Front. Electr. Electron. Eng. China (2006) 1: 31–36
DOI 10.1007/s11460-005-0014-8

RESEARCH ARTICLE

**ZHANG Ge-xiang, LI Na, JIN Wei-dong,
HU Lai-zhao**

Novel Quantum Genetic Algorithm and Its Applications

Abstract By introducing strong parallelism of quantum computing into evolutionary algorithm, a novel quantum genetic algorithm (NQGA) is proposed. In NQGA, a novel approach for updating the rotation angles of quantum logic gates and a strategy for enhancing search capability and avoiding premature convergence are adopted. Several typical complex continuous functions are chosen to test the performance of NQGA. Also, NQGA is applied in selecting the best feature subset from a large number of features in radar emitter signal recognition. The testing and experimental results of feature selection show that NQGA presents good search capability, rapid convergence, short computing time, and ability to avoid premature convergence effectively.

Keywords genetic algorithm, quantum genetic algorithm, feature selection, recognition

1 Introduction

The principles of quantum computing have been developed since quantum mechanical principles were applied to computer science about 20 years ago. A two-energy-level system can be regarded as a quantum bit (qubit), and the interaction of multiple two-energy-level systems makes nonclassical logic quantum gates possible [1,2]. Having many special merits, the study of quantum computing and quantum computers has become very hot in information science, especially after some good quantum computing algorithms, such as Grover's quantum search algorithm [3] and Shor's factorizing algorithm [4], were explored.

Translated from *Acta Electronica Sinica*, 2004, 32(3): 476–479 (in Chinese)

ZHANG Ge-xiang (✉), LI Na, JIN Wei-dong
School of Electrical Engineering,
Southwest Jiaotong University,
Chengdu 610031, China
E-mail: zhgxdylan@tom.com

HU Lai-zhao
National EW Laboratory,
Chengdu 610036, China

Quantum genetic algorithm (QGA), a new and promising genetic algorithm developed in recent years, is the product of quantum computing theory and genetic algorithm. QGA is based on the concepts of quantum computing (qubit, quantum superposition, and quantum entanglement) and quantum theory, such as quantum logic gate [1,2]. In QGA, qubit encoding is used to represent the chromosome, and evolutionary process is implemented by using quantum logic gate operation on the chromosomes. Now, much attention is paid to QGA because it has the characteristics of strong searching capability, rapid convergence, short computing time, and small population size [5–12].

Narayanan and Moore [5] presented quantum-inspired genetic algorithm (QIGA) to solve traveling salesman problem (TSP) successfully, which introduced the concepts and theory of quantum computing into genetic algorithm. By introducing qubit representation and quantum logic gate operation, Han and Kim [6] and Han *et al.* [7] presented genetic quantum algorithm (GQA) and parallel quantum genetic algorithm (PQGA) to solve an NP-hard combination optimization problem (knapsack problem). Quantum crossover [5] and quantum mutation were used to improve the performances of GQA [6] in [8]. An improved QGA based on multiqubit encoding and dynamically adjusting the rotation angle mechanism was presented to separate the blind sources [9]. Li and Jiao [10] proposed a hybrid parallel quantum evolutionary algorithm based on QGA [6] and parallel algorithm. Zhang et al. [11] presented an improved QGA by introducing population catastrophe operation and violent vibration. They also proposed a novel PQGA [12] by using a novel evolutionary strategy. The results [6–12] show that QGA and GQA are greatly superior to conventional genetic algorithm (CGA). The evolutionary strategy [6–11] is based on prior knowledge of the best solution of optimization problems. For example, in knapsack problem, the criterion of the optimal solution is that the number of "1" should be as big as possible within constraint conditions because more number "1" means bigger fitness of chromosome. However, the criterions of optimal solutions have not been gotten in continuous function optimization problems and in most practical cases.

This paper proposes a novel quantum genetic algorithm (NQGA) in which qubit phase comparison method is used to update the rotation angels of quantum logic gates, and the strategy of adjusting search grid self-adaptively is employed. Simultaneously, immigration and catastrophe operations are introduced to NQGA to strengthen search capability and to avoid premature phenomena. Testing results of several complex continuous functions and experimental results of feature selection show that NQGA is superior to GQA [6].

