Front. Mech. Eng. China (2006) 4: 479–484
DOI 10.1007/s11465-006-0064-5

RESEARCH ARTICLE

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Forecast method for used number of parts and components based on complex network

Abstract Applying directed complex network to model the main structure of a product family, according to in-degree bi-logarithmic coordinate distribution curve and distribution rule of nodes of the network, in-degree evolving rule of nodes of the network is presented and analytic expression of in-degree probability density of nodes is derived. Through the analysis of the relation between existing kinds of components and existing product numbers, an expression of the relation between kinds of components and product numbers is derived. A forecast method for the increment of component numbers and parts based on the increment of products is presented. As an example, the component numbers of an industrial steam turbine product family is forecasted, forecast result verified and forecast error analyzed.

Keywords mass customization, parts analysis, complex network, evolving rule

1 Introduction

Mass customization aims at satisfying individual customer needs while maintaining mass production efficiency [1]. In mass customization, to satisfy a customer's individual needs, the number of parts and components customized and managed by a corporation increases continually. For example, an industrial steam turbine corporation has produced more than 400 industrial steam turbines in the past 20 years. These steam turbines contain more than 90 000 kinds of parts and components. Except for the standard components, the number of parts and components manufactured by the corporation exceeds 80 000. The management of these numerous parts and components is a puzzling problem for corporations.

A network is a set of items, which we will call node or sometimes nodes, with connections between them called edges. Systems taking the form of networks abound in the world. Examples include the Internet, the World Wide Web, science collaboration network, movie actor collaboration network, metabolic networks, neural network, etc. The first time mathematicians tried to describe a network can be traced back to 1736, when they were dealing with the famous "Konigsberg seven-bridges" problem. Following the graph theory developed for centuries, research has been slow. Erdös and Rényi invented the random graph theory in 1959 [2]. After this, research has gathered momentum. Watts and Strogatz proposed small-world model in 1998 [3]. Balabási and Albert proposed their famous scale-free model in 1999 [4]. After this, the research on networks increased, an upsurge was noticed in the world, and some famous papers were published [5–8]. In network research, the focus of research is in the evolving rule and its evolving model. Besides the above three classical models, scholars proposed some other evolving models [9–10]. To date, the main research work of complex networks is focused on the complex theory, while research on the application of complex networks is very few.

At present, people usually use ABC analysis method to analyze parts and components and to manage the storage of parts and components; research on forecast method for used number of parts and components is very few. In the past, an industry steam turbine corporation mainly relied on experience and simply used the number of parts and components to manage their corporation parts and components. In this paper, we try to apply complex network theory to parts and components management of mass customization, and try to find out the intrinsical rule of relation between parts and

Translated from *Chinese Journal of Mechanical Engineering*, 2006, 42(6) (in Chinese)

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components of a product's family, and provide a new theory and a new method of parts and components analysis to customization analysis corporation.

2 Part relation network

Kinds and quantities of components of complicated mechanical products are usually varied and the relation between components is usually complicated. To describe the structure of a complicated product family, we introduce the complex network theory to this field and describe the complex relation between components within the network. The method of constructing a parts relation network is as follows: taking components as nodes and subject relations of components as edges, all edges are directed. The direction is from the parent node to its child node; the weight of an edge equals the number of child nodes contained by its parent node; hence, the structure network is a weighted directed acylic network.

Industrial steam turbine product is a kind of complicated MTO (made-to-order) product [11]. In an industrial steam turbine device, steam turbine noumenon is the most pivotal and most important part; steam turbine noumenon is the focus of industrial steam turbine. Therefore, we selected steam turbine noumenon as our research subject. According to the parts relation network construction method described above, a parts relation network of the steam turbine product family was constructed. Because standard parts and outsourced parts are not produced by the corporation, the parts relation network does not contain the standard and the outsourced parts. Eliminating the standard and the outsourced parts, the parts relation network contains 60 541 nodes and 134 513 edges. To be more clear and intuitionistic, we take only a part of the data of the steam turbine corporation product family to construct a component relation network as shown in Fig. 1.

