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RESEARCH PAPER

Evaluation of hydraulic conductivity of gap-graded granular soils based on equivalent void ratio concept

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Abstract

Gap-graded granular soils are used as construction materials worldwide, and their hydraulic conductivity depends on their relative content of coarse and fine grains, initial conditions, and particle shape. In this study, a series of constant head hydraulic conductivity tests were performed on gap-graded granular soils with different initial relative densities, fine contents, and particle shapes. The test results show that the hydraulic conductivity decreases with an increase in fine fraction and then remains approximately constant beyond the "transitional fine content." The role of the structural effect on the hydraulic conductivity is different from that on the mechanical properties (such as stiffness and shear strength). This can be attributed to the degree of filling within inter-aggregate voids, disturbance of soil structure, and densified fine bridges between coarse aggregates. The equivalent void ratio concept was introduced into the Kozeny–Carman formula to capture the effect of fines (aggregates) on the "coarse-dominated" ("fine-dominated") structure, and a simple model is proposed to capture the change of hydraulic conductivity of gap-granular soils. The model incorporates a structural variable to capture the effect of fines on "coarse-dominated" structure and coarse aggregates on "fine-dominated" structure. The performance of the model was verified with experimental data from this study and previously reported data compiled from the literature. The results reveal that the proposed model is simple yet effective at capturing the hydraulic conductivity of gap-graded granular soils with a wide range of fine contents, initial conditions, and particle shapes.

Keywords Equivalent void ratio concept · Fine fraction · Gap-graded granular soils · Hydraulic conductivity · Initial density

1 Introduction

Gap-graded granular soils are widely deposited as naturally sedimentary soils, such as in plateau deposits formed by weathering and marine sediments by particle sorting. As a typical type of transitional soil, gap-granular soil is usually composed of coarse aggregates (gravel and coarse sand) and fine particles (fine sand and silt) [8, 9], Simpson and Coop, 2012; [32, 37, 38, 42]. Gap-graded granular soils are adopted as construction and building materials worldwide, such as in subgrades, dams, and ripraps [12, 13, 20, 35]. The permeability of gap-graded soils is a crucial factor in the design of hydraulic earth structures. Internal erosion of fine particles may occur at a high rate of seepage, which

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induces failure in dams [21, 54, 55]. It has been reported that approximately half of all dam failures are associated with internal erosion [41]. Therefore, determining the permeability of gap-graded soils is crucial for evaluating the workability and safety of these geotechnical structures [14, 16, 49, 58, 60].

The hydraulic conductivity of gap-graded soils has been widely studied both experimentally and theoretically (e.g., [4–6, 10, 19, 26, 59]. It is well-established that the hydraulic conductivity is controlled by pore size and pore space distribution, which cannot be easily determined using conventional laboratory tests. As an alternative, the hydraulic conductivity is expressed as a function of the basic parameters of soil particles, such as particle size, uniformity coefficient, packing density, and particle shape [10, 16, 31, 53, 56, 57, 59, 61]. For simplicity, the void ratio is used to reflect the combined effects of particle shape, uniformity coefficient, and packing density [33]. Hence, the hydraulic conductivity can be formulated as a function of the representative particle size and void ratio [10, 33, 59]. Harmonic and geometric methods were adopted by Koltermann and Gorelick [28] to determine the representative particle size for mixtures with high and low fine fractions, respectively. The use of these two different methods leads to a discontinuity in the determined particle size in vicinity of the "transitional fine content" (ψ_t). To address this problem, Zhang et al. [59] used a power-averaging method to calculate the representative particle size of a coarse–fine mixture. Zhang et al. [59] noted that the representative particle size is correlated with the fine fraction and size of coarse (or fine) grains [26, 28, 59]. Various empirical equations have been proposed for estimating the representative particle size [16, 59]. However, these introduce additional parameters that do not have clear physical meanings and are not easy to calibrate.

In this paper, we propose a simple yet effective model, which incorporates a structure parameter correlated with the evolution of the internal structure of granular mixtures. First, the Kozeny–Carman formula was adopted as a reference model. Then, the void ratio of the mixtures is replaced by the equivalent void ratio of gap-graded granular mixtures [50, 52]. Our results reveal that the proposed model is versatile and can reproduce the behavior of gap-graded granular mixtures with a wide range of fine contents, initial conditions, and particle shape. Only one additional parameter, termed as structure parameter, is introduced, making the model suitable for practical engineering applications.

2 Experimental program

2.1 Materials and test methods

The materials used in this study were three sands (denoted as S-1, S-2, and S-3) and two gravels (denoted as G-1 and G-2). The basic physical properties of the sand and gravel (according to ASTM D422) were measured, and the results are given in Table 1. To investigate the coarse and fine fraction effect of the mixtures, the size ratio between coarse aggregates and fines is higher than 5; the median particle size of the three sand ranges from 0.12 to 0.75 mm. The two gravels had essentially uniform grading in the vicinity of 5.00 mm. D_{50} and d_{50} are used here to represent the sizes of coarse and fine grains, respectively.

The maximum and minimum void ratios of S-1 sand are 0.950 and 0.577, respectively. These values are significantly higher than those of S-3 sand, indicating that the physical properties of the two fines are distinct. The maximum and minimum void ratios of G-2 are 1.040 and 0.725, respectively, which are also much higher than those of G-1 gravel. The shapes of the sands and gravel particles were captured using scanning electron microscopy (SEM) and are shown in Fig. 1. The S-1 sand and G-2 gravel particles are angular, whereas the S-2 sand, S-3 sand, and G-1 gravel particles have a sub-round shape; therefore, the S-1 sand and G-2 gravel have a higher maximum and minimum void ratio.

2.2 Sample preparation and test procedure

Previous studies have reported that the permeability behavior of gap-graded granular soils is affected by their initial density, fine content, particle shape, and size ratio between coarse and fine particles. To prepare binary granular mixtures, S-1 sand and S-3 sand were selected as the fines and the other three (S-2 sand, G-1 gravel, and G-2 gravel) as coarse aggregates. Table 2 gives details of the constant head hydraulic conductivity tests considering the effects of the initial state, coarse fraction, particle shape, and size ratio.

Table 1 Physical properties of the tested materials

Properties	S-1 sand	S-2 sand	S-3 sand	G-1 gravel	G-2 gravel
d_{50} / D_{50}(mm)	0.12	0.70	0.75	5.00	5.00
Gs	2.62	2.64	2.64	2.63	2.63
e_{max}	0.950	0.849	0.770	0.765	1.040
e_{min}	0.577	0.499	0.491	0.551	0.725