2 Novel quantum genetic algorithm

2.1 Algorithm description

The smallest information unit in a two-state quantum computer is called a qubit. A qubit may be in the "1" state, in the "0" state, or in any superposition of the two. The state of a qubit can be represented as

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle, \tag{1}$$

where α and β are the probability amplitudes of the corresponding states and satisfy the following normalization:

$$|\alpha|^2 + |\beta|^2 = 1. \tag{2}$$

In Eq. (2), $|\alpha|^2$ gives the probability that the qubit will be found in the "0" state and $|\beta|^2$ is in the "1" state. A system with m qubits contains information on 2^m states. The linear superposition of all possible states can be represented as

$$|\psi_i = \sum_{k=1}^{2^m} C_k |S_k\rangle, \tag{3}$$

where C_k specifies the probability amplitude of the corresponding states S_k and subjects to the normalization condition $|C_1|^2 + |C_2|^2 + \cdots + |C_{2^m}|^2 = 1$.

Definition 1 The probability amplitude of one qubit is defined by a pair of complex, (α, β), as

$$[\alpha \;\; \beta]^{\mathrm{T}}, \tag{4}$$

where α and β satisfy Eq. (1) and Eq. (2).

Definition 2 The phase of a qubit is defined with an angle ζ as

$$\zeta = \arctan\left(|\beta|/|\alpha|\right), \tag{5}$$

and the product $|\alpha|\cdot|\beta|$ is represented by the symbol d, *i.e.*

$$d = |\alpha| \cdot |\beta|, \tag{6}$$

where d stands for the quadrant of qubit phase ζ. If d is positive, the phase ζ lies in the first or third quadrant; otherwise, the phase ζ lies in the second or fourth quadrant.

The probability amplitudes of m qubits are represented as

$$p = \left[\begin{array}{c|c|c|c} \alpha_1 & \alpha_2 & \cdots & \alpha_m \\ \beta_1 & \beta_2 & \cdots & \beta_m \end{array}\right] \tag{7}$$

where $|\alpha_i|^2 + |\beta_i|^2 = 1, i = 1, 2, \cdots, m.$ Hence, the phase of the ith qubit is

$$\zeta = \arctan\left(|\beta_i|/|\alpha_i|\right). \tag{8}$$

Suppose that the population size is n. Chromosomes are represented with qubits as $\boldsymbol{P}=\{\boldsymbol{p}_1, \boldsymbol{p}_2,..., \boldsymbol{p}_n\}$, where $\boldsymbol{p}_j$ (j=1, 2,..., n) is an individual of a population shown in Eq. (7). Quantum rotation gate $\boldsymbol{G}$ is chosen as quantum logic gate, and it is

$$\boldsymbol{G} = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}, \tag{9}$$

where θ is the rotation angle of the quantum rotation gate, defined as

$$\theta = k \cdot f(\alpha_i, \beta_i), \tag{10}$$

where k is a coefficient whose value influences the speed of convergence. If k is too big, the search grid of the algorithm would be large and the solutions may diverge or converge prematurely to a local optimum. If it is too small, the search grid of the algorithm would be small and the algorithm may fall in stagnant state. Hence, k is defined as a variable. In CGAs, adjustable coefficients are often relative to the maximal fitness and average fitness of the current generation. But this strategy cannot be used in NQGA because it will affect the characteristic of short computing time. Here, by taking advantage of rapid convergence of NQGA, k is defined as a variable relative to evolutionary generations to adjust the search grid of NQGA adaptively. $f(\alpha_i, \beta_i)$ is a searching direction function determining the search direction of convergence to a global optimum.

Thus, the procedure that all individuals are updated using quantum rotation gates can be described as

$$\boldsymbol{p}_j^{t+1} = \boldsymbol{G}(t) \cdot \boldsymbol{p}_j^t, \tag{11}$$

where t is evolutionary generation, $\boldsymbol{G}(t)$ stands for the tth generation quantum rotation gate, and $\boldsymbol{p}_j^t$ is the probability amplitude of an individual at tth generation.

The detailed algorithm of NQGA is described as follows.

Step 1

Initialization: choosing the population size n and the number m of qubits. A population containing n

individuals is $\boldsymbol{P}=\{\boldsymbol{p}_1, \boldsymbol{p}_2, ..., \boldsymbol{p}_n\}$, where $\boldsymbol{p}_j$ (j=1, 2,..., n) is an individual of a population shown in Eq. (7). $\alpha_i = \beta_i = 1/\sqrt{2}(i = 1, 2, \cdots, m)$, which means that all states are superposed with the same probability. The initial value of evolutionary generation g is set to 0.

