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RESEARCH ARTICLE

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De-noising of diesel vibration signal using wavelet packet and singular value decomposition

Abstract The vibration signals of diesel include excess noise that must be eliminated before extraction of characteristic parameters. Firstly, the effects of vibration-signal de-noising among Fourier transform, wavelet decomposition and wavelet packet decomposition are compared. Secondly, singular value decomposition is applied to de-noising vibration signals. Finally, a new de-noise method integrated with wavelet packet and singular value is presented. In this method, vibration signals are decomposed by wavelet packet, and the wavelet packet coefficient is de-noised by singular value decomposition again. The results indicate that the new de-noising method is the best. The SNR (signal-to-noise ratio) of the vibration signals of a diesel cylinder lid is the highest. The diesel vibration waveforms of combustion and valve become clear and the extracted characteristic parameters become more precise.

Keywords diesel, vibration signal, de-noising, wavelet packet decomposition, singular value decomposition

1 Introduction

The diesel is a complicated machine with many types of vibration sources, such as combustion, piston-liner strike, valve strike, gearing mesh, natural vibration, etc. Because vibration signals are coupled with diverse actuators and interfered with noises, they are greatly complex, possessing typical nonstationary characteristics. The diagnostic conclusion is possible only if noises are entirely filtered. The traditional Fourier transform method shows contradiction between larger SNR and higher spatial resolution. Low-pass filters undoubtedly reduce noise, but at the same time result in distortion of signals. On the other hand, high-pass filters make the edge of the waveform steeper, but less noise is eliminated [1].

The characteristics of wavelet transform are permission of signal and restraint of noise, i.e., a good filter for restraining noise. Therefore, wavelet transform is suitable for filtering background noise and is used to de-noise signals [2]. The strange attractor obtained from the reconstruction of phase space is able to reflect the dynamic character of vibration system. Then the noise of vibration signals can be effectively reduced by decomposing track matrix with singular value.

This paper draws a comparison of de-noising among Fourier transform, wavelet decomposition and wavelet packet decomposition, and then presents a new de-noising method integrated with wavelet packet and singular value, expecting to achieve better de-noising effect.

2 De-noising using wavelet packet decomposition

The wavelet transform is a localization analysis of time and frequency. The size of its window function, namely, the window's area, is invariable, but its shape is variable. Thus, it has a relatively higher frequency resolution and lower time resolution in the low frequency band but a relatively higher time resolution and lower frequency resolution in the high frequency band. In practice, however, the frequency resolution in the high frequency band is expected to be strengthened. Overcoming the disadvantage of the wavelet, the wavelet packet can orthogonally decompose signals in the whole frequency band and extract the signal's characteristics with better self-adapter capability [3].

2.1 Definition of wavelet packet

In the method of multiresolution analysis, the formula

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$L^2(R) = \underset{j\in Z}{\oplus} W_j$ indicates that multiresolution analysis decomposes Hilbert space $L^2(R)$ into sum of all orthogonal subspaces $W_j\,(j \in Z)$ according to different scales j, where W_j is the closure of the wavelet basis function $\varphi(t)$, namely wavelet subspace. In terms of binary system, the wavelet packet analysis further decomposes the wavelet subspace to enhance resolution in the high frequency band. Suppose the series $\{h_n\}_{n\in Z}$ satisfies $\sum_n h_{n-2k}h_{n-2l} = \delta_{k,l}$, $\sum_n h_n = 2^{1/2}$. Now define recursion functions $u_n = L^2(R)$, n=1,2,…. They are generated from scale function $\phi(t)$ and wavelet basis function $\varphi(t)$, hence the formula

$$\begin{cases} u_0(t) = \phi(t), u_1(t) = \varphi(t) \\ u_{2n}(t) = 2^{1/2}\sum_k h_k u_n(2t-k) \\ u_{2n+1}(t) = 2^{1/2}\sum_k g_k u_n(2t-k) \end{cases} \tag{1}$$

where $g_k = (-1)^k h_{1-k}$, namely, the two coefficients are orthogonal, while $n = 0$, the $u_0(t)$ evolves into $\phi(t)$ and $u_1(t)$ into $\varphi(t)$ .We define series $\{u_n(t)\}_{n\in Z}$ generated from Eq. (1) as wavelet packet determined by basis function $u_0(t) = \phi(t)$ [4].

