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RESEARCH ARTICLE

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Research on the valveless piezoelectric pump with Y-shape pipes

Abstract A piezoelectric pump with its Y-shape elements is presented. Two Y-shape pipes are fixed outside the chamber serving as an inlet and outlet, with the chamber and a piezoelectric vibrator being the actor. The pump has the potential to be miniaturized and integrated. Eddies occurring in the Y-shape elements are smaller, which is beneficial for the transport of living cells or long-link macromolecules. In this paper, the structure of the pump is first presented. Then, the equations on the change in volume and the mean pressure in the chamber are established, as well as the relation between the flow rate and the working frequency of the piezoelectric vibrator. Moreover, the relation between the mean pressure in the chamber and the working frequency of the piezoelectric vibrator is established. Finally, experiments are carried out to test the characteristics of the pump and to verify the correction of the theory on this pump.

Keywords piezoelectric pump, Y-shape element, Bessel function, flow resistance, flow rate

1 Introduction

Since Erik Stemme and Goran Stemme initiated the valveless piezoelectric pump with diffuser/nozzle elements [1], the valveless piezoelectric pump has attracted many researchers' attention because of its small, simple structure, and its ease to control and high transformation efficiency from electrical energy to mechanical energy with no electromagnetic pollution. Increasing achievements have been made in this pump [1–12]. Not only can this pump be an alternative to regular smaller pumps, but its application has also been broadened in the micromechanics area. There is also a good prospect for it in the areas of medical treatment and health care.

Many achievements have been made with valveless piezoelectric pumps; however, most of these have focused on the valveless piezoelectric pump with diffuser/nozzle elements [1,2,13–18]. These have two diffuser/nozzle elements fixed outside the chamber and inverted with each other. The pumping action depends on the different flow resistances when fluid flows into the two elements. One of the shortcomings of the pump stems from its two diffuser/nozzle elements. Larger eddies will occur if the angles of the elements are larger than a certain value, which hinders the usage of the pump in the transport of living cells or long-link macromolecules. There are some difficulties in processing a cone pipe, which has also limited the microminiaturization and integration of the pump.

In this paper, we will discuss the following:

First, the structure of a new piezoelectric pump with Y-shape pipes is presented [19]. Based on an analysis of the vibration of the piezoelectric vibrator, as well as the relation between the flow rate and the working frequency of the piezoelectric vibrator, equations on the change in volume and the mean pressure in the chamber are established, respectively. Furthermore, the relation between the mean pressure in the chamber and the working frequency of the piezoelectric vibrator is established. Finally, experiments are carried out to test the characteristics of the pump and to verify the correction of the theory on this pump.

2 Valveless piezoelectric pump with Y-shape pipes

The piezoelectric pump takes on a piezoelectric ceramic, which characterizes a converse piezoelectric effect, as its actor [20]. The pump can directly translate electrical energy to mechanical energy to transport fluid. The piezoelectric vibrator is usually constructed from a metal plate with a piezoelectric ceramic plate stuck to it. The metal plate can not only protect the piezoelectric ceramics, but also amplify the

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amplitude of the vibrating ceramics, which increases the flow rate of the pump.

Figure 1 shows the working principle of the piezoelectric vibrator. The piezoelectric ceramics will contract if a voltage is applied to it in the same polarized direction as shown in Fig. 1(a); the piezoelectric ceramic will extend if a voltage against the polarization direction is applied on it as shown in Fig. 1(b). The contraction and extension of the piezoelectric ceramic will deform the metal plate. The piezoelectric vibrator will protrude along its axial direction because of the extension or contraction of the ceramic, but the metal plate will not. The piezoelectric vibrator will protrude alternatively along the axial direction if an alternating voltage is applied to it.

