



# A novel criterion for assessing frost heave susceptibility of soils

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## Abstract

Frost heave susceptibility is a key index for designing the subgrade fillings of high-speed railways in cold regions. The existing methods for assessing frost heave susceptibility are mainly empirical or semi-empirical, and most of them are defined based on the fine content of soils. This study attempts to propose a new criterion based on the analytical solution of frost heave in the soil. A number of experiments are used to validate the proposed analytical model, which shows that the computed value of frost heave matches well with the measured data. The proposed model indicates that frost heave is a proportional function of the square root of time. Thus, the slope  $R$  named frost heave classification index of the proportional function is defined as a new index for frost heave susceptibility classification. A value of  $R$  less than 0.21 corresponds to a non-frost heave susceptibility condition, a value greater than 1.18 corresponds to a high frost heave susceptibility condition, and a value in the range of 0.21 and 1.18 means a frost heave susceptibility condition.  $R$  is directly related to the boundary temperatures and soil–water and soil-freezing characteristics. The parameter study shows that reducing the values of SWCC fitting parameter  $\alpha$  and cold end temperature  $T_c$  or increasing the values of permeability of frozen soil  $k_f$ , the saturated volumetric water content  $\theta_s$ , the unfrozen water content at the frost front  $\theta_u$  and the residual volumetric water content  $\theta_r$  increases the possibility of frost heave. Compared with the existing method, the new index has a clear theoretical basis, and the parameters are easily obtained. It may be a rational method for assessing frost heave susceptibility.

**Keywords** Analytical model · Frost heave · Frost heave susceptibility · Soil properties

## 1 Introduction

Frost heave susceptibility refers to the ability of a soil to display frost heave deformation, which is a crucial index to design fillings in subgrades, embankments and foundations in cold regions. To avoid frost damage in practice, it is common to replace frost heave susceptible materials with non-frost heave susceptible materials. However, in reality, frost heave occurs even in infrastructures built with non-

frost heave susceptible filling materials. For example, frost heave was reported in high-speed railway subgrades in China [44, 51, 52, 61], Norway [58], and Japan [1]. It is unusual that frost heave occurs in a graded gravel layer and Group A/B fill [8, 41, 80, 81], which is considered a non-frost heave susceptible material. Here, Group A and B refers to the fills that have a fine content (size < 0.075 mm) of less than 15% and the maximum grain diameter does not exceed 60 mm. In addition, the water content in mass is between 3 and 8%. Therefore, questions arise about the case of the frost heave even in the conventionally defined non-frost heave susceptible material. The existing frost heave susceptibility criteria have not been able to explain this puzzle. Frost heave susceptibility is influenced by soil properties and environmental factors, and it is insufficient to classify frost heave susceptibility of soils mainly only based on fine content. Therefore, a serious rethinking of the frost heave susceptibility criteria is needed.

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Casagrande [9] may be the first person to propose a classification method of frost heave susceptibility, which is mainly based on the fine content and uniformity coefficient ( $C_u$ ). This method has a remarkable influence on the particle size characteristic methods. In the past nearly ninety years, many methods (more than one hundred methods) have been proposed to identify frost heave susceptible soils, principally soil particle grading and frost heave tests. The proposed methods can be logically classified into four categories. The first category of methods for assessing frost heave susceptibility is the results of frost heave test. The frost heave test is a laboratory test to evaluate the potential for frost heave in materials in which aggregate or soil is frozen under controlled conditions. The indices include the frost heave rate [3, 30], frost heave ratio [23], amount of frost heave [57], and ratio of frost heave to the square root of the freezing index [50]. This approach is a direct way to estimate frost heave susceptibility, and the test results are accurate and reliable. However, the laboratory frost heave test is slightly expensive and time-consuming [32, 43]. Frost heave test is difficult to implement in the linear engineering of cold regions, such as high-speed railway subgrades and gas pipelines.

The second category of method for classifying frost heave susceptibility is segregation potential (SP), which was proposed in the series of studies of Konrad [35–37] and Konrad and Morgenstern [38, 39]. This kind of method is quite simple in form and reliable in theory. However, it is found that the segregation potential has difficulty in accurately expressing soil freezing characteristics because it is acquired in a steady-freezing state, but frost heave is usually a dynamic process [25]. In addition, the position and temperature of the frozen fringe are difficult to determine when conducting frost heave tests. In a further study, Konrad [36, 37] and Loranger et al. [43] established an empirical relationship between segregation potential and basic soil property parameters. This method could calculate segregation potential based on the average diameter of the fine fraction, the specific surface area of the fine fraction, the initial water content and the liquid limit of the fine fraction, which is smaller than 0.075 mm. However, this empirical approach still lacks consideration of the mechanism of frost heave and environmental conditions.

In practice, most methods are empirical or semi-empirical approaches based on experimental summaries. Frost heave susceptibility criteria based on particle size characteristics or soil–water (-ice) interaction characteristics are empirical methods. The third category of methods for assessing frost heave susceptibility is based on the fine content and particle size distribution [6, 9, 70]. A maximum particle size of 0.075 or 0.02 mm is used to determine the fine content. In these methods, some other indices will be used together, such as the uniformity coefficient,

plasticity index or mineral composition of fines. Because of its simple and easy operation, this method is very popular in many countries, such as China [22], the USA [70] and Norway [43]. It has been found that this kind of method mainly works in the country and region where it was developed. Different materials and environmental conditions cause different thresholds of the fine content. Recent studies have shown that the particle size characteristics as an indicator for assessing frost heave susceptibility are not justified, and environmental factors and permeability coefficients should be considered [45, 60]. Konrad [36] considered that frost heave susceptibility criteria based on fine content could not separate non-frost heave susceptible soil from frost heave susceptible soil. These kinds of criteria have low reliability when assessing whether the soil is non-frost or frost heave susceptible material [10, 24, 29].

The characteristics of soil–water interactions and soil–water–ice interactions constitute final category of methods for classifying frost heave susceptibility. The former indicators, such as the saturated or unsaturated hydraulic conductivities and the product of air entry suction, can be determined by conducting the standard geotechnical experiments [53, 74]. The later indicators include the frost heave stress, which refers to the stress or force exerted on soils, materials, or structures due to the expansion of pore ice or ice lens, as well as pore-water suction, which can be measured by conducting frost heave tests [26, 55]. These indices have specific physical meanings but are not easily measured and validated. Consequently, there are few further discussions and applications. The four categories of methods for assessing frost heave susceptibility are summarized in Table 1.

To explore better methods to assess frost heave susceptibility, Csathy and Townsend [14] proposed a new preliminary frost heave susceptibility criterion  $P_u$ , where the  $P_u = p_{90}/p_{70}$  value could be calculated. The notations  $p_{90}$  and  $p_{70}$  refer the pore diameters that are larger than 90 and 70 percent of the pores, respectively. The soil with  $P_u$  value less than 6 is classified as non-frost heave susceptible, while the soil with  $P_u$  value more than 6 is frost heave susceptible. Rieke [56] used a new notation, the  $R_f$  fines factor, which is closely related to the fine content, clay content in the fine fraction and liquid limit of the fine content. Through years of field frost heave tests conducted with varying water contents and water tables, Dai and Wang [17] developed a seasonal frost soil classification specifically for highway bridge foundation. In more recent studies, Cheng [12] proposed that 4 mm is used as a threshold to distinguish non-frost or frost heave susceptible soils in accordance to the “Regulations for Railway Technology Management on the track deformation limit”. Bilodeau et al. [7] proposed a model to assess segregation potential that includes three soil parameters: the fines

**Table 1** Four categories of the existing frost heave susceptibility methods

Classification method	Indices	Disadvantages	Reference
Frost heave test	Frost heave rate, frost heave ratio, frost heave and the ratio of frost heave to the square root of the freezing index	Frost heave test is tedious and expensive	[3, 23, 50, 57]
Segregation potential	The ratio of water flow velocity to temperature gradient of the frozen fringe	Only applicable for the steady state, and frost heave test needs to be conducted	[35–39, 43]
Particle size characteristics	Fine content or particle size distribution	No theory, low reliability, and strong limitations	[6, 9, 70]
Characteristics of soil–water/ice interaction	Soil–water–ice interaction characteristics, suction, unsaturated hydraulic conductivity, etc.	Lack of theory and validations; no further study	[26, 53, 55, 74]

fraction porosity, the fine particles uniformity coefficient, and the fine particles specific surface. Expanding on this, Du [19] further categorized the traditional “non-frost heave susceptible” fills into six levels and suggested that the deformation threshold for the high-speed railway subgrade is 4 mm. Cwiąkała et al. [15] considered sand equivalent to be an appropriate indicator for assessing frost heave susceptibility. These indicators are useful under specific conditions, but they remain empirical approaches and lack theoretical support. Wang et al. [72] introduced two multivariate adaptive regression splines (MARS) models which can predict frost heave susceptibility of coarse fills under a combination of fines, overburden pressure, moisture, and compaction. Laboratory results showed that the fine content has the greatest effect on frost heave susceptibility. Zhou et al. [83] recommended utilizing the freezing ring test to assess frost heave susceptibility according to the maximum frost heave rate of water nonuniformity.