2.2 Steps for wavelet packet de-noising

First, the signals are decomposed by the wavelet packet. As noise usually exists in each layer, the wavelet packet coefficient is disposed with threshold. Finally, the disposed coefficient is reconstructed and the de-noised signals are obtained [5]. The main steps are as follows:

1) Decompose signals. On the basis of choosing a wavelet basis function and scale N, signals are decomposed.

2) Calculate best tree, i.e., best wavelet packet basis function. The best tree is calculated according to a given entropy standard.

3) Quantize the disposed coefficient. A suitable threshold is chosen for each coefficient, including lower frequency coefficient, and then the coefficient is quantized.

4) Reconstruct the quantized coefficient. According to the decomposed coefficient of the Nth scale and quantized coefficient, signals are reconstructed.

The key step is to choose threshold and to quantize the coefficient, which, to some extent, will influence the quality of de-noising. There are four rules for choosing threshold: 1) SURE (Stein's unbiased risk estimate); 2) fixed threshold; 3) heuristic threshold; and 4) minimax threshold. Rules of SURE and minimax are conservative to set part of the coefficient as zero, so these two rules are very useful in extracting weak signals when high frequency of signals rarely exists in the range of noise. The other rules are more effective in de-noising signals, but possibly eliminate the useful characteristic signals.

The vibration signals of diesel include not only low frequency of combustion but also high frequency of valve vibration; hence, the rule of SURE is effective in preserving the useful high frequency signals.

2.3 De-noising comparison of different methods

The de-noising comparison of Fourier filter, wavelet and wavelet packet are seen in Fig.1. Obviously, the signal of Fig.1 (b) is nonstationary; therefore, the traditional Fourier filter de-noising is of little effect. Moreover, the high frequency signals are eliminated unexpectedly, as shown in Fig.1(c).

Figure 1(d) shows the result of wavelet de-noising, which is better than Fourier filter in low frequency, but most high frequency signals are eliminated too. Fig.1 (e) shows the result of wavelet packet de-noising, which is the same effect as wavelet de-noising in low frequency, but the useful high frequency signals are well preserved. As a result, for the signals including low frequency and high frequency, the wavelet packet de-noising is more effective. For both the decomposition of wavelet and wavelet packet, wavelet basis function is Daubechies 4, scale is 3, and soft threshold is adopted.

3 Singular value decomposition de-noising

3.1 Principle

The time series constructed from complex dynamical system include signals reflecting inherence of the system and all kinds of noise. The latter is completely brought into embedded space matrix. Therefore, the decomposition of embedded space matrix is composed of two parts: one is relate to the inherence of system, the other is the noise. The two parts are interlaced and hence in the traditional method based on a different spectrum of noise and signal it is difficult to eliminate noise. However, singular value decomposition merely eliminates noise and has no influence on the signal. The main reason is that this de-noising method is based on the different influences of smooth signal and random noise on the track matrix singular value, and not based on their spectrum peculiarity. The key to this method is the choice of phase space dimension and the number of singular value. The choice cannot be made with comparison of different phase space dimension and the number of singular value until the result is satisfied.

For any signal X, its discrete time series is X=$[x_1, x_2, \ldots, x_n]$. Then, X is divided into sections with length n and a matrix $\boldsymbol{A}$ is constructed as follows:

