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SURVEY ARTICLE

Survey on path and cycle embedding in some networks*

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Abstract To find a cycle (resp. path) of a given length in a graph is the cycle (resp. path) embedding problem. To find cycles of all lengths from its girth to its order in a graph is the pancyclic problem. A stronger concept than the pancylicity is the panconnectivity. A graph of order n is said to be panconnected if for any pair of different vertices x and y with distance d there exist xy-paths of every length from d to n. The pancyclicity or the panconnectivity is an important property to determine if the topology of a network is suitable for some applications where mapping cycles or paths of any length into the topology of the network is required. The pancyclicity and the panconnectivity of interconnection networks have attracted much research interest in recent years. A large amount of related work appeared in the literature, with some repetitions. The purpose of this paper is to give a survey of the results related to these topics for the hypercube and some hypercube-like networks.

Keywords Cycle, path, pancyclicity, hamiltonicity, panconnectivity, fault-tolerance, hypercube-like network, embedding
MSC 05C38, 90B10

1 Introduction

It is well known that a topological structure of an interconnection network can be modeled by a connected graph G. We follow Ref. [153] for graph-theoretical terminology and notation not defined here. A graph $G = (V, E)$ always means a simple and connected graph, where $V = V(G)$ is the vertex-

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set and $E = E(G)$ is the edge-set of G.

There are a lot of mutually conflicting requirements in designing the topology of an interconnection network. It is almost impossible to design a network which is optimum from all aspects. One has to design a suitable network depending on the requirements and its properties. One of the central issues in designing and evaluating an interconnection network is to study how well other existing networks can be embedded into this network. This problem can be modeled by the following graph embedding problem: given a host graph H, which represents the network into which other networks are to be embedded, and a guest graph G, which represents the network to be embedded, the problem is to find a mapping from $V(G)$ to $V(H)$ such that each edge of G can be mapped to a path in H. Two common measures of effectiveness of an embedding are the dilation, which measures the slowdown in the new architecture, and the load factor, which gauges the processor utilization [153].

A graph embedding has two main applications: to transplant parallel algorithms developed for one network to a different one, and to allocate concurrent processes to processors in the network.

The most ideal embedding is an isomorphic embedding, that is, the guest graph is isomorphic to a subgraph of the host graph, since such an embedding has both dilation and load one.

As two common guest graphs, linear arrays (i.e., paths) and rings (i.e., cycles) are two fundamental networks for parallel and distributed computation. They are suitable for developing simple algorithms with low communication cost. Many efficient algorithms were originally designed based on linear arrays and rings for solving a variety of algebraic problems, graph problems and some parallel applications, such as those in image and signal processing (see, for example, Refs. [4,105]). Thus, it is important to have an effective path and/or cycle embedding in a network. The path and/or cycle embedding properties of many interconnection networks have been investigated in the literature.

A graph G of order n is *k-pancyclic* ($k \leqslant n$) if it contains cycles of every length from k to n inclusive, and G is *pancyclic* if it is g-pancyclic, where $g = g(G)$ is the girth of G. A graph is of *pancyclicity* if it is pancyclic. The pancyclicity, which means the hamiltonicity, is an important property to determine if a topology of a network is suitable for some applications where mapping cycles of any length into the topology of the network is required.

The concept of pancyclicity, proposed first by Bondy [13], has been extended to vertex-pancyclicity [63] and edge-pancyclicity [7]. A graph G of order n is *vertex-pancyclic* (resp. *edge-pancyclic*) if any vertex (resp. edge) lies on cycles of every length from $g(G)$ to n inclusive. Obviously, an edge-pancyclic graph is certainly vertex-pancyclic.

A graph G is said to be *hamiltonian connected* if there exists a hamiltonian path between any two vertices of G [122]. A graph G of order n is said to be *panconnected* if for any pair of different vertices x and y with distance d in G, there exist xy-paths of every length from d to $n - 1$ [5,151]. A graph is of *panconnectivity* if it is panconnected. Clearly, a panconnected graph is certainly edge-pancyclic.

There exist some graphs indicating that the above concepts are not equivalent. Fig. 1 shows the containment relationships of these hamiltonian-like properties for graphs with at least three vertices.

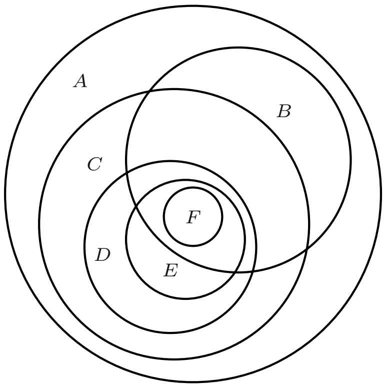


Fig. 1 Containment relationships of hamiltonian-like properties.
A: hamiltonian; B: hamiltonian-connected; C: pancyclic;
D: vertex-pancyclic; E: edge-pancyclic; F: panconnected

Since a bipartite graph contains no odd cycles, the concept of bipancyclicity is proposed. A graph G of order n is *bipancyclic* (also called *even-pancyclic* by some authors) if it contains cycles of every even length from $g(G)$ to n if n is even or $n-1$ if n is odd [121]. It is easy to see that any hamiltonian bipartite graph has no hamiltonian path between any two vertices in the same partite set. For this reason, Simmons [131] introduced the concept of hamiltonian laceability for bipartite graphs. A hamiltonian bipartite graph is *hamiltonian laceable* if there is a hamiltonian path between any two vertices in different partite sets.

Hsieh et al. [71] extended this concept to strongly hamiltonian laceability. A hamiltonian bipartite graph G is *strongly hamiltonian laceable* if it is hamiltonian laceable and there is a path of length $n-2$ between any two vertices in the same partite set. Lewinter and Widulski [106] introduced another concept, hyper hamiltonian laceability. A hamiltonian bipartite graph G is *hyper hamiltonian laceable* if it is hamiltonian laceable and for any vertex x in one partite set, there is a hamiltonian path of $G-x$ between any two vertices in the other partite set. So the hyper hamiltonian laceability is definitely also strongly hamiltonian laceability. Chang et al. [17] generalized the concept of hamiltonian laceability to *super laceability*. A connected bipartite graph G with connectivity $\kappa(G)$ is *super laceable* if for any two distinct vertices x to y from different partite sets and any integer k with $1 \leqslant k \leqslant \kappa(G)$, there exist k disjoint paths between x to y that contains all vertices of G.

Li et al. [110] generalized the concept of hamiltonian laceability to bipanconnectivity. A graph G of order n is *bipanconnected* if for any pair of different vertices x and y with distance d in G, there is an xy-path of length l for any l with $d \leqslant l \leqslant n-1$ and $l-d \equiv 0 \pmod 2$. Clearly, a bipanconnected bipartite graph is certainly hamiltonian laceable, strongly hamiltonian laceable, but not always hyper hamiltonian laceable.

Fault tolerance is also desirable in massive parallel systems that have a relatively high probability of failure. A graph G is said to be *faulty* if it has at least one faulty vertex or edge. The fault tolerance ability is a major factor in evaluating the performance of networks.

A graph G is *k-fault-tolerant hamiltonian* (resp., *connected, hamiltonian connected, pancyclic*) if $G-F$ remains hamiltonian (resp., *connected, hamiltonian connected, pancyclic*) for any $F \subset V(G) \cup E(G)$ with $|F| \leqslant k$, and is *k-vertex-fault-tolerant hamiltonian* (resp., *connected, hamiltonian connected, pancyclic*) if $G-F$ remains hamiltonian (resp., *connected, hamiltonian connected, pancyclic*) for any $F \subset V(G)$ with $|F| \leqslant k$, and *k-edge-fault-tolerant hamiltonian* (resp., *connected, hamiltonian connected, pancyclic*) if $G-F$ remains hamiltonian (resp., *connected, hamiltonian connected, pancyclic*) for any $F \subset E(G)$ with $|F| \leqslant k$ [70,91].

Use f_v and f_e to denote the numbers of faulty vertices and faulty edges in G, respectively.

A large amount of related work for several interconnection networks have appeared in the literature on the above topics, some of which are repeated. The present paper is mostly concerned about the hypercube network and its variations. We attempt to give a survey of known results on the above-mentioned topics for the hypercube network and some well-known variations of the hypercube network, from which we can find that many problems have not yet been solved.

2 Hypercubes

The n-dimensional hypercube Q_n has the vertex-set $V = \{x_1x_2\cdots x_n\colon\ x_i \in \{0,1\},\ i=1,2,\ldots,n\}$, and two vertices x and y are linked by an edge if and only if they differ exactly in one coordinate. The graphs shown in Fig. 2 are $Q_1,\ Q_2,\ Q_3$ and Q_4.

The hypercube Q_n is an n-regular n-connected bipartite graph with 2^n vertices. Q_n has a diameter of n and an average distance of about $n/2$ for a large n. Moreover, Q_n is a Cayley graph and hence vertex-transitive, and also edge-transitive. Other properties of Q_n obtained early is surveyed in Ref. [59].

Saad and Schultz [127] proved that for every even l with $4 \leqslant l \leqslant 2^n$, there is a cycle of length l in Q_n for $n \geqslant 2$. This result means that Q_n is bipancyclic for $n \geqslant 2$. Li et al. [110] improved this result by proving that Q_n is edge-bipancyclic for $n \geqslant 2$.

Theorem 2.1 (Li et al. [110]) *Q_n is bipanconnected if $n \geqslant 2$.*

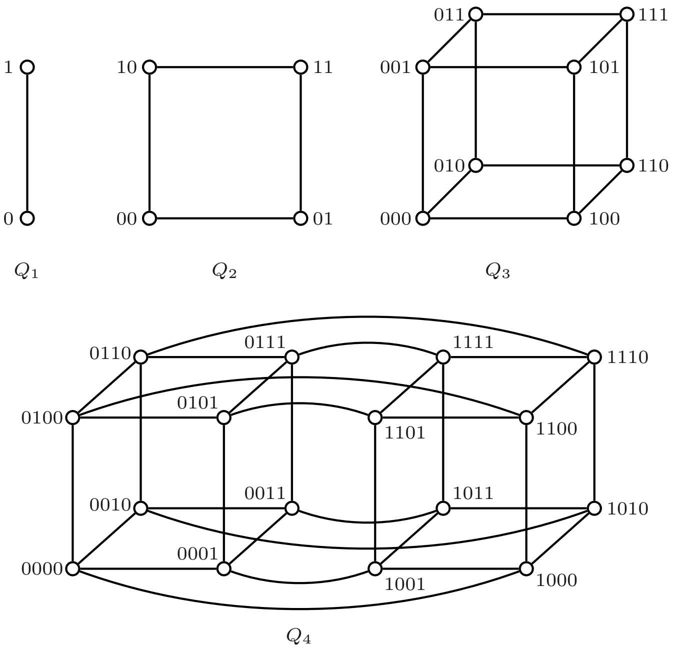


Fig. 2 Hypercubes Q_1, Q_2, Q_3 and Q_4

Recently, Tsai and Jiang [138] proposed a stronger concept than edge-bipancyclicity, called the path bipancyclicity. A bipartite graph G of order n is *k-path bipancyclic* if every path P of length m lies in a cycle of every even length from $\max\{4, 2m\}$ to n inclusive, where $1 \leqslant m \leqslant k$. From this definition, it is clear that every k-path bipancyclic graph is edge-bipancyclic where $k \geqslant 1$.