In summary, it is noted that the existing frost heave susceptibility criteria lack a physical mechanism analysis of frost heave, fundamental soil properties and environmental conditions. Although the existing frost heave susceptibility criteria are also effective in many scenarios, a significant portion of these criteria relies on empirical or semi-empirical approaches. They tend to be applicable only for specific regions and climatic conditions. When used for other regions, geological conditions or climates, their predictions may be misleading. Moreover, frost heave susceptibility is identified as a fundamental property of soils according to the existing classification methods. Recent studies have shown that frost heave susceptibility should take both the soil properties and freezing conditions into account [45, 60, 62, 65, 68]. At an earlier stage, Rieke [56] thought that the most rational frost heave susceptibility should consider fundamental soil properties and frost heave mechanisms. To date, there is no comprehensive method for rationally assessing frost heave susceptibility.

To develop a new rational method to identify the frost heave susceptibility of soil, this study first establishes three

partial differential equations that control the heat transfer and liquid water flow in soils. Then, an analytical solution of frost heave is presented by simultaneously solving the three equations. A new criterion for frost heave susceptibility is defined based on the proposed analytical model. A parametric study is then performed to better understand the new criterion. This study compares the new criterion with the existing indices to show its rationality and applicability in frost heave susceptibility classification.

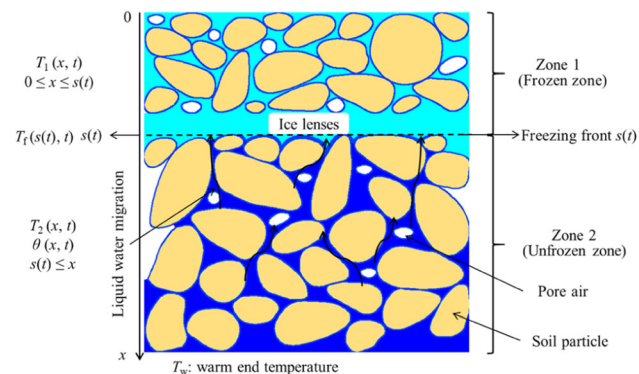
## 2 An analytical model for frost heave in a soil

### 2.1 Basic assumptions

Soil freezing is a transient-state process. The water in the soil starts to be frozen when the temperature drops below its freezing point. According to the position of the freezing front, the soil can be divided into two parts: the frozen zone (Zone 1) and unfrozen zone (Zone 2), as shown in Fig. 1. Moisture transfer within the frozen zone is considered to be negligible due to the exceedingly low permeability coefficient of frozen soil, as demonstrated in previous studies [47, 60]. The freezing front is the most active region where phase change and water migration occur. The suction generated in the freezing front is the driving force of liquid water transfer [54]. The water transfer in unfrozen region is controlled by the matric potential and temperature gradient [81]. Continued ice lens growth leads to frost heave in the soil [38].

To develop an analytical model for the frost heave process, some assumptions are made as follows:

1. The soil is homogeneous and in a semi-infinite space.
2. Compared to the unfrozen zone, liquid water migration in the frozen zone is so small that it can be neglected when computing frost heave.



**Fig. 1** Schematic diagram of the soil freezing process.  $T_1$  is the temperature profile of the frozen zone, and  $T_2$  and  $\theta$  are the temperature and volume water content profiles of the unfrozen zone, respectively.  $s$  is the position of the freezing front; and  $x$  and  $t$  are the vertical position in soil and time, respectively

3. For the unfrozen zone, the matric potential and temperature gradient are the driving forces for water transfer in the soil. The suction of the phase change at the freezing front acts as the upper boundary for water migration in the unfrozen region.
4. Frost heave is equal to the thickness of the ice lens accumulated by the water from the unfrozen zone.

### 2.2 Governing equations

The temperature in Zone 1 is defined as  $T_1(x, t)$  (°C), which varies with time  $t$  and position  $x$ . The temperature and liquid water content of Zone 2 are defined as  $T_2(x, t)$  (°C) and  $\theta(x, t)$  ( $\text{m}^3/\text{m}^3$ ), respectively. The position of the freezing front is a function of time  $t$ , which is expressed as  $s(t)$ . The freezing front divides the soil space into Zone 1 ( $0 < x < s(t)$ ) and Zone 2 ( $x > s(t)$ ).

The temperature in Zone 1 is mainly affected by heat conduction. The governing function can be expressed as

$$\frac{\partial T_1(x, t)}{\partial t} = \lambda_1 \frac{\partial^2 T_1(x, t)}{\partial x^2}, \quad 0 < x < s(t) \tag{1}$$

In Eq. (1),  $\lambda_1$  is the thermal diffusion coefficient ( $\text{m}^2/\text{s}$ ), which is assumed to be a constant.

In Zone 2, the governing function of the temperature profile can be expressed as

$$\frac{\partial T_2(x, t)}{\partial t} = \lambda_2 \frac{\partial^2 T_2(x, t)}{\partial x^2}, \quad x > s(t) \tag{2}$$

The thermal diffusion coefficient  $\lambda_2$  ( $\text{m}^2/\text{s}$ ) is a constant.

The Richards' equation is used to describe the liquid water transfer in the soil. To obtain an analytical solution of the Richards' equation, the one-parameter soil–water characteristic curve and the exponential form function of

unsaturated soil permeability are introduced [64, 76]. The equations are expressed as follows:

$$k(\psi) = k_s e^{-\alpha\psi} \tag{3}$$

$$\theta(\psi) = \theta_r + (\theta_s - \theta_r) e^{-\alpha\psi} \tag{4}$$

where  $k_s$  is the saturated permeability (m/s),  $\psi$  is the matric suction (kPa), and  $\alpha$  is the fitting parameter of the soil–water characteristic curve ( $\text{kPa}^{-1}$ ).  $\theta_s$  and  $\theta_r$  are the saturated and residual volumetric water contents ( $\text{m}^3/\text{m}^3$ ), respectively. The Richards' equation can be expressed as a wave function:

$$\frac{\partial \theta(x, t)}{\partial t} = D \frac{\partial^2 \theta(x, t)}{\partial x^2}, \quad x > s(t) \tag{5}$$

where  $D$  is the hydraulic diffusion coefficient of unsaturated soil ( $\text{m}^2/\text{s}$ ), and  $D = k_s / (\alpha g (\theta_s - \theta_r))$ . Considering that the phase-change suction at the freezing front and the capillarity suction are many times greater than the gravity potential, it is reasonable to neglect the gravity potential in the Richards' equation to generate an analytical solution.

There are four parameters,  $T_1(x, t)$ ,  $T_2(x, t)$ ,  $\theta(x, t)$ , and  $s(t)$ , in Eqs. (1), (2) and (5); thus, a fourth equation is needed. At the freezing front, there is an energy balance between latent heat and heat conduction, which can be expressed as

$$k_1 \frac{\partial T_1(x, t)}{\partial x} - k_2 \frac{\partial T_2(x, t)}{\partial x} = \rho_w v(t) L + \rho_w \theta_o L \frac{ds(t)}{dt} \tag{6}$$

where  $k_1$  and  $k_2$  are the heat conductivities of Zone 1 and Zone 2 ( $\text{Wm}^{-1} \text{K}^{-1}$ ), respectively.  $\rho_w$  is the density of bulk water ( $\text{kg}/\text{m}^3$ ), and  $v(t)$  is the velocity of water flux migration to the freezing front (m/s). Based on Darcy's law, the water flux velocity is equal to the hydraulic conductivity  $k_f$  (m/s) by the hydraulic gradient  $\partial h / \partial x$  at the freezing front.  $L$  is the latent heat when water freezes into ice (J/kg), and  $\theta_o$  is the initial water content ( $\text{m}^3/\text{m}^3$ ). For the boundary conditions, the temperatures at the cold end  $T_c$  (°C), warm end  $T_w$  (°C), and freezing front  $T_f$  (°C) are all fixed temperatures, and their expressions are  $T_1(0, t) = T_c$ ,  $T_2(\infty, t) = T_w$ , and  $T_1(s, t) = T_f = T_2(s, t)$ , respectively. The water contents at the warm side ( $\text{m}^3/\text{m}^3$ ) and freezing front ( $\text{m}^3/\text{m}^3$ ) are constant, i.e.,  $\theta(\infty, t) = \theta_s$  and  $\theta(s, t) = \theta_u$ , respectively. The initial water content is defined as a constant  $\theta_o$  ( $\text{m}^3/\text{m}^3$ ).