Step 2

According to the probability amplitudes of all individuals in $\boldsymbol{P}$, we can construct the observation state $\boldsymbol{R}$ of basic quantum states. Here $\boldsymbol{R}=\{a_1, a_2, ..., a_n\}$, where a_j (j=1, 2,..., n) is the observation state of one individual. It is a binary string with the length m, that is, $a_j=b_1, b_2, ..., b_m$, where b_k (k=1, 2,..., m) is one binary number "0" or "1". Observation states $\boldsymbol{R}$ are generated based on probability: to the probability amplitude $[\alpha_i, \beta_i]^{\mathrm{T}}$ (i=1, 2,..., $n\times m$) of every qubit in population $\boldsymbol{P}$, a random number r in the range [0,1] is generated. If $r < |\alpha_i|^2$, the corresponding observation value is "0"; otherwise, the value is "1". In the process of constructing observation state $\boldsymbol{R}$ using probability amplitude $\boldsymbol{p}$, the decoding operation of the genetic algorithm is included. After decoding, the parameter values of all optimization parameters can be obtained.

Step 3

Each individual in $\boldsymbol{R}$ is evaluated by using a fitness function.

Step 4

The best solution S_c in current generation is maintained. If S_c is more than the best solution S_o in the evolutionary process, S_o is replaced by S_c, and S_o is maintained. If a satisfactory solution is obtained or the maximum generation is arrived at, the algorithm ends. Otherwise, it continues.

Step 5

According to Eq. (10), the rotation angles of quantum rotation gates $\boldsymbol{G}$ are computed. In the process of computing, $k=0.5\times\exp(-t/t_{\max})$, where t is the remaining number of evolutionary generation g divided by the generation C_g. C_g is the generation when the population catastrophe operation is made. tmax is the maximal evolutionary generation determined by the complexity of optimization problem. $f(\alpha, \beta)$ is determined using Table 1. The quantum rotation gate in Eq. (9) is used to update the probability amplitudes of all individuals.

Step 6

To get good individuals, immigration operation is introduced to make the optimal solution searching easy. Immigration operation can make the algorithm jump out of the suboptimal solution. When immigration operation is made in every M_g generation, a new generation population is generated using the method of initialization. Each α is a random value from 0 to 1, and the corresponding β is $\pm\sqrt{1-|\alpha|^2}$. The probability amplitudes of several qubits of the best individual in Step 4 are replaced by the ones in the new population.

Table 1 Look-up table of function $f(\alpha, \beta)$

$d_1>0$	$d_2>0$	$f(\alpha, \beta)$	
		$\lvert\zeta_1\rvert>\lvert\zeta_2\rvert$	$\lvert\zeta_1\rvert<\lvert\zeta_2\rvert$
True	True	+1	−1
True	False	−1	+1
False	True	−1	+1
False	False	+1	−1

$d_1=\alpha_1\cdot\beta_1$, $\zeta_1=\arctan(\beta_1/\alpha_1)$, where α_1, β_1 is the probability amplitude of the best solution; $d_2=\alpha_2\cdot\beta_2$, $\zeta_2=\arctan(\beta_2/\alpha_2)$, where α_2, β_2 is the probability amplitude of the current solution.

Step 7

If the best solution maintained is not changed in many generations, such as C_g generations, then the population catastrophe operation should be executed. Population catastrophe operation can make the algorithm jump out of the local solution and avoid the stagnant state. Hence, it improves the search capability. In this operation, the best individual in Step 4 is replaced by the best individual in the new population.

Step 8

Evolutionary generation g increases, and the algorithm goes back to Step 2.

The reasons that NQGA has the characteristics of rapid convergence, good search capability, short running time, and ability to avoid premature phenomena lie in the following points.

(a) Qubit representation can make the individuals have the information of different superposition states in the evolutionary process. Thus, the population diversity can be maintained well and the selection pressure can be avoided effectively.
(b) The rotation angles of quantum logic gates vary adaptively, that is, the rotation angles change gradually from big to small. In the beginning, the angles vary greatly to search the solution in a big range. Then, as the evolutionary generation increases, the angles decrease gradually to guarantee the algorithm searching the solution in the local solution space. When population catastrophe operation happens, the rotation angles of quantum logic gates increase again. Therefore, the updated method is simple, valid, and practical.
(c) Immigration operation introduces high-quality genes into the population so that the most satisfying solution is easy to obtain and the local values can be avoided, and population catastrophe operation can make the algorithm get rid of the evolutionary stagnant state or jump out of local minimums as soon as possible.

