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SHORT COMMUNICATION

Vertical dynamic response of a pile embedded in a poroelastic soil layer overlying rigid base

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Abstract
This paper presents an analytical solution to investigate the vertical dynamic response of an elastic pile embedded in poroelastic soil overlying a rigid base. The pile top is subjected to vertical harmonic loading. The soil is modeled as a poroelastic soil layer underlying the base of the pile and a series of independent infinitesimally thin layers along the shaft of the pile. Using Biot's poroelastodynamic theory and the method of Hankel integral transformation, the vertical dynamic response describing by the complex impedance is obtained. Following the verification of the derived solution against an existing solution, arithmetical examples are provided to parametrically analyze the influence of relevant parameters on the vertical dynamic impedance.

Keywords Dynamic impedance · Pile · Poroelastic soil · Vertical vibration

List of symbols

a_0	Dimensionless frequency
K_v	Vertical impedance
$\mathrm{Re}(K_v)$	Real part of vertical impedance, denoting stiffness of the system
$\mathrm{Im}(K_v)$	Imaginary part of vertical impedance, denoting damping of the system

1 Introduction

The problem of the dynamic interactions between the soil and the foundation plays an important role in the analysis of structures subjected to seismic loading and machine vibrations. Reissner [11] first studied a rigid cylindrical foundation resting on the half-space subjected to a vertical loading. To consider the contribution of the soil surrounding the foundation to its dynamic response, several simplified models including the Winkler model [8], the plane-strain continuum model [10, 16] and the three-dimensional model [5–7, 13, 14] were developed to research the dynamic response of foundations embedded in the soil.

The studies mentioned above represent the soil as an elastic single-phase material; however, in certain cases it is more appropriate to consider the soil as a two-phase fluid-saturated medium. Based on Biot's poroelastodynamic theory [2], Cai and Hu [3] investigated the vertical dynamic response of a rigid foundation embedded in poroelastic half-space, and then Cai et al. [4] extended their solution to the analysis of the vertical vibration of a rigid foundation embedded in poroelastic soil overlying rigid bedrock. Zheng et al. [15] proposed an analytical solution of a pile embedded in a poroelastic half-space to a vertical harmonic loading which allows considering the compressibility of the elastic pile. However, as indicated by Cai et al. [4, 16], in many cases of practical interest the thickness of the substratum soil layer being infinite may no longer be appropriate and may lead to substantial errors.

The objective of this paper is to propose an analytical solution on the vertical vibration of an elastic pile embedded in poroelastic soil overlying a rigid base. Following the method by Zheng et al. [15], the soil is divided

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into two parts: a poroelastic soil layer underlying the pile base and a series of infinitesimally thin independent layers surrounding the pile shaft. The base resistance and shaft reactions are assumed to be uncoupled and calculated, respectively. The dynamic response of the soil-pile system is quantified by the dynamic impedance of the pile top.

2 Formulation of the governing equations

Owing to the difficulty in obtaining an exact analytical solution for the dynamic response of an embedded pile, several simplifying assumptions proposed by Baranov [1] were extended and adopted by Novak and Beredugo [10], Cai and Hu [3, 4] and Zheng et al. [15]. Following their method, the soil is divided into two independent parts and it is assumed that the soil along the pile shaft is composed of a series of infinitesimally thin independent layers with Lame's constants $\bar{\lambda}_s = \lambda_s(1+i\beta)$ and $\bar{G}_s = G_s(1+i\beta)$, while the soil underlying the pile base is represented by a poroelastic soil layer with Lame's constants $\bar{\lambda}_b = \lambda_b(1+i\beta)$ and $\bar{G}_b = G_b(1+i\beta)$, where β is the damping ratio of the soil, $i=\sqrt{-1}$, as depicted in Fig. 1. The stress gradient in the vertical direction and radial displacements for the independent thin layers is ignored, the interfaces between the thin layers are permeable, the contact surface between the pile base and the soil is smooth and permeable, and the pile shaft is assumed perfectly bonded to its surrounding soil [15]. The cylindrical pile with a radius of R and a length of h is modeled as a one-dimensional elastic rod [15]. A harmonic loading $Pe^{i\omega t}$ acted at the pile top is transferred to the soil via the pile shaft resistances $f_s(z,t)$ and the base resistance $f_b(z,t)$, where P is the amplitude of the loading, and ω denotes the exciting frequency. A cylindrical coordinate system located at the center of the pile base is defined such that the z-axis is normal to the base.