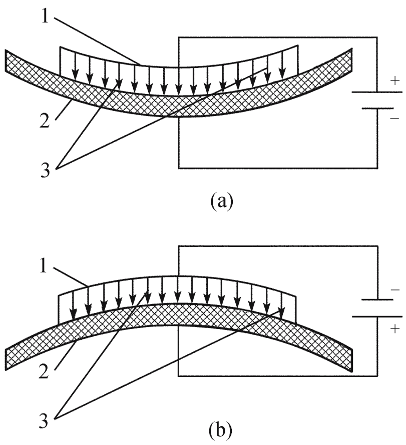

Fig. 1 Working principle of the piezoelectric vibrator
(a) The voltage direction is the same as the polarized direction; (b) The voltage direction is against the polarized direction
1. Piezoelectric ceramic plate; 2. metal plate; 3. polarized direction of a piezoelectric ceramic plate

The Y-shape pipe has an angle of θ as shown in Fig. 2. Two branch pipes i and i' are symmetrical to each other from the trunk pipe j. Its shape is just like a 'Y', therefore, it is called a Y-shape pipe for convenience. The branch pipes i and i' have the same parameters in this research: their diameter is d_i, and their cross section is A_i; the diameter of the trunk pipe is d_j, and its cross section is A_j. The Y-shape pipe is defined as a positive pipe if the branch pipes i and i' work as an inlet and the trunk pipe j works as an outlet; it is defined as a negative pipe if the branch pipes i and i' work as an outlet and the trunk pipe j works as an inlet. The flow resistance coefficient of the positive pipe differs from that of the negative pipe. The difference depends on the different angles of θ. For convenience, to illustrate the left and right Y-shape pipes, as well as to define the flow resistance coefficients, $j=1$ and $i=2$ express the left Y-shape pipe; $j=3$ and $i=4$ express the right one. When $i=1$ and $j=2$, the flow resistance coefficient of its trunk pipe j corresponding to the positive pipe is ζ_L, and that of the negative pipe is ζ_L'. When $j=3$ and $i=4$, the flow resistance coefficient of its trunk pipe j corresponding to the positive pipe is ζ_R', and that of the negative pipe is ζ_R.

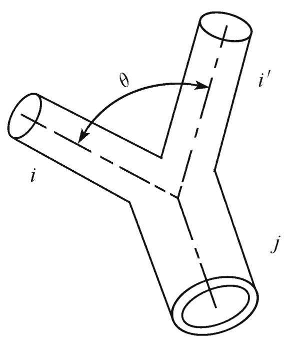

Fig. 2 Y-Shape pipe

The cross section of the Y-shape pipe can be processed into a square. Compared with the circle Y-shape pipe, the square Y-shape pipe is easily processed and has a smaller structure. The height and width of the cross section of the square Y-shape pipe can be varied according to the needs on the condition of a constant section area, which makes the usage of the square Y-shape pipe flexible in more areas.

The valveless piezoelectric pump with Y-shape pipes is composed of a piezoelectric vibrator, a body, a chamber and two Y-shape pipes. The two Y-shape pipes opposite each other are fixed to the left and right outside the chamber and works as an outlet and inlet, respectively. The pump can be categorized as a three dimensional structure (shown in Fig. 3) and a planar structure (shown in Fig. 4). When the vibrator protrudes upwards, the volume of the chamber expands and fluid flows into the chamber from the positive pipe and the negative pipe, as a suction stroke. When the vibrator protrudes downwards, the volume of the chamber contracts and the fluid will flow out of the chamber from the negative pipe and the positive pipe, as an exhaust stroke. The Y-shape pipes change functions: the positive and the negative pipes during the suction stroke will be changed to negative and positive pipes during the exhaust stroke, respectively. In a cycle of suction and exhaust, a net flow is produced because of the different flow resistances of the positive and negative pipes.

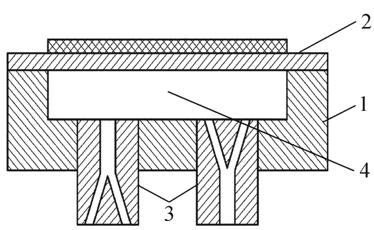

Fig. 3 Valveless piezoelectric pump with Y-shape pipes (three dimensional structure)
1. Body; 2. piezoelectric vibrator; 3. a pair of Y-shape pipes; 4. chamber

3 Analysis on flow during one pumping cycle

A pumping cycle of the piezoelectric pump is composed of four time stages. Each has a one-fourth cycle. The horizontal

