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RESEARCH ARTICLE

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Nature of complex number and complex-valued neural networks

Abstract We discuss the nature of complex number and its effect on complex-valued neural networks (CVNNs). After we review some examples of CVNN applications, we look back at the mathematical history to elucidate the features of complex number, in particular to confirm the importance of the phase-and-amplitude viewpoint for designing and constructing CVNNs to enhance the features. This viewpoint is essential in general to deal with waves such as electromagnetic wave and lightwave. Then, we point out that, although we represent a complex number as an ordered pair of real numbers for example, we can reduce ineffective degree of freedom in learning or self-organization in CVNNs to achieve better generalization characteristics. This merit is significantly useful not only for wave-related signal processing but also for general processing with frequency-domain treatment through Fourier transform.

Keywords electromagnetic wave, lightwave, coherence, adaptive processing in sensing and imaging, learning logic, neural hardware

1 Introduction

Complex-valued neural networks (CVNNs) [1–6] extend the application fields steadily. We have various application systems employing CVNNs in the field of, for example, ultrasonic fault detection to find defects in metals and other materials [7], blind separation based on principal component analysis (PCA) in sonar [8] and voice processing [9], radars including ground penetrating radars to visualize plastic landmines [10–15] and satellite radars to estimate landscape information [16] and/or land-use classification [17], blur-compensation image processing [18], filtering and other time-sequential signal processing [19,20], frequency-domain multiplexed (FDM) microwave signal processing [21] and pulse beamforming in ultra-wideband (UWB) communications [22], FDM neural networks and learning logic circuits using lightwave [4,23–25] and fast adaptive three-dimensional holographic movie generation for optical tweezers [26,27], and developmental learning of motion control in combination with reinforcement learning [28]. In parallel, general associative memories [29] and independent component analysis (ICA) neural networks [30, 31] are also making progress in their improvement.

In the case of linear processing with a simple network structure, we often use the complex-valued least mean square (LMS) algorithm [32]. Neural networks, in general, conduct nonlinear processing. Regarding the nonlinearity to be employed, we have a series of discussions including several milestone papers [33]. The pros and cons of respective nonlinearities basically depend on the nature of the signal to be treated. We often deal with wave-related complex signals [1,3]. When we observe a wave signal by using coherent detection, or a baseband complex signal generated through Hilbert transform, we obtain the complex amplitude, i.e., the phasor, inevitably. The CVNNs are compatible with such wave phenomena. This is the most significant feature of the CVNNs. Actually, in the very early stage of the CVNN research, a pioneering idea and a basic experiment was reported concerning this important feature. That is, in 1992, M. Takeda and T. Kishigami [34] pointed out the fact that the electromagnetic field in a phase-conjugate resonator is formulated in the same manner as that of an associative memory, and that the resonant system realizes a quite fast recall. In this case, the limitation in the energy supply causes amplitude saturation, which realizes the neural nonliearity in the signal amplitude in a natural way.

In such wave information processing or wave control,

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it is essentially important to deal directly with phase (or phase difference) and amplitude. The reason lies in the facts that the amplitude corresponds to the wave energy (e.g., number of photons of lightwave), and that the phase difference represents time course and/or position change. From this viewpoint, the so-called amplitude-phase-type nonlinearity is consistent with wave [1,4,33], as is often the case in signal processing widely in electronics.

In this paper, with such application background, we examine what gives rise to the merits of the CVNN. As a result, we find that the weight multiplication at synapses yields the phase rotation as well as the amplitude amplification or attenuation. This type of multiplication reduces ineffective degree of freedom in the learning or self-organization to enhance the generalization characteristics in comparison with double-dimensional real-number networks, in spite of the fact that a complex number can be represented as an ordered pair of real numbers. The network dynamics consisting of this elemental rotation and amplification/attenuation leads to significant merits in total, originating from the consistency with the wave-related phenomena and information.

2 Application examples of CVNNs

In this section, we review two examples of applications of feedforward CVNNs to provide the readers with physical picture of wave-related systems. One example is a lightwave learning logic circuit in which the logic functions are variable depending on optical frequency. In other words, it is a FDM logic circuit.

Figure 1 shows the basic idea of the frequency multiplexed learning logic circuit [24]. This logic utilizes the ultra-wide frequency band of optics based on the FDM. For example, a learning optical element works as an AND logic gate at a certain optical frequency, but as an XOR at another frequency. We employ a frequency-dependent learning process based on steepest-descent and backpropagation learning in CVNNs [1,4,21].