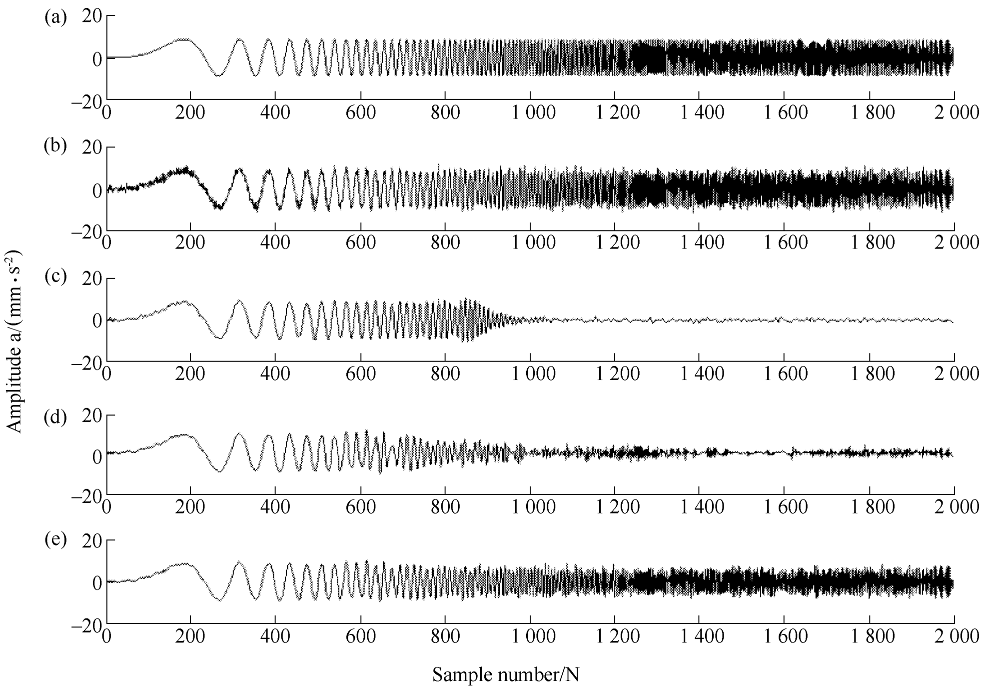

Fig. 1 Comparison of de-noising among Fourier filter, wavelet and wavelet packet. (**a**) Heavy sine, (**b**) Heavy sine superposed with Gauss White Noise, SNR is 6, (**c**) Fourier filter, (**d**) wavelet de-noising, (**e**) wavelet packet de-noising

$$A = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \\ x_{n+1} & x_{n+2} & \cdots & x_{2n} \\ \vdots & \vdots & & \vdots \\ x_{(m-1)n+1} & x_{(m-1)n+2} & \cdots & x_{mn} \end{bmatrix}$$

where $N = mn$, N is sample number.

Matrix $\boldsymbol{A}$ is decomposed by singular value method according to the formula $\boldsymbol{A} = \boldsymbol{USV}'$, where $\boldsymbol{U}$ is a $m \times m$ dimension matrix, $\boldsymbol{V}$ is $n \times n$ dimension matrix and $\boldsymbol{UU}' = \boldsymbol{I}$, $\boldsymbol{VV}' = \boldsymbol{I}$. $\boldsymbol{S}$ is a $m \times n$ dimension diagonal matrix, whose diagonal elements are $s_1, s_2, \cdots, s_p$, $p = \min(m, n)$, $s_1 \geqslant s_2 \geqslant \cdots \geqslant s_p$, where, $s_1, s_2, \cdots, s_p$ are called singular values of matrix $\boldsymbol{A}$; $\boldsymbol{U}$ is called left singular matrix and $\boldsymbol{V}$ is called right singular matrix. Matrix $\boldsymbol{A}$ can be rewritten as $\boldsymbol{A} = \sum_{i=1}^{p} \boldsymbol{u}_i s_i \boldsymbol{v}_i'$, where $\boldsymbol{u}_i, \boldsymbol{v}_i$ is respectively the column vector of $\boldsymbol{U}$ and $\boldsymbol{V}$. The signal energy can be expressed by $E = \sum_{i=1}^{p} s_i^2$. In terms of singular value decomposition theory, for different numbers of singular values, if the square sum of singular values satisfies the formula $\sum_{i=1}^{q} s_i^2 / \sum_{i=1}^{p} s_i^2 \approx 1$, the information of the original signal mostly exists in the signal expressed by $\boldsymbol{A}' = \sum_{i=1}^{q} \boldsymbol{u}_i s_i \boldsymbol{v}_i'$, where p and q are the numbers of singular value, and $q \leqslant p$. If n is the basic period of main pattern of the signal, the main information of the original signal mostly exists in the signal expressed by $\boldsymbol{A}' = \boldsymbol{U}_1 \boldsymbol{S}_1 \boldsymbol{V}_1'$. Matrix $\boldsymbol{A}'$ is an optimal approach to $\boldsymbol{A}$, where $\boldsymbol{A}$ is a reconstructed track matrix without noise. For $\boldsymbol{A}$, the noise is greatly reduced and the fault characteristic is extruded.