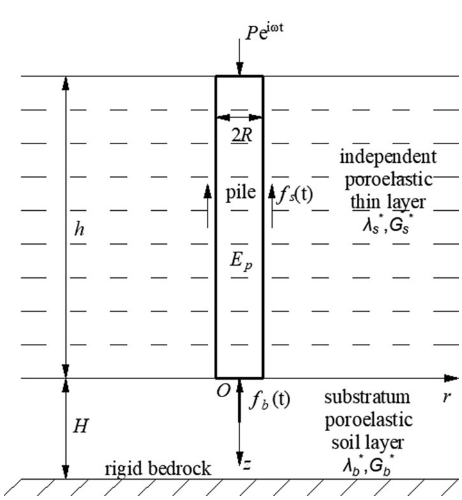

Fig. 1 Sketch of computational model

In the axisymmetric problem the governing equation of the soil underlying the pile base is given by Biot [2] as:

$$\bar{G}_b\left(\nabla^2 - \frac{1}{r^2}\right)u_{rb} + (\bar{\lambda}_b + \bar{G}_b)\frac{\partial e}{\partial r} - \alpha\frac{\partial p_f}{\partial r} = \rho\ddot{u}_{rb} + \rho_f\ddot{w}_{rb} \tag{1}$$

$$\bar{G}_b\nabla^2 u_{zb} + (\bar{\lambda}_b + \bar{G}_b)\frac{\partial e}{\partial z} - \alpha\frac{\partial p_f}{\partial z} = \rho\ddot{u}_{zb} + \rho_f\ddot{w}_{zb} \tag{2}$$

$$-\frac{\partial p_f}{\partial r} = \rho_f\ddot{u}_{rb} + m\ddot{w}_{rb} + b\dot{w}_{rb} \tag{3}$$

$$-\frac{\partial p_f}{\partial z} = \rho_f\ddot{u}_{zb} + m\ddot{w}_{zb} + b\dot{w}_{zb} \tag{4}$$

$$-p_f = M\left(\frac{\partial \dot{w}_{rb}}{\partial r} + \frac{\dot{w}_{rb}}{r} + \frac{\partial \dot{w}_{zb}}{\partial z}\right) + \alpha M\dot{e} \tag{5}$$

where u_{zb} and u_{rb} are the vertical and radial displacements of the solid matrix underlying the pile base, respectively; w_{zb} and w_{rb} are the vertical and radial displacements of the fluid phase relative to the solid matrix, respectively; ρ and ρ_f are the density of the soil and pore fluid, respectively; $m=\rho_f/n$ with n being the porosity; p_f is the pore fluid pressure; b is a parameter accounting for the internal friction between the solid phase and pore fluid; α and M are Biot's parameters accounting for the compressibility of the soil; $e=\frac{\partial u_{rb}}{\partial r}+\frac{u_{rb}}{r}+\frac{\partial u_{zb}}{\partial z}$. A dot over the variable denotes differentiation with respect to the time t.

The constitutive relations for the poroelastic soil are:

$$\sigma_z = \lambda e + 2G\frac{\partial u_z}{\partial z} - \alpha p_f \tag{6}$$

$$\tau_{zr} = G\left(\frac{\partial u_z}{\partial r} + \frac{\partial u_r}{\partial z}\right) \tag{7}$$

The governing equation of pile motions is [15]:

$$E_pA_p\frac{\partial^2 u_p}{\partial z^2} - f_s = \rho_pA_p\ddot{u}_p \tag{8}$$

where E_p, A_p, ρ_p and u_p are the Young's modulus, sectional area, density and vertical displacement of the pile.

2.1 Solution of the governing equations

The problem under consideration is assumed to be time-harmonic with a factor $e^{i\omega t}$, but in the following term $e^{i\omega t}$ is suppressed from all variables for convenience.