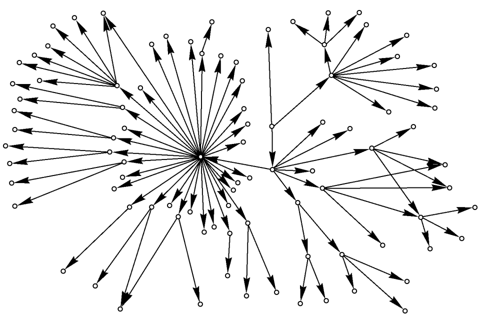

Fig. 1 Example of parts relation network

3 In-degree distribution curve of nodes

In a directed network, the in-degree of a node is the number of edges converging to it, and the out-degree of a node is the number of edges diverging from it. In parts relation network, a node's in-degree denotes the kind number of parent components of the node, and out-degree of a node denotes the kind number of child components or parts of the node. Analyzing the parts relation network through programming, we can obtain the in-degree distribution curve, as shown in Fig. 2. As seen in Fig. 2, we can draw a conclusion that the in-degree distribution of a parts relation network follows a power law.

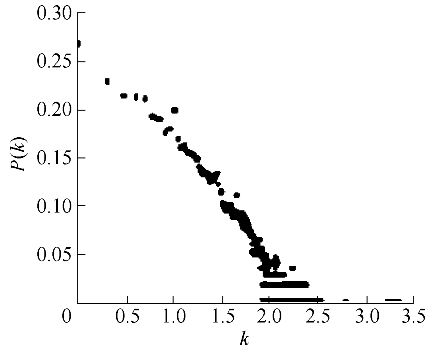

Fig. 2 In-degree distribution curve

4 In-degree evolving rule

In complex network theory, if a network has growth and preferential linking characters, its degree distribution follows a power law. Here, we first make a qualitative analysis of the parts relation network. In a customization corporation of mechanical product, it is difficult to realize completely ATO (assemble-to-order) production because the degree of customization of a mechanical product is usually high. When a customer orders a new product, the new customized product usually needs some universal and standard components and parts; it also requires some new components or parts to satisfy its particular characteristic. Hence, the number of components and parts increases continually; it means that the nodes of the parts relation network increase continually. Hence, we can assume that the parts relation network has growth characteristic. On the other hand, when engineers design a new product, they usually tend to select the existing universal components and parts to reduce design workload, shorten design period, and decrease product cost. Hence, we can assume that the parts relation network has preferential linking characteristic. Therefore, we can assume that the parts relation network has the two generic mechanisms of scale-free network.

Research on the evolving rule of network has been developed well recently. The scale-free model proposed by Barabási and Albert is the simplest model that captures power law degree distribution. The theoretical exponent of the scale-free network's degree distribution is –3. But the exponent of most actual networks' degree distribution curve has a value between –1 and –3; some networks' degree distribution even deflect from the power-law. To explain the difference between practical network and scale-free network, and to explore the different evolving rule of actual network, some scholar proposed some amendatory models, such as nonlinear preferential attachment model [12], initial attractiveness model [13], aging and cost model [14], etc. While researching on the evolving rule of parts relation network, we consult other network's evolving rule and by combining the characteristics of the actual network, an in-degree evolving rule is proposed as follows: 1) Start with a small number m_0 of lone nodes, and every node has initial attractiveness A; 2) A new node and m new edges are added to the network at every time step, and the new node has initial attractiveness A too. The end node of a new edge is selected according to preferential linking rule. When a new edge is added to the network, the probability of an existing node to be selected as end node of the new edge is in direct ratio to its in-degree. The expression is given as follows:

$$\Pi(k_i) = \frac{k_i}{\sum_j k_j}$$

where k_i is in-degree of node i, $\sum_j k_j$ is the sum of in-degree of all nodes. After N time step, we can obtain a network with $N + m_0$ nodes and $N \times m$ edges.

According to the above evolving rule, we can obtain the dynamical equation of in-degree as follows:

$$\frac{\partial k_i}{\partial t} = m \frac{k_i + A}{\sum_j k_j} \tag{1}$$

where t is time.

Now we try to obtain the solution to the in-degree dynamical equation.

Substituting $\sum_j k_j = (m + A)t$ into Eq. (1) leads to

$$\frac{\partial k_i}{\partial t} = m \frac{k_i + A}{(m + A)t} \tag{2}$$

From Eq. (2) we can obtain

$$k_i = C \cdot t^{\frac{m}{m+A}} - A \tag{3}$$

Initial limit condition is: $k_i(t_i) = A$; from the initial limit condition we can obtain

$$C = \frac{2A}{(t_i)^{\frac{m}{m+A}}}$$

Substituting $C = \frac{2A}{(t_i)^{\frac{m}{m+A}}}$ into Eq. (3) leads to

$$k_i = 2A \cdot \left(\frac{t}{t_i}\right)^{\frac{m}{m+A}} - A \tag{4}$$

Using Eq. (4), we can write the probability that a node has an in-degree $k_i(t)$ smaller than k, $P(k_i(t) < k)$ as

$$P(k_i(t) < k) = p\left(t_i > \left(\frac{2A}{k+A}\right)^{\frac{m+A}{m}} \cdot t \right) \tag{5}$$