3.2 Example

Vibration signals on diesel cylinder lid are collected with sample frequency 20 kHz and sample number 1 024×60. The collected time domain data are reconstructed into phase space, with dimensions 52×52. Then the matrix is decomposed and singular value is $s_1, s_2, \cdots, s_{52}$. The relation between the number of singular value and energy percent is that while the number of singular value is 52, 22, 18, 15, 14, 13, 12, 11, 11, 10, 9, 8, the corresponding energy percent (%) is 100, 99, 98, 97, 96, 95, 94, 93, 92, 91, 90, 89. The number of singular value is decreased to 22 when energy percent is 99 %, but the de-noising is not ideal, as shown in Fig. 2 (b). When energy percent is 90 %, the number of singular value is decreased to 9, and the de-noising is comparatively ideal, as shown in Fig. 2 (c). If the number of singular value is less than 9, some useful information will be simultaneously eliminated.

4 A new de-noising method integrated with wavelet packet and singular value

From the above analysis, we can conclude that the 3 de-noising

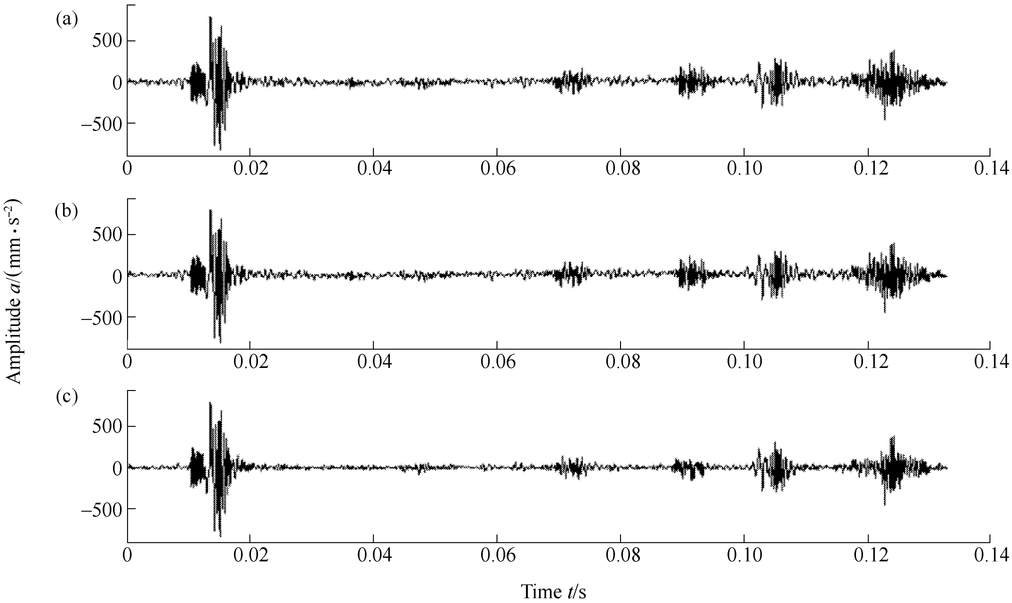

Fig. 2 Singular value de-noising effect of vibration signals of diesel cylinder lid. (**a**) Original signal, (**b**) While the number of singular value is 22, (**c**) While the number of singular value is 9