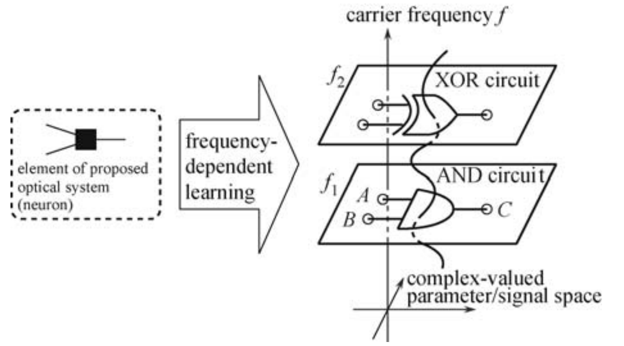

Fig. 1 Basic idea of the frequency-controlled optical learning logic circuit [24]

Figure 2 shows the experimental setup where we use a spatial light modulator (SLM) as a signal and weight generator. Figure 3 displays learning results showing the logic outputs for continuous input signals. We can obtain a set of results, AND and XOR, as we intended, with natural generalization characteristics. We can also estimate the realizable logic density in the frequency domain [25]. Figure 4 illustrates that such a FDM learning logic circuit can be equivalent with a number of conventional logic. This feature realizes a high flexibility in hardware in the future.

Besides the context mentioned above, coherent neural hardware will increase its significance gradually and steadily also from the viewpoint of quantum electron circuits. Modern microelectronics pursues ultimate miniaturization. The nano-electronics subjects to stochasticity both in space and in time. We can no longer say that a bit is 1 or 0 definitely. This fact leads to a failure of bit processing, i.e., symbol processing. We have inevitably to shift ourselves into the framework of pattern processing, which is represented by neural networks.

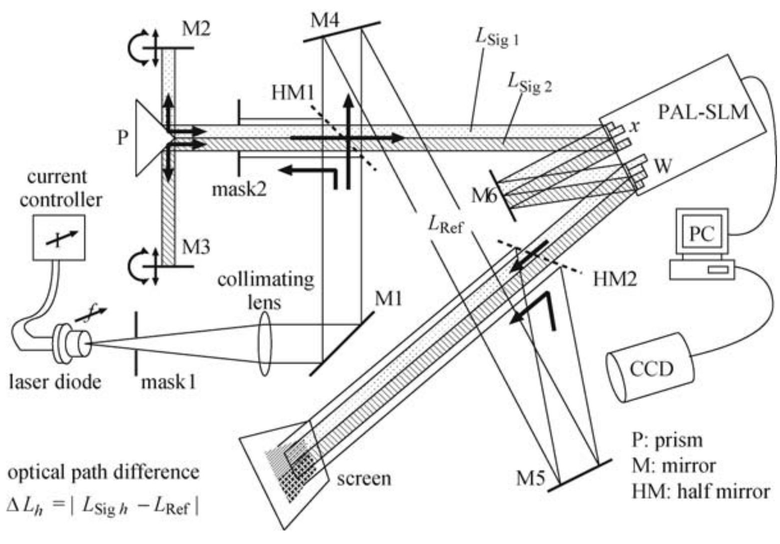

Fig. 2 Optical setup with two optical-path difference neural circuits [24]