Assuming that we add the nodes as equal time intervals to the network, the t_i values have a constant probability density

$$P(t_i) = \frac{1}{m_0 + t} \tag{6}$$

Substituting Eq. (6) into Eq. (5) we obtain

$$\begin{aligned} p\left(t_i > \left(\frac{2A}{k+A}\right)^{\frac{m+A}{m}} t \right) &= 1 - \int_0^{\left(\frac{2A}{k+A}\right)^{\frac{m+A}{m}} \cdot t} p(t_i)\mathrm{d}t_i \\ &= 1 - \frac{t}{m_0 + t}\left(\frac{2A}{k+A}\right)^{\frac{m+A}{m}} \end{aligned} \tag{7}$$

From Eq. (7) we can obtain

$$\begin{aligned} P(k,t) &= \frac{\partial P(k_i(t) < k)}{\partial k} \\ &= \frac{t}{m_0 + t} \cdot \frac{m + A}{m} A^{\frac{m+A}{m}} k^{-\frac{2m+A}{m}} \end{aligned} \tag{8}$$

Predicting that asymptotically ($t \to \infty$), we obtain

$$P(k) \approx \frac{m + A}{m} A^{1+\frac{A}{m}} k^{-2-\frac{A}{m}} = \left(1 + \frac{A}{m}\right) A^{1+\frac{A}{m}} k^{-\gamma} \tag{9}$$

where $\gamma = 2 + \frac{A}{m}$. We can draw a conclusion that probability density of in-degree follows a power law. Because m is positive, and while A is positive, γ is between 2 and ∞. If selecting appropriate initial attractiveness A and appropriate m, we can obtain any y between 2 and ∞.

Equation (9) is a universal computed formula of the in-degree evolving rule. According to different actual networks, by choosing appropriate parameters, we can obtain the in-degree probability density computed formula of the actual network. Analyzing bi-logarithmic probability density distribution curve of in-degree of the actual network in Fig.2, we can obtain the slope of the in-degree distribution curve; the slope is approximately –2.3, which means that the exponent of probability density distribution curve is 2.3, i.e., $\gamma \approx 2.3$. In Eq. (9), assuming m=3.4, $A = 1.0$, we can obtain

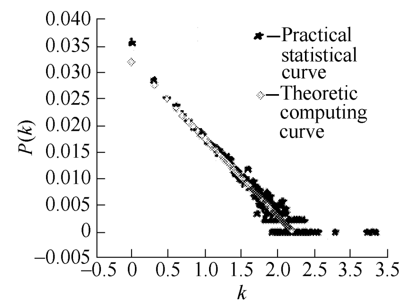

Fig. 3 In-degree comparison in log-log scale of theoretic computing curve and practical

$\gamma \approx 2.3$. We calculate $P(k)$ using theoretical approaches and compare the theoretical result with the actual statistical result as shown in Fig. 3.

As seen in Fig. 3, we can draw a conclusion that the evolving rule can depict and emersion repeat in-degree distribution rule well and truly; hence we can assume the evolving coincides with the practical.

5 Computing method of a node's in-degree increment

According to the in-degree evolving rule of nodes, it is easy to obtain the relationship between the in-degree increment of every existing node and the increment of the network's nodes. When the number of network's nodes is N_1, assuming the in-degree of an existing node is S_1, we can calculate the in-degree increment of the existing node while the number of network's nodes increases to N_2. According to the in-degree evolving rule, assuming the increment of network's nodes is 1 at each time step, we can calculate the relevant in-degree increment of the existing node step by step.

When the number of network's nodes increases from N_1 to N_1+1, the in-degree increment of the existing node is

$$\Delta S_1 = \frac{S_1}{(m+A)(N_1+1)} \tag{10}$$

At this time, the in-degree of the existing node equals to $S_1 + \Delta S_1$.

When the number of network's nodes increases from N_1+1 to N_1+2, the in-degree increment of the existing node is

$$\Delta S_2 = \frac{S_1+\Delta S_1}{(m+A)(N_1+2)} \tag{11}$$

At this time, the in-degree of the existent node equals to $S_1+\Delta S_1+\Delta S_2$.

When the number of network's nodes increases from N_1+2 to N_1+3, the in-degree increment of the existing node is

$$\Delta S_3 = \frac{S_1+\Delta S_1+\Delta S_2}{(m+A)(N_1+3)} \tag{12}$$

At this time, the in-degree of the existing node equals to $S_1+\Delta S_1+\Delta S_2+\Delta S_3$.