Theorem 2.2 (Tsai and Jiang [138]) *Q_n is $(2n-4)$-path bipancyclic for $n \geqslant 3$.*

Next, we consider a path or a cycle embedding under the case that faulty vertices and/or faulty edges appear in Q_n. In the first place, we consider the case of only faulty edges. It is clear that the condition $f_e \leqslant n-2$ is necessary to guarantee that Q_n is f_e-edge-fault-tolerant hamiltonian. Alspach et al. [6] showed that Q_n contains $\lfloor n/2 \rfloor$ edge-disjoint hamiltonian cycles. This result implies that Q_n is $(\lfloor n/2 \rfloor - 1)$-edge-fault-tolerant hamiltonian. Later, it was shown that Q_n is $(n-2)$-edge-fault-tolerant hamiltonian for $n \geqslant 2$ by several authors, for example, Chen and Shin [26], Leu and Kuo [107], Litifi et al. [104] and Sen et al. [128]. Li et al. [110] improved this result by proving that Q_n is $(n-2)$-edge-fault-tolerant edge-bipancyclic for $n \geqslant 2$. Although the condition $f_e \leqslant n-2$ is necessary for Q_n to be f_e-edge-fault-tolerant hamiltonian, Sengupta [129] showed that Q_n is $(n-1)$-edge-fault-tolerant hamiltonian if $n \geqslant 4$ and any vertex is incident with at least two fault-free edges. Xu et al. [154] further improved these results by proving the following theorem.

Theorem 2.3 (Xu et al. [154]) *Q_n is $(n-1)$-edge-fault-tolerant edge-6-bipancyclic provided $n \geqslant 3$ if any vertex is incident with at least two fault-free edges.*

Obviously, from Theorem 2.3, we have when $f_e \leqslant n-1$ and all faulty edges are not incident with the same vertex if $f_e = n-1$, then a fault-free cycle of length 2^n in Q_n is a hamiltonian cycle, which is a result of Sengupta [129]. If $f_e \leqslant n-2$, then any vertex is incident with at least two fault-free edges since Q_n is n-regular, which satisfies the hypothesis in Theorem 2.3. Also, any edge e in Q_n lies on exactly $n-1$ cycles of length four. If $f_e \leqslant n-2$, then every edge in Q_n must lie on a fault-free cycle of length four. Thus, Theorem 2.3 implies the result of Li et al. [110]. The results of Sengupta [129] and Li et al. [110] can also be improved by the following result.

Theorem 2.4 (Xu et al. [157]) *For any two different vertices x and y with distance d in Q_n, if $f_e \leqslant n-2$ and $n \geqslant 2$, then Q_n contains a fault-free xy-path of length l for every l with $d+2 \leqslant l \leqslant 2^n - 1$, where l and d have the same parity.*

If we restrict all faulty edges not to be incident with the same vertex if $f_e = n-1$, then Theorem 2.4 can be improved as follows.

Theorem 2.5 (Jing et al. [96]) *For any two different vertices x and y with distance d in Q_n, if $f_e \leqslant n-1$ $(n \geqslant 4)$ and all faulty edges are not incident with the same vertex if $f_e = n-1$, then Q_n contains a fault-free xy-path of length l for every l with $d+4 \leqslant l \leqslant 2^n - 1$, where l and d have the same parity.*

In 1989, Esfahanian [46] proved that Q_n is $(2n-3)$-edge-fault-tolerant connected if every vertex is incident with at least one fault-free edge and $n \geqslant 2$. Thus, it is quite natural to consider a path or a cycle embedding in Q_n for more faulty edges. Chan and Lee [15] showed that Q_n is $(2n-5)$-edge-fault-tolerant hamiltonian provided $n \geqslant 3$ and every vertex is incident with at least two fault-free edges; but there exists a Q_n with $2n-4$ faulty edges and where every vertex is incident with at least two fault-free edges not containing a hamiltonian cycle. Tsai [136] improved this result by showing that Q_n is $(2n-5)$-edge-fault-tolerant 4-bipancyclic provided $n \geqslant 3$ and any vertex is incident with at least two fault-free edges. This is improved by Tsai and Lai [139], Shih et al. [130], independently, by showing that Q_n is $(2n-5)$-edge-fault-tolerant edge-6-bipancyclic and any vertex is incident with at least two fault-free edges for $n \geqslant 3$. Recently, these results have been further improved as follows.

Theorem 2.6 (Wang, Wang and Xu [149]) *If Q_n has at most $(2n-5)$ faulty edges and every vertex is incident with at least two fault-free edges, then for any two distinct vertices x and y with distance d, there exists a fault-free xy-path of length l for every l with $d+4 \leqslant l \leqslant 2^n - 1$, where l and d have the same parity and $n \geqslant 3$.*

In case of considering only faulty vertices, Provost and Melhem [126] developed an algorithm that is able to find a fault-free cycle of length $3\times 2^{n-2}$ in Q_n if $f_v = 1$, or a fault-free cycle of length 2^{n-1} if $f_v = 2$. This result is significantly improved by Chan and Lee [16] by showing that Q_n contains a fault-free cycle of length at least $2^n - 2f_v$ if $f_v \leqslant \lfloor (n+1)/2 \rfloor$. Yang et al. [164] further showed that Q_n contains a fault-free cycle of length at least $2^n - 2f_v$ if $1 \leqslant f_v \leqslant n-2$. Fu [53] improved this result by tolerating more faults up to $2n-4$.

Theorem 2.7 (Fu [53]) *Q_n contains a fault-free cycle of length at least $2^n - 2f_v$ if $f_v \leqslant 2n-4$ and $n \geqslant 3$.*

In the case where both faulty vertices and faulty edges are considered, Tseng [145] showed that Q_n contains a fault-free cycle of length at least $2^n - 2f_v$ if $f_e \leqslant n-4$ and $f_v + f_e \leqslant n-1$. Sengupta [129] generalized this result by showing that Q_n contains a fault-free cycle of length $2^n - 2f_v$ if $f_v > 0$ or $f_e \leqslant n-2$, and $f_v + f_e \leqslant 2n-4$.

Sun et al. [133] showed that $Q_n - F$ is hyper hamiltonian if $|F| = f_{av} + f_e \leqslant n-3$ for $n \geqslant 3$, where f_{av} is the number of disjoint pairs of adjacent vertices in Q_n. Hsieh [65] has improved the result of Sun et al. by showing that there exists a fault-free cycle of length at least $2^n - 2f_v$ in Q_n if $f_e \leqslant n-2$ and $f_e + f_v \leqslant 2n-4$ for $n \geqslant 3$. Hsieh and Shen [80] proved that every fault-free edge of Q_n lies on a cycle of every even length from 4 to $2^n - 2f_v$ in Q_n if $f_v + f_e \leqslant n-2$ and $n \geqslant 3$. Recently, Tsai [137] has improved the two results and the result of Xu et al. [154] (i.e., Theorem 2.3 above) by proving the following theorem.

Theorem 2.8 (Tsai [137]) *If $f_e + f_v \leqslant n-2$ and $n \geqslant 3$, then every fault-free edge and fault-free vertex of Q_n lie on a fault-free cycle of every even length from 4 to $2^n - 2f_v$. If $f_e + f_v = n-1$ and every fault-free vertex is incident with at least two fault-free edges, then every fault-free edge and fault-free vertex of Q_n for $n \geqslant 4$ lie on a fault-free cycle of every even length from 6 to $2^n - 2f_v$. Furthermore, Q_n for $n \geqslant 5$ has a fault-free cycle of every even length from 4 to $2^n - 2f_v$ if $f_e \leqslant n-2$ and $f_e + f_v \leqslant 2n-4$.*

Du et al. [43] obtained the following result which can tolerate more edge-faults.

Theorem 2.9 (Du et al. [43]) *Q_n $(n \geqslant 3)$ contains a fault-free cycle of length at least $2^n - 2f_v$ provided that $f_v + f_e \leqslant 2n-4$, $f_e \leqslant 2n-5$ and each vertex is incident with at least two non-faulty edges.*

As regards to fault-tolerant panconnectivity, Fu [54] showed that for any two distinct fault-free vertices x and y with distance d in Q_n, if d is odd (or even), then there exists a fault-free xy-path with length at least $2^n - 2f_v - 1$ (or $2^n - 2f_v - 2$) when $f_v \leqslant n-2$ and $n \geqslant 3$. Since Q_n is bipartite, the path of length $2^n - 2f_v - 1$ (or $2^n - 2f_v - 2$) turns out to be the longest if all faulty nodes belong to the same partite set. Kueng et al. [102] and Ma et al. [113], respectively, improved this result by showing the following two theorems.

Theorem 2.10 (Kueng et al. [102]) *If $f_v \leqslant 2n-5$ and every vertex has* *at least two fault-free neighbors, then for any two distinct fault-free vertices x and y with distance d in Q_n, there exists a fault-free xy-path of length at least $2^n - 2f_v - 1$ (resp. $2^n - 2f_v - 2$) if d is odd (resp. even) distance and $n \geqslant 3$.*

Theorem 2.11 (Ma et al. [113]) *For any two distinct fault-free vertices x and y with distance d in Q_n, there exists a fault-free xy-path of length l with $f_v + f_e \leqslant n-2$ for each l satisfying $d+2 \leqslant l \leqslant 2^n - 2f_v - 1$, where l and d have the same parity and $n \geqslant 3$.*

The bounds on path length l and faulty set size $f_v + f_e$ for a successful embedding are tight. That is, the result does not hold if $l < d+2$ or $l > 2^n - 2f_v - 1$ or $f_v + f_e > n-2$.

Lastly, we consider the hamiltonian laceability of Q_n. Harary and Lewinter [60] proved that Q_n is strongly hamiltonian laceable if and only if $n \geqslant 2$. Lewinter and Widulski [106] proved that Q_n is hyper hamiltonian laceable if and only if $n \geqslant 3$. Theorem 2.4 shows that Q_n is $(n-2)$-edge-fault-tolerant hamiltonian laceable and strongly hamiltonian laceable for $n \geqslant 2$. This result was also obtained by Tsai et al. [141], independently. Hsieh and Kuo [74], and Tsai et al. [141] showed that Q_n is $(n-3)$-edge-fault-tolerant hyper hamiltonian laceable for $n \geqslant 3$. Sun et al.[1)] proved that Q_n is $(n-3)$-fault-tolerant hamiltonian laceable and strongly hamiltonian laceable and hyper hamiltonian laceable for $n \geqslant 3$ if $f_{av} + f_e \leqslant n-3$. Chang et al. [17] generalized the concept of the hamiltonian laceability to *super laceability* for bipartite graphs. A connected bipartite graph G with connectivity $\kappa(G)$ is *super laceable* if for any two distinct vertices x to y from different partite sets and any integer k with $1 \leqslant k \leqslant \kappa(G)$, there exist k disjoint paths between x to y that contains all vertices of G.

Theorem 2.12 (Chang et al. [17]) *Q_n is super laceable, moreover, $(n-2)$-edge-fault-tolerant super laceable for any $n \geqslant 1$.*

We conclude this section with an interesting result obtained by Chen [27].