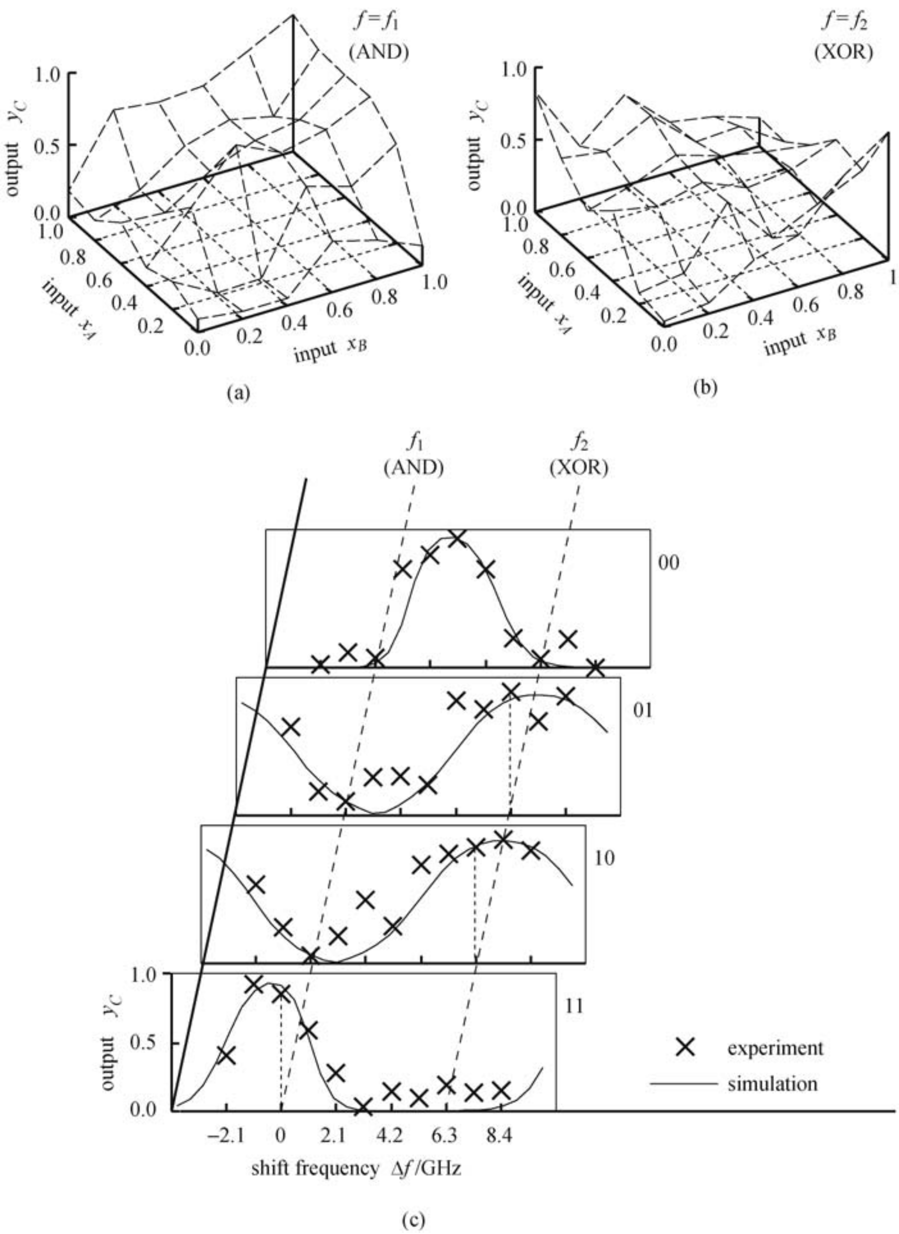

Fig. 3 Experimental results. (a) Input generalization characteristic at $f=f_1$ (AND); (b) input generalization characteristic at $f=f_2$ (XOR); (c) frequency-domain generalization characteristics (see detailed parameters in Ref. [24])

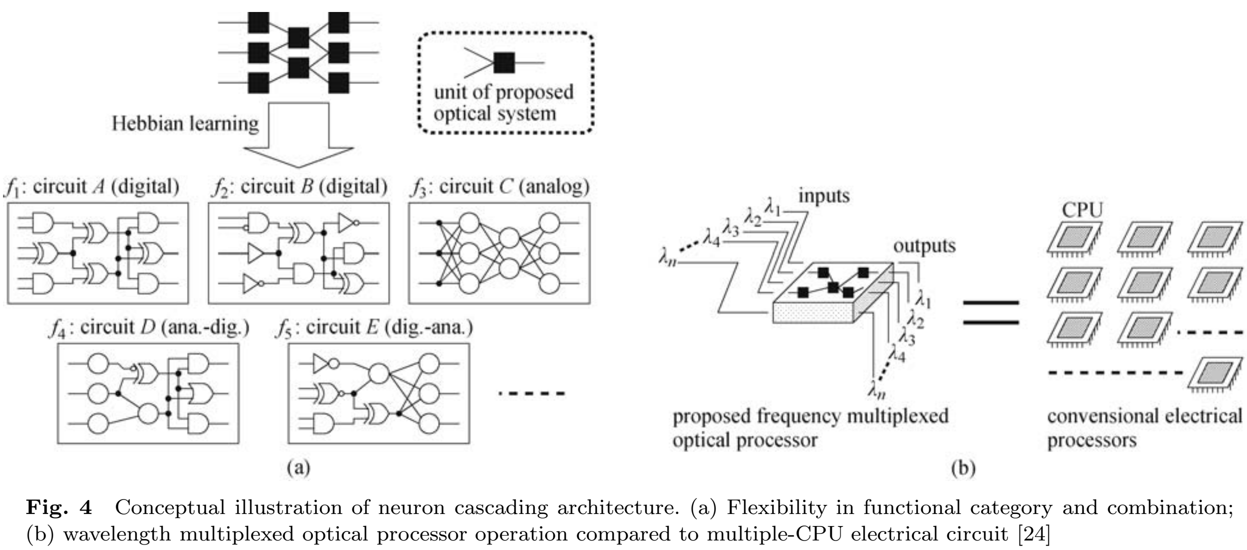

Fig. 4 Conceptual illustration of neuron cascading architecture. (a) Flexibility in functional category and combination; (b) wavelength multiplexed optical processor operation compared to multiple-CPU electrical circuit [24]