### 2.3 Theoretical derivation

The temperature in Zone 1 ( $0 < x < s(t)$ ) is solved first. By satisfying the boundary conditions  $T_1(0, t) = T_c$  and  $T_1(s, t) = T_f$ , a general solution of Eq. (1) can be expressed as

$$T_s = T_1(s, t) = T_c + m \operatorname{erf}\left(\frac{s}{2\sqrt{\lambda_1 t}}\right) \tag{7}$$

here, the unknown parameters  $T_s$ ,  $T_c$ , and  $m$  are all constants; thus, it is deduced that the Gaussian error function  $\operatorname{erf}(x)$  must be a constant, as follows:

$$\frac{s}{2\sqrt{\lambda_1 t}} = \beta \tag{8}$$

where  $\beta$  is constant. Substituting Eq. (8) into Eq. (7) and eliminating the constant  $m$ , the temperature profile of Zone 1 can be obtained as

$$T_1(x, t) = T_c + \frac{T_f - T_c}{\operatorname{erfc}(\beta)} \operatorname{erf}\left(\frac{x}{2\sqrt{\lambda_1 t}}\right) \tag{9}$$

An analytical expression of Eq. (2) satisfying the boundary conditions  $T_f = T_2(s, t)$  and  $T_2(\infty, t) = T_w$  can be written as

$$T_2(x, t) = T_w - \frac{T_w - T_f}{\operatorname{erfc}(\beta\sqrt{\lambda_1}/\sqrt{\lambda_2})} \operatorname{erfc}\left(\frac{x}{2\sqrt{\lambda_2 t}}\right) \tag{10}$$

An analytical expression of the volumetric water content in Eq. (5) can be written as

$$\theta(x, t) = \theta_s - \frac{\theta_s - \theta_u}{\operatorname{erfc}(\beta\sqrt{\lambda_1}/\sqrt{D})} \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) \tag{11}$$

To obtain the relationship between unfrozen water content  $\theta_u$  and the metric suction  $\psi$ , Eq. (4) is used at the freezing front:

$$\theta_u(\psi) = \theta_r + (\theta_s - \theta_r)e^{-\alpha\psi} \tag{12}$$

Combining Eqs. (11) and (12), the  $\psi$  profile can be obtained as

$$\psi = -\frac{1}{\alpha} \ln \left\{ 1 - \frac{\theta_s - \theta_u}{\theta_s - \theta_r} \frac{\operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)}{\operatorname{erfc}(\beta\sqrt{\lambda_1}/\sqrt{D})} \right\} \tag{13}$$

The hydraulic gradient can be obtained by taking the derivative of Eq. (13):

$$\frac{\partial h}{\partial x} = \frac{1}{\alpha g} \left( \frac{\theta_s - \theta_u}{\theta_u - \theta_r} \right) \left[ \frac{e^{-\frac{x}{2\sqrt{Dt}}\beta}}{\operatorname{erfc}(\beta\sqrt{\lambda_1}/\sqrt{D})} \right] \frac{1}{\sqrt{\pi Dt}} \tag{14}$$

Substituting Eqs. (8), (9), (10) and (14) into Eq. (6) generates a new form of the energy balance equation, as shown in Eq. (15). The unknown parameter  $\beta$  can be determined by solving Eq. (15).

$$k_1 \frac{T_f - T_c}{\operatorname{erf}(\beta)} \frac{e^{-\beta^2}}{\sqrt{\pi\lambda_1}} - k_2 \frac{T_w - T_s}{\operatorname{erfc}(\beta\sqrt{\lambda_1}/\sqrt{\lambda_2})} \frac{e^{-\frac{\lambda_1}{2}\beta^2}}{\sqrt{\pi\lambda_2}} = \theta_o \rho_w L \beta \sqrt{\lambda_1} + \rho_w L k_f \frac{\partial h}{\partial x} \tag{15}$$

Frost heave in soil with a certain amount of fine content is a result of ice lens formation fed by water migration to the frozen zone. The integration of the thickness of the ice lenses can be defined as the amount of frost heave. The analytical expression of frost heave is obtained by integrating the water flux at the freezing front:

$$H = 1.09 \times \int_0^t v(t) dt = 1.09 \times \int_0^t k_f \frac{\partial h}{\partial x} dt = 2.18 \times k_f \left[ \frac{1}{\alpha g} \left( \frac{\theta_s - \theta_u}{\theta_u - \theta_r} \right) \left[ \frac{e^{-\frac{x}{2\sqrt{Dt}}\beta}}{\operatorname{erfc}(\beta\sqrt{\lambda_1}/\sqrt{D})} \right] \frac{1}{\sqrt{\pi D}} \right] \sqrt{t} \tag{16}$$

Equation (16) shows an expression for frost heave relating to the SFCC parameters permeability of frozen soil ( $k_f$ ) and unfrozen water content at the frost front ( $\theta_u$ ), SWCC fitting parameter ( $\alpha$ ), saturated volumetric water content ( $\theta_s$ ), residual volumetric water content ( $\theta_r$ ), thermal diffusion coefficient of frozen soil ( $\lambda_1$ ), hydraulic diffusion coefficient of unsaturated soil ( $D$ ), and the constant  $\beta$ . These parameters are all soil properties that have clear physical meanings. They are easily obtained from laboratory testing or from the literature.

It is interesting to find that frost heave is a proportional function of the square root of time in Eq. (16). Equation (16) can be transformed into a new form (Eq. (17)) to show the slope of the proportional function:

$$\frac{H}{\sqrt{t}} = 2.18 k_f \frac{1}{\alpha g} \left( \frac{\theta_s - \theta_u}{\theta_u - \theta_r} \right) \left[ \frac{e^{-\frac{x}{2\sqrt{Dt}}\beta}}{\operatorname{erfc}(\beta\sqrt{\lambda_1}/\sqrt{D})} \right] \frac{1}{\sqrt{\pi D}} \tag{17}$$

The above analytical solution shows the relation of frost heave to the soil properties and boundary temperatures, which can provide a theoretical basis for classifying frost heave susceptibility. Equation (17) provides an explicit solution, whereas the segregation potential (SP) proposed by Konrad and Morgenstern [38] provides an implicit solution. It is worth noting that Eq. (17) is not an empirical solution, unlike methods based on the fine content [70].

### 3 Results and analysis

#### 3.1 Model validation

Three kinds of soils in the literature, i.e., silt, clay and coarse-grained soil, are chosen to verify the proposed model. The frost heave test conditions are shown in Table 2.

The input parameters of the analytical model are given in Table 3. The parameters can be obtained in the literature, such as saturated hydraulic conductivity ( $k_s$ ), permeability of frozen soil ( $k_f$ ), soil–water characteristic curve (SWCC),

**Table 2** Test conditions of frost heave

Case No	Soil type	Size $D/H$ (mm)	Temperature $T_c/T_\infty$ (°C)	Initial water content $\theta_o$ (m <sup>3</sup> /m <sup>3</sup> )	Reference
1	Devon silt	97/104	– 3.4/1.1	0.42	[38]
2		97/76	– 2.5/1.1	0.42	
3	Tibetan red clay	160/160	– 3/2	0.44	[75]
4	Coarse-grained soil	198/200	– 12/2	0.155	[21]

**Table 3** Input parameters for the model validation

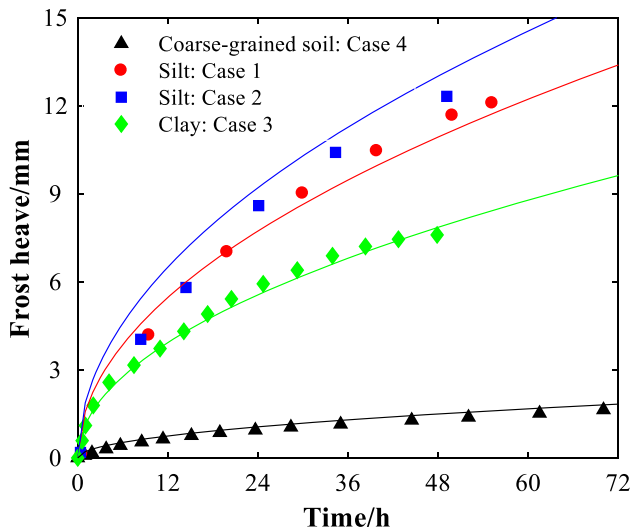
Parameters	Case 1	Case 2	Reference	Case 3	Reference	Case 4	Reference
$T_c$ (°C)	– 3.4	– 6.2	[38]	– 3	[75]	– 12	[21]
$T_\infty$ (°C)	1.1	1.1		2		2	
$T_f$ (°C)	– 0.1	– 0.1		– 0.51		– 0.23	[71]
$\theta_o$ (m <sup>3</sup> /m <sup>3</sup> )	0.42	0.42		0.44		0.155	[21]
$\theta_\infty$ (m <sup>3</sup> /m <sup>3</sup> )	0.42	0.42		0.44		0.35	
$k_1$ (W/K*m)	1.76	1.76		1.53	[42]	1.40	
$k_2$ (W/K*m)	1.47	1.47		1.23		1.15	
$\lambda_1$ (m <sup>2</sup> /s)	8.34e–7	8.34e–7	[48]	8.23e–7	[48]	7.65e–7	[48]
$\lambda_2$ (m <sup>2</sup> /s)	4.73e–7	4.73e–7		4.42e–7		5.28e–7	
$L$ (J/kg)	334,000	334,000	Constant	334,000	Constant	334,000	Constant
$\beta$	0.1408	0.1956	Solving equations	0.0727	Solving equations	0.3818	Solving equations
$k_s$ (m/s)	1e–9	9e–10	[38]	1e–8	[79]	1e–4	[40]
$k_f$ (m/s)	7.3e–12	7.8e–12		3.43e–10	[67]	3.86e–9	[67]
$\alpha$ (kPa <sup>–1</sup> )	0.002	0.002	[4, 78]	0.003	[75, 79]	0.12	[11, 40]
$\theta_s$ (m <sup>3</sup> /m <sup>3</sup> )	0.42	0.42		0.44		0.35	
$\theta_r$ (m <sup>3</sup> /m <sup>3</sup> )	0.08	0.08		0.14		0.04	
$D$ (m <sup>2</sup> /s)	1.5e–7	1.35e–7		1.1e–6		2.7e–4	
$\theta_u$ (m <sup>3</sup> /m <sup>3</sup> )	0.10	0.10	[60]	0.27	[60]	0.06	[60]