Theorem 2.13 (Chen [27]) *Let $n > h \geqslant 2$, $F \subset E(Q_n)$ with $|F| < n-h$, and $E_0 \subset E(Q_n) \setminus F$ with $|E_0| = h$. If the subgraph induced by E_0 consists of pairwise vertex-disjoint paths, then in the graph $Q_n - F$ all edges of E_0 lie on a cycle of every even length l with*

$$2^{h-1}(n+1-h) + 2(h-1) \leqslant l \leqslant 2^n.$$

Chen [27] also gave an example to show that when $h = 2$ the result in Theorem 2.13 is optimal in the following sense. Q_n contains two edges such that any cycle in Q_n passing through them is of length at least $2n$, and edge subsets E_0 and F with $|E_0| = 2$ and $|F| = n-2$ such that no Hamilton cycle passes through the two edges of E_0 in $Q_n - F$.

1) Sun C -M, Hung C -N, Huang H -M, Hsu L -H. Hamiltonian laceability of faulty hypercubes. 2008

3 Folded hypercubes

The n-dimensional folded hypercube, denoted by FQ_n, is a graph obtained from Q_n by adding all complementary edges, which join a vertex $x = x_1x_2\cdots x_n$ to another vertex $\overline{x} = \overline{x}_1\overline{x}_2\cdots\overline{x}_n$ for every $x \in V(Q_n)$, where $\overline{x}_i = 1 - x_i$. The graphs shown in Fig. 3 are FQ_3 and FQ_4, respectively.

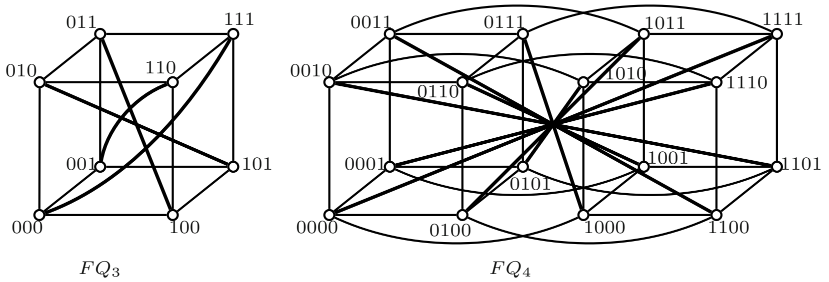


Fig. 3 FQ_3 and FQ_4 (thick lines represent the complementary edges)

The folded hypercube FQ_n, proposed by El-Amawy and Latifi [45], is an $(n+1)$-regular $(n+1)$-connected graph with 2^n vertices. Like Q_n, FQ_n is a Cayley graph and hence vertex-transitive. However, FQ_n has a diameter of $\lceil n/2 \rceil$, superior to Q_n. Xu and Ma [155] showed that FQ_n is bipartite if and only if n is odd, the length of a shortest odd cycle is $n+1$ if n is even, and obtained the following result on the pancyclicity of FQ_n.

Theorem 3.1 (Xu and Ma [155]) *For $n \geqslant 2$, FQ_n is edge-even-pancyclic; and if n is even, then FQ_n is also edge-$(n+1)$-odd-pancyclic.*

As regards to the panconnectivity, Ma and Xu [119] obtained the following result.

Theorem 3.2 (Ma and Xu [119]) *For any two vertices x and y with distance d, FQ_n contains an xy-path of every length l with $h \leqslant l \leqslant 2^n - 1$ provided l and h have the same parity, where $h \in \{d, n+1-d\}$.*

By Theorem 3.2, if n is odd then FQ_n is bipanconnected, and if n is even then for any two different vertices x and y with distance d in FQ_n, there is an xy-path of every length l with $n - d + 1 \leqslant l \leqslant 2^n - 1$ and every length l' with $d \leqslant l' \leqslant n - d$ provided l' and d have the same parity. This result is the best since the length of the shortest odd cycle is $n+1$ in FQ_n if n is even.

In the case where only faulty edges are considered, Wang [148] showed that FQ_n is $(n-1)$-edge-fault-tolerant hamiltonian for $n \geqslant 2$. It is clear that if FQ_n has at most $(n-1)$ faulty edges, then each vertex is incident with at least two fault-free edges since FQ_n is $(n+1)$-regular. Thus, the following theorems generalize the result of Wang.

Theorem 3.3 (Xu, Ma and Du [157]) *For $n \geqslant 3$, FQ_n is $(n-1)$-edge-fault-tolerant edge-even-pancyclic; if n is even, FQ_n is also $(n-1)$-edge-fault-tolerant edge-$(n+1)$-odd-pancyclic.*

Theorem 3.4 (Ma, Xu and Du [120]) *For $n \geqslant 3$, FQ_n is $(2n-3)$-edge-fault-tolerant hamiltonian if each vertex is incident with at least two fault-free edges.*

In the case where both faulty vertices and faulty edges are considered, Hsieh [67] used Theorem 3.3, and Hsieh et al.[1)], independently, showed that FQ_n contains a fault-free cycle with length at least $2^n - 2f_v$ if $f_v + f_e \leqslant n-1$ for $n \geqslant 4$. Fu [56] improved the number of faults tolerated and showed the following result.

Theorem 3.5 (Fu [56]) *FQ_n contains a fault-free cycle with length at least $2^n - 2f_v$ if $f_v + f_e \leqslant 2n-4$ and $f_e \leqslant n-1$, where $n \geqslant 3$.*

For panconnectivity and laceability of FQ_n, Hsieh and Kuo [75] showed that FQ_n is strongly hamiltonian-laceable when n is odd, and is hamiltonian-connected when $n = 1$ or n $(\geqslant 2)$ is even. Recently, Hsieh [66] has improved this result as follows.

Theorem 3.6 (Hsieh [66]) *FQ_n is $(n-2)$-edge-fault-tolerant hamiltonian-connected if $n(\geqslant 2)$ is even, $(n-1)$-edge-fault-tolerant strongly hamiltonian-laceable if $n(\geqslant 1)$ is odd, and $(n-2)$-edge-fault-tolerant hyper hamiltonian-laceable if $n(\geqslant 3)$ is odd.*

For more faulty edges, considering that any vertex is incident with at least three fault-free edges is a necessary condition for a graph to be hamiltonian-connected, and any vertex is incident with at least two fault-free edges is a necessary condition for a graph to be hamiltonian-laceable, Chen [28] showed the following result.

Theorem 3.7 (Chen [28]) *FQ_n is $(2n-5)$-edge-fault-tolerant hamiltonian-connected if n $(\geqslant 4)$ is even and any vertex of FQ_n is incident with at least three fault-free edges, and $(2n-4)$-edge-fault-tolerant strongly hamiltonian-laceable if n $(\geqslant 3)$ is odd and any vertex is incident with at least two fault-free edges.*

In the case where both faulty vertices and faulty edges are considered, Chen [28] obtained the following result.

Theorem 3.8 (Chen [28]) *If $f_v + f_e \leqslant n-2$ and $n \geqslant 2$, then for any two distinct fault-free vertices x and y with distance d, FQ_n contains a fault-free xy-path of every length l with*

$$d + 2 \leqslant l \leqslant 2^n - 2f_v - 1$$

provided l and d have the same parity, and a fault-free xy-path of every length l with

$$n - 1 \leqslant l \leqslant 2^n - 2f_v - 1$$

provided n $(\geqslant 2)$ is even.

1) Hsieh S -Y, Kuo C -N, Huang H -L. Fault-tolerance ring embedding on folded hypercubes with faulty elements. Parallel Computing, a manuscript, 2007-09-05

4 Crossed cubes

Two binary strings $x = x_2x_1$ and $y = y_2y_1$ are *pair-related*, denoted by $x \sim y$, if and only if $(x, y) \in \{(00, 00), (10, 10), (01, 11), (11, 01)\}$.

The n-dimensional crossed cube, denoted by CQ_n, is such a graph, its vertex-set is the same as Q_n, two vertices $x = x_n \cdots x_2x_1$ and $y = y_n \cdots y_2y_1$ are linked by an edge if and only if there exists j $(1 \leqslant j \leqslant n)$ such that

(a) $x_n \cdots x_{j+1} = y_n \cdots y_{j+1}$,

(b) $x_j \neq y_j$,

(c) $x_{j-1} = y_{j-1}$ if j is even, and

(d) $x_{2i}x_{2i-1} \sim y_{2i}y_{2i-1}$ for each $i = 1, 2, \ldots, \lceil j/2 \rceil - 1$.

The graphs shown in Fig. 4 are CQ_3 and CQ_4.

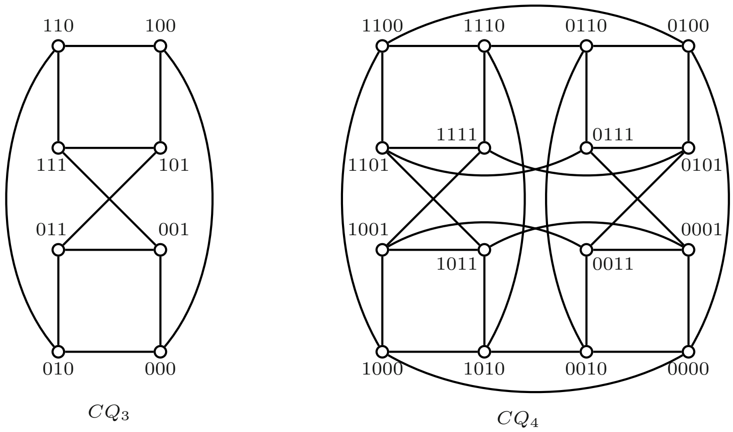


Fig. 4 Crossed cubes CQ_3 and CQ_4

The crossed cube CQ_n was first proposed by Efe [44]. Like Q_n, CQ_n is an n-regular n-connected graph with 2^n vertices. Furthermore, CQ_n has a diameter of $\lceil (n+1)/2t \rceil$, superior to Q_n. Moreover, CQ_n is not vertex-transitive if $n \geqslant 5$ proved by Kulasinghe and Bettayeb [103] and not edge-transitive if $n \geqslant 3$ proved by Huang and Xu [92]. This lack of symmetry removes the crossed cubes from the class of Cayley graphs if $n \geqslant 5$. For $n = 1, 2, 3, 4$, CQ_n is a Cayley graph.

Efe [44], Chang et al. [18] and Huang et al. [88], independently, showed that CQ_n is pancyclic for $n \geqslant 2$. This result was generalized by several authors, independently.

Theorem 4.1 (Fan et al. [49], Hu et al. [86], Ma and Xu [116], Yang et al. [166]) *CQ_n is edge-pancyclic for $n \geqslant 2$.*

In the case where only faulty edges are considered, Hung et al. [95] showed that CQ_n is $(2n-5)$-edge-fault-tolerant hamiltonian if every vertex is incident with at least two fault-free edges for $n \geqslant 3$.

In the case where both faulty vertices and faulty edges are considered, Huang et al. [90] and Chen et al. [29] showed, independently, that CQ_n is $(n-2)$-fault-tolerant hamiltonian for $n \geqslant 3$. Yang et al. [161] improved this by showing the following result.

Theorem 4.2 (Yang et al. [161]) *CQ_n is $(n-2)$-fault-tolerant pancyclic for $n \geqslant 3$.*

As regards to panconnectivity of CQ_n, Fan et al. [50] showed that for any two different vertices x and y in CQ_n, there exists an xy-path of every length from $\lceil (n+1)/2 \rceil + 1$ to $2^n - 1$ for $n \geqslant 3$. This result was improved by several authors, independently.