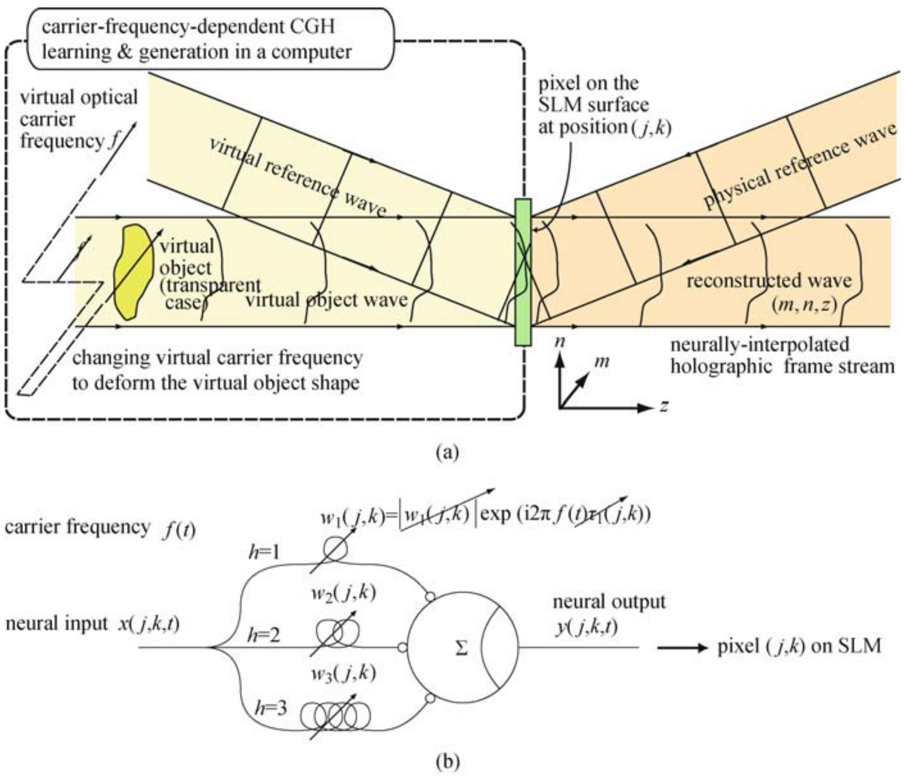

Fig. 5 Schematic diagram of CGH and coherent neural network. (a) Schematic diagram of CGH; (b) coherent neuron with multiple input connections [26]

Furthermore, when we make progress into subnanometer electronics, we also face to the quantum nature of electrons more explicitly, where the wave phenomena play significantly important roles. Consequently, the coherent neural networks can become the most fundamental framework of information processing hardwre.

The second example is an application in computer-generated hologram (CGH) aiming at three-dimensional movie (3D-movie) applications such as optical tweezers in biology. Figure 5(a) shows a schematic illustration of carrier frequency-dependent CGH, in which the SLM yields a frequency-variable image as a reconstructed lightwave. That is, the SLM generates a movie, a series of images, when we change the optical frequency. A computer generates such a frequency-dependent CGH through calculation. However, the calculation cost often becomes very high since we have a number of frames (images) per second. To reduce the cost, we employ a coherent neural network shown in Fig. 5(b) to interpolate the frames by utilizing the generalization nature [26].

The coherent neural network learns the delay and transmittance of the neural connections. After a learning process at a set of sparse frequency points, the network generates interpolating images, thanks to the neural generalization ability in the optical frequency domain. In other words, the generalization ability reduces the number of the learning points.

Figure 6 shows (upper in each black panel at time t) an example of a series of master and interpolating CGHs with (lower) reconstructed image showing a moving sharp spot. We found that the image quality is high enough in spite of the drastically reduced calculation cost. A resulting movie is available in Ref. [26]. Methods to improve the quality of such neural CGH were also reported [27].