Applying the same analogy as given above, when the number of network's nodes increases from N_1 to N_2, the increment of the existing node at every time step, ΔS_1, $\Delta S_2, \cdots, \Delta S_{N_2-N_1}$ can be obtained. When the number of network's nodes increases to N_2, the in-degree of the existing node equals to $S_1+\Delta S_1+\Delta S_2+\cdots+\Delta S_{N_2-N_1}$.

Programming according to the calculating method, when the number of network's nodes increases, we can obtain the relevant in-degree increment of every existing node.

6 Forecast methods for used number of parts and components

In mass customization, the traditional analysis method of parts and components is the ABC analysis method [11]; it is based on the statistical result of used number of parts and components, without taking into account the complicated structure relation of parts and components and its intrinsic evolving rule. Therefore, it cannot reflect the actual circs completely. It is important for the corporation to search for a more scientific and more rational analysis and management method for parts and components.

In practice, the corporation focuses on the following: if the used numbers of every existing part and component are known, how do we forecast the increment of every existing part and component while the increment of the product increases. We try to solve this problem by using the in-degree evolving rule of parts relation network. Before solving the problem, two issues need to be solved: one is the relation between the product number and kind number of part and component and the other is the relation between the used number of a part or a component and the in-degree of node matching the part or the component. In the following, we will analyze these two problems.

6.1 Relation between product number and kind of part and component

Analyzing the existing product number and existing kind number of part and component, we can obtain the relation between product number and kind number of part and component. The relation between the above two objects of a steam turbine product family is obtained as shown in Fig.4.

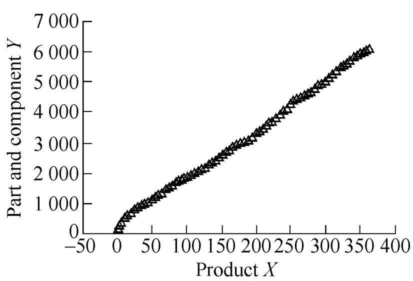

Fig. 4 Relation between product number and kind number of part and component

From Fig. (4), we can conclude that when the product number exceeds a certain value, the relation between the kind of part and number of product follows a linear law. Analyzing the relation between product number and kind number of part and component, we can obtain the formula between the two objects as given below:

$$Y = 162x + 2\,646 \tag{13}$$

where Y is the kind number of part and component in product family, and x is the product number of the product family.

6.2 Relation between increment of node's in-degree and increment of part or component

From the practical meaning of part relation network of product family, we can deduce that the used number increment of a part or a component is equal to the weight sum of the new in-degree of the part or component node. The formula used for calculation is given as follows

$$\Delta S = \sum_{i=1}^{n} w_i \quad (14)$$

where ΔS is the increment of the used number of the appointed part or component, w_i is weight of the i th new in-degree of the node, and n is the total increment of in-degree of the node.

As seen in Eq.(14), only when the weight of all in-degrees has the same value, it is convenient to calculate the increment of the used number of a part or a component through the in-degree increment, and the used number increment of a part or a component is equal to the in-degree increment multiplied by the weight of the all new edges. If the weight of all edges is 1, then the increment of the used number of a node is equal to the increment of its in-degree. Therefore, we need to analyze the weight distribution rule of different part nodes and component nodes, firstly. As an example, we analyze a steam turbine product family. Results show that the in-degree weight distribution rule of different kind parts or components is different; the results also show that the in-degree weight of almost all components is 1, and the in-degree weight of the vane parts has no established rule. The in-degree weight of most other parts is 1 too. To be more intuitionistic, here we compare the analysis results of different parts, as shown in Table 1.

Table 1 Weight comparison with different node style

Node style	Total number of node	Number of node whose in-degree weight is 1
Component	18 810	18 559
Other part	17 081	14 393
Vane part	24 291	602

As seen in Table 1, in the three different kinds of parts and components, the rule of weight of edge expressing component and not vane part is good, 98.7 % of the component's weight is 1, and 84.3 % of other parts is 1. Analyzing the in-degree increment of parts and components, we can obtain the used number increment of these parts and components. As an example, we forecast used number increments of components.

6.3 Forecast of used number of components

Taking components as our research subject, according to the in-degree evolving rule, we can obtain the relationship between the used number of components and the increment of the product. We take average product number of a corporation in a year as the product increment. According to experience, product number in a year is 26. From Eq. (13), we can obtain the increment of parts and components as 4 212, i.e., the evolving step is 4 212. Because the existing kind number of parts and components is known, i.e., the node number of the network is known, the in-degree of every existing node in the network is also known. Programming according to the evolving rule, we can obtain the in-degree increment of every node, because the increment of a component is equal to its in-degree increment. Therefore, we can forecast the used number increment of components for the next year, and provide a theoretical basis for the corporation to arrange for its storage.