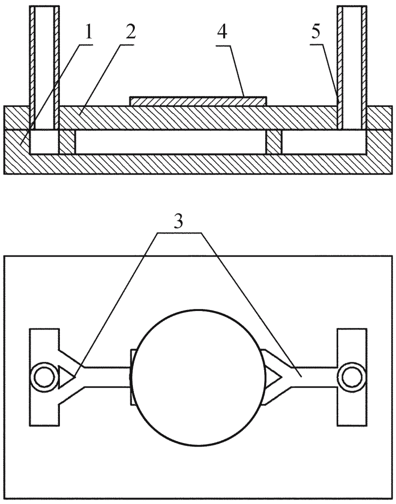

Fig. 4 Valveless piezoelectric pump with Y-shape pipes (planar structure)
1. Body; 2. plate; 3. a pair of Y-shape pipes; 4. piezoelectric vibrator; 5. inlet/outlet

position (time t = 0) is supposed to be the initial position of the piezoelectric vibrator. The four stages are as follows: the piezoelectric vibrator moves up from its horizontal position to its highest position; the vibrator moves back from its highest position to its horizontal position; the vibrator moves down from its horizontal position to its lowest position; and the vibrator moves back to its horizontal position from its lowest position. The vibrator will move cyclically according to the four stages. If the pump structure is manufactured, the suction quantities or exhaust quantities during each pumping stage are governed by the maximal deformation (w_{max}) during each stage. The total quantity of fluid flowing out or into the chamber during each stage is distributed to each branch of the Y-shape pipe according to a certain proportion. Let us suppose that each mean pressure during the four stages is p_i (i = 1, 2, 3, 4), the variations of the pressure in the chamber from one stage to another can be expressed by the variations of the mean pressure from one stage to another because of the small variation of the chamber volume and the short period of each stage. The amplitude of pressure in the chamber can generally infer the input of energy to the vibrator. The flow equations of the inlets and outlets of the two Y-shape pipes can be established and then the flow can be analyzed according to the mean pressures during each stage if the mean pressures are known.

During the first and fourth stages, the chamber volume increases and the pressure decreases, because the pump works as a suction. During the second and third stages, the pump works as an exhaust with a raised pressure and a reduced volume. The pressures at the points of the two Y-shape pipes connected to the chamber are approximately expressed as p_i. Suppose the pressures of the left end of the left Y-shape pipe and the right end of the right Y-shape pipe, which all connect the outside, are p_L and p_R, respectively, the energy equation of the two ends connecting the outside during the ith stage can be expressed as

$$\begin{cases} Z_L + \dfrac{p_L}{\rho g} + \dfrac{\alpha_1 (v_{Li})^2}{2g} = Z_L' + \dfrac{p_i}{\rho g} + \dfrac{\alpha_2 (v_{Li}')^2}{2g} + \tau_i h_{wi} \\ Z_R + \dfrac{p_R}{\rho g} + \dfrac{\alpha_1 (v_{Ri}')^2}{2g} = Z_R' + \dfrac{p_i}{\rho g} + \dfrac{\alpha_2 (v_{Ri})^2}{2g} + \tau_i h_{wi}' \\ (i = 1-4) \end{cases} \tag{1}$$

where

Z_L— the water head of the left end of the left Y-shape pipe working as an exhaust

Z_L'— the water head of the left end of the left Y-shape pipe working as a suction

Z_R— the water head of the right end of the right Y-shape pipe working as an exhaust

Z_R'— the head of the right end of the right Y-shape pipe working as a suction

here, $Z_L = Z_L' = Z_R = Z_R' = 0$

p_L— the pressure of the left end of the left Y-shape pipe connecting outside

p_R— the pressure of the right end of the right Y-shape pipe connecting outside

here, p_L and p_R are omitted because they are smaller than p_i in the chamber.

α_1, α_2 — the kinetic energy coefficients

here, α_1 and α_2 are all 1.

p_i— pressure in the chamber during the ith pumping stage

v_{Li}, v_{Li}' —mean flow velocities according to the cross section A_2 and cross section A_1 of the left Y-shape pipe, respectively, during the ith pumping stage

v_{Ri}, v_{Ri}' —mean flow velocities according to cross section A_4 and cross section A_3 of the right Y-shape pipe, respectively, during the ith pumping stage

h_{wi}, h_{wi}'— loss water heads along the flow directions of the left and right Y-shape pipes

τ_i— a function considering the effectiveness of the two branches of a Y-shape pipe, defined as

$$\tau_i = \begin{cases} \dfrac{1}{2} & (i = 1,4) \\ -\dfrac{1}{2} & (i = 2,3) \end{cases} \tag{2}$$