3 What is the complex number?

3.1 Geometric and intuitive definition

In the old days history, the definition of the complex number changed gradually [35]. In the 16th century, Cardano tried to work with imaginary roots in dealing with quadratic and cubic equations. Afterward, Euler used complex numbers in his calculations intuitively and correctly. It is said that by 1728 he knows the transcendental relationship $i \log i = -\pi/2$. The Euler formulae appear in his book as

$$\cos x = \frac{e^{ix} + e^{-ix}}{2} \quad \text{and} \quad \sin x = \frac{e^{ix} - e^{-ix}}{2i}. \qquad (1)$$

It is also believed that, in early 1749, Euler already had a visual concept of complex numbers as points of plane. He described a number x on a unit circle as $x = \cos g + i \sin g$ where g is an arc of the circle. In 1798, Wessel described representation of the points of a plane by complex

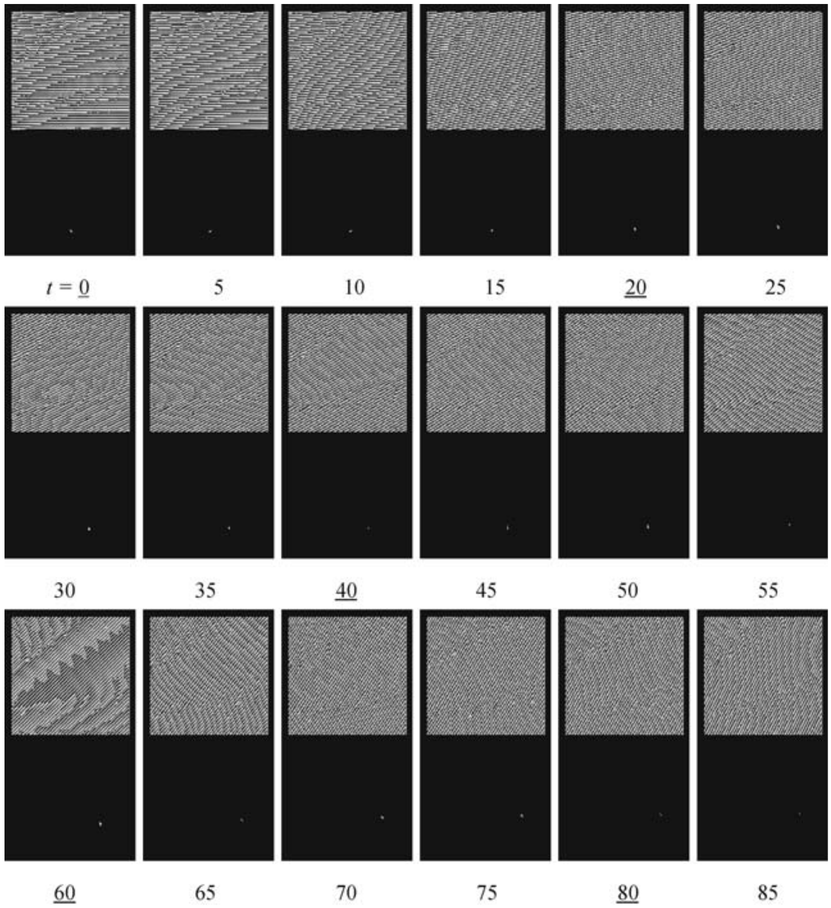

Fig. 6 Simulation result of interpolation when a single point is intended to move on an arc. In each image: (Upper) CGH phase images in gray scale, and (Lower) brightness on screen. Underlined time t in the movie: learning points t_p in time. (See details in Ref. [26])

numbers to deal with directed line segments. Argand also interpreted $\sqrt{-1}$ as a rotation through a right angle in the plane, and justified this idea on the ground that two $\sqrt{-1}$ rotations yields a reflection, i.e., -1. Gauss was in full possession of the geometrical theory by 1815. Furthermore, he proposed to call $+1$, -1, and $\sqrt{-1}$ as direct, inverse, and lateral unity, instead of positive, negative, and imaginary or "impossible" elements, to enhance the substantiality of imaginary number.

3.2 Definition as ordered pair of real numbers

The geometrical representation is intuitively simple and visually understandable, but may be weak in strictness. In 1835, Hamilton presented the formal definition of the complex number as an "ordered pair of real numbers", which also led to the discovery of quaternions, in his article entitled "Theory of conjugate functions, or algebra as the science of pure time". He defined addition and multiplication in such a manner that the distributive, associative, and commutative laws hold. The definition as the ordered pair of real numbers is algebraic, and can be stricter than the intuitive rotation interpretation.

At the same time, the fact that a complex number is defined by two real numbers may lead present-day neural-network researchers to consider a complex network equivalent to just a doubled-dimension real-number network in essence. However, in this paper, the authors would like to clarify the merit by focusing on the rotational function even with this definition.