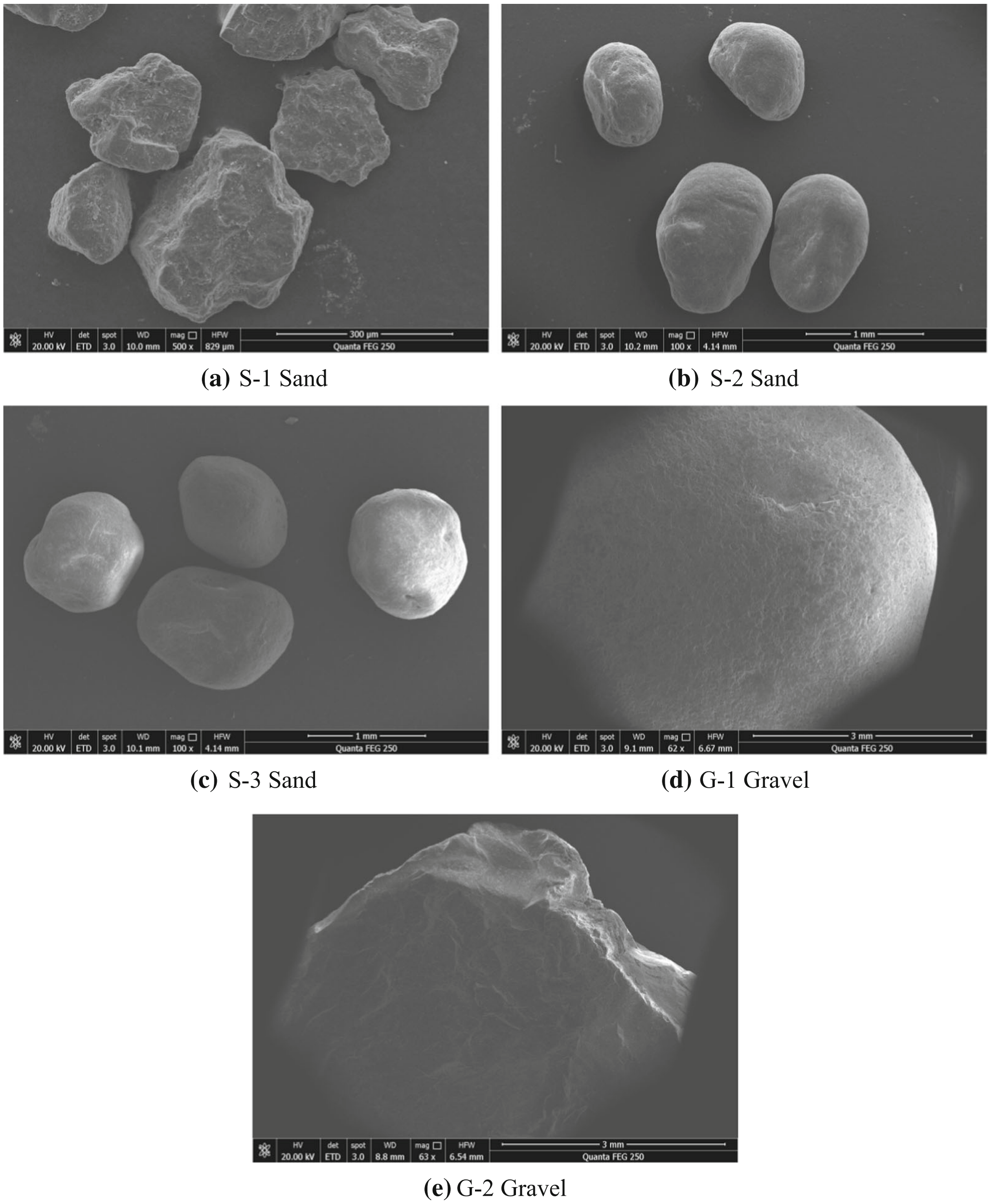


Fig. 1 Electron microscope scanning results of test materials: **a** S-1 sand; **b** S-2 sand; **c** S-3 sand; **d** G-1 gravel; **e** G-2 gravel

To investigate the influence of the fine fraction and initial density, S-2 sand was selected as the coarse aggregate and S-1 sand added to it to prepare the mixture M-1. A full range of fine fractions and three initial relative densities of the mixtures (40, 65, and 85%) were considered. As shown in Table 2, the coarse aggregates in both M-1 and M-2 have a sub-round shape; however, the sizes of the coarse aggregates differ between them (D_{50}/d_{50} = 5.8 for M-1 and 41.7 for M-2); therefore, the effect of size ratio can be investigated. In addition, to consider the influence of particle shape, the sub-round gravel (G-1) and angular gravel (G-2) were mixed with fines, respectively, to prepare mixtures. Note that large inter-connected pores exist in the mixtures with coarse-dominated structures (M-2, M-3, M-4, and M-5), resulting in extremely high hydraulic conductivities. In such cases, the hydraulic conductivity value cannot be measured precisely using a constant head permeameter. Therefore, among the mixtures with gravel aggregates, only those with fine-dominated structures were tested (60, 80, and 100%).

The permeameter had an inner diameter of 10 cm and a height of 40 cm. Three manometers were fixed on the side of the permeameter, and the distance between adjacent piezometers was 10 cm. To produce gap-graded

Table 2 Properties of gap-graded granular mixtures and details of the tests

Test number	Materials Coarse fine	Fine fraction (%)	Relative density (%)	Maximum void ratio	Minimum void ratio
M-1	S-2 S-1	0	40, 65, 85	0.849	0.499
		10		0.770	0.463
		20		0.728	0.437
		40		0.690	0.421
		60		0.754	0.462
		80		0.854	0.512
		100		0.950	0.577
M-2	G-1 S-1	60	65	0.653	0.381
		80		0.801	0.506
		100		0.950	0.577
M-3	G-2 S-1	60	65	0.659	0.380
		80		0.813	0.500
		100		0.950	0.577
M-4	G-1 S-3	60	50	0.614	0387
		80		0.653	0.454
		100		0.770	0.491
M-5	G-2 S-3	60	50	0.643	0.398
		80		0.670	0.450
		100		0.770	0.491

specimens, the fines and aggregates were homogeneously mixed. Then, the specimens were compacted inside a cylindrical mold for the desired density using the moist tamping technique. The specimens were then saturated, and a constant head permeameter was used to measure the hydraulic conductivity according to ASTM D2434 [3]. Distilled water was allowed to flow from top to bottom. The head difference between adjacent piezometers was recorded when the seepage field stabilized. All tests were performed at room temperature, and the hydraulic conductivity was corrected according to the actual temperature. After the constant head hydraulic conductivity tests, the soils from three different locations were sampled and their fine fraction measured. Most of the measured data are close to the prescribed values, with a relative error of less than 4%.Query All specimens were tested under saturated conditions of saturation, and a discussion of the effect of the degree of saturation is beyond the scope of this paper. It should be noted that the unsaturated property of soils should be considered in applications such as radioactive waste disposal, dams, and reservoir construction [15, 36], in which the soils undergo a change in the degree of saturation [24, 25].

2.3 Test result and analysis

The maximum and minimum void ratios of the different gap-graded samples were measured and are given in Table 2. Figure 2 shows the change in the maximum (minimum) void ratio with the fine fraction. At low fine contents, the void ratio decreases with the increase in the fine fraction as the fines fill the inter-aggregate pores. When the void ratio reaches its minimum value, the inter-aggregate space is completely filled with fines, and the fine content at this point is noted as the "transitional fine content" [34, 62]. Beyond this point, the void ratio increases with a further increase in the fine fraction. Note that the "transitional fine content" determined from the $e_{\max}$ and $e_{\min}$ curves is not always consistent because this method is

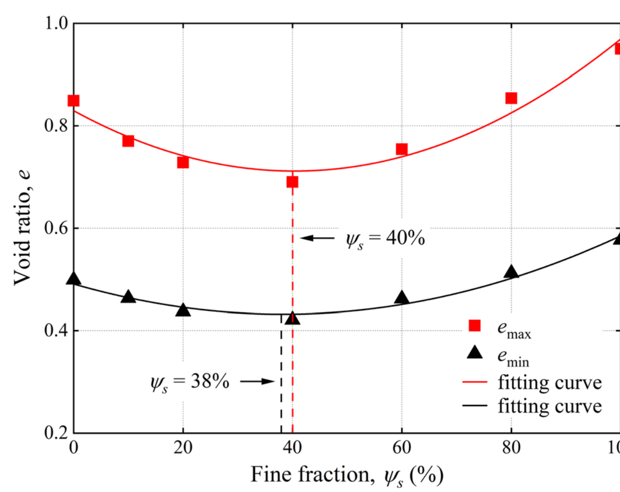


Fig. 2 Change of maximum (minimum) void ratio with fine fraction

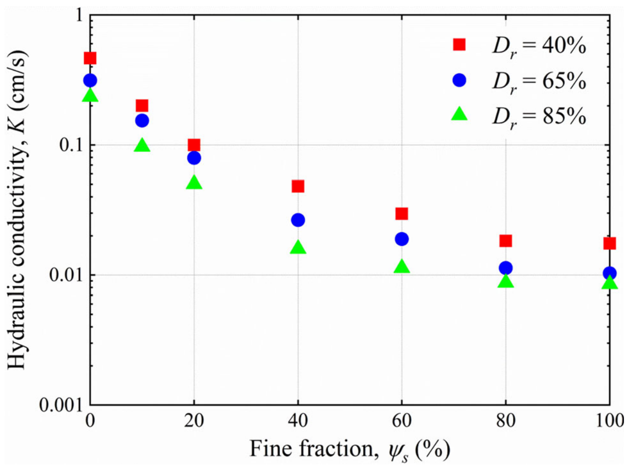


Fig. 3 Change of measured hydraulic conductivity with fine content (M-1 mixture)

empirical. Thus, an average value is used. For the gap-graded mixture (M-1), the "transitional fine content" is obtained as 38 and 40%, respectively, from the minimum points of the $e_{\min}$–ψ_s and $e_{\max}$–ψ_s curves.