It can be seen from Eq. (1) that h_{wi} and h_{wi}' are positive values, and fluid flows into the chamber during the first and fourth pumping stages; h_{wi} and h_{wi}' are negative values, and fluid flows out the chamber during the second and third pumping stages. The loss water head during each pumping stage can be expressed as

$$\begin{cases} h_{wi} = \zeta_L \dfrac{(v_{Li}')^2}{2g} \\ h_{wi}' = \zeta_R \dfrac{(v_{Ri}')^2}{2g} \end{cases} \quad (i = 1,4) \tag{3}$$

$$\begin{cases} h_{wi} = \zeta'_{L} \dfrac{(v'_{Li})^2}{2g} \\ h'_{wi} = \zeta'_{R} \dfrac{(v'_{Ri})^2}{2g} \end{cases} \quad (i = 2,3) \tag{4}$$

The continuing equation for the fluid flowing in the left and right Y-shape pipes can be expressed as

$$\begin{cases} 2v_{Li} \cdot A_2 = v'_{Li} \cdot A_1 \\ 2v_{Ri} \cdot A_4 = v'_{Ri} \cdot A_3 \end{cases} \quad (i = 1-4) \tag{5}$$

The mean velocities flowing through A_1 and A_3 during each pumping stage can be concluded from Eqs. (1) to (5) and in consideration of the negative pressure occurring in the chamber. They are as follows:

Left Y-shape pipe

$$\begin{cases} v'_{Li} = k_1 \cdot \left(\left| \dfrac{p_i}{e_1} \right| \right)^{\frac{1}{2}} \quad (i = 1,4) \\ v'_{Li} = k_1 \cdot \left(\left| \dfrac{p_i}{e_2} \right| \right)^{\frac{1}{2}} \quad (i = 2,3) \end{cases} \tag{6}$$

Right Y-shape pipe

$$\begin{cases} v'_{Ri} = k_1 \cdot \left(\left| \dfrac{p_i}{e_3} \right| \right)^{\frac{1}{2}} \quad (i = 1,4) \\ v'_{Ri} = k_1 \cdot \left(\left| \dfrac{p_i}{e_4} \right| \right)^{\frac{1}{2}} \quad (i = 2,3) \end{cases} \tag{7}$$

In Eqs. (6) and (7),

$k_1 = (2/\rho)^{1/2}$ ρ is the density of the fluid

$u_1 = [A_1/(2A_2)]^2 - 1$, $u_2 = 1 - [A_3/(2A_4)]^2$

$e_1 = u_1 - \zeta_L/2$, $e_2 = u_1 + \zeta'_L/2$, $e_3 = u_2 - \zeta_R/2$, $e_4 = u_2 - \zeta'_R/2$

It can be seen from Eqs. (6) and (7) that the mean flowing velocities passing through A_1 and A_3 of the left and right Y-shape pipes can be calculated in given conditions, which are the flow resistance coefficients of the Y-shape pipes and the size of the pump structure. It lays a foundation for the analysis of the mean flow velocity and the flow rate of the pump.

4 Vibration of piezoelectric vibrator and change of chamber volume

The piezoelectric vibrator protrudes to force the fluid into the chamber when the piezoelectric pump is working. Cyclical applied force, which is called vibration resistance, is produced with the change in pressure in the chamber and the fluid motion. The vibration resistance distributes uniformly on the vibrator surface, which is supposed as $q = q_0 e^{jwt}$, because of the small displacement of the vibrator. The pre-processing conditions that are considered as the static deformation from its weight are omitted. The vibration deformation is center symmetrical, and both the speed and the vibration deformation around the border of the vibrator are zero because of its fixed border. A differential equation group with a supposed vibration function $w(r, t)$ and the border conditions can be expressed as [18]

$$\begin{cases} \Delta\Delta w + \dfrac{1}{\gamma^2} \dfrac{\partial^2 w}{\partial t^2} = \dfrac{q}{D} \\ D = Eh^3/[12(1-\mu^2)] \\ \gamma = \sqrt{D/\rho_0} \\ q = q_0 \exp(jwt) \\ \rho_0 = h\rho_1 \\ w(r,t)\big|_{r=R} = \dfrac{\partial w}{\partial r}\Big|_{r=R} = 0 \end{cases} \tag{8}$$

where

$\Delta = \nabla^2$— Laplace operator

ρ_0 — the density of the piezoelectric vibrator, and $\rho_0 = h \cdot \rho_1$

h — the thickness of the piezoelectric vibrator

R, ρ_1 —the radium and the density of the piezoelectric vibrator, respectively

E, μ — the modulus of elasticity and the Poisson ratio

q_0— the amplitude of the vibration resistance, usually coming from the experiment