Based on the definition of the complex number as an ordered pair of real numbers, we represent a complex number z as

$$z \equiv (x, y), \tag{2}$$

where x and y are real numbers. Then, the addition and multiplication of z_1 and z_2 are defined in *complex domain* as

$$(x_1, y_1) + (x_2, y_2) \equiv (x_1 + x_2, y_1 + y_2), \tag{3}$$

$$(x_1, y_1) \cdot (x_2, y_2) \equiv (x_1 x_2 - y_1 y_2, x_1 y_2 + y_1 x_2). \tag{4}$$

As a reference, the addition and multiplication (as a step in the calculation of inner product, for example) of *two-dimensional real values* is expressed as

$$(x_1, y_1) + (x_2, y_2) = (x_1 + x_2, y_1 + y_2), \tag{5}$$

$$(x_1, y_1) \cdot (x_2, y_2) = (x_1 x_2, y_1 y_2). \tag{6}$$

In the comparison, the addition process is identical. Contrarily, the complex multiplication seems quite artificial, but this definition (4) brings the complex number with its unique function, that is, the angle rotation, as well as amplitude amplification/attenuation, which are the result of the intermixture of the real and imaginary components.

It is easily verified that the commutative, associative, and distributive laws hold. We have the unit element $(1, 0)$ and the inverse of z ($\neq 0$), which is

$$z^{-1} \equiv \left(\frac{x}{x^2+y^2}, \frac{-y}{x^2+y^2} \right) = \left(\frac{x}{|z|^2}, \frac{-y}{|z|^2} \right), \tag{7}$$

where $|z| \equiv \sqrt{x^2+y^2}$.

3.3 Real 2×2 matrix representation

We can also use real 2×2 matrices, instead of the ordered pairs of real numbers, to represent complex numbers [35,36]. With every complex number $c = a + ib$, we associate the $\boldsymbol{C}$-linear transformation

$$T_c : \boldsymbol{C} \to \boldsymbol{C}, \qquad z \mapsto cz = ax - by + i(bx + ay), \tag{8}$$

which includes a special case of $z \to iz$ that maps 1 into i, i into -1, ..., with a rotation with right angle each. In this sense, this definition is a more precise and general version of Argand's interpretation of complex numbers. If we identify $\boldsymbol{C}$ with $\boldsymbol{R}^2$ by

$$z = x + iy = \begin{pmatrix} x \\ y \end{pmatrix}, \tag{9}$$

it follows that

$$T_c \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} ax - by \\ bx + ay \end{pmatrix} = \begin{pmatrix} a & -b \\ b & a \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}. \tag{10}$$

In other words, the linear transformation T_c determined by $c = a + ib$ is described by the matrix $\begin{pmatrix} a & -b \\ b & a \end{pmatrix}$. Generally, a mapping represented by a 2×2 matrix is non-commutative. However, in the present case, it becomes *commutative*. By this real matrix representation, the imaginary unit i in $\boldsymbol{C}$ is given as

$$I \equiv \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad I^2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = -E. \tag{11}$$

In the days of Hamilton, we did not have matrices yet. Even present, it is very rare to define complex numbers in terms of real 2×2 matrices [35(Chapter 3, §2, 5.),36]. The introduction of complex numbers through 2×2 matrices has the advantage, over introducing them through ordered pairs of real numbers, that it is unnecessary to define an ad hoc multiplication. What is most important is that this matrix representation clearly expresses the function specific to the complex numbers. That is, the rotation and amplification or attenuation as

$$\begin{pmatrix} a & -b \\ b & a \end{pmatrix} = r \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, \tag{12}$$

where r and θ denote amplification/attenuation of amplitude and rotation angle applied to signals, respectively, in the multiplication calculation. On the other hand, addition is rather plain. The complex addition function is identical to that in the case of doubled-dimension real numbers.

In summary, the phase rotation and amplitude amplification/attenuation are the most important features of complex numbers. The significance is described in the following sections.

4 Complex-valued neural networks

4.1 Synapse and network function

In wave-related adaptive processing, we often obtain excellent performance with learning or self-organization based on the CVNNs. As already mentioned, the reason depends on situations. However, the discussion in Sect. 3 suggests that the origin lies in the complex rule of arithmetics. That is to say, the merit arises from the functions of the four fundamental rules of arithmetics of complex numbers, in particular the multiplication, rather than the representation of the complex numbers, which can be geometric, algebraic, or in matrices. Moreover, the essence of the complex numbers also lies in the characteristic multiplication function, the phase rotation, as overviewed in Sect. 3 [1].

Let us consider a very simple case shown in Fig. 7(a) where we have a single-layered 2-input 2-output feedforward neural network in real number. For simplicity, we omit the possible nonlinearity at the neurons, i.e., the activation function is the identity function, where the neurons have no threshold. We assume that the network should realize a mapping that transforms an input $\boldsymbol{x}^{\mathrm{IN}}$ to an output $\boldsymbol{x}^{\mathrm{OUT}}$ in Fig. 7(b) through supervised learning that adjusts the synaptic weights w_{ji}. Simply, we have only a single teacher pair of input and output signals. Then, we can describe a general input-output relationship as

$$\begin{pmatrix} x_1^{\mathrm{OUT}} \\ x_2^{\mathrm{OUT}} \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x_1^{\mathrm{IN}} \\ x_2^{\mathrm{IN}} \end{pmatrix}. \tag{13}$$

We have a variety of possible mapping obtained by the learning because the number of parameters to be determined is larger than the condition, i.e., the learning task is an ill-posed problem. The functional difference emerges as the difference in the generalization characteristics. For example, learning can result in a degenerate mapping shown in Fig. 7(c), which is often unuseful in practice.