and soil freezing characteristic curve (SFCC). For the Devon silt samples,  $T_c$ ,  $T_w$ ,  $T_f$ ,  $k_1$ ,  $k_2$ ,  $k_s$ ,  $\theta_o$ , and  $k_f$  were measured directly in Konrad and Morgenstern [38]. The SWCC fitting parameters  $\theta_s$ ,  $\theta_r$ ,  $\alpha$ , and hydraulic diffusion coefficient of unsaturated soil  $D$  are calculated from Azmatch et al. [4] and Zhang et al. [78].  $\lambda_1$  and  $\lambda_2$  are calculated from Mitchell and Soga [48].  $\theta_u$  is 25% of the initial moisture content for silt [60]. For the proposed model,  $L$  is the latent heat when water freezes to ice, which is a constant.  $\beta$  is calculated by solving the energy balance equation.

The parameters  $T_c$ ,  $T_w$ ,  $T_f$ ,  $\theta_o$ ,  $\theta_\infty$ ,  $\theta_s$ ,  $\theta_r$ , and  $\alpha$  of Case 3 were measured directly from Xue [75]. The saturated hydraulic conductivity  $k_s$  is given by Zhang et al. [79], and the permeability coefficient of frozen soil  $k_f$  is calculated according to the model proposed by Teng et al. [67]. Therefore,  $D$  is calculated from Xue [75] and Zhang et al. [79]. The thermodynamic parameters  $k_1$  and  $k_2$  are obtained

from Liu [42], and the other two thermodynamic parameters  $\lambda_1$  and  $\lambda_2$  are calculated from Mitchell and Soga [48].  $\theta_u$  is 60% of the initial moisture content for clay [60].

The parameters  $T_c$ ,  $T_w$ ,  $k_1$ ,  $k_2$ ,  $\theta_\infty$ , and  $\theta_o$  of coarse-grained soil are measured directly from Gao [21]. The temperature of the freezing front is given by Wang [71]. The thermodynamic parameters  $\lambda_1$  and  $\lambda_2$  are calculated from Mitchell and Soga [48]. The saturated hydraulic conductivity  $k_s$  is  $1.0e^{-4}$  m/s [40], and the permeability coefficient  $k_f$  of frozen soil is calculated according to the model proposed by Teng et al. [67]. The SWCC parameters  $\theta_s$ ,  $\theta_r$ , and  $\alpha$  are calculated from Chen et al. [11], and the parameter  $D$  is calculated from Chen et al. [11] and Leng et al. [40]. For the coarse-grained soil, the unfrozen water content in the frozen state is very low,  $\theta_u$  of 10% of initial water content is observed in sand with initial degree of saturation 50% [60]. Coarse-grained soil Case 4 has 10% fine content and low initial saturation, and its unfrozen



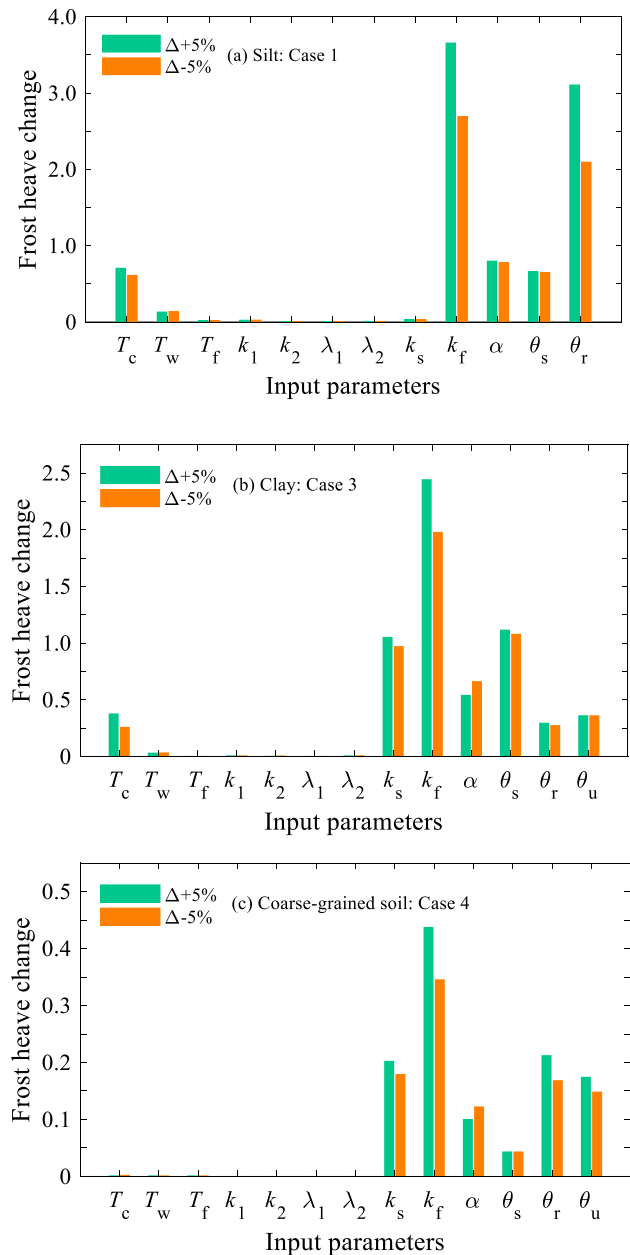
**Fig. 2** Comparison between the experimental and calculated results of frost heave. Solid points are the experimental data, and solid lines are the predicted results: **a** Case 1 and Case 2 are silt [ 38]; **b** Case 3 is clay [ 75]; **c** Case 4 is coarse-grained soil [ 21]

water content is taken as 0.06. All the input parameters are presented in Table 3.

The comparison between the measured and predicted frost heave is shown in Fig. 2. The calculated results are in good agreement with the measured data for the four cases, where frost heave displays rapid initial growth followed by a gradual stabilization. Among the four cases considered, silt experiences greater frost heave compared to clay when supplied with sufficient water. The coarse-grained soil with low fine content has low permeability coefficient when freezing, resulting in a slower increase in frost heave. Notably, the growth trends and Eq. (16) reveal that frost heave follows a power function in relation to time.

### 3.2 Sensitivity analysis of the model input parameters

Due to the presence of multiple input parameters in the proposed model, errors are inevitably introduced during the equation-solving process. Meanwhile, many parameters make it challenging to use the model effectively for predicting frost heave. To address this, conducting sensitivity analysis on the proposed model becomes necessary in order to determine the controlling factors. In this study, a controlling single-factor variable approach is employed, where one parameter is varied by  $\pm 5\%$  of the common range while keeping the remaining input parameters constant (as indicated in Table 3). Subsequently, the change in frost heave is compared for each parameter change. It is worth noting that sensitivity analysis is not performed for the constant ( $L$ ), the non-independent variables ( $D$ ,  $\beta$ ), and the



**Fig. 3** Sensitivity analysis of input parameter for the proposed model

initially explicit variables ( $\theta_o$ ,  $\theta_\infty$ ). The results of the sensitivity analysis are presented in Fig. 3.