Theorem 4.3 (Fan et al. [48,50], Xu et al. [156], Yang et al. [166]) *For any two vertices x and y with distance d in CQ_n with $n \geqslant 2$, CQ_n contains an xy-path of every length l from d to $2^n - 1$ except for $d+1$.*

In Theorem 4.3, the length $l = d+1$ has to be removed. In fact, it is easy to find that for any two integers $n \geqslant 2$ and l with $1 \leqslant l \leqslant \lceil (n+1)/2 \rceil - 1$, there always exist two distinct vertices x and y in CQ_n with distance l and no xy-path of length $l+1$ in CQ_n. Recently, Hsu and Lai[1)] have given a necessary and sufficient condition to check the existence of the path of length $d_{CQ_n}(x, y) + 1$, called the nearly shortest path, for any two distinct vertices x, y in CQ_n. Moreover, only some pair of vertices have no nearly shortest path and give a construction scheme for the nearly shortest path if it exists.

As regards to fault-tolerant panconnectivity of CQ_n, Huang et al. [90] and Chen et al. [29], independently, showed that CQ_n is $(n-3)$-fault-tolerant hamiltonian connected for $n \geqslant 3$. Recently, Ma et al. [115] have improved this result as follows.

Theorem 4.4 (Ma et al. [115]) *If $f_v + f_e \leqslant n-3$, then for any two distinct fault-free vertices x and y in CQ_n and for each l with $2^{n-1} - 1 \leqslant l \leqslant 2^n - f_v - 1$, there exists a fault-free xy-path of length l for $n \geqslant 3$.*

In Theorem 4.4, the lower bound on l and the upper bound of $f_v + f_e$ for a successful embedding are tight for some n. In other words, the result may not hold if $l \leqslant 2^{n-1} - 2$ or $f_v + f_e \geqslant n - 2$.

For more faults, we can state only the following result.

Theorem 4.5 (Hsieh and Lee [76]) *If each vertex is incident to at least two fault-free edges, then CQ_n is $(2n-5)$-edge-fault-tolerant hamiltonian.*

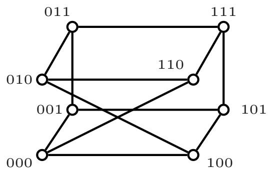
5 Twisted cubes

In the literature, there are several twisted cubes, for example, see a brief survey [40]. The n-dimensional twisted cube, denoted by TQ_n, was proposed by Hilbers et al. [62]. The authors only consider TQ_n for odd value of n

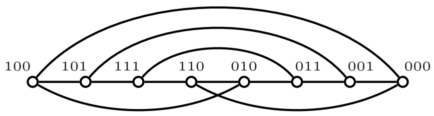
1) Hsu H -C, Lai P -L. Constructing the nearly shortest path in crossed cubes. 2008

exclusively. TQ_n is a variant of Q_n, and has the same vertex-set of Q_n. To form TQ_n, we remove some edges from Q_n and replace them with edges that span two dimensions in such a manner.

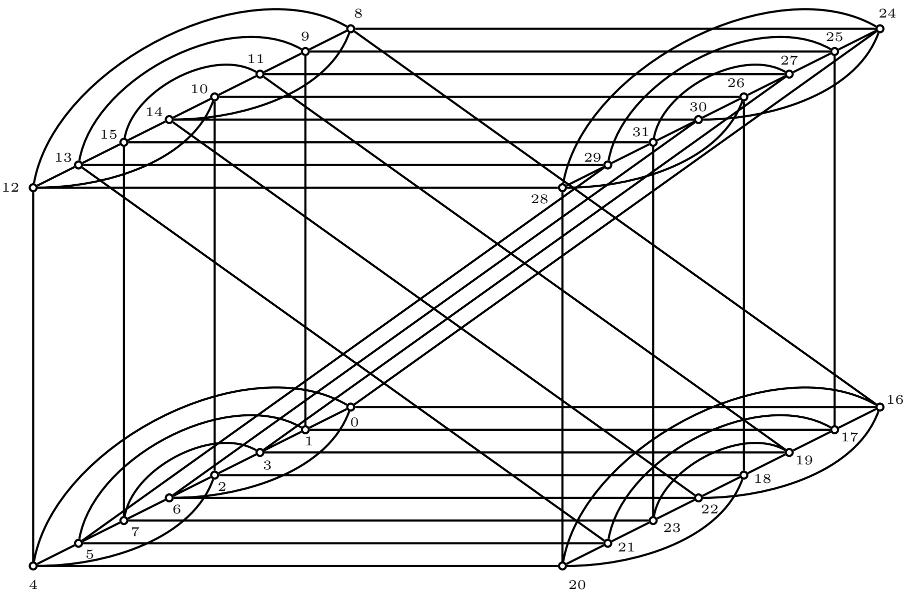
To be precise, for a vertex $x = x_{n-1}x_{n-2}\cdots x_1x_0$, we define the parity function $P_i = x_i \oplus x_{i-1} \oplus \cdots \oplus x_0$, where $\oplus$ is the exclusive-or operation. If $P_{2j-2}(x) = 0$ for some $1 \leqslant j \leqslant \lfloor n/2 \rfloor$, we divert the edge on $(2j-1)$-th dimension to a vertex $y = y_{n-1}y_{n-2}\cdots y_1y_0$ such that $y_{2j}y_{2j-1} = \overline{x}_{2j}\overline{x}_{2j-1}$ and $y_i = x_i$ for $i \neq 2j$ or $2j-1$. TQ_3 and TQ_5 are shown in Fig. 5.



TQ_3



another drawing of TQ_3



TQ_5

Fig. 5 Twisted cubes TQ_3 and TQ_5

The twisted cube TQ_n is an n-regular graph with 2^n vertices. Furthermore, TQ_n has a diameter of $\lceil (n+1)/2 \rceil$, superior to Q_n.

For any odd integer n ($\geqslant 3$), Chang et al. [19] and Huang et al. [88], independently, showed that TQ_n is pancyclic. This result is improved by Xu and Ma[1)] as TQ_n is vertex-pancyclic. Later, Fan et al. [52] and Xu et al.[2)],

1) Xu J -M, Ma M -J. Vertex-pancyclicity of some hypercube-like networks. 2005

2) Xu M, Hu X -D, Xu J -M, Zhou M -J, Ma M -J. Edge-pancyclicity and hamiltonian connectivity of twisted cubes. 2005

respectively, generalized these to the following result.

Theorem 5.1 (Fan et al. [52] and Xu et al.[2)]) *TQ_n is edge-pancyclic for any odd integer n ($\geqslant 3$).*

In the case where only faulty edges is considered, Fu [57] showed that TQ_n is $(2n-5)$-edge-fault-tolerant hamiltonian if every vertex is incident with at least two fault-free edges for any odd integer $n \geqslant 3$. Li et al. [110] showed the following result.

Theorem 5.2 (Li et al. [110]) *TQ_n is $(n-2)$-edge-fault-tolerant pancyclic for any odd integer $n \geqslant 3$.*

In the case where both faulty vertices and faulty edges are considered, Huang et al. [91] and Chen et al. [29], independently, showed that TQ_n is $(n-2)$-fault-tolerant hamiltonian for any odd integer $n \geqslant 3$. This result was improved by Chang et al. [23] and Yang et al. [162], independently.

Theorem 5.3 (Chang et al. [23] and Yang et al. [162]) *TQ_n is $(n-2)$-fault-tolerant pancyclic for any odd integer $n \geqslant 3$.*

As regards to fault-tolerant panconnectivity of TQ_n, we have known the following results.

Theorem 5.4 (Chen et al. [29], Huang et al. [91]) *TQ_n is $(n-3)$-fault-tolerant hamiltonian connected for any odd integer $n \geqslant 3$.*

Theorem 5.5 (Fan et al. [51]) *If $f_v + f_e \leqslant n-3$, then for any two fault-free vertices x and y in TQ_n and for each l with $2^{n-1}-1 \leqslant l \leqslant 2^n - f_v - 1$, there exists a fault-free xy-path of length l for any odd integer $n \geqslant 3$.*

For more faults, we can state only the following result.

Theorem 5.6 (Hsieh and Lee [76]) *If each vertex is incident to at least two fault-free edges, then TQ_n is $(2n-5)$-edge-fault-tolerant hamiltonian.*

6 Locally twisted cubes

The n-dimensional locally twisted cube LTQ_n ($n \geqslant 2$), proposed by Yang et al. [165], is defined recursively as follows:

(a) LTQ_2 is a graph isomorphic to Q_2.

(b) For $n \geqslant 3$, LTQ_n is built from two disjoint copies of LTQ_{n-1} according to the following steps. Let $0LTQ_{n-1}$ denote the graph obtained by prefixing the label of each vertex of one copy of LTQ_{n-1} with 0, let $1LTQ_{n-1}$ denote the graph obtained by prefixing the label of each vertex of the other copy LTQ_{n-1} with 1, and connect each vertex $x = 0x_2x_3\ldots x_n$ of $0LTQ_{n-1}$ with the vertex $1(x_2+x_n)x_3\ldots x_n$ of $1LTQ_{n-1}$ by an edge, where '+' represents the modulo 2 addition. The graphs shown in Fig. 6 are LTQ_3 and LTQ_4.

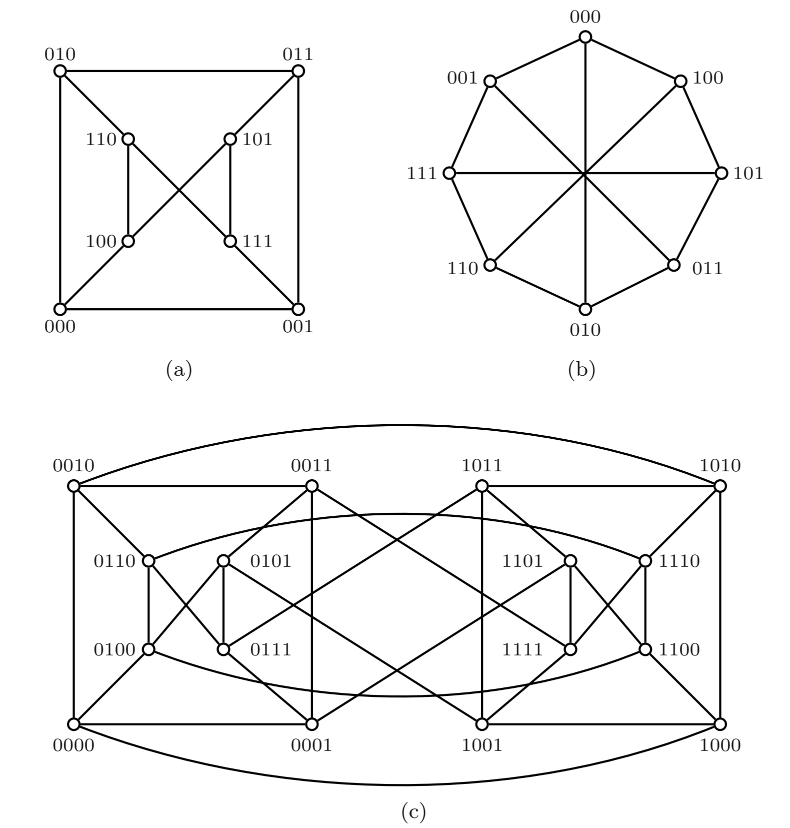


Fig. 6 LTQ_n for $n = 3, 4$.
(a) Ordinary drawing of LTQ_3, (b) symmetric drawing of LTQ_3, (c) LTQ_4

For $n \geqslant 3$, Yang et al. [167] showed that LTQ_n is pancyclic. Xu and Ma[1)] improved this result by proving that LTQ_n is vertex-pancyclic. Further, Ma and Xu [117], Hu et al. [86] improved these results.