2.2 Performance test

We choose two typical continuous functions to test the performances of NQGA.

1. Multipeak function:

$$f_1 = 10 + \frac{\sin(1/x)}{(x-0.16)^2 + 0.1}, \quad x \in [0.01, 1]. \tag{12}$$

Function f_1 has a lot of local optimums in its whole solution space. The optimal solution is f_1=19.8949 at x=0.1275.

2. Two-dimensional multimodal function:

$$f_2 = \cos(2\pi x_1)\cos(2\pi x_2)e^{-(x_1^2+x_2^2)/10}, \quad 1- \le x_1, x_2 \le 1. \tag{13}$$

This function is a well-known multimodal test function. It has 13 local optimums in its whole solution space. The optimal solution is f_2=1 at x_1=x_2=0.

For comparison, NQGA and GQA are used to optimize the two functions. Population size n=10, the number of qubits is 15, maximal evolutionary generation is 1,000, and the process ends at estimation error 0.0001. In NQGA, M_g and C_g are 40 and 100, respectively. The statistical results of 100 tests are shown in Table 2.

From Table 2, conclusions can be drawn that the mean generation of NQGA is less than that of GQA. The more complex the function is, the more obvious the difference is. The success rate of NQGA is much higher than that of GQA, which indicates that immigration and catastrophe operations in NQGA are very helpful in making NQGA jump out of local optimums. The mean time and mean generation of NQGA are a little shorter than those of GQA, which are also the results of the immigration and catastrophe operations of NQGA.

3 Application example

Feature selection is a process of extracting the most discriminatory information and removing the irrelevant and redundant information from a large number of measurable attributes [13]. Good features can enhance within-class pattern similarity and between-class pattern dissimilarity [14]. The minimum number of relevant and significant features can simplify the design of classifiers without

Table 2 The statistical results of 100 tests using NQGA and GQA

Functions	Algorithms	Mean generation	Mean time (s)	Success rate (%)
f_1	GQA	177.17	3.5289	87.00
	NQGA	139.00	3.2617	100.00
f_2	GQA	358.21	13.2192	76.00
	NQGA	249.07	10.7918	100.00

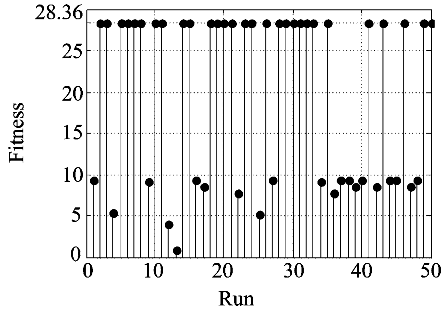

Fig. 1 The fitness values of 50 runs using GQA

degrading its performances. Hence, feature selection is important in data analysis, and it is a crucial step in pattern recognition, machine learning, and data mining [13,14]. It is a typical combinatorial optimization in which the choice of good evaluation criterion and searching strategy is a key problem.

In the following example, the evaluation criterion is given first, and then NQGA is used to select the best feature subset from the original feature set with 16 radar emitter signal features.

Suppose that the maximum within-class clustering of the ith class is represented by C_{ii}. We define C_{ii} as

$$C_{ii} = \max\left\{\left[\frac{1}{M_i^q}\sum_{k=1}^{M_i^q}\left|\left|x_{ik}^q - E(X_i^q)\right|\right|^p\right]^{\frac{1}{p}}\right\}, \tag{14}$$

where q=1, 2,...,N, N is the number of features, M_i^q is the number of samples of the qth feature of the ith class, x_{ik}^q is the kth sample value of the qth feature of the ith class, $X_i^q = \left[x_{i1}^q \quad x_{i2}^q \quad \cdots \quad x_{iM_i^q}^q\right]$, $E(X_i^q)$ is the expectation of X_i^q, and p ($p\geq1$) is an integer. Similarly, the maximum within-class clustering C_{jj} of the jth class has the same form as C_{ii}.