The deformation function of the piezoelectric vibrator can be obtained from the function group above:

$$w(r,t) = \frac{q_0}{w^2 h\rho_1 \times G(r)}[G(r) - G(R)]\cos wt \tag{9}$$

where

$$G(r) = I_1\left(R\sqrt{\frac{w}{r}}\right)J_0\left(r\sqrt{\frac{w}{r}}\right) + J_1\left(R\sqrt{\frac{w}{r}}\right)I_0\left(r\sqrt{\frac{w}{r}}\right) \tag{10}$$

$$I_n(\beta) = \sum_{k=0}^{\infty} \frac{(\beta/2)^{n+2k}}{k!(k+n)!} \tag{11}$$

$I_0(r\sqrt{w/r})$ and $I_1(R\sqrt{w/r})$ are the values of $I_n(r\sqrt{w/r})$ when n = 0 and n = 1, respectively; and J_0 and J_1 are zero order Bessel function and one order Bessel function, respectively.

If the time varies from t_1 to t_2, the difference ∇V of the chamber volume of the piezoelectric pump can be expressed as

$$\nabla V(t_1 - t_2) = 2\pi \int_0^R r \cdot [w(r,t_1) - w(r,t_2)]\, dr \tag{12}$$

where $w(r, t)$ is the deformation function of the piezoelectric vibrator; and R is the radium of the vibrator. Supposing that the vibrator is at a horizontal position at the initial time

($t = 0$) and moves up when it starts, and the amplitude of every point keeps its constant value, the difference ∇V of the chamber volume coming from the deformation of each pumping stage is

$$\nabla V = 2\pi \int_0^R r \cdot w\left(r, \frac{T}{4}\right) \mathrm{d}r \tag{13}$$

where $w(r, T/4)$ is the deformation amplitude of every point on the piezoelectric vibrator during each pumping stage.

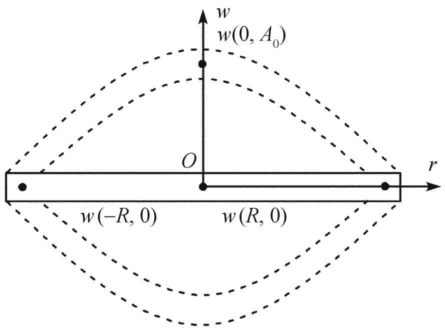

Fig. 5 Coordinates for the vibration deformation of the vibration

The above equation is theoretical only. Actually, the working frequency of the piezoelectric vibrator is not very high in application, (from a few ten Hz to a few hundred Hz). The deformed surface is close to the revolved parabolic surface [19]. For the convenience of calculation, the function of the revolved parabolic surface is adopted to calculate the difference of the chamber volume coming from the vibration deformation during each pumping stage. In the coordinate plan w–r for the piezoelectric vibrator, supposing the amplitude of the central vibration deformation is $w(0, T/4) = A_0$ and the border of the vibrator is fixed, there is $w(R, T/4) = w(-R, T/4) = 0$. The deformed amplitudes of every point of the vibrator, when the vibrator is working, are shown in Fig. 5. The deformed curve has to pass through the three points, which are the point of $w(0, T/4) = A_0$, the point of $w(R, T/4) = 0$, and the point of $w(-R, T/4) = 0$. The equation of the vibration deformation amplitude can be obtained from three points.

$$w\left(r, \frac{T}{4}\right) = -\frac{A_0}{R^2} \cdot r^2 + A_0 \tag{14}$$

Substituting Eq. (14) into Eq. (13), the difference ∇V of the chamber volume during each pumping stage can be approximately obtained

$$\nabla V = 2\pi \int_0^R r \cdot \left(-\frac{A_0}{R^2} \cdot r^2 + A_0\right) \mathrm{d}r = \frac{\pi}{2} A_0 \cdot R^2 \tag{15}$$

Using Eq. (15) to approximately calculate the difference ∇V of the chamber volume during each pumping stage is much simpler than Eq. (13), and it has little error.