The effects of the fine content and initial density on hydraulic conductivity are given in Fig. 3. It can be seen that the hydraulic conductivity of the mixtures decreases with increasing initial density and fine content until the "transitional fine content." In mixtures with low fine fractions, the inter-aggregate pores get increasingly filled with fine grains with the increase in the fine content. As a result, the original seepage channel of the coarse-grained structure is continuously blocked, and the size of inter-connected pores decreases, leading to a sharp decrease in the hydraulic conductivity of the mixed soils. However, the hydraulic conductivity of the mixtures showed only a minor change beyond the "transitional fine content" (Figs. 3, 4), which is consistent with the results of previous studies [26, 28]. Coarse aggregates became dispersed in the fine-grained matrix, and the void ratio and tortuosity increased as the fine content increased. Consequently, the change in the hydraulic conductivity of the mixtures was negligible, which is similar to the case of pure fines.

The change in hydraulic conductivity with varying fine fraction can be interpreted based on the Kozeny–Carman formula: for fine fractions less than the "transitional fine content," both the void ratio and representative particle size of mixtures decrease as the fine content increases, resulting in a decrease in the hydraulic conductivity. After the fine fraction reaches the "transitional fine content," the void ratio increases and representative particle size decreases with increase in fine content. As a result, the hydraulic conductivity undergoes only minor changes owing to the offset mechanism between the void ratio and representative particle size.

The effect of the aggregate shape on the hydraulic conductivity and fine content is shown in Fig. 4. It is evident that the hydraulic conductivity of the mixtures is affected by the shape of the aggregates, and this effect vanishes as the fine fraction increases. For a given initial relative density and fine content, the void ratio of soils with angular gravel (G-2) is higher than that of those with sub-round gravel (G-1) resulting in a higher permeability. The effect of the size ratio on the hydraulic conductivity is shown in Fig. 5. Note that the particle shape of M-1 resembles that of M-2, and the two mixtures have the same relative density (65%). However, M-1 has a higher maximum (minimum) void ratio, leading to a higher hydraulic conductivity. In this study, only the vertical permeability of granular materials is measured owing to technical constraints. Hence, the shape effect is associated with vertical physical properties. Note that packing inhomogeneity may be induced during sedimentation, sample preparation,

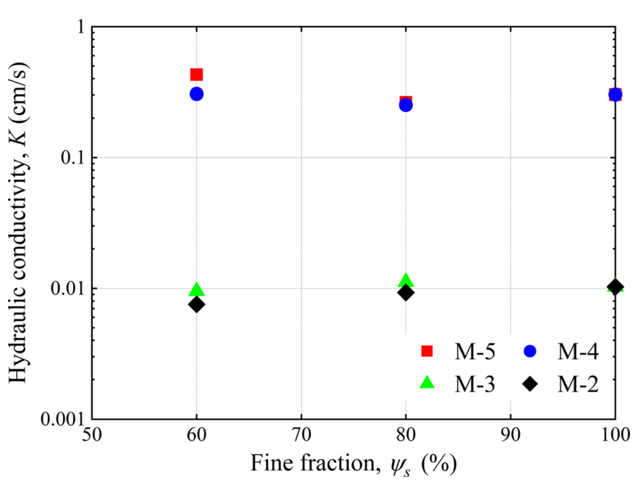


Fig. 4 Change of measured hydraulic conductivity with fine content (M-2, M-3, M-4, and M-5 mixtures, effect of particle shape)

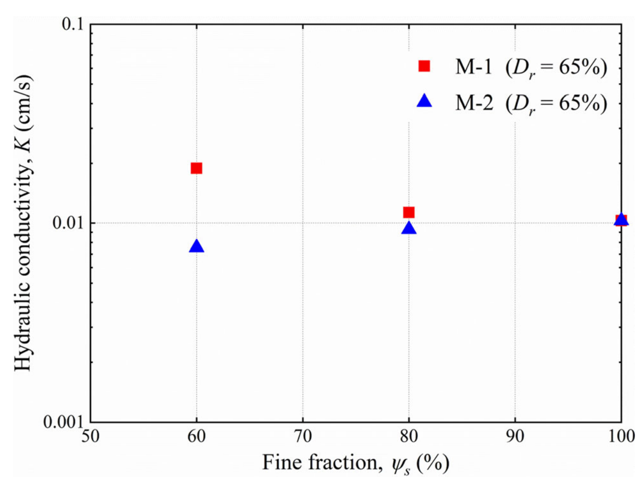


Fig. 5 Change of measured hydraulic conductivity with fine content (M-1 and M-2 mixtures: effect of size ratio)

wetting, and loading processes for soils comprised of ellipsoidal and flat particles [24, 25]. Permeability anisotropy arises because of nonuniform packing structure, and the shape factor has a strong influence on the permeability anisotropy of granular materials [27].

3 Estimating the hydraulic conductivity of gap-graded granular soils

3.1 Kozeny–Carman formula

The Kozeny–Carman formula has been widely adopted for estimating the hydraulic conductivity of granular soils [2, 11, 16, 40, 59]. In the original Kozeny–Carman formula, the hydraulic conductivity is formulated as a function of the void ratio, fluid properties, and geometric properties of soils. Some geometric properties, such as the specific surface, cannot be easily measured using conventional laboratory tests. It can be calculated from the representative particle size (d_{rep}) exploiting the analogy between real coarse particles and spherical particles. Therefore, the Kozeny–Carman formula has been further modified by researchers [7, 18]:

$$K = \frac{\gamma_w}{180 \cdot \mu} \cdot d_{\mathrm{rep}}^2 \cdot \frac{e^3}{1+e} \tag{1}$$

where K is the hydraulic conductivity, e is the void ratio, γ_w is the unit weight of fluid, and μ is the fluid viscosity. The modified form has been validated extensively on the permeability of gap-graded granular soils in previous studies [26, 28, 59], where various methods (such as the geometric mean, harmonic mean, and power-averaging mean) were adopted to determine the representative particle size of fine–coarse mixtures. However, as discussed earlier, some of the parameters do not have clear physical meanings and are not easy to calibrate. In this paper, a simple yet effective concept, termed as “equivalent void ratio concept,” is introduced herein to reproduce the hydraulic conductivity of gap-graded granular soils. The modified version of the Kozeny–Carman formula Eq. (1) is adopted as the reference model.

3.2 Equivalent void ratio concept

The mode of the internal structure is critical for modeling the hydraulic conductivity of gap-graded granular mixtures. The structure of gap-graded granular mixtures is related to the relative contents of coarse aggregates and fines. Monkul and Ozden [34] noted that there a threshold fine fraction, the “transitional fine content,” exists, which distinguishes the relative dominance of fines and coarse aggregates. Mixtures with low fine contents have coarse-dominated structures, however, after exceeding the “transitional fine content,” such mixtures transition from a coarse-dominated to fine-dominated structure [34, 43, 44, 47, 62].

The “transitional fine content” depends on the physical properties of fines and coarse aggregates and generally varies between 20 and 50%. Various experimental and calculation methods can be used to determine the “transitional fine content,” and the values obtained from different methods are not always consistent (Thevanayagametal et al., 2002; [17, 62]. In this study, an empirical method is adopted to determine the transitional fine content based on the $e_{\max}$–ψ_s and $e_{\min}$–ψ_s curves.

For gap-graded granular soils with a coarse-dominated structure, that is, with a fine fraction lower than the transitional fine content ψ_t, Thevanayagam et al. [52] proposed a conceptual model to compute the equivalent void ratio:

$$e_{\mathrm{eq}} = \frac{e + (1-\lambda)\psi_s}{1-(1-\lambda)\psi_s} \tag{2}$$

where e_{eq} is the equivalent void ratio, e is the overall void ratio of gap-graded granular soil, and λ is a structural parameter, which describes the influence of fines on the coarse-grained structure. The value of λ is related to the physical properties of coarse and fine particles, for example, the particle shape and size ratio between coarse and fine particles.