The minimum distance D_{ij} between the ith class and the jth class is

$$D_{ij} = \min\left\{\left|\left|E(X_i^q) - E\left(X_j^q\right)\right|\right|\right\}. \tag{15}$$

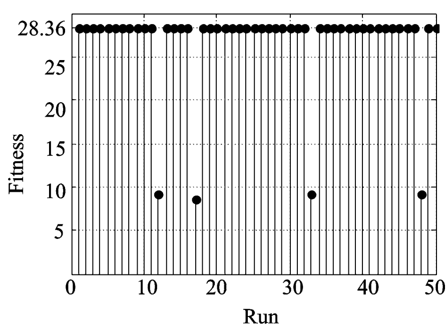

Fig. 2 The fitness values of 50 runs using NQGA

Table 3 Statistical results of GQA and NQGA

Algorithms	Average generation	Average time (s)	Average fitness	Maximal fitness
GQA	465.78	22.59	19.77	28.36
NQGA	486.84	26.08	26.81	28.36

Thus, the between-class separability S_{ij} between the *i*th class and the *j*th class is defined as

$$S_{ij} = \frac{D_{ij}}{C_{ii} + C_{jj}}. \tag{16}$$

Assume that there is a total of H classes to be recognized, the criterion function of NQGA is represented by

$$f = \frac{2}{H(H-1)} \sum_{i-1}^{H-1} \sum_{j=i+1}^{H} (S_{ij}/q). \tag{17}$$

Obviously, the bigger the f is, the better the selected feature subset is.

In our prior work, 16 features have been extracted from 10 radar emitter signals using different approaches [15–17]. The features are fractal dimensions, including information dimension, box dimension, and correlation dimension, two resemblance coefficient features, Lempel–Ziv complexity, approximate entropy, wavelet entropy, and eight energy distribution features based on wavelet packet decomposition. The 10 radar emitter signals are binary phase shift-key radar emitter signal (BPSK), quaternary phase shift-key radar emitter signal (QPSK), multiple phase shift-key radar emitter signal (MPSK), linear frequency modulation radar emitter signal (LFM), nonlinear frequency modulation radar emitter signal (NLFM), continuous wave radar emitter signal (CW), Frequency Diversity emitter signal (FD), frequency shift-key radar emitter signal (FSK), Intra-Pulse Frequency Encode radar emitter signal (IPFE), and Chirp Step-Frequency radar emitter signal (CSF), respectively. NQGA and GQA are used to search the optimal feature subset from 16 features using the introduced evaluation criterion. In the experiment, the population size is 10, the number of qubits is 16, the fitness function is shown in Eq. (17), and the maximal evolutionary generation is 1,000. M_g and C_g are 40 and 100, respectively. The experiments are carried out 50 times. The fitness function values of GQA and NQGA in the searching process are shown in Fig. 1 and Fig. 2, respectively. The statistical results are shown in Table 3.

The best feature subsets obtained by NQGA and GQA are composed of three features, and the maximal fitness values obtained by NQGA and GQA are identical. In Fig. 1 and Fig. 2, the time spent in obtaining the maximal fitness value using NQGA is more than that of GQA. Table 3 shows that the average fitness value of NQGA is much bigger than that of GQA. Because of immigration and catastrophe operation in NQGA, the average generation and average time of NQGA are a little bigger than those of GQA.

Neural network (NN) classifiers are designed to test the performances of the selected feature subset. For comparison, selected and original feature sets are used to train NN classifiers to recognize 10 different radar emitter signals. The structures of NN classifiers are 3–15–10 and 16–25–10. For the hidden layer and output layer, *tansig* and *logsig*, respectively, are chosen as the transfer functions. We choose the resilient backpropagation (RPROP) algorithm [18] as the training algorithm of the NN. The ideal outputs of NN are "1", the output tolerance is 0.05, and training error is 0.001. Recognition results are shown in Table 4. All numbers in Table 4 are percentages. From Table 4, we can see that the accurate recognition rate is 99.21% using the selected feature subset and 96.21% using the original feature set. This demonstrates the effectiveness of the proposed approach.

4 Conclusions

Quantum computing has the characteristics of strong parallelism. How to combine the theory of quantum computing with the principles of genetic algorithm is still a promising research problem. By introducing qubit chromosome representation, quantum phase comparison method, immigration operation, and population catastrophe, this paper proposes an NQGA based on the concepts and principles of quantum computing. Because qubit can represent basic states and their superposition states simultaneously, NQGA only needs a small population size instead of degrading its performances. The evolutionary operation that quantum rotation gates are updated using the quantum phase comparison method is simple and valid. Immigration operation and population catastrophe can enhance the search capability of NQGA and make the algorithm jump out of local minimums. Therefore, NQGA has the characteristics of good search capability, rapid convergence, short computing time, and ability to avoid premature convergence effectively, which are also shown in the experiment results from algorithm performance testing and feature selection in radar emitter signal recognition. Experimental results also show that NQGA is superior to GQA.