Theorem 6.1 (Hu et al. [86], Ma and Xu [117]) *LTQ_n is edge-pancyclic for $n \geqslant 2$.*

As regards to fault-tolerant pancyclicity and panconnectivity of LTQ_n, we have known only the following results.

Theorem 6.2 (Chang, Ma and Xu [22]) *LTQ_n is $(n-2)$-fault-tolerant pancyclic for $n \geqslant 3$.*

Theorem 6.3 (Ma and Xu [118]) *For any two different vertices x and y with distance d in LTQ_n ($n \geqslant 3$), there exists an xy-path of every length l from d to $2^n - 1$ except for $d + 1$.*

For more faults, we can state only the following result.

1) See footnote 1) on p. 229

Theorem 6.4 (Hsieh and Lee [76]) *If each vertex is incident to at least two fault-free edges, then LTQ_n is $(2n-5)$-edge-fault-tolerant hamiltonian.*

7 Möbius cubes

The n-dimensional Möbius cube, denoted by MQ_n, is such an undirected graph, its vertex set is the same as the vertex set of Q_n, the vertex $X = x_1x_2\cdots x_n$ connects to n other vertices Y_i, $(1 \leqslant i \leqslant n)$, where each Y_i satisfies one of the following equations:

$$Y_i = \begin{cases} x_1x_2\cdots x_{i-1}\overline{x}_ix_{i+1}\cdots x_n, & x_{i-1}=0, \\ x_1x_2\cdots x_{i-1}\overline{x}_i\overline{x}_{i+1}\cdots \overline{x}_n, & x_{i-1}=1. \end{cases}$$

From the above definition, X connects to Y_i by complementing the bit x_i if $x_{i-1}=0$ or by complementing all bits of $x_i,\ldots,x_n$ if $x_{i-1}=1$. The connection between X and Y_1 is undefined, so we can assume that x_0 is either equal to 0 or equal to 1, which gives us slightly different network topologies. If we assume $x_0=0$, we call the network a '0-Möbius cube'; and if we assume $x_0=1$, we call the network a '1-Möbius cube', denoted by 0-MQ_n and 1-MQ_n, respectively. The graphs shown in Fig. 7 are 0-MQ_4 and 1-MQ_4.

The Möbius cubes MQ_n was first proposed by Cull and Larson [39]. Like Q_n, MQ_n is an n-regular n-connected graph with 2^n vertices and $n2^{n-1}$ edges. Moreover, MQ_n has a diameter of $\lceil (n+2)/2\rceil$ for 0-MQ_n $(n\geqslant 4)$ and $\lceil (n+1)/2\rceil$ for 1-MQ_n $(n\geqslant 1)$. However, for $n\geqslant 4$, MQ_n is neither vertex-transitive nor edge-transitive.

Cull and Larson [39] first proved the existence of hamiltonian cycles in MQ_n by proving that in an n-dimensional 0-Möbius or 1-Möbius cube, there are 2^{n-k} disjoint cycles of length 2^k for any $k\geqslant 2$. Huang et al.[88] and Fan [47], independently, showed that MQ_n is pancyclic for $n\geqslant 2$. This result was improved as follows.

Theorem 7.1 (Xu and Xu [160], Hu et al. [86]) *MQ_n is edge-pancyclic for $n\geqslant 2$.*

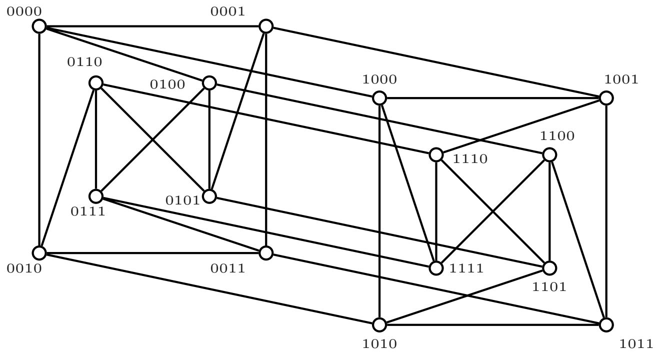
Fan [47] proved that MQ_n is hamiltonian connected for $n\geqslant 3$. This result is generalized by the following.

Theorem 7.2 (Xu et al. [156]) *For any two different vertices x and y with distance d in MQ_n $(n\geqslant 3)$, there exists an xy-path of every length l from d to 2^n-1 except for $d+1$.*

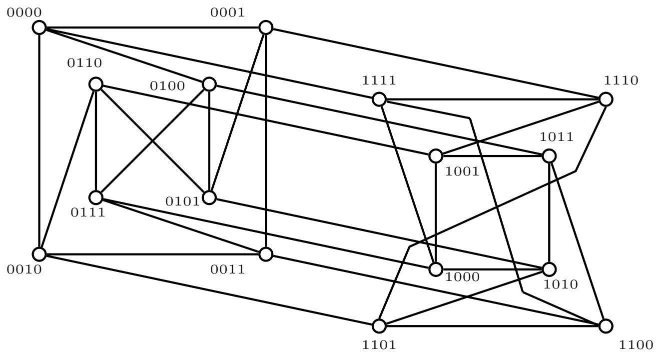
In the case where faulty vertices and faulty edges are considered, respectively, we can state the following two results.

Theorem 7.3 (Hsieh and Chen [69]) *MQ_n is $(n-2)$-edge-fault-tolerant pancyclic for $n\geqslant 2$.*

Theorem 7.4 (Yang et al. [168]) *MQ_n is $(n-2)$-vertex-fault-tolerant pancyclic for $n\geqslant 2$.*



(a)



(b)

Fig. 7 (a) 0-Möbius cube 0-MQ_4, (b) 1-Möbius cube 1-MQ_4

In the case where both faulty vertices and faulty edges are considered, Huang et al. [89] showed that MQ_n is $(n-2)$-fault-tolerant hamiltonian for $n \geqslant 3$. This result is improved as follows.

Theorem 7.5 (Hsieh and Chang [68]) *MQ_n is $(n-2)$-fault-tolerant pancyclic for $n \geqslant 2$.*

As regards to the fault-tolerant panconnectivity of MQ_n, Hsieh and Chen [69] proved that MQ_n contains a fault-free hamiltonian path if $f_e \leqslant n-1$. Huang et al. [89] and Chen et al. [29], independently, proved that MQ_n is $(n-3)$-fault-tolerant hamiltonian connected for $n \geqslant 3$. Recently, Fan et al.[1)] have improved this result by showing the following theorem.

Theorem 7.6 (Fan et al.[1)]) *If $f_v \leqslant n-3$, then for any two distinct fault-free vertices x and y in MQ_n, there exists a fault-free xy-path of every length l from $2^{n-1}-1$ to $2^n - f_v - 1$ for $n \geqslant 3$.*

This result is tight in the sense that the two bounds on path length l and faulty size f_v. That is, the result does not hold if $l \leqslant 2^{n-1}-2$ or $f_v \geqslant n-2$.

1) Fan J, Jia X, Lin X. Fault-tolerant embedding of paths in Möbius cubes. 2008

8 Augmented cubes

The n-dimensional augmented cube AQ_n ($n \geqslant 1$), proposed by Choudum and Sunitha [36–38], can be defined recursively as follows: AQ_1 is a complete graph K_2 with the vertex set $\{0, 1\}$. For $n \geqslant 2$, AQ_n is obtained by taking two copies of the augmented cube AQ_{n-1}, denoted by AQ^0_{n-1} and AQ^1_{n-1}, and adding $2 \times 2^{n-1}$ edges between the two as follows.

Let

$$V(AQ^0_{n-1}) = \{0u_{n-1} \ldots u_2u_1 : \ u_i = 0 \text{ or } 1\},$$
$$V(AQ^1_{n-1}) = \{1u_{n-1} \ldots u_2u_1 : \ u_i = 0 \text{ or } 1\}.$$

A vertex $u = 0u_{n-1} \ldots u_2u_1$ of AQ^0_{n-1} is joined to a vertex $v = 1v_{n-1} \ldots v_2v_1$ of AQ^1_{n-1} if and only if either

(i) $u_i = v_i$ for $1 \leqslant i \leqslant n-1$; in this case, v (resp. u) is called a hypercube neighbor of u (resp. v), setting $v = u^h$ or $u = v^h$, or

(ii) $u_i = \overline{v}_i$ for $1 \leqslant i \leqslant n-1$; in this case, v (resp. u) is called a complement neighbor of u (resp. v), setting $v = u^c$ or $u = v^c$.

The graphs shown in Fig. 8 are the augmented cubes AQ_1, AQ_2 and AQ_3, respectively.

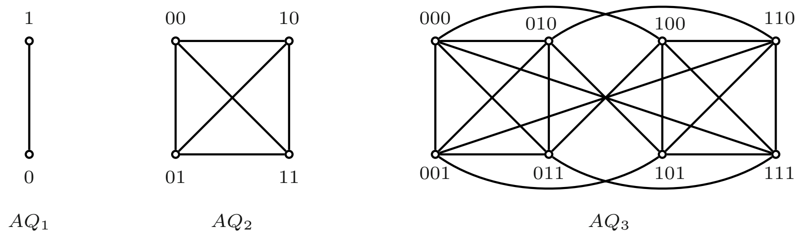


Fig. 8 Augmented cubes AQ_1, AQ_2 and AQ_3

Obviously, AQ_n is a $(2n-1)$-regular graph with 2^n vertices. It has been shown by Choudum and Sunitha [36–38] that AQ_n is vertex-symmetric, $(2n-1)$-connected for $n \neq 3$ (AQ_3 is 4-connected) and has diameter $\lceil n/2 \rceil$, the wide-diameter and fault-diameter $\lceil n/2 \rceil + 1$ ($n \geqslant 5$). At the same time, they showed that AQ_n is pancyclic for $n \geqslant 2$. This result was improved as follows.

Theorem 8.1 (Hsieh and Shiu [81]) *AQ_n is vertex-pancyclic for $n \geqslant 2$.*

Hsu et al. [83] proved that AQ_n ($n \geqslant 1$) is hamiltonian-connected, and they also showed that AQ_n is $(2n-3)$-fault-tolerant hamiltonian and $(2n-4)$-fault-tolerant hamiltonian connected for $n \geqslant 4$. Ma et al. [114] and Wang et al. [150] improved these results as follows.

Theorem 8.2 (Ma et al. [114]) *AQ_n is panconnected for $n \geqslant 1$ and $(2n-3)$-edge-fault-tolerant pancyclic for $n \geqslant 2$.*

Theorem 8.3 (Wang et al. [150]) *AQ_n is $(2n-3)$-fault-tolerant pancyclic for $n \geqslant 4$.*

In the case where both faulty vertices and/or faulty edges are considered, Xu and Wang[1)] showed the following result.