Figure 3 clearly shows the effects of parameter changes on the absolute frost heave change. For the silt, the hydraulic conductivity of the frozen soil  $k_f$  is the most sensitive parameter, followed by the residual water content  $\theta_r$ . The other controlling input parameters for silt may be the SWCC fitting parameter  $\alpha$ , the saturated volumetric water content  $\theta_s$ , the temperature of cold end  $T_c$ , and the temperature of warm end  $T_w$ . The most sensitive parameter for clay is also the hydraulic conductivity of frozen soil  $k_f$ , followed by the saturated volumetric water content  $\theta_s$  and

**Table 4** Test conditions of the one-dimensional frost heave experiments

Source	Num	Temperature $T_c/T_w$ (°C)	Category	Time (h)	FH (mm)	IWC (kg/kg)	FHS
[2]	1	− 17/4	SC	120.8	7.7	Saturation	FHS-gray
	2		MH	121.6	14		HFHS-red
	3		SM	126.6	16.3		HFHS-red
[5]	4	− 3/12	CGS	93.2	4.8	10%	FHS-gray
	5	− 2/12			4.2	10%	FHS-gray
	6	− 3/12			0.7	9.60%	NFHS-green
[13]	7	− 5/2	Expansive	68.9	11.3	21%	HFHS-red
	8	− 5/1	Soil	68.7	11.4	21%	HFHS-red
	9	− 10/2		70.2	14.2	Saturation	HFHS-red
[16]	10	− 5/2	Silt	121.5	32	10%	HFHS-red
[20]	11	− 5/5	Silt	115	15.2	Saturation	HFHS-red
[21]	12	− 12/2	CGS	284	5.7	8%	FHS-gray
	13			281.1	3.7	8%	FHS-gray
	14			286	2.6	8%	NFHS-green
	15			284.8	1.7	8%	NFHS-green
	16			287	1	6%	NFHS-green
	17			286.4	0.8	4%	NFHS-green
	18	− 7.7/12.2	CGS	120	1.4	7%	NFHS-green
[24]	19	− 10.9/10.3		48	0.8	3%	NFHS-green
	20	− 3.4/1.1	Silt	55.2	12.1	Saturation	HFHS-red
[38]	21	− 4.8/1.1		75.9	16.9		HFHS-red
	22	− 2.5/1.1		70.4	14.3		HFHS-red
	23	− 6.2/1.1		49.2	12.3		HFHS-red
	24	− 21.5/5.0	Clay	70	41.1	Saturation	NFHS-red
[46]	25	− 23.0/7.5		64.9	36.8		NFHS-red
	26	− 3/1	CGS	48	0.8	7%	NFHS-green
[71]	27	− 5/1		48.2	1.9		FHS-gray
	28	− 7/1		48.1	2.2		FHS-gray
	29	− 3/2	Clay	48	7.6	Saturation	HFHS-gray
[75]	30	− 5/insulation	CGS	56.1	0.8	4%	NFHS-green
	31	− 10/insulation		32	0.9		NFHS-green
	32	− 15/insulation		20	0.7		NFHS-green

Note: CGS means coarse-grained soil, FH means frost heave, and IWC means initial water content.

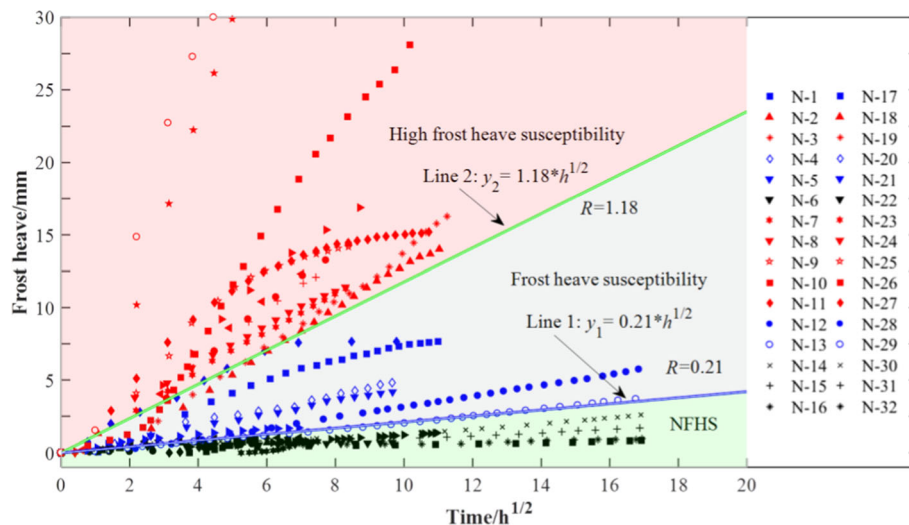
the saturated hydraulic conductivity of frozen soil  $k_s$ . The SWCC fitting parameter  $\alpha$ , the unfrozen water content  $\theta_u$ , residual water content  $\theta_r$ , and the temperature of cold end  $T_c$  also have obvious effect on frost heave. For the coarse-grained soil, water retention and permeability characteristics are controlling factors for frost heave. In general, the controlling factors of the proposed model are the parameters of water retention properties, permeability properties, and boundary temperatures. These parameters must be given special consideration in the application of model. The thermodynamic parameters have very limited effects on frost heave. For fine grained soil, the boundary temperatures are also sensitive factors. However, for coarse-

grained soils, parameters of water retention properties and permeability properties are main sensitive factors.

### 3.3 A new criterion for frost heave susceptibility classification

Equation (16) shows that frost heave is proportional to the square root of time. The slope of the proportional function is defined as  $R$ ,  $R = H/t^{1/2}$ , i.e., the ratio of frost heave ( $H$ /mm) to the square root of time ( $t/h$ ).  $R$  is a function of soil property parameters and boundary conditions.  $R$  is named as frost heave classification index, and its dimension is  $\text{mm/h}^{1/2}$ . To obtain appropriate threshold values for  $R$ , a





**Fig. 4** New frost heave susceptibility classification

substantial number of one-dimensional frost heave test results are used to determine the threshold values. The expression for frost heave and time is converted into the form of Eq. (16). This study collects 32 groups of experimental results, which cover coarse-grained soils, silt and clay. The test conditions are listed in Table 4. The 32 groups of experimental results are plotted in Fig. 4.

It can be observed that frost heave is nearly proportional to the square root of time in Fig. 4. Different soils show different growth trends, where the trend of silt is fast, the trend of coarse-grained soil is slow and the trend of clay is moderate. In Fig. 4, three distinct regions can be found that are divided by the experimental results, including two dense regions and a sparse region. The green region is a dense one, which means minor values of  $R$  and slight frost heave behavior. The red dense region indicates large values of  $R$  and severe frost heave behavior. The gray sparse region is moderate. To separate the frost heave behavior regions, two critical thresholds can be determined from a statistical point of view. The threshold values of  $R$  are 0.21 and 1.18. Consequently, the critical gradation line 1, which separates non-frost heave susceptible and frost heave susceptible soils, is defined as  $y_1 = 0.21 * h^{1/2}$ , where  $y_1$  is frost heave (mm) and  $h$  is time (hour). The other critical gradation line 2 is defined as  $y_2 = 1.18 * h^{1/2}$ . A negligible frost heave in ASTM D5918 corresponds to a value less than 1 mm/day, which converts to an  $R$  value of 0.204. A medium frost heave rate in ASTM D5918 is 4 to 8 mm/day, which corresponds to an  $R$  value of 1.224. This shows that the critical values of slope  $R$  are close to the definition in ASTM D5918. Therefore, the new frost heave susceptibility classification can be divided into three levels. The  $R$  value of less than 0.21 corresponds to non-frost heave susceptibility. High frost heave susceptibility means

the  $R$  value of more than 1.18. The value of  $R$  in the range of 0.21 to 1.18 indicates frost heave susceptibility.

In Table 4, frost heave susceptibility (FHS) of the results is determined using new index  $R$ . The notation HFHS-red indicates that the experimental result falls in the red region, which shows the specimen is high-frost heave susceptible soil. Similarly, the notations FHS-gray means that the experimental result falls in the gray region, which is a frost heave susceptible region. The notation NFHS-green means that the specimen is non-frost heave susceptible soil and the experimental result falls in the gray region.

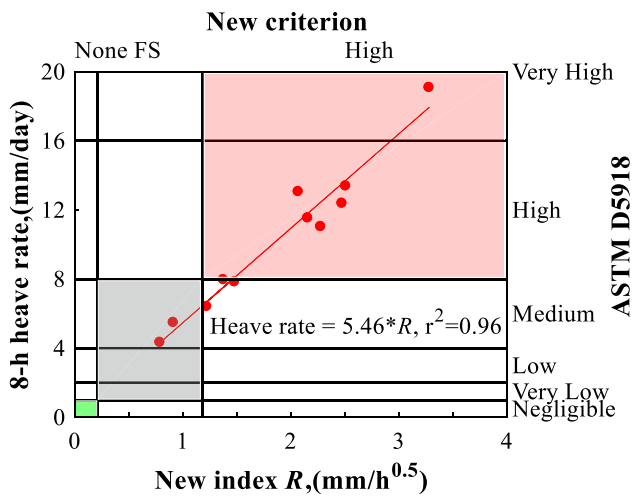
### 3.4 Comparison of $R$ and the method of ASTM D5918

To better understand the new criterion  $R$ , a comparison is made between  $R$  and ASTM D5918 [3] for evaluating frost heave susceptibility. The experimental results were selected from Johnson [34], and the frost heave tests were performed according to ASTM D5918. The test materials were from various road construction projects and quarries, and presented a wide variety of soil types. The values of the new index  $R$  were obtained by fitting frost heave (mm) and time (hour) using the equation  $y = R * h^{1/2}$ , with the coefficient of determination ( $R^2$ , denoted as  $r^2$ ) calculated. Table 5 displays the classifications of frost heave susceptibility based on the new index  $R$  and the method of ASTM D5918. It can be found that the  $r^2$  ( $R^2$ ) is basically close to or greater than 0.9 for the 11 groups of test data.