When the fine faction is greater than the “transitional fine content,” coarse–fine mixtures exhibit a fine-dominated structure. Thevanayagam [50] proposed the following equation to determine the equivalent void ratio in this case:

$$e_{\mathrm{eq}} = \frac{e}{\psi^s + (1-\psi_s)/R_d^{\eta}} \tag{3}$$

where η is a structure parameter describing the influence of coarse grains on the fine-grained structure, and its value depends on the grain characteristics and packing of fine matrix. R_d is the size ratio between coarse and fine particles (D_{50}/d_{50}), where D_{50} and d_{50} are the median sizes of the coarse and fine grains, respectively.

3.3 Estimating hydraulic conductivity

In this study, the equivalent void ratio is incorporated into the Kozeny–Carman formula (reference model) to develop a simple model for predicting the hydraulic conductivity of gap-graded granular soils. A gap-graded granular mixture can have one of two types of structures depending on the relative content of fines and coarse aggregates. Therefore, different parameter values should be adopted for the reference model based on the fine fraction. In the case of the coarse-dominated structure, the representative particle size of the mixtures is equal to that of the pure coarse materials (denoted as $d_{\mathrm{rep}}(L)$).

$$K = \frac{\gamma_w}{180\mu} d_{\mathrm{rep}}^2(L) \frac{e_{\mathrm{eq}}^3}{1 + e_{\mathrm{eq}}} \quad (4)$$

The contribution of fines to the coarse-grained structure is captured by the structural parameter λ in Eq. (2). As reported previously [52], Goudarzyn et al., 2016; [46], λ varies between 0 and 1. However, this range is based on analyses of the mechanical properties of granular soils. It will be discussed in the next section that the value of λ is usually greater than 1 because the mechanism governing its change with varying permeability is different from that with varying mechanical properties (such as shear strength and stiffness).

For gap-graded granular soils with a fine-dominated structure, the representative particle size is equal to that of the pure fine materials (denoted as $d_{\mathrm{rep}}(S)$), and the hydraulic conductivity of the mixtures is expressed as

$$K = \frac{\gamma_w}{180\mu} d_{\mathrm{rep}}^2(S) \frac{e_{\mathrm{eq}}^3}{1 + e_{\mathrm{eq}}} \quad (5)$$

The coarse fraction effect on the fine-grained structure is captured by the structural parameter η in Eq. (3). Equations (2)–(5) estimate the hydraulic conductivity of the gap-graded granular soils, and the overall hydraulic conductivity is computed by substituting Eqs. (2) and (3) into Eqs. (4) and (5), respectively. The unit weight γ_w and viscosity μ of a fluid are constant at a given temperature. The permeability of granular mixtures depends on the fine fraction, overall void ratio, and structure parameters.

4 Validation of proposed model

The representative particle size ($d_{\mathrm{rep}}(L)$, $d_{\mathrm{rep}}(S)$) can be determined according to the particle size distribution of the coarse and fine grains. The model has two parameters: the structure parameters λ and η. λ describes the fine fraction effect on the coarse-grained structure and η the coarse fraction effect on the fine-grained structure. Both can be determined by trial and error based on the test data. In this section, the model predictions are compared with the laboratory data of 19 gap-graded mixtures: five of which are the gap-graded soils used in this study and the rest from values reported in the literature [26, 59], Lee and Koo, 2013; Choo et al., 2017; [1].

4.1 Materials

The gap-graded granular materials considered in this study were a series of coarse–fine mixtures with varying initial densities (40, 65, and 85%), particle shape (sub-round aggregates and angular aggregates), and size ratios (5.8 and 41.7). The values of the physical and model parameters for the gap-graded mixtures are given in Table 3. Note that the structure parameters (λ and η) can be assumed to be constant for a given gap-graded granular soil regardless of its fine fraction. In addition, for a given gap-graded mixture, parameter λ appears to be fairly independent of the initial density. However, η decreases with increase in initial relative density.

The experimental data and model predictions are compared in Figs. 6 and 7. The hydraulic conductivity of the mixtures was reproduced well by the model based on the equivalent void ratio concept. Thus, that the proposed model can effectively predict the hydraulic conductivity of gap-graded granular soils.

4.2 Materials used in previous studies

Experimental data of 14 gap-graded mixtures reported in the literature were used for further validation of the proposed model. Materials included artificially crushed sands of different sizes (Choo et al., 2017), glass beads of different sizes and Accusand [59], and glass beads of different sizes [26], Lee and Koo 2013; [1]. Details of the tests and the physical properties of the materials are given in Table 4.

(1). Choo et al. 2017: Four artificially crushed sands (S-4, S-5, S-6, and S-7) from the same parent rock but with different particle sizes were used to prepare granular mixtures. The median particle size d_{50} of the sand ranged from 0.17 to 1.13 mm and the roundness of the materials from 0.17 to 0.20, indicating that the shape of all the material particles was angular. S-4 sand was selected as the coarse aggregate, and the

Table 3 Values of parameters of gap-graded granular mixtures

Test number	Size ratio	d_{rep}(L)	d_{rep}(S)	Initial D_r (%)	Model parameters	
	D_{50}/d_{50}	(mm)	(mm)		λ	η
M-1	5.83	0.62	0.10	40	1.59	1.00
				65	1.57	0.85
				85	1.56	0.64
M-2	41.7	–	0.10	65	–	0.18
M-3	41.7	–	0.10	65	–	0.32
M-4	6.67	–	0.60	50	–	0.32
M-5	6.67	–	0.60	50	–	0.45

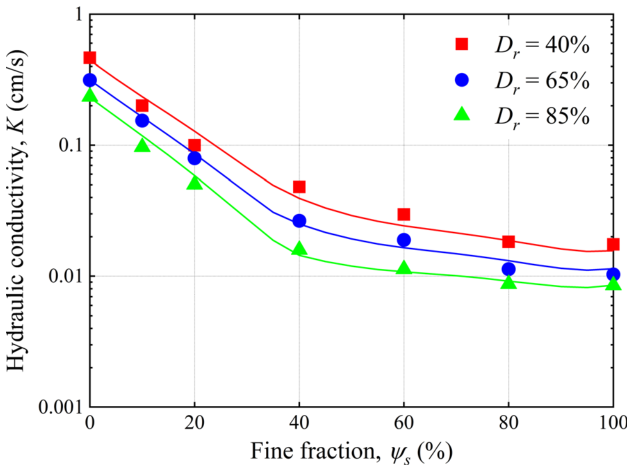


Fig. 6 Comparison of the measured and predicted hydraulic conductivity of gap-graded granular mixtures (M-1 mixture)

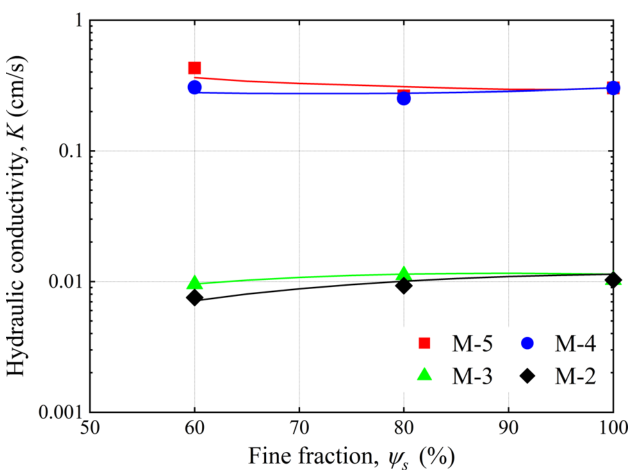


Fig. 7 Comparison of the measured and predicted hydraulic conductivity of gap-graded granular mixtures (M-2, M-3, M-4, and M-5 mixtures)

three other finer grains (S-5, S-6, and S-7) were added to prepare granular mixtures for hydraulic conductivity tests.

(2). [59]: Glass beads of different sizes were selected as coarse aggregates and mixed with Accusand to prepare gap-graded granular soils with various fine contents. Glass beads have almost uniform particle sizes ranging from 2 to 50 mm, and Accusand has a narrow particle size distribution with a median of 0.71 mm.

(3). [26]: Glass beads of different sizes (fine, medium, and coarse) were used. The particle sizes of the fine, medium, and coarse glass beads ranged from 0.148 to 0.177 mm, 0.350 to 0.420 mm, and 0.590 to 0.710 mm, respectively. They were mixed in pairs to prepare the granular mixtures.