Table 4 Statistical result comparison of GQA and NQGA (%)

Type	BPSK	QPSK	MPSK	LFM	NLFM	CW	FD	FSK	IPFE	CSF
Feature subset selected	97.90	78.17	100	90.87	100	98.47	98.78	97.91	100	91.67
Original feature set	99.01	93.75	100	100	100	100	100	100	100	100

Acknowledgements This work was supported by the National Natural Science Foundation of China (No. 60572143) and by the National EW Lab Pre-research Foundation (No. NEWL51435QT220401).

References

1. Hey T., Quantum computing: an introduction, Comput. Control Eng. J., 1996, 10(3): 105–112
2. Narayanan A., Quantum computing for beginners, Fumio Harashima. Proceedings of the 1999 Congress on Evolutionary Computation, Piscataway: IEEE Press, 1999, 2231–2238
3. Grover L. K., Quantum mechanics helps in searching for a needle in a haystack, Phys. Rev. Lett., 1997, 79(2): 325–328
4. Shor P. W., Algorithms for quantum computation: discrete logarithms and factoring, Proceedings of the 35th Annual Symposium on Foundations of Computer Science, Piscataway: IEEE Press, 1994, 124–134
5. Narayanan A. and Moore, M., Quantum-inspired genetic algorithm, Toshio Fukuda, Proceedings of IEEE International Conference on Evolutionary Computation, Piscataway: IEEE Press, 1996, 61–66
6. Han K. and Kim J.-H., Genetic quantum algorithm and its application to combinatorial optimization problems, Proceedings of the 2000 IEEE Conference on Evolutionary Computation, Piscataway: IEEE Press, 2000, 1354–1360
7. Han K.-H., Park K.-H., Lee C.-H. and Kim J.-H., Parallel quantum-inspired genetic algorithm for combinatorial optimization problems, Proceedings of the IEEE Conference on Evolutionary Computation, Piscataway: IEEE Press, 2001, 1442–1429
8. Li B. and Zhuang Z., Genetic algorithm based on quantum probability representation, Lect. Notes Comput. Sci., 2002, 2412: 500–505
9. Yang J.-A., Li B. and Zhuang Z., Research of quantum genetic algorithm and its application in blind source separation, J. Electron., 2003 20(1): 62–68 (in Chinese)
10. Li Y. and Jiao L., An effective method of image edge detection based on parallel quantum evolutionary algorithm, Signal Process., 2003, 19(1): 69–74 (in Chinese)
11. Zhang G., Jin W. and Li N., An improved quantum genetic algorithm and its application, Lect. Notes Comput. Sci., 2003, 2639: 449–452
12. Zhang G., Jin W., Hu L., A novel parallel quantum genetic algorithm, Pingzhi Fan, Proceedings of the Fourth International Conference on Parallel and Distributed Computing, Applications and Technologies, Piscataway: IEEE Press, 2003, 693–697
13. Chakraborty B., Genetic algorithm with fuzzy fitness function for feature selection, Proceedings of the IEEE International Symposium on Industrial Electronics, Piscataway: IEEE Press, 2002, 315–319
14. Sural S. and Das P. K., A genetic algorithm for feature selection in a neuro-fuzzy OCR system, Proceedings of Sixth International Conference on Document Analysis and Recognition, Piscataway: IEEE Press, 2001, 987–991
15. Zhang G., Rong H., Jin W. and Hu L., Radar emitter signal recognition based on resemblance coefficient features, Lect. Notes Comput. Sci., 2004, 3066: 665–670
16. Zhang G., Hu L. and Jin W., Complexity feature extraction of radar emitter signals, Proc of the Third Asia-Pacific Conf. on Environmental Electromagnetics, Piscataway: IEEE Press, 2003, 495–498
17. Zhang G., Jin W. and Hu L., Fractal feature extraction of radar emitter signals, Proc. of the Third Asia-Pacific Conf on Environmental Electromagnetics, Piscataway: IEEE Press, 2003, 161–164
18. Riedmiller M. and Braun H., A direct adaptive method for faster back propagation learning: the RPROP algorithm, Proc. of the IEEE Int. Conf. on Neural Networks, Piscataway: IEEE Press, 1993, 586–591