2.2 Pile base resistance

The pile base resistance $f_b(z,t)$ can be calculated by employing the solution for a foundation resting on the soil surface. The governing equations of the substratum soil can be solved via Hankel integral transformations with respect to the radial coordinate r. The pth-order Hankel transform of a function $f(r,z)$ with respect to r is defined as [12]:

$$\tilde{f}^{p}(\xi,z)=\int_0^\infty rf(r,z)J_p(\xi r)\mathrm{d}r \tag{9}$$

where $J_p(\xi r)$ is the Bessel function of the first kind of order p; and ξ is the Hankel transform parameter.

Applying zeroth-order Hankel transforms on p_f, u_{zb}, w_{zb} and σ_{zb} as well as first-order Hankel transforms on u_{rb} and τ_{rzb} yields:

$$\tilde{p}_f^0=A_1\mathrm{e}^{-\chi_1 z}+A_2\mathrm{e}^{\chi_1 z}+B_1\mathrm{e}^{-\chi_2 z}+B_2\mathrm{e}^{\chi_2 z} \tag{10}$$

$$\tilde{u}_{zb}^0=\beta_1A_1e^{-\chi_1 z}-\beta_1A_2e^{\chi_1 z}+\beta_2B_1e^{-\chi_2 z}-\beta_2B_2e^{\chi_2 z}+C_1e^{-\chi_3 z}-C_2e^{\chi_3 z} \tag{11}$$

$$\xi\tilde{u}_{rb}^1=\beta_3A_1\mathrm{e}^{-\chi_1 z}+\beta_3A_2\mathrm{e}^{\chi_1 z}+\beta_4B_1\mathrm{e}^{-\chi_2 z}+\beta_4B_2\mathrm{e}^{\chi_2 z}+\chi_3C_1\mathrm{e}^{-\chi_3 z}+\chi_3C_2\mathrm{e}^{\chi_3 z} \tag{12}$$

$$\tilde{w}_{zb}^0=-\beta_5A_1\mathrm{e}^{-\chi_1 z}+\beta_5A_2\mathrm{e}^{\chi_1 z}-\beta_6B_1\mathrm{e}^{-\chi_2 z}+\beta_6B_2\mathrm{e}^{\chi_2 z}-\gamma_1C_1\mathrm{e}^{-\chi_3 z}+\gamma_1C_2\mathrm{e}^{\chi_3 z} \tag{13}$$

$$\tilde{\sigma}_{zb}^0=\beta_7A_1\mathrm{e}^{-\chi_1 z}+\beta_7A_2\mathrm{e}^{\chi_1 z}+\beta_8B_1\mathrm{e}^{-\chi_2 z}+\beta_8B_2\mathrm{e}^{\chi_2 z}-2G_b\chi_3C_1\mathrm{e}^{-\chi_3 z}-2G_b\chi_3C_2\mathrm{e}^{\chi_3 z} \tag{14}$$

$$\tilde{\tau}_{rzb}^1=-G_b(\beta_9A_1\mathrm{e}^{-\chi_1 z}-\beta_9A_2\mathrm{e}^{\chi_1 z}+\beta_{10}B_1\mathrm{e}^{-\chi_2 z}-\beta_{10}B_2\mathrm{e}^{\chi_2 z}+\beta_{11}C_1\mathrm{e}^{-\chi_3 z}-\beta_{11}C_2\mathrm{e}^{\chi_3 z}) \tag{15}$$