Table 4 Properties of materials from the literature

Unmixed materials	G_s	d_{50}(mm)	e	K (cm/s)	Sources
S-4 sand	2.65	1.13	0.885	0.967	Choo et al. 2017
S-5 sand	2.65	0.71	0.820	0.318	
S-6 sand	2.65	0.47	0.870	0.155	
S-7 sand	2.65	0.17	0.820	0.021	
20/30 Accusand	2.66	0.71	0.567	0.194	[59]
2-mm glass beads	2.50	2.00	0.536	1.790	
5-mm glass beads	2.50	5.00	0.570	10.30	
14-mm glass beads	2.50	14.0	0.616	25.30	
50-mm glass beads	2.50	50.0	0.751	116.0	
Fine sand	2.50	0.16	0.698		[26]
Medium sand	2.50	0.39	0.686		
Coarse sand	2.50	0.65	0.639	0.029	
				0.126	
				0.343	
0.2-mm glass beads	2.50	0.20	0.631	0.112	Lee and Koo 2013
0.5-mm glass beads	2.50	0.50	0.623	0.390	
0.7-mm glass beads	2.50	0.70	0.631	0.718	
0.6–0.8-mm glass beads	2.50	0.70	0.592	0.600	[1]
10-mm glass beads	2.50	10.0	0.675	31.20	
30-mm glass beads	2.50	30.0	0.715	207.5	

(4). Lee and Koo 2013: Glass beads of different sizes (0.2, 0.5, and 0.7 mm) were mixed to prepare gap-graded granular mixtures according to the desired fine content.

(5). [1]: Glass beads of size 10–30 mm were selected as the aggregate and mixed with fines (glass beads of size 0.6–0.8 mm) to prepare gap-graded granular mixtures.

The values of the model parameters for the above mixtures are listed in Table 5. For comparison, the model proposed by Zhang et al. [59] was also adopted to predict the hydraulic conductivity of granular mixtures from the literature. In this model, the representative particle size is empirically correlated with the volume fraction of fines, which varies with the packing density. The Kozeny–Carman formula was adopted by Zhang et al. [59] as a reference and a power-averaging method used to estimate the

Table 5 Values of parameters of gap-graded granular mixtures from the literature

Mixtures	D_{50}/d_{50}	ψ_t (%)	d_{rep}(L)	d_{rep}(S)	Parameters		
Coarse	Fine			(mm)	(mm)	λ	η
S-4 sand	S-5 sand	1.58	44	0.69	0.44	1.14	0.95
	S-6 sand	2.40	39	0.69	0.28	1.46	0.55
	S-7 sand	6.76	38	0.69	0.11	1.78	0.37
2-mm glass beads	20/30 Accusand	2.82	40	1.90	0.55	1.33	0.75
5-mm glass beads		7.04	33	4.20	0.55	1.57	0.40
14-mm glass beads		19.70	44	5.90	0.55	1.54	0.25
50-mm glass beads		70.40	45	9.90	0.55	1.36	0.70
Coarse sand	Medium sand	1.67	45	0.63	0.35	1.15	1.00
Medium sand	Fine sand	2.44	43	0.35	0.16	1.23	0.55
Coarse sand	Fine sand	4.06	41	0.63	0.16	1.43	0.43
0.7-mm glass beads	0.5 mm glass beads	1.40	30	0.92	0.69	1.04	0.76
0.5-mm glass beads	0.2 mm glass beads	2.50	32	0.69	0.36	1.15	0.61
10-mm glass beads	0.6–0.8 mm glass beads	14.30	36	5.56	0.91	1.58	0.18
30-mm glass beads	0.6–0.8 mm glass beads	42.90	42	13.28	0.91	1.58	0.15

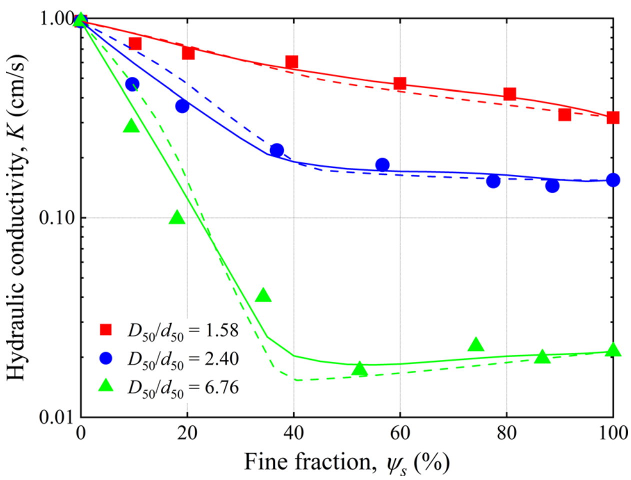


Fig. 8 Comparison of the measured and predicted hydraulic conductivity of gap-graded granular mixtures (data from Choo et al. 2017; Solid lines: this work; Dashed lines: [59]

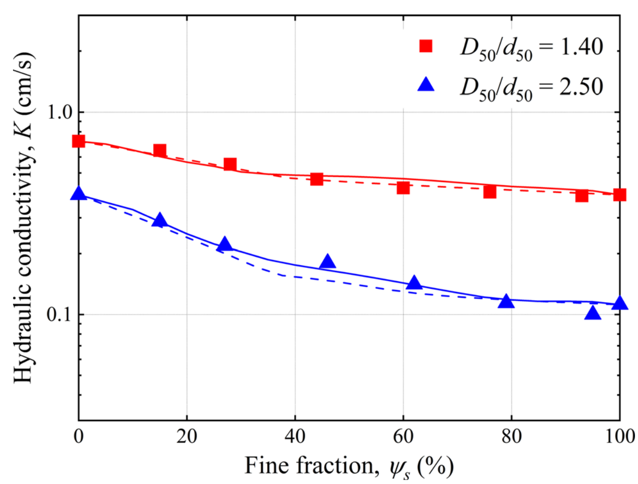


Fig. 9 Comparison of the measured and predicted hydraulic conductivity of gap-graded granular mixtures (data from Lee and Koo 2013; Solid lines: this work; Dashed lines: [59]

effective diameter of the granular mixtures. The predictions of the two models were compared against the experimental data, as shown in Figs. 8, 9, 10, 11, 12. It can be seen that both models reproduce the changing mode of two stages: the hydraulic conductivity first decreases, then remains almost constant as the fine fraction exceeds the "transitional fine content," and finally approaches that of pure fines. This is consistent with the evolution of the internal structure of gap-graded granular mixtures.

To evaluate the accuracy of the hydraulic conductivity determined by the proposed model, the experimental data and model predictions are plotted in Fig. 13. The prediction for mixtures with a low size ratio (D_{50}/d_{50} smaller than 6.76) varies within the narrow range of 0.77–1.30 times the measured data, even though the experimental data were obtained by different researchers using various testing methods. Because the size ratio is relatively low, the fines cannot move freely during the fall head permeability test. Therefore, the specimen remains unchanged during the testing process, and the internal structure can be assumed to be a soil element (Fig. 14a). However, for the coarse-dominated structure with a high size ratio (higher than 14.3), there is a remarkable difference between the