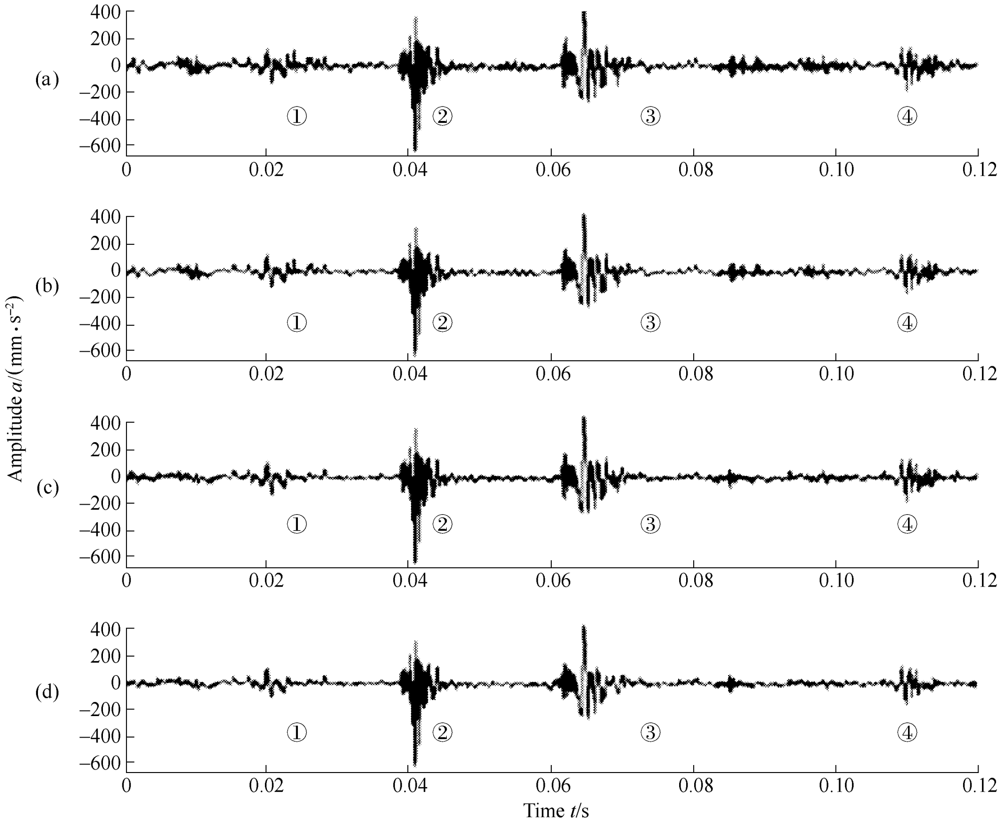

Fig. 3 De-noising comparison of vibration signals of diesel cylinder lid using wavelet packet, singular value and the new method. (**a**) Original signal, (**b**) Using wavelet packet, (**c**) Using singular value, (**d**) Using the new method integrated with wavelet packet and singular value

method integrated with wavelet packet and singular value have better effects. Fig.3 (a) is the vibration time domain of diesel cylinder lid of mode PZ12V190. Although the vibration waveforms of different phases can be roughly identified, the characteristic of the waveform is not extruded for the influence of noise.

According to the theoretical analysis and many experiments, the vibration signals of diesel are decomposed by wavelet packet while basis function is daubechies 4 and scale is 5. Then the wavelet packet coefficient is decomposed by singular value decomposition again, where the energy percent is 90 %, that is, the first 9 singular values are used to reconstruct the signal. The de-noised waveform is show in Fig. 3 (d). By analyzing, the sections ①～④ are respectively related to the adjacent cylinder combustion vibration, current cylinder inlet valve vibration, combustion vibration and outlet valve vibration. These four phase signals are used to diagnose faults; hence, the useful signals and the others are regarded as noise. Hereby, the SNR of signal de-noised by the method integrated wavelet packet and singular value is 5.77. The SNR of Fig. 3 (a) to Fig. 3(c) is respectively 3.74, 4.02 and 4.82. It is obvious that the effect of the new de-noising method is the best.

5 Conclusions

1) By comparing de-noising among Fourier filter, wavelet and wavelet packet, the last one is found to be the best. Not only the low frequency signals but also the useful high frequency signals are well preserved.

2) When the vibration energy remains 90 %, the number of the singular value is 9, and the singular value method shows good de-noising effect. If the number of singular value is less than 9, useful information will simultaneously be eliminated.

3) The new method integrated with wavelet packet and singular value brings about a better de-noising result. Using the method, the diesel vibration waveform of combustion and valve becomes clear and the extracted characteristic parameters become more precise.

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