where $\chi_1=\sqrt{\frac{d_1-\sqrt{d_1^2-4d_2}}{2}}$;$\chi_2=\sqrt{\frac{d_1+\sqrt{d_1^2-4d_2}}{2}}$; $\chi_3=\sqrt{\xi^2+\frac{(\rho_f\gamma_1-\rho)\omega^2}{\bar{G}_b}}$;$d_1=\xi^2-(\gamma_2+\gamma_3)$;$d_2=\gamma_4-(\gamma_2+\gamma_5)\xi^2$;$\gamma_1=\frac{\rho_f\omega^2}{m\omega^2-b\omega i}$; $\gamma_2=\frac{m\omega^2-b\omega i}{M}-\xi^2$; $\gamma_3=\frac{\alpha^2-2\alpha\gamma_1-\rho\gamma_1/\rho_f}{(\bar{\lambda}_b+2\bar{G}_b)(m\omega^2-b\omega i)}$; $\gamma_4=\frac{\rho\omega^2(m\omega^2-b\omega i)-\rho_f^2\omega^4}{(\bar{\lambda}_b+2\bar{G}_b)M}$;$\gamma_5=\frac{(m\omega^2-b\omega i)(\alpha^2-2\alpha\gamma_1-\rho/\rho_f)}{\bar{\lambda}_b+2\bar{G}_b}$; $\gamma_6=\frac{(\chi_1^2+\gamma_2)\gamma_1}{(\gamma_1-\alpha)\rho_f\omega^2}$; $\gamma_7=\frac{(\chi_2^2+\gamma_2)\gamma_1}{(\gamma_1-\alpha)\rho_f\omega^2}$; $\beta_1=\frac{\chi_1((\bar{\lambda}_b+\bar{G}_b)\gamma_6+\gamma_1-\alpha)}{\bar{G}_b(\chi_1^2-\chi_3^2)}$; $\beta_2=\frac{\chi_2((\bar{\lambda}_b+\bar{G}_b)\gamma_7+\gamma_1-\alpha)}{\bar{G}_b(\chi_2^2-\chi_3^2)}$; $\beta_3=\gamma_6+\beta_1\chi_1$; $\beta_4=\gamma_7+\beta_2\chi_2$; $\beta_5=\beta_1\gamma_1+\frac{\chi_1}{m\omega^2-b\omega i}$; $\beta_6=\beta_2\gamma_2+\frac{\chi_2}{m\omega^2-b\omega i}$; $\gamma_7=\lambda\gamma_6-2\bar{G}_b\beta_1\chi_1-\alpha$; $\beta_8=\lambda\gamma_7-2\bar{G}_b\beta_2\chi_2-\alpha$; $\beta_9=\frac{\xi^2\beta_1+\chi_1\beta_3}{\xi}$; $\beta_{10}=\frac{\xi^2\beta_2+\chi_2\beta_4}{\xi}$; $\beta_{11}=\frac{\xi^2+\chi_3^2}{\xi}$; A_1, A_2, B_1, B_2, C_1 and C_2 are arbitrary coefficients.

As mentioned earlier, the boundary conditions can be expressed as:

$$u_{zb}(r,0)=u_0,\ 0\le r\le R \tag{16}$$

$$\sigma_{zb}(r,0)=0,\ R\le r<\infty \tag{17}$$

$$p_f(r,0)=0,\ 0\le r<\infty \tag{18}$$

$$\tau_{zrb}(r,0)=0,\ 0\le r<\infty \tag{19}$$

$$u_{rb}(r,H)=0,\ 0\le r<\infty \tag{20}$$

$$u_{zb}(r,H)=0,\ 0\le r<\infty \tag{21}$$

$$w_{zb}(r,H)=0,\ 0\le r<\infty \tag{22}$$

where u_0 is the vertical displacement of the pile base.

The arbitrary coefficients can be determined by the aforementioned boundary conditions. Substituting Eqs. (16)–(22) into Eqs. (10)–(15) yields:

$$\tilde{u}_{zb}^0(\xi,0)=f(\xi)\tilde{\sigma}_z^0(\xi,0) \tag{23}$$

where $f(\xi)=k_1.((I_1-I_2k_2I_4^{-1}I_3k_2)^{-1}-(I_2-I_1k_3I_3^{-1}I_4k_3)^{-1}).k_4$;$k_1=\{\beta_3,\ \beta_4,1\}$;$k_2=\mathrm{diag}\{\mathrm{e}^{-\chi_1 H},\mathrm{e}^{-\chi_2 H},\ \mathrm{e}^{-\chi_3 H}\}$; $k_3=\mathrm{diag}\{\mathrm{e}^{\chi_1 H},\mathrm{e}^{\chi_2 H},\mathrm{e}^{\chi_3 H}\}$;$k_4=\{1,0,0\}^{\mathrm{T}}$;$I_1=\begin{bmatrix}\beta_7&\beta_8&-2\bar{G}_b\chi_3\\\beta_9&\beta_{10}&\beta_{11}\\1&1&0\end{bmatrix}$;$I_2=\begin{bmatrix}\beta_7&\beta_8&-2\bar{G}_b\chi_3\\-\beta_9&-\beta_{10}&-\beta_{11}\\1&1&0\end{bmatrix}$;$I_3=\begin{bmatrix}\beta_1&\beta_2&1\\\beta_3&\beta_4&\chi_3\\\beta_5&\beta_6&\gamma_1\end{bmatrix}$;$I_4=\begin{bmatrix}-\beta_1&-\beta_2&-1\\\beta_3&\beta_4&\chi_3\\-\beta_5&-\beta_6&-\gamma_1\end{bmatrix}$.