To show a clear correlation between the new criterion  $R$  and well-recognized criterion ASTM D5918 for assessing frost heave susceptibility, the results in Table 5 are visually presented in Fig. 5. According to the frost heave susceptibility classification threshold values of the two

**Table 5** Comparison of frost heave susceptibility classification between new index *R* and ASTM D5918

Soil	Soil type (USCS)	ASTM D5918 (2013)		New index <i>R</i>		
		Heave rate (mm/day)	Grade	<i>R</i> (mm/h <sup>0.5</sup> )	Grade	r <sup>2</sup>
Well graded sand with silt and gravel	SW-SM	13.4	High	2.51	High	0.91
Poorly graded sand with silt and gravel	SP-SM	11.5	High	2.16	High	0.93
Silt with sand subgrade	ML	11.0	High	2.28	High	0.90
Lean clay subgrade	CL	12.4	High	2.48	High	0.88
Limestone subbase	GP-GM	6.4	Medium	1.23	High	0.96
Clayey sand subgrade	SC	7.8	Medium	1.48	High	0.95
Crushed limestone subbase	GP-GM	8.0	Medium	1.38	High	0.91
Clayey sand subgrade	SC	13.1	High	2.07	High	0.90
Sandy lean clay subgrade	CL	4.3	Medium	0.79	Frost	0.94
Loess	ML	19.1	Very high	2.77	High	0.88
Sand lean clay subgrade	CL	5.5	Medium	0.92	Frost	0.78



**Fig. 5** Comparison of criteria of assessing frost heave susceptibility criteria, five horizontal lines of ASTM D5918 and two vertical lines of *R* are plotted in Fig. 5. The classifications

of ASTM D5918 ranging from “Very low to Low” and “High to Very high” are defined as “frost heave susceptibility” and “high frost heave susceptibility” to correspond to the new criterion *R*. The three highlighted areas in red, gray, and green correspond to high-frost, frost, and non-frost heave susceptibility, respectively. Figure 5 shows that the frost heave susceptibility is almost in the same class ranges. This indicates that the threshold values of *R* are correct. Furthermore, the fitting function reveals a strong proportionality between the heave rate measured by ASTM D5918 and *R*. This suggests that when assessing frost heave susceptibility, both the new criterion and ASTM D5918 exhibit similar trends in value growth for the respective indexes. Therefore, the new criterion *R* proves to be a rational index for determining frost heave susceptibility classification. Unlike ASTM D5918, which offers more detailed classification but frost heave test is slightly expensive and time-consuming, the new index *R* is supported by theory and incorporates fundamental soil property parameters and temperature boundary parameters.

**Table 6** The values of the input parameters of silt

Silt: Case 1		
Parameter	Control value	Comparative value
$T_c/^\circ\text{C}$	- 3.4	- 5, - 10, - 15, - 20, - 25
$T_w/^\circ\text{C}$	1.1	5, 10, 15, 20, 25
$\alpha/\text{kPa}^{-1}$	0.002	0.008, 0.004, 0.001, 0.0005, 0.0002
$\theta_s/-$	0.42	0.40, 0.45, 0.50, 0.55, 0.60
$\theta_f/-$	0.08	0.04, 0.055, 0.070, 0.10, 0.12
$\theta_v/-$	0.10	0.12, 0.14, 0.16, 0.18, 0.20
$k_f/(\text{m/s})$	$7.34 \times 10^{-12}$	$7.34 \times 10^{-10}$ , $7.34 \times 10^{-11}$ , $7.34 \times 10^{-13}$ , $7.34 \times 10^{-14}$ , $7.34 \times 10^{-15}$
$\lambda_1/(\text{m}^2/\text{s})$	$7.6 \times 10^{-7}$	$1.76 \times 10^{-6}$ , $1.45 \times 10^{-6}$ , $1.15 \times 10^{-6}$ , $5.4 \times 10^{-7}$ , $2.93 \times 10^{-7}$

**Table 7** Physical values of the comparative parameters of clay

Clay: case 3		
Parameter	Control value	Comparative value
$T_c/^\circ\text{C}$	- 3	- 5, - 10, - 15, - 20, - 25
$T_w/^\circ\text{C}$	2	5, 10, 15, 20, 25
$\alpha/\text{kPa}^{-1}$	0.003	0.005, 0.0015, 0.00085, 0.00055, 0.00035
$\theta_s/-$	0.44	0.50, 0.56, 0.62, 0.68, 0.74
$\theta_r/-$	0.14	0.08, 0.10, 0.12, 0.16, 0.18
$\theta_u/-$	0.27	0.20, 0.22, 0.24, 0.26, 0.30
$k_f/(\text{m/s})$	$3.43 \times 10^{-10}$	$3.43 \times 10^{-9}$ , $3.43 \times 10^{-11}$ , $3.43 \times 10^{-12}$ , $3.43 \times 10^{-13}$ , $3.43 \times 10^{-14}$
$\lambda/(\text{m}^2/\text{s})$	$7.6 \times 10^{-7}$	$1.76 \times 10^{-6}$ , $1.45 \times 10^{-6}$ , $1.15 \times 10^{-6}$ , $5.4 \times 10^{-7}$ , $2.93 \times 10^{-7}$

**Table 8** The values of the analytic parameters of coarse-grained soil

Coarse-grained soil: case 4		
Parameter	Control value	Comparative value
$T_c/^\circ\text{C}$	- 12	- 2, - 5, - 15, - 20, - 25
$T_w/^\circ\text{C}$	2	5, 10, 15, 20, 25
$\alpha/\text{kPa}^{-1}$	0.12	0.09, 0.18, 0.24, 0.32, 0.41
$\theta_s/-$	0.35	0.38, 0.40, 0.42, 0.45, 0.48
$\theta_r/-$	0.04	0.01, 0.02, 0.03, 0.05, 0.055
$\theta_u/-$	0.06	0.05, 0.07, 0.08, 0.09, 0.10
$k_f/(\text{m/s})$	$3.86 \times 10^{-9}$	$3.86 \times 10^{-10}$ , $3.86 \times 10^{-11}$ , $3.86 \times 10^{-12}$ , $3.86 \times 10^{-13}$ , $3.86 \times 10^{-14}$
$\lambda/(\text{m}^2/\text{s})$	$7.6 \times 10^{-7}$	$1.76 \times 10^{-6}$ , $1.45 \times 10^{-6}$ , $1.15 \times 10^{-6}$ , $5.4 \times 10^{-7}$ , $2.93 \times 10^{-7}$

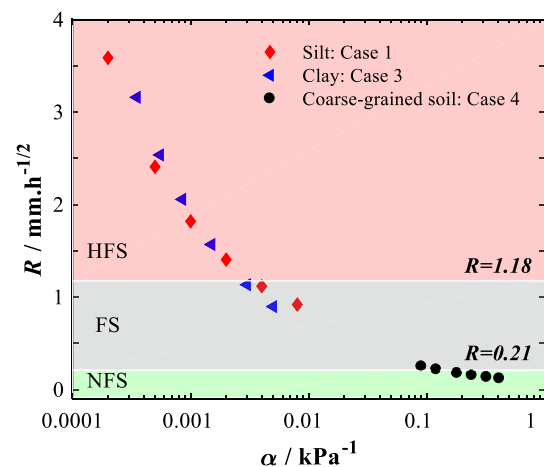
Consequently, the new index  $R$  introduces a novel approach to analyze frost heave susceptibility of soil.

### 3.5 Parametric study

The soil properties and freezing conditions have an important effect on frost heave. To better understand frost heave susceptibility, a parametric study is conducted in this study. The parameters include boundary temperatures  $T_c$  and  $T_w$ , SWCC fitting parameter  $\alpha$ , saturated and residual volume water contents  $\theta_s$  and  $\theta_r$ , unfrozen water content at the freezing front  $\theta_u$ , hydraulic conductivity of frozen soil  $k_f$ , and thermal diffusion coefficient of frozen soil  $\lambda_1$ .  $\alpha$  is the desaturation coefficient, and smaller pores in soil indicate a smaller value of  $\alpha$ . Thus,  $\alpha$  is larger in coarse-grained soil than in fine-grained soil. Another two key parameters  $\theta_u$  and  $k_f$  depend on the soil type and initial water content. A higher initial water content and fine-grained soil often lead to a higher  $\theta_u$ . The  $k_f$  of silt tends to be larger than that of clay and coarse-grained soil. The specific control and detailed comparative values of the parameters of silt, clay, and coarse-grained soil are listed in Tables 6, 7 and 8, respectively. It is noted that the other input parameters are the same as those listed in Table 3.