Theorem 8.4 (Xu and Wang[1)]) *If $f_v + f_e \leqslant 2n - 5$, then for any two distinct fault-free vertices x and y with distance d in AQ_n, there exists a fault-free xy-path of length l for every l with $d + 2 \leqslant l \leqslant 2^n - f_v - 1$.*

9 Balanced hypercubes

The n-dimensional balanced hypercube, denoted by BQ_n and proposed by Huang and Wu [87,152], has 4^n vertices. Each vertex has a unique n-component vector on $\{0, 1, 2, 3\}$ for an address, also called an n-bit string. A vertex $(a_0, a_1 \ldots, a_{n-1})$ connects to the following $2n$ vertices:

$$\begin{cases} ((a_0+1) \pmod 4), a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1}), \\ ((a_0-1) \pmod 4), a_1, \ldots, a_{i-1}, a_i, a_{i+1}, \ldots, a_{n-1}), \end{cases}$$

$$\begin{cases} ((a_0+1) \pmod 4), a_1, \ldots, a_{i-1}, (a_i + (-1)^{a_0}) \pmod 4), a_{i+1}, \ldots, a_{n-1}), \\ ((a_0-1) \pmod 4), a_1, \ldots, a_{i-1}, (a_i + (-1)^{a_0}) \pmod 4), a_{i+1}, \ldots, a_{n-1}) \end{cases}$$

for $1 \leqslant i \leqslant n-1$.

Figure 9 shows BQ_1 and BQ_2. The balanced hypercube BQ_n is an $2n$-regular, vertex-transitive bipartite graph with 4^n vertices. Wu and Huang [152] showed that BQ_n contains all cycles of length 4^l, $2 \times 4^{l-1}$ for $1 \leqslant l \leqslant n$. Xu et al. [158] improve this result by showing the following theorem.

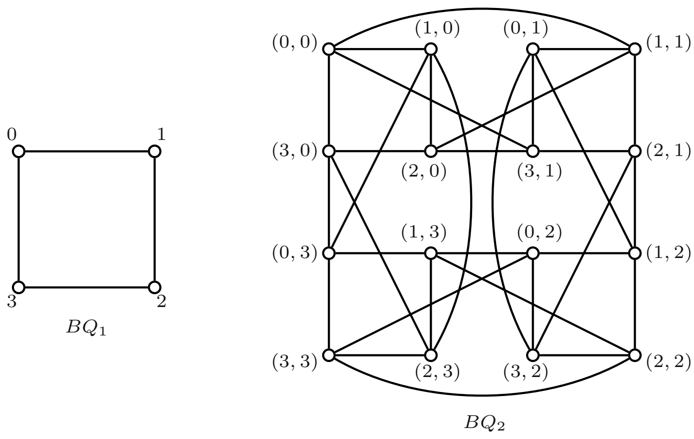


Fig. 9 Balanced hypercubes BQ_1 and BQ_2

Theorem 9.1 (Xu et al. [158]) *BQ_n is edge-bipancyclic and hamiltonian laceable.*

1) Xu J -M, Wang H -L. Fault-tolerant panconnectivity of augmented cubes. 2008

We do not know any other results on this network.

10 Star graphs

An n-dimensional star graph, denoted by S_n, proposed by Akers and Krishnamurthy [2], is an undirected graph consisting of $n!$ vertices labelled with $n!$ permutations on a set of the symbols $1, 2, \ldots, n$. There is an edge between any two vertices if and only if their labels differ only in the first and another position. The graphs shown in Fig. 10 are S_2, S_3 and S_4.

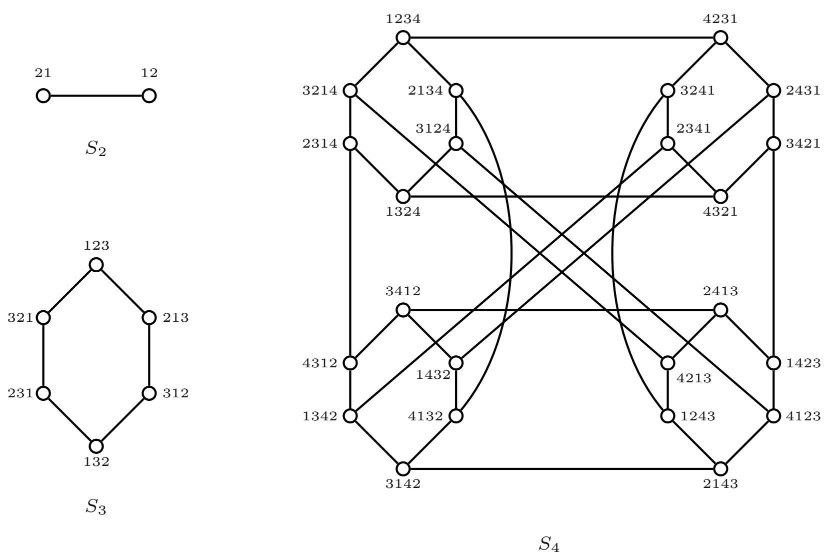


Fig. 10 Star graphs S_2, S_3 and S_4

Like the hypercube, the star graph is a vertex- and edge-transitive graph [2]. Moreover, it has been proved [3] that S_n is a Cayley graph on the symmetry group S_n with respect to the generating set $\{t_1, t_2, \ldots, t_{n-1}\}$, where $t_i = (1, i+1)$ $(1 \leqslant i \leqslant n-1)$ denotes a permutation that exchanges two symbols in the first and the $(i+1)$-th position, which implies that S_n is $(n-1)$-regular. Furthermore, S_n is bipartite since each edge connects an odd permutation with an even permutation, and contains no cycles of length 4.

Jwo et al. [97] showed that S_n is bipancyclic for $n \geqslant 3$. In the case where only faulty edges is considered, Tseng et al. [146] proved that S_n is $(n-3)$-edge-fault-tolerant hamiltonian for $n \geqslant 4$, and Li [108] showed that S_n is $(n-3)$-edge-fault-tolerant bipancyclic for $n \geqslant 3$. Xu et al. [159] improved these results as follows.

Theorem 10.1 (Xu et al. [159]) *For $n \geqslant 3$, S_n is $(n-3)$-edge-fault-tolerant edge-bipancyclic.*

We now consider faulty vertices. Since S_n is a bipartite graph with two partite sets of equal size, any xy-path has maximal length $(n! - 1)$ if the distance between them is odd and $(n! - 2)$ if distance between them is even. Tseng et al. [146] showed that S_n with $f_v \leqslant n - 3$ can embed a fault-free cycle of length at least $n! - 4f_v$ for $n \geqslant 4$. Hsieh et al. [72] showed that S_n with $f_v \leqslant n - 5$ can embed a fault-free path of length $n! - 2f_v - 2$ (resp. $n! - 2f_v - 1$) between two arbitrary distinct fault-free vertices of even (resp. odd) distance for $n \geqslant 6$.

Theorem 10.2 (Hsieh [64]) *S_n with $f_v \leqslant n-3$ can embed a fault-free path of length $n! - 2f_v - 2$ (resp. $n! - 2f_v - 1$) between two arbitrary distinct fault-free vertices of even (resp. odd) distance for $n \geqslant 4$.*

Since S_n is regular of degree $n - 1$ and is bipartite with two partite sets of equal size, this result is optimal (in the worst case) with respect to both the length of the embedded path and the number of tolerable vertex faults.

Hsieh et al. [71] proved that S_n with $n \geqslant 4$ is strongly hamiltonian laceable. In Ref. [73], they also proved that S_n is $(n - 4)$-edge-fault-tolerant hamiltonian laceable and is $(n - 3)$-edge-fault-tolerant hamiltonian laceable exclusive of two exceptions in which at most two vertices are excluded for $n \geqslant 6$. Li et al. obtained stronger results.

Theorem 10.3 (Li et al. [109]) *For $n \geqslant 4$, S_n is $(n-3)$-edge-fault-tolerant hamiltonian laceable, $(n-3)$-edge-fault-tolerant strongly hamiltonian laceable and $(n - 4)$-edge-fault-tolerant hyper hamiltonian laceable.*

Theorem 10.4 (Lin et al. [111]) *S_n is super laceable if and only if $n \neq 3$.*

For more faulty edges, Fu [55] showed that S_n is $(2n - 7)$-edge-fault-tolerant hamiltonian if every vertex is incident with at least two fault-free edges for $n \geqslant 4$. Tsai, Fu and Chen [144] have obtained a stronger result.

Theorem 10.5 (Tsai, Fu and Chen [144]) *S_n is $(2n-7)$-edge-fault-tolerant strongly hamiltonian laceable if every vertex is incident with at least two fault-free edges for $n \geqslant 4$.*

Recently, Hsieh and Wu [82] improved the result of Fu [55] by increasing faulty edges from $2n - 7$ to $3n - 10$.

Theorem 10.6 (Hsieh and Wu [82]) *S_n is $(3n - 10)$-edge-fault-tolerant hamiltonian if every vertex is incident with at least two fault-free edges for $n \geqslant 4$.*

11 Pancake graphs

The n-dimensional pancake graph, denoted by P_n and proposed by Akers and Krishnameurthy [3], is a graph consisting of $n!$ vertices labelled with $n!$ permutations on a set of the symbols $1, 2, \ldots, n$. There is an edge from vertex

i to vertex j if and only if j is a permutation of i such that

$$i = i_1 i_2 \cdots i_k i_{k+1} \cdots i_n,$$

$$j = i_k \cdots i_2 i_1 i_{k+1} \cdots i_n,$$

where $2 \leqslant k \leqslant n$. The pancake graphs P_2, P_3, and P_4 are shown in Fig. 11 for illustration.

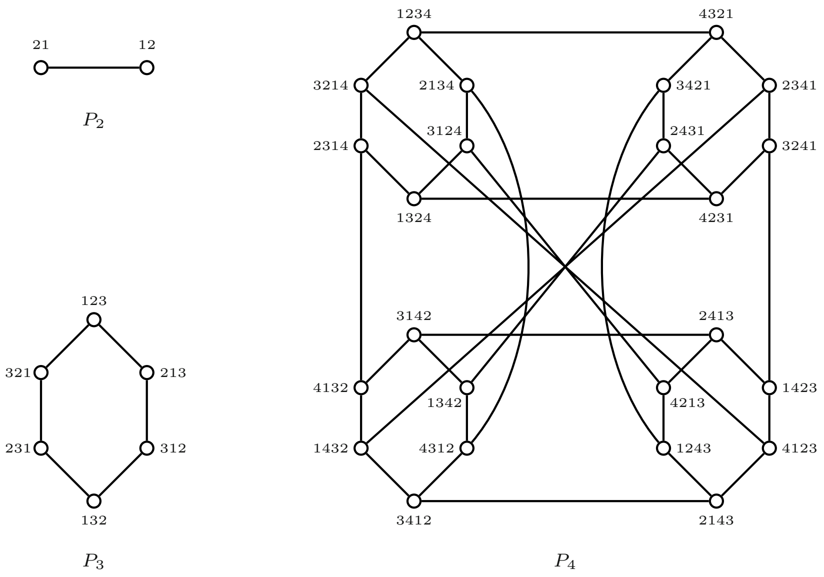


Fig. 11 Pancake graphs P_2, P_3 and P_4

The pancake graph P_n is $(n-1)$-regular and $(n-1)$-connected and contains no cycles of length four [111]. Moreover, P_n is a Cayley graph and, hence, is vertex transitive, but not edge-transitive [3]. It was shown in Ref. [61] that the diameter of the pancake graph is bounded above by $3(n+1)/2$. It is still an open problem to compute the exact diameter of the pancake graph.