By virtue of Eqs. (16)–(17), the following dual integral equations can be obtained:

$$\int_0^\infty\xi^{-1}[1+B(\xi)]N(\xi)J_0(\xi r)\mathrm{d}\xi=\frac{u_0}{L},\quad 0\le r\le R \tag{24}$$

$$\int_0^\infty N(\xi)J_0(\xi r)\mathrm{d}\xi=0,\quad R\le r\le\infty \tag{25}$$

where $N(\xi)=\xi\tilde{\sigma}_z^0(\xi,0)$; $B(\xi)=\frac{\xi g(\xi)}{L}-1$; $L=\lim_{\xi\to\infty}\xi g(\xi)=-(1-v)$; v denotes the Poisson's ratio of the solid phase.

Following the method proposed by Noble [9], we define $N(\xi)$ as:

$$N(\xi)=\frac{2\xi u_0}{\pi L}\int_0^R\Phi(x)\cos(\xi x)\mathrm{d}x \tag{26}$$

Substituting Eq. (26) into Eqs. (24)–(25), the following Fredholm integral equation of the second kind can be obtained:

$$\Phi(x)+\frac{1}{\pi}\int_0^R K(x,y)\Phi(y)\mathrm{d}y=1 \tag{27}$$

where $K(x,y)=2\int_0^\infty H(\xi)cos(\xi x)cos(\xi y)\mathrm{d}\xi$.

Now the base resistance $f_b(t)$ can be calculated as:

$$f_b=\bar{G}_b\int_0^{2\pi}\mathrm{d}\theta\int_0^R\sigma_z(r,0)rdr=2\pi\bar{G}_b\tilde{\sigma}_z^0(0,0) \tag{28}$$

Substituting Eqs. (26) into (28) the pile base resistance is obtained:

$$f_b=\frac{4\bar{G}_bu_0}{1-v}\int_0^R\Phi(x)\mathrm{d}x \tag{29}$$

2.3 Pile shaft resistance

In the following the solution for pile shaft resistance $f_s(t)$ is derived. As mentioned earlier, the gradient of normal stresses is ignored; therefore, the governing equations of the poroelastic soil along the pile shaft can be rearranged as [15]:

$$\bar{G}_s\left(\frac{\partial^2 u_{zs}}{\partial r^2}+\frac{1}{r}\frac{\partial u_{zs}}{\partial r}\right)=\rho_s\ddot{u}_{zs}+\rho_f\ddot{w}_{zs} \tag{30}$$

$$\rho_f\ddot{u}_{zs}+m\ddot{w}_{zs}+b\dot{w}_{zs}=0 \tag{31}$$

where u_{zs} and w_{zs} are the vertical displacement of the solid matrix and that of the porous fluid relative to the solid matrix at the side of the pile shaft.

Substituting Eq. (31) into Eq. (30), we can obtain the general solution for the vertical displacement of the solid phase:

$$u_{zs}=DK_0(qr)+EI_0(qr) \tag{32}$$

where $q=\sqrt{(\gamma_1\rho_f-\rho)\omega^2/\bar{G}_s}$, $I_0(qr)$ and $K_0(qr)$ are zeroth-order modified Bessel functions of the first and second kinds, respectively; D and E are arbitrary coefficients.