#### 3.5.1 Effect of the fitting parameter $\alpha$

The parameter  $\alpha$  is changed from 0.0002 to 0.41, while the other input parameters are the same as those in Case 1, Case 3, and Case 4, which are listed in Table 3. Figure 6 shows that the  $R$  values gradually decrease with increasing  $\alpha$  for the three soils. It is easy to find that the value of  $R$  for



**Fig. 6** Effect of the fitting parameter  $\alpha$  on frost heave susceptibility

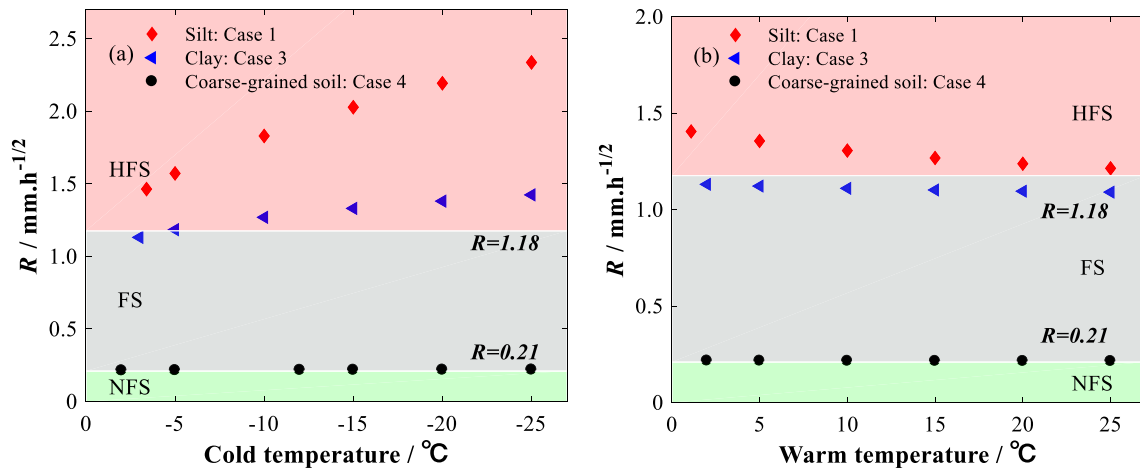


Fig. 7 Effect of the boundary temperatures on frost heave susceptibility

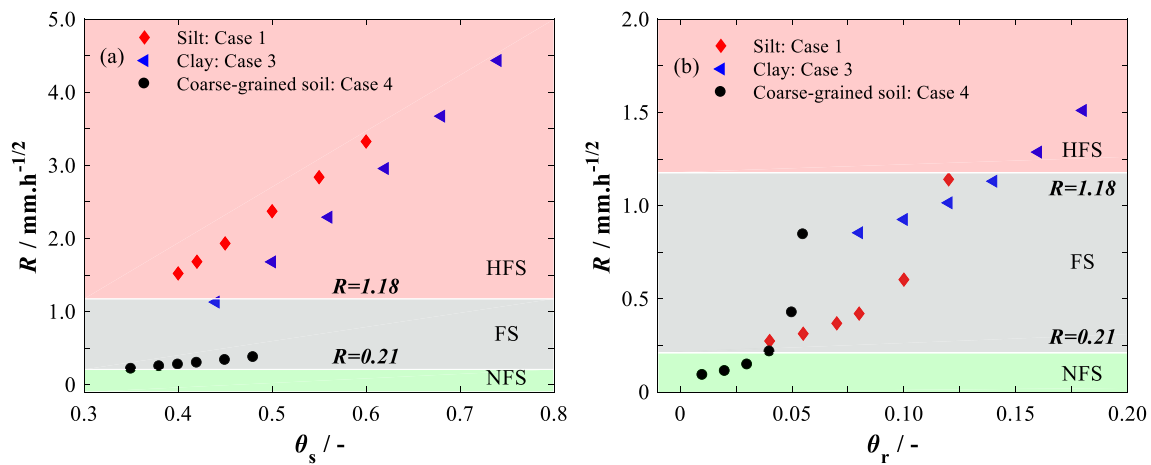


Fig. 8 Effect of saturated and residual volumetric water contents on frost heave susceptibility

coarse-grained soil is very small. However, when  $\alpha$  is less than 0.1, the soil shows a noticeable potential for frost heave. The frost heave susceptibility of silt and clay is obviously sensitive to  $\alpha$ . With decreasing  $\alpha$  from 0.01 to 0.0002, their classification changes from frost to high frost heave susceptibility. The values of  $R$  of fine-grained soil have an almost linear relationship with the logarithm of  $\alpha$ . The new index  $R$  is a specific parameter used to describe the frost heave behavior under certain conditions. The smaller  $\alpha$  is, the stronger the water retention capacity. This means that soils would have a large amount of unfrozen water under freezing. This is beneficial for liquid water migration.

### 3.5.2 Effect of the boundary temperatures

A negative temperature is one of the indispensable conditions for frost heave. Figure 7 plots the relationship between frost heave susceptibility and boundary

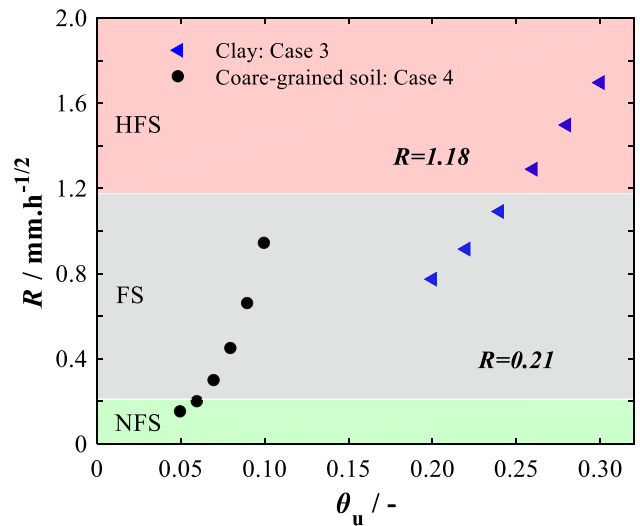


Fig. 9 Effect of unfrozen volumetric water content at the frost front on frost heave susceptibility

temperature. As the freezing temperature decreases, the frost heave susceptibility of silt and clay becomes stronger, and the increasing trend of silt is more obvious than that of clay. For the warm end, a higher temperature leads to lower frost heave susceptibility, and silt is also more sensitive than clay. However, temperatures do not work for coarse-grained soils. In general, the freezing temperature has a stronger influence than the warm end temperature, with little influence on clay and a great influence on silt. A lower freezing temperature leads to a thinner frozen fringe. Meanwhile, the resistance of water transfer will become small, and the amount of water flow will increase. A higher warm end temperature decreases the latent heat released by migrating and in situ water into ice. This inhibits the frost heave behavior but is not obvious [38].

### 3.5.3 Effect of $\theta_s$ , $\theta_r$ , and $\theta_u$

Figure 8a shows that the value of  $R$  increases with increasing  $\theta_s$  for the three kinds of soil. A linear relationship can be observed between  $R$  and  $\theta_s$ . The value of coarse-grained soils has a limited increase when  $\theta_s$  increases from 0.35 to 0.48. However, the  $R$  value of fine-grained soil seems to be more sensitive to  $\theta_s$ . The increasing tendencies of fine-grained soils are similar. At a certain  $\theta_s$ , the value of  $R$  of silt is greater than that of clay.

The residual volumetric water content  $\theta_r$  is also a key influencing factor for frost heave susceptibility. A higher  $\theta_r$  results in a greater value of  $R$ , as shown in Fig. 8b. For coarse-grained soil and silt, the value of  $R$  gradually increases with increasing  $\theta_r$ , but when  $\theta_r$  reaches a certain critical value, the value of  $R$  increases sharply. Especially for coarse-grained soil, the frost heave behavior will be significant when  $\theta_r$  is greater than 0.05. For clay, the value

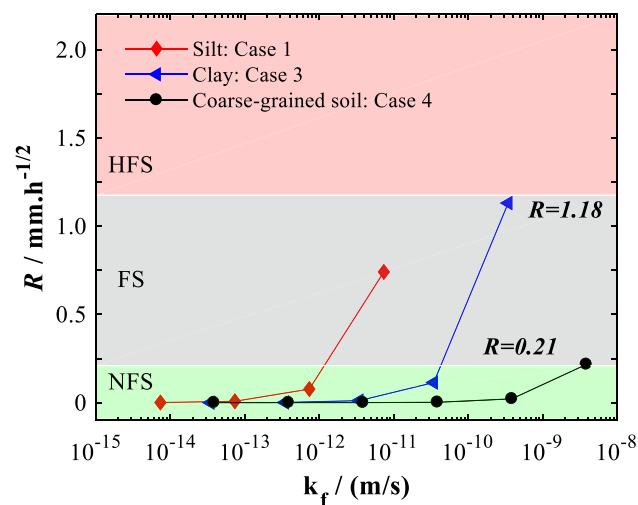


Fig. 10 Effect of the permeability of frozen soil  $k_f$  on frost heave susceptibility

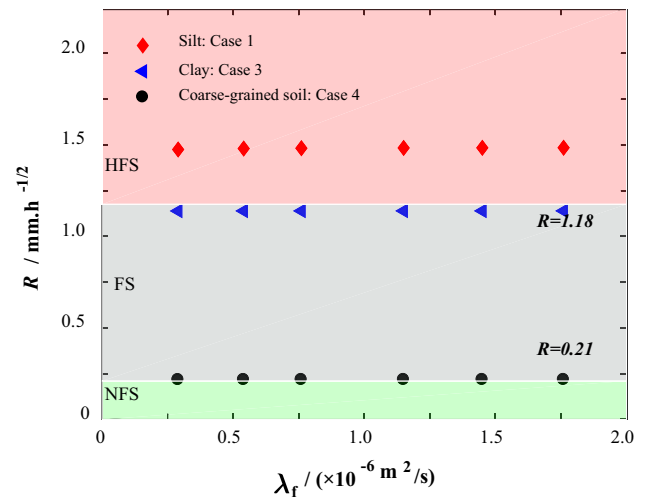


Fig. 11 Effect of thermodynamic parameters on frost heave susceptibility

of  $R$  increases steadily. It is worth noting that  $\theta_s$  and  $\theta_r$  have a significant influence on  $R$  for the three kinds of soils, but  $\theta_s$  does not have a significant influence on  $R$  for coarse-grained soil. Higher  $\theta_s$  and  $\theta_r$  values indicate the strong water retention capacity of soils. This would increase the liquid water transfer under freezing conditions.