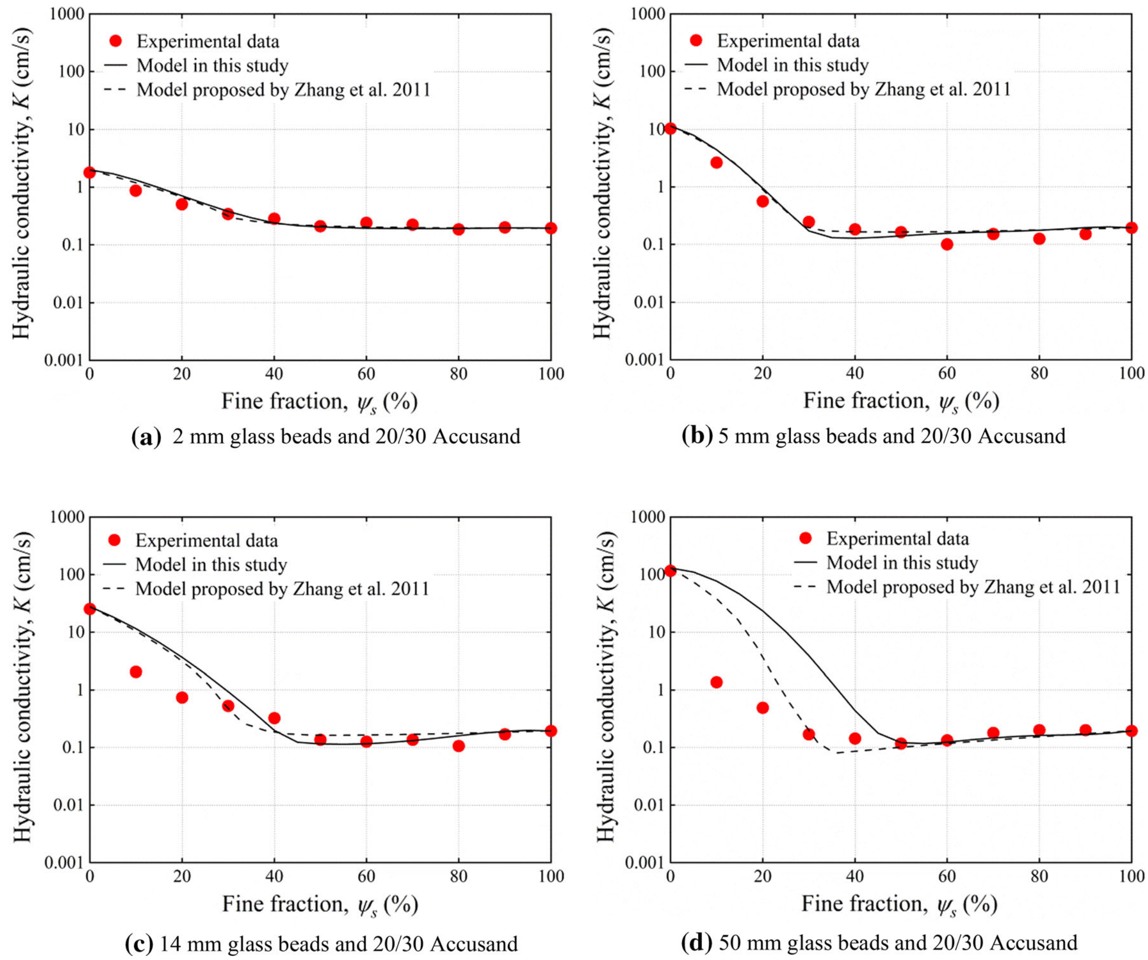
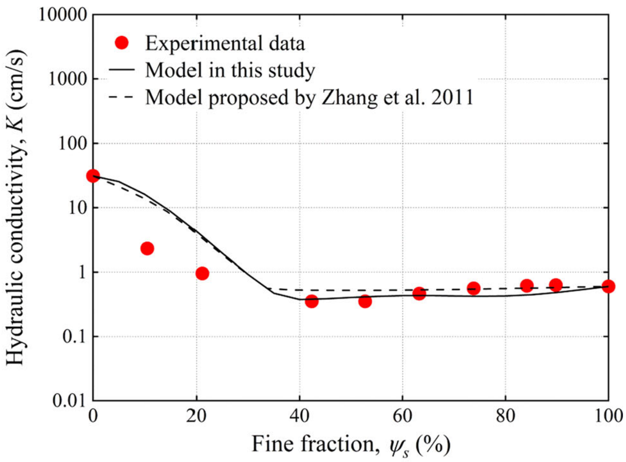


Fig. 10 Comparison of the measured and predicted hydraulic conductivity of gap-graded granular mixtures (data from [59]: **a** 2-mm glass beads and 20/30 Accusand; **b** 5-mm glass beads and 20/30 Accusand; **c** 14-mm glass beads and 20/30 Accusand; **d** 50-mm glass beads and 20/30 Accusand

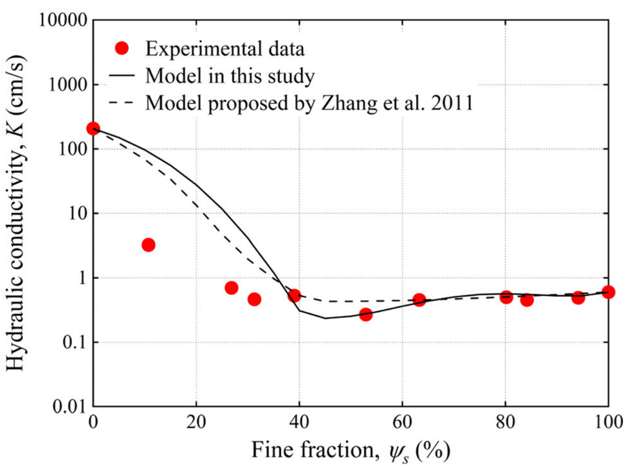
measured data and model prediction (Fig. 13b), with the measured hydraulic conductivity obviously overestimated by the two models (Figs. 10c, d, and 11). This is due to the change in the internal structure of the granular mixtures during the tests; when the fines are relatively smaller than the coarse aggregates, the former can flow freely within the channels consisting of inter-connected inter-aggregate pores. As a result, the fine grains tend to gather at the bottom of the specimen (Fig. 14b; "clogging effect"). It is thus not reasonable to assume the specimens to be a soil element, and the hydraulic conductivity is controlled by the bottom part of the specimens, where the hydraulic conductivity is relatively low. Note that both models assume elementary tests; therefore, their predictions are higher than the measured conductivity in the case of mixtures with high size ratios. In addition, a comparison between the two models and experimental data reveals that our proposed model is simple yet more effective than that of Zhang et al. for capturing the hydraulic conductivity of granular mixtures.

5 Effect of internal structure

The hydraulic properties of gap-graded granular mixtures depend on the internal structure and the changes it undergoes during the testing process. The structure of soils depends on the physical properties of fines, coarse aggregates, and mixtures, such as the initial density, size ratio (D_{50}/d_{50}), particle shape, and fine fraction. Gap-graded mixtures can be classified into two types based on the relative content of fines and coarse aggregates: coarse-dominated structure below the "transitional fine content" and fine-dominated structure beyond it. The effect of the structure on the hydraulic conductivity is considered by incorporating structural variables, which capture the effect of fines on "coarse-dominated" and of coarse aggregates on "fine-dominated" structures.



(a) 10 mm and 0.6-0.8 mm glass beads

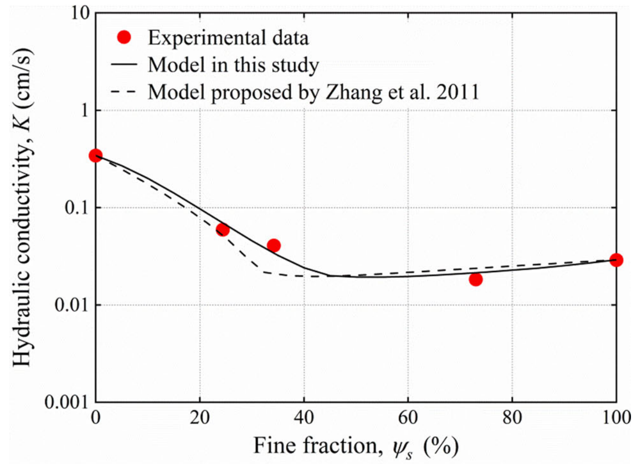


(b) 30 mm and 0.6-0.8 mm glass beads

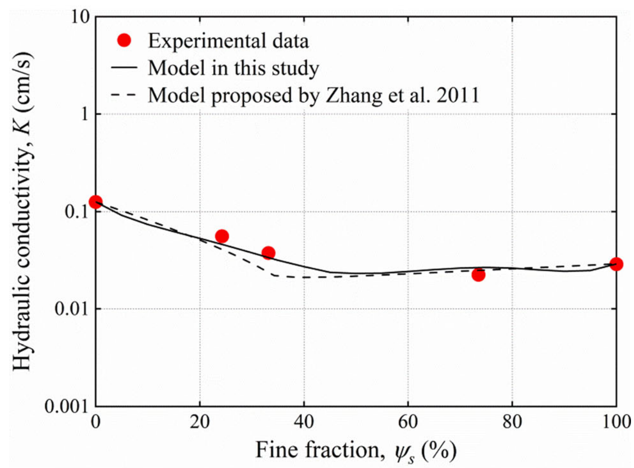
Fig. 11 Comparison of the measured and predicted hydraulic conductivity of gap-graded granular mixtures (data from [1]: **a** 10-mm and 0.6–0.8-mm glass beads, **b** 30-mm and 0.6–0.8-mm glass beads