The assumptions mentioned earlier can be translated to the following boundary conditions:

$$u_{zs}|_{r\to\infty}=0 \tag{33}$$

$$u_p=u_{zs}|_{r=R} \tag{34}$$

$$f_s=-2\pi R\tau_{rz}|_{r=R} \tag{35}$$

Substituting Eqs. (32)–(34) into Eq. (32) we obtain:

$$D=\frac{u_p}{K_0(qR)} \tag{36}$$

$$E=0 \tag{37}$$

According to Eqs. (32)–(37), the shaft resistance $f_s(t)$ can be obtained:

$$f_s=2\pi R\bar{G}_s u_p q\frac{K_1(qR)}{K_0(qR)} \tag{38}$$

2.4 Vertical dynamic impedance

The dynamic response presented in this paper is described by the vertical complex impedance of the pile top K_v, which is defined as the amplitude of the axial load that leads to a unit displacement at the pile top. The real part of the impedance $\mathrm{Re}(K_v)$ represents the dynamic stiffness of the soil-pile system, and the imaginary part $\mathrm{Im}(K_v)$ corresponds to the material and radiation damping. Substituting Eq. (38) into Eq. (8) yields:

$$\frac{\partial^2 u_p}{\partial z^2}-\delta^2 u_p=0 \tag{39}$$

where $\delta=\sqrt{(2\bar{G}_s qK_1(qR)/RK_0(qR)-\rho_p\omega^2)/E_p}$.

The general solution of Eq. (40) is:

$$u_p=J\mathrm{e}^{-\delta z}+K\mathrm{e}^{\delta z} \tag{40}$$

where J and K are arbitrary coefficients to be determined from the boundary conditions at the head and base of the pile, viz:

$$E_pA_p\frac{\partial u_p}{\partial z}|_{z=0}+f_b=0 \tag{41}$$

$$E_pA_p\frac{\partial u_p}{\partial z}|_{z=-h}+P=0 \tag{42}$$

Substituting Eqs. (41)–(42) into (40) yields:

$$J=\frac{P\chi}{E_pA_p\delta(\chi e^{\delta h}-e^{-\delta h})} \tag{43}$$

$$K=\frac{P}{E_pA_p\delta(\chi e^{\delta h}-e^{-\delta h})} \tag{44}$$

where $\chi=\frac{(1-v)E_pA_p\delta+4\bar{G}_b\int_0^R\Phi(x)\mathrm{d}x}{(1-v)E_pA_p\delta-4\bar{G}_b\int_0^R\Phi(x)\mathrm{d}x}$.

According to Eqs. (40) and (43)–(44), the vertical impedance K_v of the soil-pile system is obtained:

$$K_v=\frac{P}{u_p(-h)}=\frac{E_pA_p\delta(\chi e^{\delta h}-e^{-\delta h})}{\chi e^{\delta h}+e^{-\delta h}} \tag{45}$$

3 Verification and discussion

In this section arithmetical examples are presented to verify the accuracy of the derived solution and discuss the sensitivity of the vertical impedance on the relative stiffness of the soil and contributions of soil surrounding the shaft and underlying the base of the pile to the dynamic response. The analysis presented is based on the plots that describe the variation of the vertical impedance with the dimensionless frequency $a_0=\omega R\sqrt{\rho/G_b}$. Unless otherwise mentioned, the following parameters are adopted: $v=0.3$, $\alpha=0.95$, $\beta=0.05$, $n=0.4$, $M^*=M/G_b=10$, $\rho_p^*=\rho_p/\rho=1.25$, $\rho_f^*=\rho_f/\rho=0.53$, $h^*=h/R=10$, $H^*=H/R=1$, $E_p^*=E_p/G_b=1000$, $b^*=bR\sqrt{1/\rho G_b}=10$, $G_s=G_b$.

First we consider a pile embedded in half-space by setting $H^*=50$, which is large enough to simulate the half-space [4, 16], to compare the derived results against these of Zheng et al. [15]. As shown in Fig. 2, the present solution matches well with that of Zheng et al. [15]. In addition, Fig. 3 shows a comparison of the impedance presented with the solution at the hand, against the results given by Zheng et al. [14]. The present solution is degenerated to the Zheng et al.'s solution [14] by setting $\alpha = 1$, $M^* \to \infty$ as well as $b^* = 10^{-6}$ and $\rho_f^* = 10^{-6}$, to eliminate the effect of pore pressures on wave propagation [3, 4, 15]. Observe that there exist small differences in a relative low-frequency range of $0 < a_0 \leq 0.2$ owing to the ignorance of the normal stress gradient for thin layers in the present solution. On the other hand, the present solution agrees well with that of Zheng et al. [14] in higher frequencies.