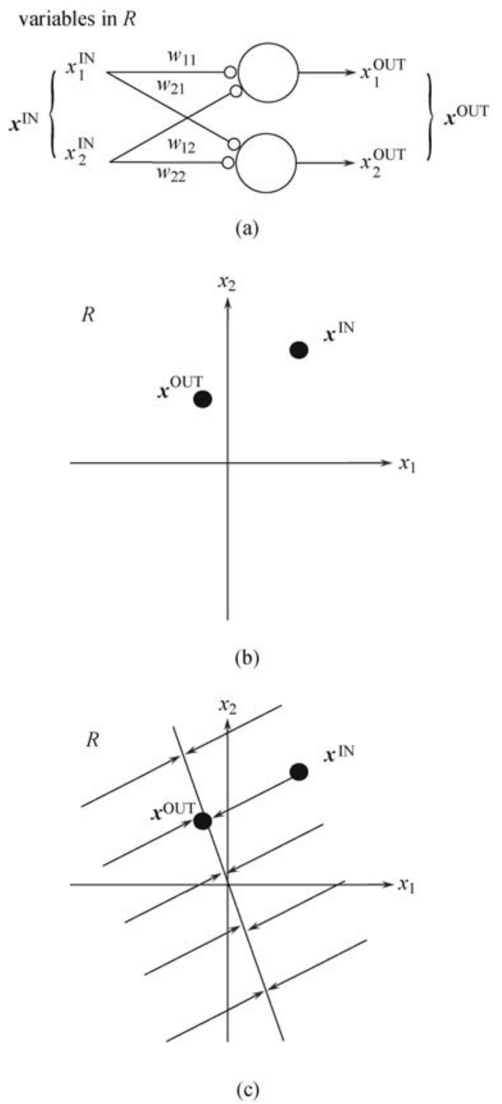

Fig. 7 A simple linear feedforward network to learn a mapping. (a) A real-valued single-layered two-input two-output feedforward network; (b) a task to learn a mapping that maps $\boldsymbol{x}^{\rm IN}$ to $\boldsymbol{x}^{\rm OUT}$; (c) a possible but degenerate solution that is often unuseful

Next, let us consider the mapping learning task in the one-dimensional complex domain, which transforms a complex value $\boldsymbol{x}^{\rm IN}=(x_1^{\rm IN},x_2^{\rm IN})$ to another complex value $\boldsymbol{x}^{\rm OUT}=(x_1^{\rm OUT},x_2^{\rm OUT})$. Figure 8(a) shows the complex-valued network, where the weight is a single complex value. The situation is expressed just like in Eq. (13) as

$$\begin{pmatrix} x_1^{\rm OUT} \\ x_2^{\rm OUT} \end{pmatrix} = \begin{pmatrix} |w|\cos\theta & -|w|\sin\theta \\ |w|\sin\theta & |w|\cos\theta \end{pmatrix} \begin{pmatrix} x_1^{\rm IN} \\ x_2^{\rm IN} \end{pmatrix}, \quad (14)$$

where $\theta \equiv \arg(w)$. The degree of freedom is reduced, and the arbitrariness of the solution is also reduced. Figure 8(b) illustrates the result of the learning. The mapping is a combination of phase rotation and amplitude attenuation. This example is truly an extreme. The dynamics of a neural network is determined by various parameters such as network structure, input–output data dimensions, and teacher signal numbers. However, the above characteristics of phase rotation and amplitude modulation are embedded in the complex-valued network as a universal elemental process of weighting.

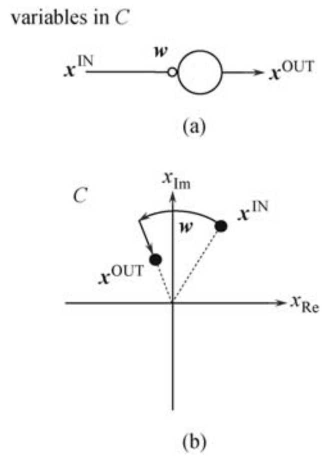

Fig. 8 Another simple linear feedforward network to learn the same task given in Fig. 7(b). (a) CVNN seemingly identical to Fig. 7(a); (b) a solution obtained in this small degree-of-freedom case

The essential merit of neural networks in general lies in the high degree of freedom in learning and self-organization. However, if we know *a priori* that the objective quantities include "phase" and/or "amplitude," we can reduce possibly harmful portion of the freedom by employing a CVNN, resulting in a more meaningful generalization characteristics. The "rotation" in the complex multiplication works as an elemental process at the synapse, and realizes the advantageous reduction of the degree of freedom. This feature corresponds not only to the geometrical intuitive definition of complex numbers but also to the Hamilton's definition by ordered pairs of real numbers, or the real 2×2 matrix representation.