5 Pressure variation of chamber and flow rate of pump

The sucked volume of fluid during the first and fourth pumping stages can be obtained if the mean flow velocities of the two Y-shape pipes during each pumping stage are known, and similarly with the exhaust volume during the second and third pumping stages. The time of every stage is T/4 (T is the vibrating period), as a result, the sucked or exhausted volumes during each stage can be expressed, respectively, as [20–21]

$$\begin{cases} V_{\mathrm{L}i} = T \cdot v'_{\mathrm{L}i} \cdot A_1 / 4 \\ V_{Ri} = T \cdot v'_{Ri} \cdot A_3 / 4 \end{cases} \quad (i = 1-4) \tag{16}$$

where $V_{\mathrm{L}i}$ and $V_{\mathrm{R}i}$ are the volume flowing in or out the left and right Y-shape pipes during the ith stage, respectively. Supposing $k_2 = k_1 \cdot A_1$, $k_3 = k_1 \cdot A_3$ and k_1 is the same as the aforementioned. Substituting Eqs. (6) and (7) into Eq. (16), we get [18,19]

$$\begin{cases} V_{\mathrm{L}i} = \dfrac{T}{4} \cdot \left(\left|\dfrac{p_i}{e_1}\right|\right)^{\frac{1}{2}} \cdot k_2 \\ V_{\mathrm{R}i} = \dfrac{T}{4} \cdot \left(\left|\dfrac{p_i}{e_3}\right|\right)^{\frac{1}{2}} \cdot k_3 \\ V_{\mathrm{L}j} = \dfrac{T}{4} \cdot \left(\left|\dfrac{p_j}{e_2}\right|\right)^{\frac{1}{2}} \cdot k_2 \\ V_{\mathrm{R}j} = \dfrac{T}{4} \cdot \left(\left|\dfrac{p_j}{e_4}\right|\right)^{\frac{1}{2}} \cdot k_3 \end{cases} \quad (i = 1, 4; j = 2, 3) \tag{17}$$

The pressure in the chamber is negative during the first and fourth stages of the suction stages, and cavitations occur in the chamber to some degree with fluid flowing into the chamber [22]. The cavitations are expected to affect the pump. If the cavitations are omitted, the sum of the volumes flowing in the chamber from the left and the right Y-shape pipes is theoretically equal to the varied volume of the chamber. It can be expressed by the matrix

$$\begin{bmatrix} V_{\mathrm{L1}} & V_{\mathrm{R1}} \\ V_{\mathrm{L2}} & V_{\mathrm{R2}} \\ V_{\mathrm{L3}} & V_{\mathrm{R3}} \\ V_{\mathrm{L4}} & V_{\mathrm{R4}} \end{bmatrix} \cdot \begin{bmatrix} 1 \\ \\ \\ 1 \end{bmatrix} = \nabla V \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{18}$$

Considering Eq. (17), Eq. (18) can be rewritten as

$$\frac{T}{4} \cdot \begin{bmatrix} [|p_1/e_1|]^{\frac{1}{2}} & [|p_1/e_3|]^{\frac{1}{2}} \\ [|p_2/e_2|]^{\frac{1}{2}} & [|p_2/e_4|]^{\frac{1}{2}} \\ [|p_3/e_2|]^{\frac{1}{2}} & [|p_3/e_4|]^{\frac{1}{2}} \\ [|p_4/e_1|]^{\frac{1}{2}} & [|p_4/e_3|]^{\frac{1}{2}} \end{bmatrix} \cdot \begin{bmatrix} k_2 \\ \\ \\ k_3 \end{bmatrix} = \nabla V \cdot \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} \tag{19}$$