Figure 4 depicts the effect of the substratum soil H^* on the vertical dynamic impedance K_v. A special case of a half-space is also considered for comparison. Notice that the amplitudes of the impedance tend to decrease with the increase of the value H^* at the resonance frequencies. However, the substratum thickness has a negligible influence on the dynamic response when $H^* > 1$.

Figure 5 presents the effect of the poroelastic parameter of the soil b^* on the vertical dynamic impedance K_v. It should be noted that the value of b^* is inversely proportional to the permeability of poroelastic soil. A special case of a single-phase medium is also presented for comparison. Observe that the parameter b^* only dominates the oscillation amplitudes of both the stiffness and damping at the resonance frequencies but has little influence on the resonance frequency, and the amplitudes of the impedance K_v at the resonance frequencies decrease slightly with the increase of the value of b^*. The behavior of the pile embedded in the single-phase medium is close to the pile embedded in the poroelastic soil with low permeability.

Figure 6 shows the effect of the relative stiffness of the soil layers G_s/G_b on the impedance K_v. We can observe that the amplitudes of both the real and imaginary components of the impedance K_v at the resonance frequencies decrease significantly with the increase of the relative stiffness G_s/G_b. Nevertheless, the relative stiffness G_s/G_b has a trivial effect on the resonance frequencies.

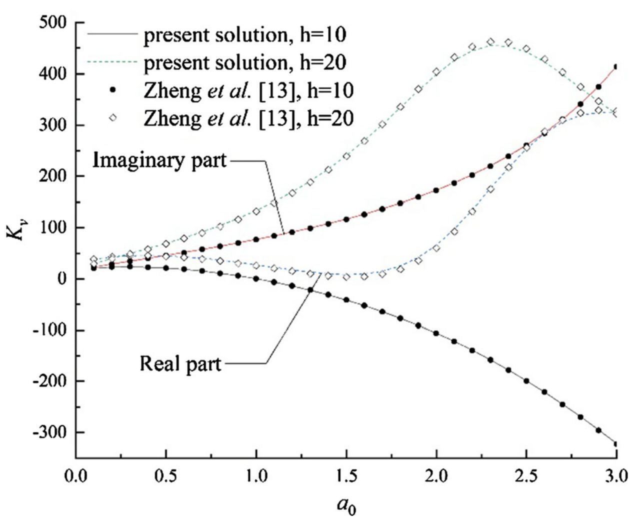

Fig. 2 Comparison of dynamic impedance for the special case of a pile embedded in half-space against the solution of Zheng et al. [15]

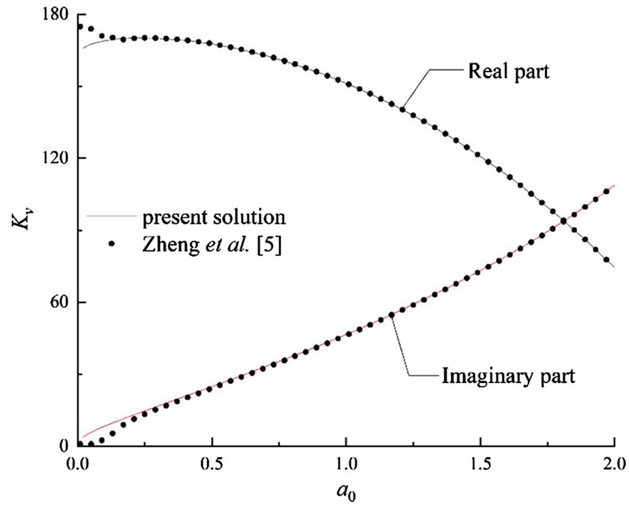

Fig. 3 Comparison of dynamic impedance for the special case of an end-bearing pile embedded in elastic soil against the solution of Zheng et al. [14]