The unfrozen water content  $\theta_u$  is important for determining the hydraulic conductivity of frozen soil and is a key boundary parameter in Eq. (5) [66, 73]. The hydraulic conductivity of frozen soil is provided in Konrad and Morgenstern [38], eliminating the need to calculate  $k_f$  based on  $\theta_u$ . Consequently, the parameter study for  $\theta_u$  in relation to silt is not conducted. Figure 9 shows that the  $R$  value increases with the increasing  $\theta_u$ , which corresponds to a higher hydraulic conductivity of frozen soil. This relationship exhibits an exponential tendency for coarse-grained soil, indicating that unfrozen water content is a highly influential factor in assessing frost heave susceptibility. In contrast, a linear tendency is observed for clay. As the unfrozen water content ( $\theta_u$ ) increases, the  $R$  value of coarse-grained soils transitions from the non-frost heave susceptible zone to the frost heave susceptible zone. Fine-grained soils have broad unfrozen water content, and there is a wider range of values of  $R$ . This means that fine-grained soils have stronger frost heave susceptibility.

### 3.5.4 Effect of the permeability of frozen soil $k_f$

The effect of  $k_f$  on the value of  $R$  is shown in Fig. 10. The value of  $R$  increases with increasing of  $k_f$ ; when  $k_f$  is small enough, the values of  $R$  of the three kinds of soils are close to zero. There may be a critical threshold of  $k_f$  for the tendency of  $R$ . For silt and clay, the values of  $R$  increase

**Table 9** Comparison between the new criterion and existing frost heave susceptibility methods

Criteria	Source	Main indicator	Methods	Expression	Basic items	Universal	Disadvantages
Particle characteristic	[6, 9, 70]	Fines content particle size distribution	Experiment	Empirical summary	Basic geotechnical tests	No	No theoretical support, low reliability, strong limitations,
SP	[35]	Segregation potential	Theory	Specific formula	Frost heave test	Yes	Only for steady state, hard to determine the position of frozen fringe
Frost heave tests	[3, 23, 57]	Frost heave ratio frost heave rate frost heave	Experiment	Specific category	Frost heave tests	No	Long period, high cost, advanced test equipment
Soil–water–ice interaction	[53, 55]	$K_s$ Suction	Experiment	Empirical summary	Related tests	No	Lack of theory and validation
This study	–	Slope $R$	Theory	Specific formula	Basic test of soil properties or frost heave test	Yes	–

Note:  $K_s$  means saturated hydraulic conductivity

sharply when  $k_f$  is  $10^{-12}$  and  $4 \times 10^{-11}$  m/s, respectively. From the limited data, the critical threshold of  $k_f$  for coarse-grained soil may be  $4 \times 10^{-9}$  m/s.  $k_f$  is a direct factor determining liquid water transfer. As shown in Fig. 10, the value of  $R$  of silt is more sensitive to  $k_f$  than those of clay and coarse-grained soil. Corresponding to reality, the frost heave behavior of silt is severe under a sufficient water supply.

### 3.5.5 Effect of thermodynamic parameters

Figure 11 shows that  $\lambda_f$  of frozen soil hardly affect the value of  $R$ . These types of parameters mainly influence the temperature profile and the moving velocity of the frost front. However, they have little influence on  $k_f$  and the hydraulic gradient. Therefore, the frost heave behavior could be neglected.

## 4 Discussion

The new criterion  $R$  for accessing frost heave susceptibility has some features.  $R$  is obtained based on reasonable assumptions and theoretical derivation. In addition,  $R$  is very simple and is easily measured, and can reflect the influences of environmental conditions and soil properties on frost heave behavior. More specifically, the new criterion  $R$  has a specific mathematical expression with parameters of the SWCC and SFCC of soil (see Eq. (17)). The SWCC and SFCC are the water retention capacity of the soil and hydraulic conductivity of the frozen soil, respectively. According to Ye et al. [77], gravel with a fine content less than 8% is not susceptible to frost heave in Germany; this content is 5% in the USA and 15% from the

Ministry of Communications in China. However, the relationship between the particle characteristic criteria and fine content has not yet been established. The particle characteristic criteria are empirical methods, which are supported by extensive frost heave test data. Because of their simple easy-to-obtain parameters and good practical effect, they are mainstream methods at present.

Different fine contents and particle size distributions change the pore distribution, hydraulic conductivity, and water retention capacity of soil [27, 28, 31, 59, 69]. These key parameters will influence the unfrozen water content and hydraulic conductivity of the frozen fringe, thereby determining the frost heave susceptibility. The expression of  $R$  shows these effects, but particle characteristic criteria do not. The new criterion  $R$  might explain why the frost heave susceptibility of silt is strong, that of clay is medium and that of coarse-grained soil is low under the same water supply conditions. That is, the criterion  $R$  is a more comprehensive and rational method than particle characteristics for assessing frost heave susceptibility.

Segregation potential is reliable in theory with consideration of the Clausius–Clapeyron equation and Darcy's law, and this method is quite simple. However, the velocity of liquid water transfer and the temperature gradient of the frozen fringe do not have a clear relationship with the fundamental soil properties and freezing conditions. Therefore, the frost heave test has been used to study segregation potential [33]. Konrad [36, 37] and Loranger et al. [43] have developed empirical models that include the water content and other properties of fines. The only insufficiency is that there is no relationship between the theoretical and empirical models. The frost heave test is direct and reliable for assessing frost heave susceptibility, and it will be more widely promoted when test apparatuses

are not sophisticated, expensive, and time-consuming. For the new method, when assessing frost heave susceptibility, more attention should be given to the SWCC and the SFCC. The related parameters can be obtained from laboratory tests and the SWCC and SFCC databases [18, 49, 63]. The comparison between the  $R$  method and the existing methods is shown in Table 9.

As listed in Table 9, the input parameters of the new index  $R$  are easy to obtain by conducting basic tests of the soil properties, as mentioned above. Another way to obtain the index  $R$  is to conduct the frost heave test. By fitting the proportional function between frost heave and the square root of time,  $R$  is obtained. In summary, the new criterion for frost heave susceptibility classification mainly has three advantages: theoretical basis, easily obtained parameters, and comprehensive consideration of soil properties and environmental conditions. There are more than 100 methods for assessing frost heave susceptibility. The abundance of methods means the lack of a successful comprehensive method. The new simple index  $R$  may be a comprehensive method to classify frost heave susceptibility.

## 5 Conclusion

An analytical model for frost heave is derived by simultaneously solving three partial differential equations, which control the heat transfer and liquid water flow. The proposed model provides a clear expression among frost heave, environmental conditions and soil properties. A number of frost heave experiments are used to validate the proposed model, which shows that the computed results match well with the experimental results. However, the proposed model has certain limitations: (1) it does not consider the effects of water table; (2) it cannot accurately predict frost heave behavior of soil or materials with challenging-to-measure property parameters such as SWCC and SFCC parameters; and (3) it cannot consider the ice lens initiation and cannot predict the positions of ice lenses.

The proposed model indicates that frost heave is a proportional function of the square root of time. Thus, the slope  $R$  of the proportional function is defined as a new criterion for frost heave susceptibility classification. When  $R$  is less than 0.21, it represents a non-frost heave susceptibility condition, a value from 0.21 to 1.18 represents a frost heave susceptibility condition, and a value greater than 1.18 represents a high-frost heave susceptibility condition. The new indicator  $R$  has a theoretical basis and can directly reflect the influences of boundary temperatures and soil–water or soil–freezing characteristics. These parameters related to  $R$  have specific physical meanings and are easily measured in the laboratory.

Parametric analysis shows that the parameters  $\alpha$ ,  $k_f$ , and  $\theta_u$  have obvious influences on frost heave susceptibility. The decrease in  $\alpha$  and  $T_c$  leads to an increase in frost heave susceptibility. Increasing  $k_f$ ,  $\theta_s$ ,  $\theta_r$ , and  $\theta_u$  leads to severe frost heave susceptibility. The thermodynamic parameters of soils have less influence on frost heave susceptibility.

Compared with the existing methods, the new index  $R$  has three advantages: (1) a clear theoretical derivation, (2) input parameters easily obtained by conducting basic tests of soil properties, and (3) taking soil properties and environmental conditions into account. The new indicator  $R$  may be a comprehensive and rational method for assessing frost heave susceptibility.

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