According to Eq. (19), the mean pressure has to be derived strictly to

$$\begin{cases} p_1 = p_4 \\ p_2 = p_3 \end{cases} \tag{20}$$

Considering the relatively negative pressure occurring in the chamber, the mean pressure in the chamber can be solved from Eq. (19)

$$\begin{cases} \left(|p|\right)^{\frac{1}{2}} = \dfrac{4 \cdot \nabla V \cdot f}{\left[k_2 \cdot \left(|e_1|\right)^{-\frac{1}{2}} + k_3 \cdot \left(|e_3|\right)^{-\frac{1}{2}}\right]} \\ \left(|p_2|\right)^{\frac{1}{2}} = \dfrac{4 \cdot \nabla V \cdot f}{\left[k_2 \cdot \left(|e_2|\right)^{-\frac{1}{2}} + k_3 \cdot \left(|e_4|\right)^{-\frac{1}{2}}\right]} \end{cases} \tag{21}$$

where f is the frequency of the input voltage, $f = 1/T$.

During one stage of the four stages in a working period of the pump, suppose the flow flowing into the chamber is a positive flow and that flowing out of the chamber is a negative flow, the volume V_L flowing in the chamber from the left Y-shape pipe in one period is

$$V_L = V_{L1} - V_{L2} - V_{L3} + V_{L4} \tag{22}$$

According to the flow continuity, the volume V_R flowing out the chamber through the right Y-shape pipe in one period is equal to V_L

$$\begin{cases} V_R = -V_{R1} + V_{R2} + V_{R3} - V_{R4} \\ |V_L| = |V_R| \end{cases} \tag{23}$$

Then the flow rate Q of the pump can be expressed as

$$Q = |V|/T \tag{24}$$

According to Eqs. (19), (22) and (24) and the volume flowing in or out of the chamber through the left Y-shape pipe, the flow rate Q_1 can be calculated as

$$Q_1 = \frac{|V_L|}{T} = \frac{k_2}{2} \cdot \left| \left(\left| \frac{p_1}{e_1} \right| \right)^{\frac{1}{2}} - \left(\left| \frac{p_2}{e_2} \right| \right)^{\frac{1}{2}} \right| \tag{25}$$

According to Eqs. (19), (23) and (24) and the volume flowing in or out of the chamber through the right Y-shape pipe, the flow rate Q_2 can be calculated as

$$Q_2 = \frac{|V_R|}{T} = \frac{k_3}{2} \cdot \left| \left(\left| \frac{p_2}{e_4} \right| \right)^{\frac{1}{2}} - \left(\left| \frac{p_1}{e_3} \right| \right)^{\frac{1}{2}} \right| \tag{26}$$

Based on the analysis above,

$$Q = Q_1 = Q_2 \tag{27}$$

If the two Y-shape pipes have the same structure and sizes: $d_1 = d_3$, $d_2 = d_4$, $\zeta_L = \zeta'_R$, $\zeta'_L = \zeta_R$, $k_2 = k_3$, $A_1 = A_3$, the mean pressure in the chamber and the flow rate of the pump can be simplified as

$$\left(|p_1|\right)^{\frac{1}{2}} = \left(|p_2|\right)^{\frac{1}{2}} = \frac{4 \cdot \nabla V \cdot f}{k_2 \cdot \left[\left(|e_1|\right)^{-\frac{1}{2}} + \left(|e_2|\right)^{-\frac{1}{2}}\right]} \tag{28}$$

$$Q = \frac{k_2}{2} \cdot \left(|p_1|\right)^{\frac{1}{2}} \cdot \left[\left|\left(|e_1|\right)^{-\frac{1}{2}} - \left(|e_2|\right)^{-\frac{1}{2}}\right|\right] \tag{29}$$

6 Theoretical calculation and comparison with experiment

With the theoretical derivation above, the pressure variation in the chamber and the flow rate of the pump can be calculated. The two Y-shape pipes have the same structure and size in this research: $d_2 = 2$ mm, $d_1 = 5$ mm, $R = 25$ mm, and $\theta = 90°$. The flow resistance coefficients are taken as $\zeta_L = 2$, $\zeta'_L = 1$. The driving frequency varies from 30Hz to 80Hz. We carried out the experiments using water with the temperature at 25°C. The piezoelectric pump with Y-shape pipes in the experiment is shown in Fig. 6. To reduce any errors from manufacturing and fixture, a straight pipe with a diameter of $d_3 = 5$ mm is taken as a substitute for the right Y-shape pipe and the flow rate from the experiment is doubled. The geometric parameters and working parameters are shown in Table 1.