5.1 Coarse-dominated structure

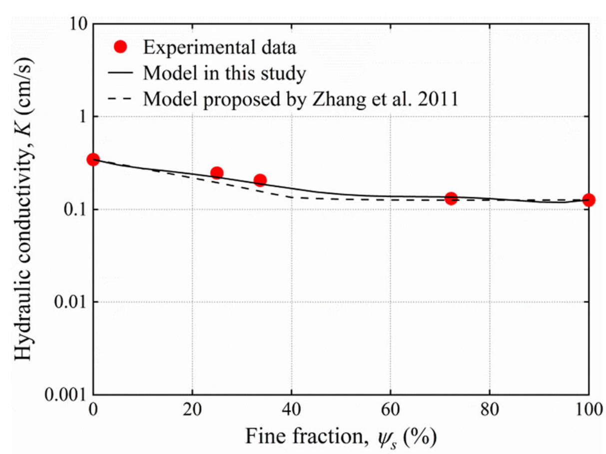
For coarse-dominated structures, a proportion of the fines are usually wedged between aggregates, and these fines participate in the transmission of the inter-aggregate force. This increases the stiffness and shear strength of gap-graded mixtures [12, 23, 45, 51, 52]. As noted by Thevanayagam et al. [52], the fines associated with coarse-dominated structures can be classified into two: active and non-active particles, according to their contribution to the inter-aggregate force skeleton. The structure parameter λ denotes the proportion of fines that are effective in the inter-aggregate force chains. λ ranges from 0 to 1. A value 0 indicates that all the fines are restricted within inter-aggregate pores, and none can participate in the transmission of the inter-aggregate force. A value of 1 indicates that all the fines participate in the inter-aggregate force chain (Fig. 15).



(a) Coarse and fine sand



(b) Medium and fine sand



(c) Coarse and medium sand

Fig. 12 Comparison of the measured and predicted hydraulic conductivity of gap-graded granular mixtures (data from [26]: **a** Coarse and fine sand, **b** Medium and fine sand; **c** Coarse and medium sand

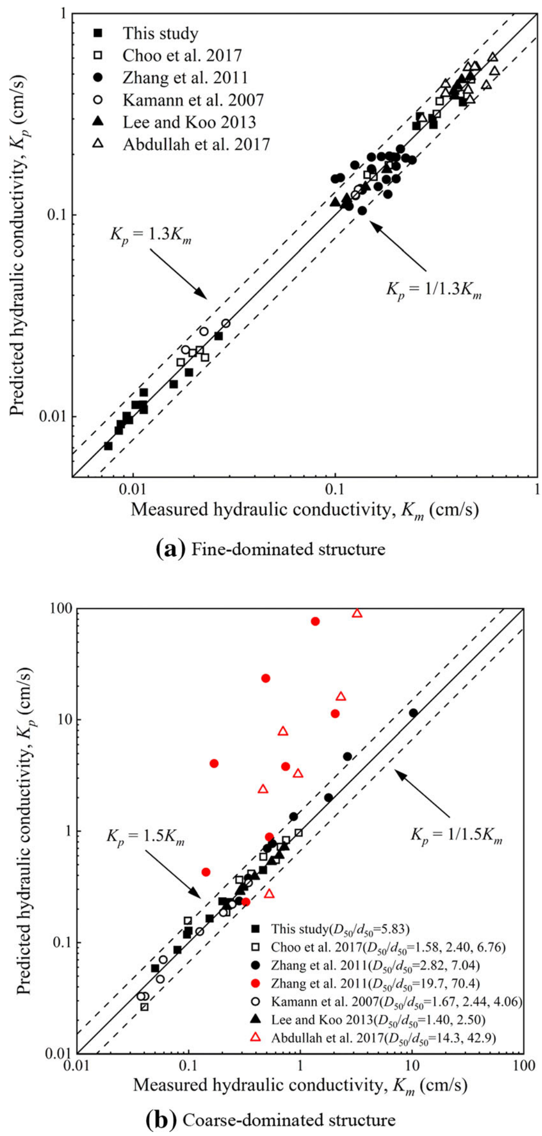


Fig. 13 Correlation between the model prediction and experimental data for granular mixtures from the literature: **a** Fine-dominated structure; **b** Coarse-dominated structure

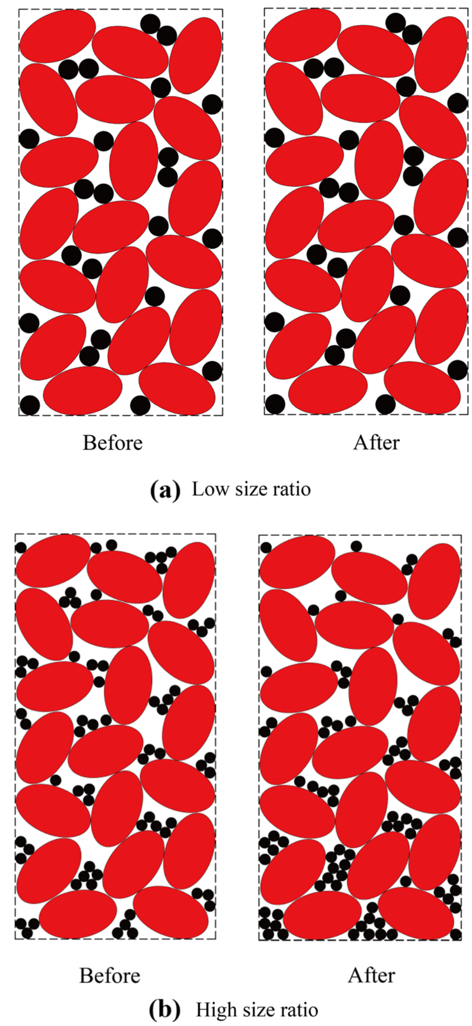


Fig. 14 Effect of size ratio on the clogging phenomenon: **a** low size ratio; **b** high size ratio

The original version of the equivalent void ratio concept is based on an analysis of the effective stiffness of granular mixtures [39, 50–52]. However, the effect of the gap-graded structure on hydraulic conductivity is different from that on the mechanical properties, and the mechanism governing this difference has seldom been reported. The effect of fines on the hydraulic properties of coarse-dominated mixtures is captured through the structure parameter λ. The value of λ depends on the filling state of fines in the inter-aggregate space, and three characterized values are identified:

(1). When λ is 0, the equivalent void ratio equals the inter-aggregate void ratio of the mixtures, and no fines skeleton exists within the inter-aggregate skeleton.

(2). When λ is 1, the equivalent void ratio equals the void ratio of the mixtures, that is, the fines behave as coarse aggregates with the same volume.

(3). Suppose that the equivalent void ratio is relatively low, i.e., $e_{eq} \approx 0$, λ is derived as

$$\lambda = 1 + \frac{e}{\psi_s} \tag{6}$$

Thus, the effect of the fines is ignored if λ lies between 0 and 1, which is unrealistic. In fact, the equivalent void ratio is lower than the void ratio of mixtures, which is consistent with the fact that the hydraulic conductivity of mixtures decreases with increasing fine fraction. Therefore, the structure parameter λ should be greater than 1. In this case, a proportion the fines are confined within the inter-aggregate pores, which block the original seepage channel of the