Theorem 11.1 (Kanevsky and Feng [99]) *P_n contains cycles of every length from 6 to $n!$ except for $n! - 1$ for $n \geqslant 4$.*

Theorem 11.2 (Hung et al. [94]) *P_n is $(n-3)$-fault hamiltonian and $(n-4)$-fault hamiltonian connected for $n \geqslant 4$.*

In particular, the fact that $P_n - F$ is hamiltonian when F consists of only a single vertex implies the existence of a cycle of length $n! - 1$. As a simple consequence, Theorem 11.2 improves Theorem 11.1. Combining Theorem 11.1 with Theorem 11.2, we have P_n is pancyclic for $n \geqslant 4$. The first result in Theorem 11.2 is improved as follows.

Theorem 11.3 (Tsai, Fu and Chen [143]) *P_n is $(2n-7)$-fault hamiltonian for $n \geqslant 4$.*

12 Bubble-sort graphs

A bubble-sort graph B_n, proposed by Akers and Krisnamurthy [3], has $n!$ vertices labelled by distinct permutations on $\{1, 2, \ldots, n\}$. Two vertices $x = x_1x_2 \cdots x_n$ and $y = y_1y_2 \cdots y_n$ in B_n are adjacent if and only if $x_i = y_{i+1}$ and $x_{i+1} = y_i$ for some i and $x_j = y_j$ for all $j \neq i$ or $i + 1$. Fig. 12 shows the bubble-sort graphs B_2, B_3 and B_4.

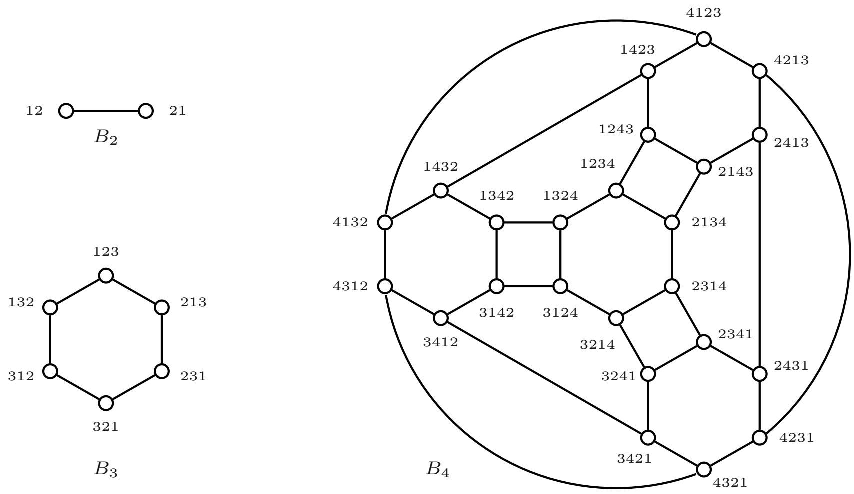


Fig. 12 Bubble-sort graphs B_2, B_3 and B_4

B_n is bipartite, $(n-1)$-regular and vertex transitive. A very important property of the bubble-sort graph is a recursive structure. We define a subgraph $B_n(i)$ in B_n for any fixed i, $1 \leqslant i \leqslant n$, as the induced subgraph by the set of vertices $\{x \mid x[n] = i\}$, where $x[n]$ represents the n-th element of the label of vertex x in B_n. By the definition of the bubble-sort graph, $B_n(i)$ is isomorphic to B_{n-1}. Hence, B_n is partitioned into n subgraphs each of which is isomorphic to B_{n-1}.

Also, by the definition, B_n is a Cayley graph on the symmetric group on $\{1, 2, \ldots, n\}$ with the set of transpositions $\{(1, 2), (2, 3), \ldots, (n-1, n)\}$ as the generating set. Tchuente [134] proved that Cayley graphs on the symmetric group on $\{1, 2, \ldots, n\}$ generated by transpositions are hamiltonian laceable for $n \geqslant 4$, which implies the following result.

Theorem 12.1 (Tchuente [134]) *B_n is hamiltonian laceable for $n \geqslant 4$.*

Recently, Kikuchi and Araki [100] have shown the following result.

Theorem 12.2 (Kikuchi and Araki [100]) *B_n is edge-bipancyclic for $n \geqslant 5$, $(n-3)$-edge-fault-tolerant bipancyclic for $n \geqslant 4$.*

Theorem 12.3 (Araki and Kikuchi [8]) *B_n is $(n-3)$-edge-fault-tolerant strongly hamiltonian laceable for $n \geqslant 4$.*

13 (n, k)-Stars

We have seen that the number of vertices is $n!$ for an n-star graph S_n, and there is a large gap between $n!$ and $(n+1)!$ for expanding an S_n to an S_{n+1}. To remedy this drawback, Chiang and Chen [34] proposed the (n, k)-star graph, denoted by $S_{n,k}$, with vertex set

$$\{u_1u_2\cdots u_k:\ u_i \in \{1, 2, \ldots, n\},\ u_i \neq u_j \text{ for } i \neq j\}.$$

Adjacency is defined as follows: a vertex $u_1u_2\cdots u_i\cdots u_k$ is adjacent to

(1) the vertex $u_iu_2\cdots u_1\cdots u_k$, where $2 \leqslant i \leqslant k$ (i.e., we swap u_i with u_1), and

(2) the vertex $xu_2u_3\cdots u_k$, where $x \in \{1, 2, \ldots, n\} - \{u_i:\ 1 \leqslant i \leqslant k\}$.

Figure 13 shows a $(4, 2)$-star graph $S_{4,2}$.

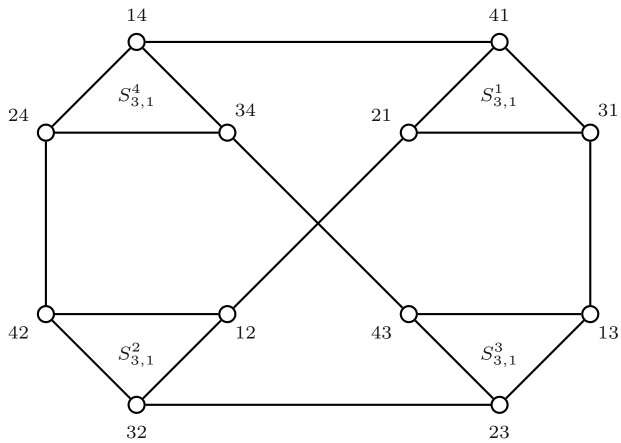


Fig. 13 $(4, 2)$-star graph $S_{4,2}$

By definition, $S_{n,n-1} \cong S_n$ and $S_{n,1} \cong K_n$. Thus, $S_{n,k}$ is a generalization of S_n. It has been shown that $S_{n,k}$ is an $(n-1)$-regular $(n-1)$-connected vertex-transitive graph with $n!/(n-k)!$ vertices, diameter $2k-1$ for $k \leqslant \lfloor n/2 \rfloor$ and $\lfloor (n-1)/2 \rfloor + k$ for $k \geqslant \lfloor n/2 \rfloor + 1$.

For $n-k=1$, $S_{n,n-1} \cong S_n$ [34], which is known to be hamiltonian if and only if $n > 2$ and hamiltonian connected if and only if $n = 2$ [1].

Theorem 13.1 (Chen et al. [30]) *$S_{n,k}$ is 3-vertex-pancyclic when $1 \leqslant k \leqslant n-4$ and $n \geqslant 6$, and 6-vertex-pancyclic when $n-3 \leqslant k \leqslant n-2$.*

Theorem 13.2 (Hsu et al. [84]) *$S_{4,2}$ is 1-fault-tolerant hamiltonian and hamiltonian connected. For two integers n and k with $n > k \geqslant 1$, $S_{n,k}$ is $(n-3)$-fault-tolerant hamiltonian, and $(n-4)$-fault-tolerant hamiltonian-connected.*

14 Arrangement graphs

The arrangement graph was proposed by Day and Tripathi [41] as a common generalization of star graphs and alternating group graphs. Given two positive integers n and k with $n > k$, the (n,k)-arrangement graph $A_{n,k}$ is the graph with vertex-set $V = \{p\colon\ p = p_1p_2\cdots p_k$ with $p_i \in \{1,2,\ldots,n\}$ for $1 \leqslant i \leqslant k$ and $p_i \neq p_j$ if $i \neq j\}$ and edge-set $E = \{(p,q)\colon\ p,q \in V$ and p,q differ in exactly one position$\}$. Fig. 14 shows $A_{4,2}$.

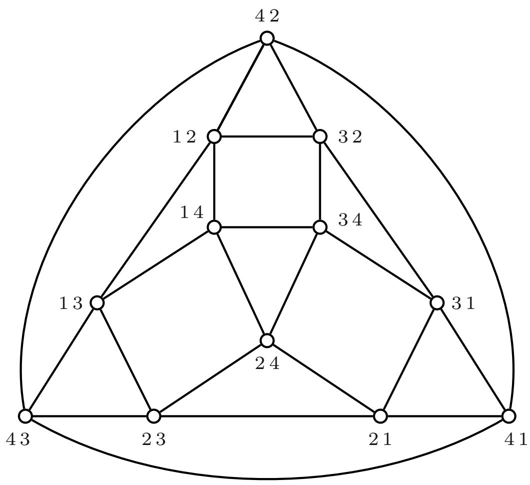


Fig. 14 Structure of $A_{4,2}$

The (n,k)-arrangement graph $A_{n,k}$ is a regular graph of degree $k(n-k)$ with $n!/(n-k)!$ vertices and diameter $\lfloor 3k/2 \rfloor$. $A_{n,1}$ is isomorphic to a complete graph K_n and $A_{n,n-1}$ is isomorphic to a star graph S_n. Moreover, $A_{n,k}$ is vertex-transitive and edge-transitive [41].

Since the arrangement graph $A_{n,n-1}$ is isomorphic to a star graph S_n, which is bipartite, we consider $k \leqslant n-2$ below.

Theorem 14.1 (Day and Tripathi [42]) *$A_{n,k}$ is pancyclic for $n-k \geqslant 2$.*

Hsieh et al. [70] studied the existence of hamiltonian cycles in faulty arrangement graphs, Lo and Chen [112] studied edge fault hamiltonian connectivity of the arrangement graph. These results have been generalized by Hsu et al. [85].

Theorem 14.2 (Hsu et al. [85]) *$A_{n,k}$ is $(k(n-k)-2)$-fault-tolerant hamiltonian, and $(k(n-k)-3)$-fault-tolerant hamiltonian-connected for $n > k \geqslant 1$.*

For panpositionable hamiltonicity of $A_{n,k}$, Teng et al. [135] obtained the following result.

