	0.372 2	249.196	0.419 5	12.72%
8	7.46	355.254	0.400 7	377.597	0.425 9	6.29%
9	6.63	502.359	0.397 9	518.418	0.410 6	3.20%
10	5.97	654.879	0.378 1	666.265	0.384 7	1.74%
11	5.43	800.856	0.347 4	816.399	0.354 2	1.94%
12	4.97	927.158	0.309 8	958.278	0.320 2	3.36%
13	4.59	1029.000	0.270 4	1085.000	0.285 2	5.44%
14	4.26	1103.000	0.232 0	1196.000	0.251 7	8.43%
15	3.98	1146.000	0.196 0	1278.000	0.218 7	11.52%
16	3.73	1172.000	0.165 2	1315.000	0.185 3	12.20%
17	3.51	1186.000	0.139 4	1299.000	0.152 7	9.53%
18	3.32	1156.000	0.114 5	1251.000	0.123 8	8.22%
19	3.14	1076.000	0.090 6	1174.000	0.098 9	9.11%
20	2.98	973.044	0.070 2	1090.000	0.078 7	12.02%
21	2.84	890.332	0.055 5	1002.000	0.062 5	12.54%
22	2.71	840.045	0.045 6	938.176	0.050 9	11.68%
23	2.60	827.004	0.039 2	902.227	0.042 8	9.10%
24	2.49	829.487	0.034 6	886.582	0.037 0	6.88%
25	2.39	832.379	0.030 8	890.901	0.032 9	7.03%

existing blades, the designed ones have obviously much better aerodynamic performance.

Acknowledgements This study was supported by the National Natural Science Foundation of China (Grant No. 50676053), the Natural Science Foundation of Guangdong Province (No. 06027186), and the Science and Technology Development Plan of Guangdong Province (No. 2005B10201024).

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