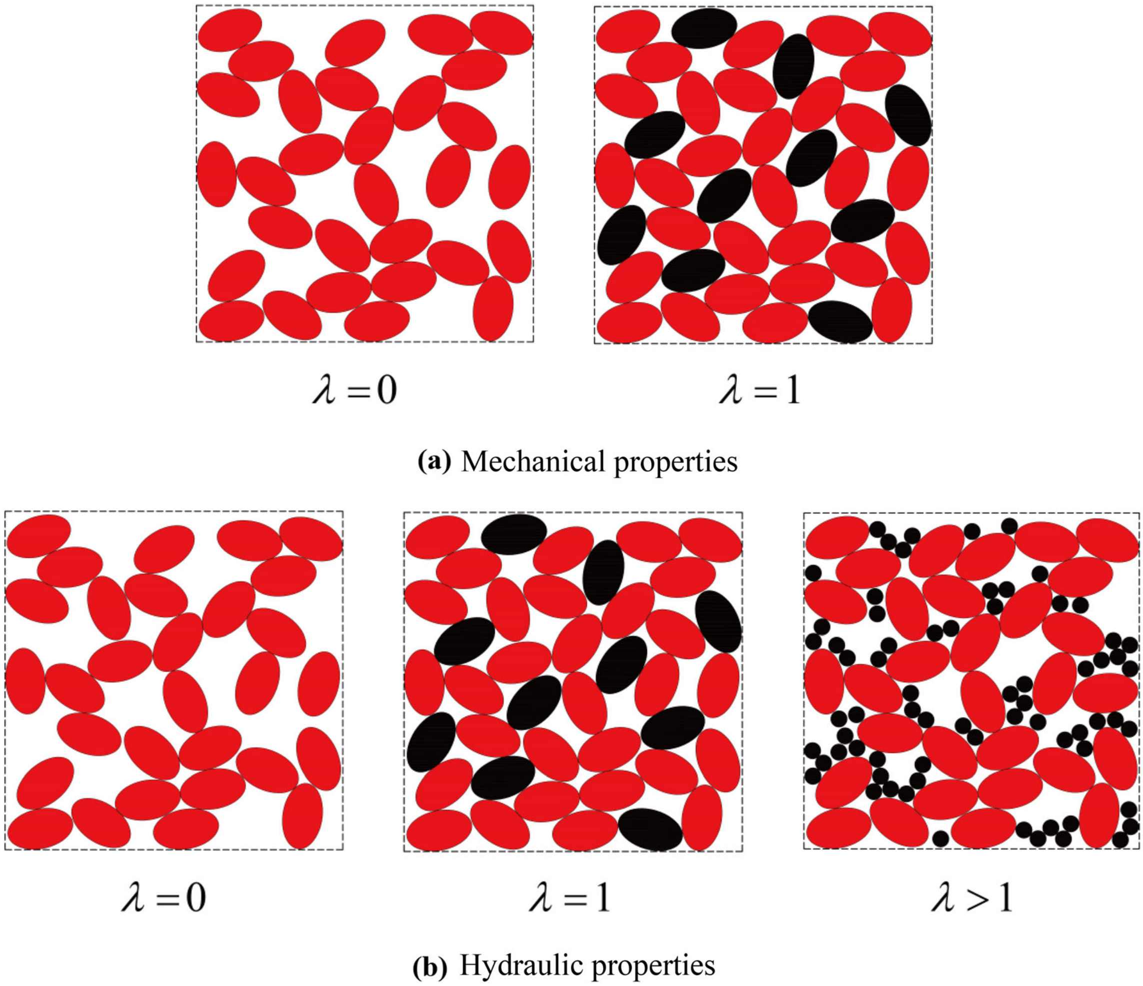


Fig. 15 Schematic figure for internal structure corresponding to different values of structural parameters: **a** Mechanical properties; **b** Hydraulic properties

coarse-grained structure, leading to a decrease in hydraulic conductivity (Fig. 15).

5.2 Fine-dominated structure

For fine-dominated structures, the coarse aggregates act like reinforced particulates within the fines, which increases the shear stiffness of the mixtures. Shi et al. [47] reported that the reinforcing effect in granular mixtures is different from that in cement-based materials, where the evolution of the interfacial transition zone plays a crucial role [22, 29, 30, 48]. The mechanisms governing the reinforcing effect in gap-graded soils are different from those in traditional composites. Shi et al. [44] reported that two phenomena may contribute to the reinforcing effect in binary coarse–fine mixtures: (1) partial contact between coarse aggregates and (2) densified fines acting like a bridge to transmits loading between coarse aggregates. This reinforcing effect leads to an increase in stiffness and shear strength with increasing coarse fraction.

The effect of coarse aggregates on the hydraulic properties of fine-dominated structures is more complicated. As shown in Fig. 4, the hydraulic conductivity may either decrease or increase slightly with increasing coarse fraction. The following phenomena may be responsible for the change in hydraulic conductivity of binary mixtures: (1) the rising coarse fraction leads to a decrease in the specific surface; hence, the hydraulic conductivity increases; (2) the original fine-dominated granular soil either becomes loose (disturbed structure, increases the hydraulic conductivity) or densified (fine bridge, reduces the hydraulic conductivity). The hydraulic conductivity of mixtures relies on the combined effect of these phenomena.

The structure parameters (λ and η) change with varying size ratio. The parameters are calibrated, and the results are presented in terms of the structure parameters (λ and η) and size ratio (D_{50}/d_{50}) in Fig. 16. It can be seen that λ increases as the size ratio increases, while η shows the opposite trend.

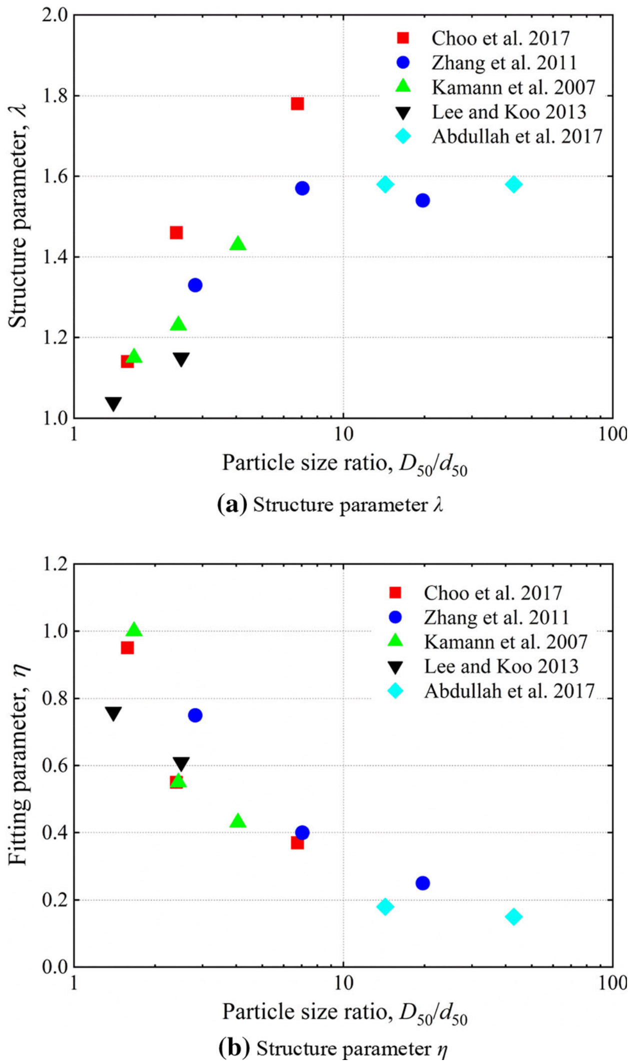


Fig. 16 Change of structure parameters with the size ratio: **a** Structure parameter λ, **b** structure parameter η

6 Conclusions

Gap-graded granular soils are distributed worldwide and are widely used as construction materials. A series of constant head hydraulic conductivity tests were performed on gap-graded granular soils with different initial relative densities, fine contents, and particle shapes to create a comprehensive database for future work. A model was proposed to estimate the hydraulic conductivity of gap-graded granular soils. The conclusions of the study and features of the model are summarized as follows:

(1). The hydraulic conductivity decreases with an increase in fine content up to the "transitional fine content" and then remains approximately constant with a further increase in fine fraction. The hydraulic conductivity of mixtures is also affected by the size ratio and particle shape, and this effect vanishes with an increase in fine fraction.

(2). The equivalent void ratio concept was introduced into the modified Kozeny–Carman formula. It incorporates a structural variable to consider the effect of fines on "coarse-dominated" and coarse aggregates on "fine-dominated" structures.

(3). The effect of the gap-graded structure on hydraulic conductivity is different from that on mechanical properties; the mechanism governing the difference is interpreted according to the degree of filling (coarse-dominated structure), disturbance of structure, and densified fine bridge (fine-dominated structure).

(4). A simple yet effective model with only one structure parameter was proposed for gap-graded materials. The performance of the model was verified using experimental data. The model was found to be versatile and able to capture the hydraulic conductivity of gap-graded granular soils with a wide range of fine contents, initial conditions, and particle shape.

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Data availability All data generated or analyzed during this study are included in this published article.

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