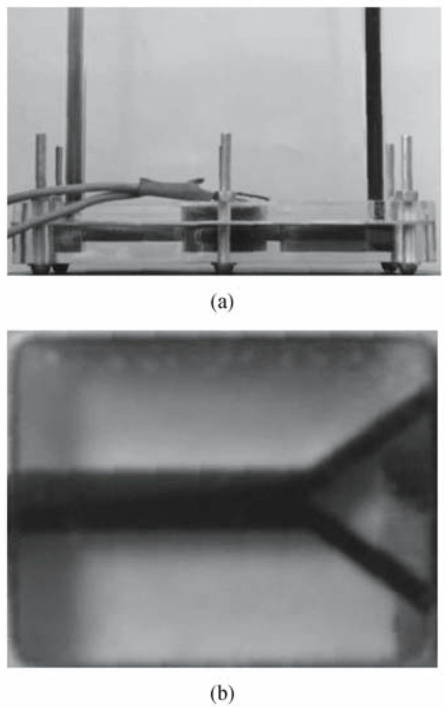

Fig. 6 The valveless piezoelectric pump with Y-shape pipes and its Y-shape pipes
(a) The valveless piezoelectric pump with Y-shape pipes; (b) Y-shape pipe

Table 1 Geometric parameters and working parameters of the pump

Driving voltage/V	Driving frequency/ Hz	Diameters /mm	Diameter of vibrator/mm
110	30–80	$d_1 = 5$ $d_2 = 2$ $d_3 = 5$	50

The theoretical and experimental curves of Q–f regarding flow rate and frequency, and the curve relating to the theoretical mean pressure are shown in Figs. 7 and 8, respectively.

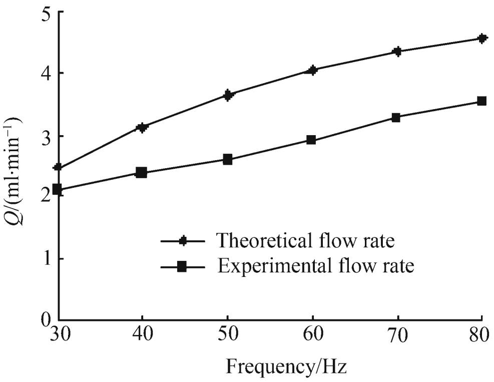

Fig. 7 Theoretical and experimental flow rate Q

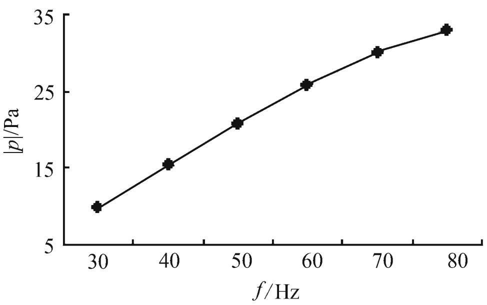

Fig. 8 Mean pressure in the chamber

It can be seen from Fig. 7 that the theoretical and experimental values have the same tendency. The theoretical value is bigger than the experimental one because the effect of cavitations or the errors from the experiments are not considered; the cavitations will reduce the flow rate to some extent. The difference between the theoretical value and the experiment also reflects the effect of cavitations on the pump. It can be seen from Fig. 8 that the mean pressures in the chamber during the four pumping stages are equal to each other, that is $|p_1| = |p_2|$. It can be seen from Figs. 7 and 8 that the flow rate increases and the mean pressure rises accompanying a rise in the driving frequency. The cavitations occur during the suction stage when the pressure in the chamber is negative. These cavitations will affect the flow rate, and these effects will increase with the rise of negative pressure. Therefore, the effects will increase with the increase in driving frequency.

7 Conclusions

1) A new valveless piezoelectric pump with Y-shape pipes is presented. The pump is apt to be further miniaturized.

2) The vibration of the vibrator is analyzed. Equations on volume variation and mean pressure in the chamber are established. Furthermore, the equation on the relationship between the flow rate and the driving frequency, and the equation on the relationship between the pressure variation in the chamber and the driving frequency are also established.

3) A valveless piezoelectric pump with Y-shape pipes has been manufactured. Experiments have been carried out to test the characteristics of the pump and to verify its theory. The theory on valveless piezoelectric pump has been further perfected.

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