Although we considered a small feedforward network in this section, the conclusion is applicable also to other CVNNs such as complex-valued Hebbian-rule based network and complex correlation learning networks, where the weight is updated by the multiplication results. The elemental process of phase rotation and amplitude modulation results in the network behavior consistent with phase rotation and amplitude modulation in total.

The nature is a great advantage when we deal with not only waves such as electromagnetic wave and lightwave, but also arbitrary signals with the Fourier synthesis principle, or in the frequency domain through the

Fourier transform.

4.2 Nonlinearity of the neuron activation function

The rotation at the synapses is the most fundamental specific nature in CVNNs. The neuron nonlinearity can be another issue.

The complex LMS is the most widely used basis of adaptive processing of complex signals [32]. The introduction of nonlinearity into the neuron activation function once seemed to have a serious problem in the differentiability in the complex domain. Liouville's theorem in complex analysis states that every entire (holomorphic) function must be constant. It follows that, if we introduce some nonlinearity, we have to abandon the differentiability. This fact was considered to be a big problem at around 1990s because some researchers believed that the indifferentiability should lead directly to the impossibility to obtain and/or analyze the dynamics of the CVNNs.

However, the concern was found to be a trifle because neural dynamics are generally described by partial differentiation in terms of a number of variables associated with the neurons. Actually, nowadays, we calculate partial differentials in terms of real and imaginary parts, or phase and amplitude, to determine neural dynamics in CVNNs. This manner is practically effective.

At the same time, it is true that we discard the conformal mapping nature of the holomorphic function. However, when we utilize a conformal mapping function, we often concentrate upon the mapping structure itself, rather than a combination with some nonlinearity. Additional nonlinearity should rather be hindrance. Accordingly, the non-holomorphy is not a big problem again.

In complex-valued associative memories, researchers investigated the requirements on the nonlinearity to determine an effective energy function [37]. As a result, we have two types of possibility. One is to apply nonlinearity to real and imaginary parts, respectively, and to combine them to yield a complex output [38,39]. Another is to employ nonlinear functions for the phase and amplitude, respectively [4].

In other CVNNs, we may have possibilities to employ other nonlinearity depending on the objects, i.e., what type of processing we aim at. Even in such cases, the above-mentioned two types of nonlinearity will be the most promising candidates since we normally consider that a direct extension of the real sigmoid function works well also widely in complex domain.

4.3 Amplitude and phase or real and imaginary in nonlinearity

When we deal with wave information or wave itself, the real and imaginary axes are essentially less meaningful than amplitude and phase (or phase difference) because the real and imaginary axes are determined relatively to an arbitrarily determined phase reference. An example is the coherent detection in communications receiver, where we prepare a local oscillator (LO) with a phase-locked loop (PLL) locked to some reference to be used for demodulation, that is, extraction of real and imaginary signals. The receiver determines the real and imaginary parts, which never exist beforehand [1,4]. Instead, the difference of two phase values are meaningful itself, which corresponds to time course and/or position difference. In this sense, the phase difference represents certain information directly. The amplitude, orthogonal to phase, is also meaningful, signifying energy or power of the wave. Accordingly, the amplitude–phase nonlinearity is more suitable for wave-related processing. Actually, based on the amplitude–phase nonlinearity, we have proposed new adaptive systems such as the optical learning logic circuits realizing frequency-multiplexed operation [25] and the fast method to yield CGH for three-dimensional movies [26,27] reviewed in Sect. 2.

5 Conclusion

We looked back at the history of the complex number to elucidate and discuss its features. We found that the phase rotation in the complex multiplication is the most important characteristic. It follows that, in the neural network construction, we have to focus upon the phase and amplitude of the signals to be treated to emphasize the merit of the CVNNs. This nature is a great merit in dealing with wave-related information or wave itself such as electromagnetic wave, lightwave, sound wave, and ultrasonic wave. The advantage is useful not only for pure sinusoidal wave but also for arbitrary signals in combination with the concept of Fourier synthesis and/or in the treatment in the frequency domain through the Fourier transform. The most important merit of the CVNNs lies in this point.

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