To verify the method, we apply the method to forecast the increment of the actual product of a corporation and compare the theoretically forecast increment and the actual increment. We select two different product numbers as our initial product number, and give the two cases the same product increment. We forecast the increment of every component of the two cases, and the comparison of forecast results of the two cases is shown in Table 2.

Table 2 Forecast result comparison between two different cases

Items	Case 1	Case 2
Initial product number	310	336
Product increment	26	26
Initial part number of products	16 078	17 605
Part number of products whose absolute forecast error is less than 2	15 911	17 473
Average forecast error	17.7%	15.6%

In the above evolving process, the actual increment of most components is zero, and the theoretically forecast increment of these components is very little, almost equal to zero. The average forecast result of these components is 0.04. As for a corporation, only when the component increment reaches a relatively great value, it has obvious practical meaning. Therefore, we choose these components, whose increment is more than 1, for further analysis. According to the different initial product numbers, we analyze and compare the absolute value of average error and the relative error of the part whose increment is larger than 1; the results of two different initial product numbers is shown in Table 3.

Table 3 Forecast error comparison with parts whose increment is larger than 1

Initial product number	310	336
Average absolute value of forecast error	0.24	0.21
Average of actual increment of used number of part	1.357	1.345
Relative value of forecast error	17.7%	15.6%

As seen in Table 3, when the initial product number increases, the forecast precision increases too. This is in accord with the law of statistics. So, as the historical data increases, the forecast method proposed by this paper will become increasingly precise. Here, we choose some components as examples and compare the theoretically forecast increment and the actual increment; the results are shown in Table 4. It is necessary to highlight that the initial product number is 336 and the increment of the product is 26 in Table 4.

Table 4 Actual increment and theory increment comparison with some example component

Coding	Part name	Actual increment	Theory increment
25143000606	Adjuster noumenon	3	1.62
21645006903	Critical protector	2	2.23
23710204001	Dentiform joint coupling	1	0.67
22020000900	Assembly of spring seat	2	2.27
25885040103	Assembly of atmoseal device	1	0.75
28600080200	Shutoff device of critical	1	0.57
20101160000	Damper	1	1.44
25143000606	Assembly of drive	1	0.35
22569150011	Assembly of exhaust cylinder	1	0.47
21634240400	Manual jigger	1	0.63
28465000200	Amplifier	3	2.13
22528106102	Under half rough of outer cylinder	1	0.10
22524106100	Upper half rough of outer cylinder	1	0.10
23729000500	Joint coupling HBS56	2	3.44
21366000900	Airproof assembly of slide pole	2	2.61
22131001000	Oil filter	4	2.90

In short, we can summarize the steps to obtain the increment of components as follows: 1) constructing the parts relation network of a product; 2) analyzing the in-degree evolving rule of node; 3) confirming the computing method of in-degree increment of node; 4) analyzing the relation between product number and parts and components number; 5) confirming component increment according to product increment; 6) confirming in-degree increment of existing parts and components according to kind increment of parts and components; and 7) confirming used number increment of parts or components according its in-degree increment.

7 Conclusions

Parts relation network of a product family has obvious growth and preferential linking characteristics. Its in-degree distribution follows a power law; the exponent is –2.3. The evolving rule proposed by this paper can describe and repeat the in-degree distribution rule of parts relation network of a product family. It will provide precise meaning for people to research parts relation network thoroughly. The in-degree increment calculation method based on the in-degree evolving rule proposed by this paper is feasible. When the node number increases, we can calculate the increment of every existing node of the network. The relation between kind number of parts and number of products and the relation between used number of parts or components and the in-degree weight of the edge is the basic requirement for forecasting used number of parts and components. Analysis of the result shows that, in parts relation network of a steam turbine product, the weight of most of the components and other parts is equal to 1. So, the forecast method for parts and components is suitable for these parts and components. It will provide a scientific basis for the corporation to manage its storage. Though the forecast method has some error, the error is within the range accepted by engineering. The error will reduce as the product increases; hence the forecast method has important practical meaning. Although we have considered the steam turbine product of a corporation as our example, the research methods and the conclusions have universal applicability and can provide helpful guidance for other corporations to forecast their used number of parts and components.

Acknowledgments This work was supported by the National Natural Science Foundation of China (Nos. 60374057 and 50575204) and the High-tech Research and Development Program of China (No. 2005AA411910).

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