Theorem 14.3 (Teng et al. [135]) *$A_{n,k}$ is panpositionable hamiltonian and panconnected for $k \geqslant 1$ and $n-k \geqslant 2$.*

15 Alternating group graphs

An n-dimensional alternating group graph, denoted by AG_n, proposed by Jwo et al. [98] and further investigated by Cheng et al. [31–33], is an undirected graph with vertices labelled with even permutations on a set of the symbols $1, 2, \ldots, n$. There is an edge between two vertices p and q if and only if q can be obtained from p by rotating the symbols in positions $1, 2$, and i from left to right for some $i = 3, 4, \ldots, n$. Fig. 15 depicts examples of AG_3 and AG_4.

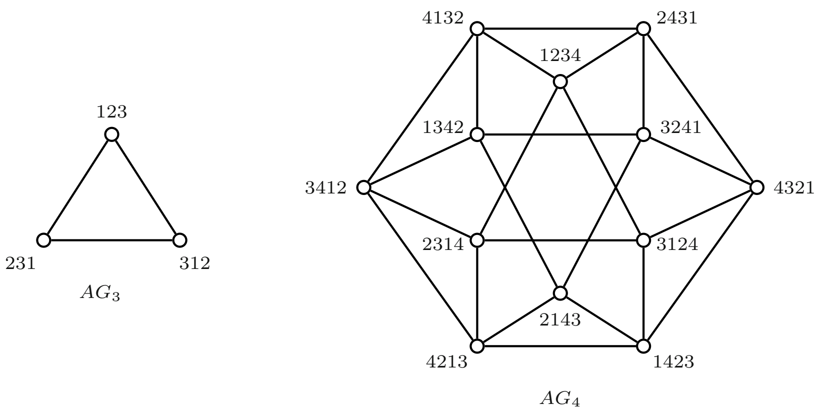


Fig. 15 Examples of alternating group graphs

It is easy to see that AG_n is $(2n-4)$-regular, has $n!/2$ vertices and $(n-2)n!/2$ edges. Moreover, AG_n, which belongs to the class of Cayley graphs, has been shown to be vertex-transitive, edge-transitive, maximal connectivity, and has a small diameter and average distance. Furthermore, Chiang and Chen [35] showed that AG_n is isomorphic to the $(n, n-2)$-arrangement graph $A_{n,n-2}$.

Theorem 15.1 (Jwo et al. [98]) *AG_n is pancyclic and hamiltonian-connected for $n \geqslant 3$.*

Theorem 15.2 (Chang et al. [20]) *AG_n is panconnected for $n \geqslant 3$, $(n-2)$-vertex-fault-tolerant hamiltonian and $(n-3)$-vertex-fault-tolerant hamiltonian-connected for $n \geqslant 4$.*

Since AG_n is isomorphic to the $(n, n-2)$-arrangement graph $A_{n,n-2}$, by Theorem 14.2, the following result holds.

Theorem 15.3 (Hsu et al. [85]) *AG_n is $(2n-6)$-fault-tolerant hamiltonian and $(2n-7)$-fault-tolerant hamiltonian-connected for $n \geqslant 4$.*

Recently, Tsai, Chen and Fu [142] have further shown that the alternating group graph remains pancyclic, even if there are up to $2n-6$ edge faults for $n \geqslant 3$.

Theorem 15.4 (Tsai, Chen and Fu [142]) *AG_n is $(2n-6)$-edge-fault-tolerant edge-pancyclic for $n \geqslant 3$.*

For vertex faults, we can state the following result.

Theorem 15.5 (Chang et al. [21]) *AG_n is $(n-2)$-vertex-fault-tolerant pancyclic, $(n-3)$-vertex-fault-tolerant vertex-pancyclic, $(n-4)$-vertex-fault-tolerant edge-4-pancyclic for $n \geqslant 4$.*

Teng et al.[1)] have proposed a new concept called *panpositionable hamiltonicity*. The panpositionable hamiltonian property advances the hamiltonicity further. A hamiltonian graph G of order n is panpositionable if for any two different vertices x and y of G and for any integer l satisfying $d_G(x,y) \leqslant l \leqslant n - d_G(x,y)$, there exists a hamiltonian cycle C of G such that $d_C(x,y) = l$. The following result is obtained.

Theorem 15.6 (Teng et al.[1)]) *AG_n is panpositionable hamiltonian if $n \geqslant 3$.*

16 *k*-ary *n*-cubes

The *n-dimensional undirected toroidal mesh*, denoted by $Q(k_1,\ldots,k_n)$, is defined as the cartesian products $C_{k_1} \times C_{k_2} \times \cdots \times C_{k_n}$, where C_{k_i} is a cycle of length k_i ($\geqslant 3$) for each $i = 1,2,\ldots,n$ and $n \geqslant 2$. It is clear that $Q(k_1,\ldots,k_n)$ has $k_1 \cdots k_n$ vertices and has girth $g = \min\{4, k_i, 1 \leqslant i \leqslant n\}$, and is bipartite if and only if k_i is even for each $i = 1,2,\ldots,n$. By properties of cartesian products (see Ref. [153]), we obtain immediately that $Q(k_1,\ldots,k_n)$ is a $2n$-regular and $2n$-connected Cayley graph with diameter d, where

$$d = \sum_{i=1}^{n} \left\lfloor \frac{k_i}{2} \right\rfloor.$$

For fault-tolerant hamiltonicity and hamiltonian-connectivity of $Q(k_1,\ldots, k_n)$, we only know the following result.

Theorem 16.1 (Kim and Park [101]) *If $m \geqslant 3$, $n \geqslant 3$ and n is odd, then $Q(m,n)$ is 2-fault-tolerant hamiltonian and 1-fault-tolerant hamiltonian connected.*

Assume that $k_i = k \geqslant 3$ for each $i = 1,2,\ldots,n$ and $n \geqslant 2$ below. The n-dimensional undirected toroidal mesh $Q(k,k,\ldots,k)$ is called the *k-ary n-dimensional cube*, or *k-ary n-cube* for short, denoted by Q_n^k.

Bettayeb [12] and Bose et al. [14] showed that Q_n^k is hamiltonian, respectively. Recently, Huang[2)] has proved that Q_n^k is strongly hamiltonian laceable if k is even. Ashir and Stewart [10] showed that Q_n^k contains a cycle of some given length. Wang et al. [147] showed that Q_n^k is hamiltonian-connected

1) Teng Y -H, Tan J J M, Hsu L -H. Panpositionable hamiltonicity of the alternating group graphs. 2006

2) Huang C -H. Strongly Hamiltonian laceability of the even k-ary n-cube. 2008

when k is odd. Hsieh, Lin and Huang [79] showed that Q_n^3 is panconnected and edge-pancyclic. Hsieh and Lin further showed the following two results.

Theorem 16.2 (Hsieh and Lin [78]) *If k is even, then any edge in Q_n^k lies on a cycle of every length from k to k^n.*

Theorem 16.3 (Hsieh and Lin [77,78]) *If k is odd then for any two distinct vertices x and y in Q_n^k, there exists an xy-path of every length l for $\lfloor k/2 \rfloor n \leqslant l \leqslant k^n - 1$; if k is even then Q_n^k is bipanconnected.*

Stewart and Xiang [132] also obtained the second conclusion in the above theorem that if k is even then Q_n^k is bipanconnected, and strengthened the result of Hsieh et al. [79] that Q_n^3 is panconnected and edge-pancyclic as follows.

Theorem 16.4 (Stewart and Xiang [132]) *If k is odd, then Q_n^k is edge-bipancyclic; and m-panconnected, where $m = (n(k-1) + 2k - 6)/2$, and $(k-1)$-pancyclic.*

A graph G of order υ is called *bipancycle-connected* if each pair of vertices x and y in G is contained by a cycle of each even length from the length of the smallest cycle that contains x and y to υ, and called *strictly m-pancycle-connected* for $m < \upsilon$ if each pair of vertices in G is contained by a cycle of each length from m to υ.

Theorem 16.5 (Fang[1)]) *If k is even then Q_n^k is bipancycle-connected; and if k is odd then Q_n^k is strictly m-pancycle-connected, where $m = nk - n$.*

The lower bound $nk - n$ in the above theorem may be reached.

In the case where both faulty vertices and/or faulty edges are considered, Yang, Tan and Hsu [163] obtained the following result.

Theorem 16.6 (Yang, Tan and Hsu [163]) *If k is odd, then Q_n^k is $(2n-2)$-fault-tolerant hamiltonian and $(2n-3)$-fault-tolerant-hamiltonian connected.*

Since Q_n^k is regular of degree $2n$, the degrees of fault-tolerance $2n-3$ and $2n-2$ respectively, are optimal in the worst case. For more faults, Ashir and Stewart [11] obtained the following result.

Theorem 16.7 (Ashir and Stewart [11]) *If every vertex is incident with at least two fault-free edge, then Q_n^k is $(4n-5)$-fault-tolerant hamiltonian.*

This result is optimal in the worst case, that is, there are situations where the number of faults is $4n-4$ and every vertex is incident with at least two fault-free edge and no Hamilton cycle exists. At the same time, Ashir and Stewart [11] also remarked that given a faulty Q_n^k, the problem of deciding whether there exists a Hamilton cycle is NP-complete.

1) Fang J -F. The bipancycle-connectivity and the m-pancycle-connectivity of the k-ary n-cube. 2008

17 Remarks and comments

The topological structure of an interconnection network can be modeled by a graph whose vertices represent components of the network and whose edges represent links between components. An n-dimensional hypercube or n-cube Q_n is one of the most efficient networks for parallel computation. It has many desirable and attractive features such as regularity, recursive structure, vertex and edge symmetry, maximum connectivity, and effective routing and broadcasting algorithms, and becomes the first choice for the topological structure of parallel processing and computing systems. However, the hypercube has its own intrinsic drawbacks, such as its large diameter. As a result of a focused attention, several variations of the hypercube have been proposed to improve some properties such as diameter; some of these variations have been mentioned in this paper.

In interconnection networks, the simulation of one architecture by another is important. The problem of simulating one network by another is modeled as a graph-embedding problem. Path or cycle networks are suitable for designing simple algorithms with low communication costs. These have motivated a great deal of research on embedding paths or cycles into various other interconnection networks. We have seen from this survey that the path-embedding and cycle-embedding problems for the hypercubes have been studied in depth. However, the same problems for variations of the hypercube have not been studied much although some known results have been mentioned in this survey.

It has be observed that the hypercube and its variations mentioned above are of recursive structures, and so all proof proceeds of known results apply induction on order by making good use of the recursive structure of the networks. In the process of proofs, two key obstacles are generally encountered. The one is the induction base for small order, which is often used by simple observation, direct verification, or indirect verification by a computer search. The other is to construct a required path or cycle by the induction hypothesis, which relates to some structural properties of the networks. Thus, the authors deem that the key of studying the path-embedding and cycle-embedding problems for variations of the hypercube is to investigate the structural properties of these networks.

In this paper, we survey many results on hamiltonicity, hamiltonian-connectivity, pancyclicity, vertex-pancyclicity, edge-pancyclicity, and panconnectivity for the hypercube network and its variations. In fact, these notions have been investigated in the context of some other networks, for example, in recursive circulant networks [9,24,124], Butterfly networks [25,93,140], cube-connected cycle networks [58], hypercube-like networks [76,86,123,125], and so on.

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