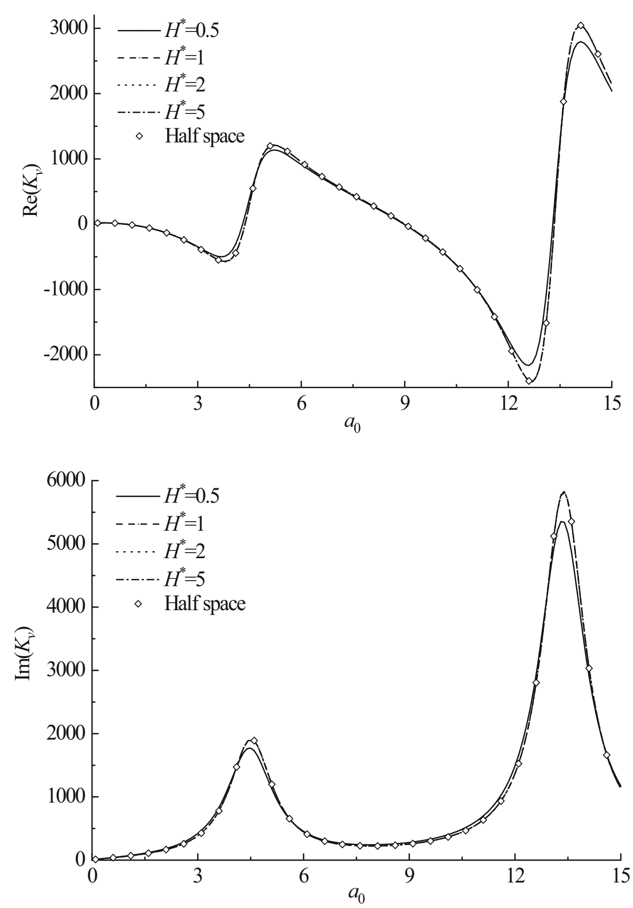

Fig. 4 Effect of substratum thickness H^* on the dynamic impedance

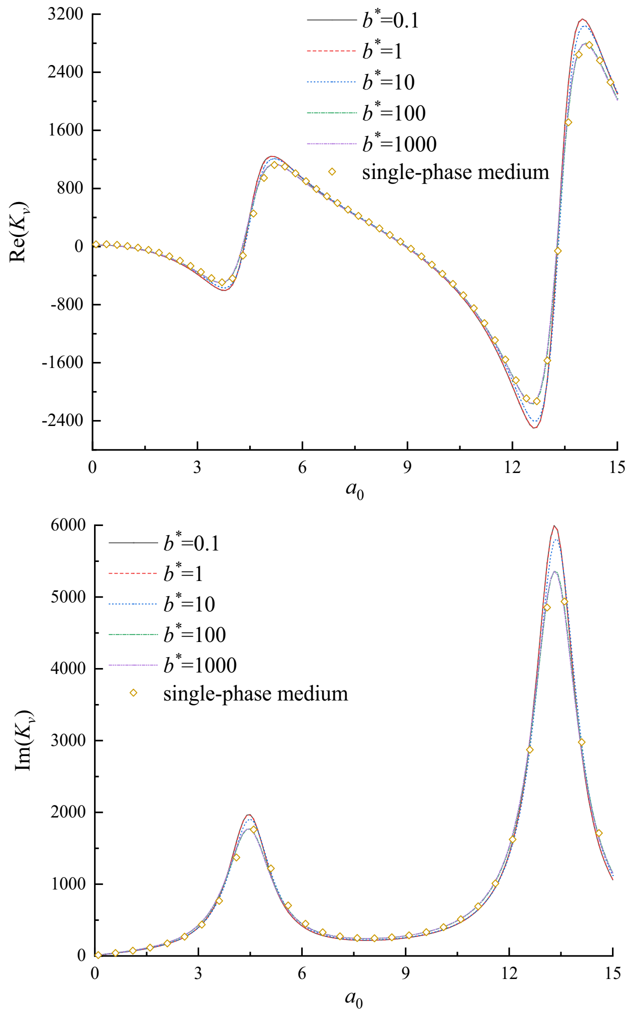

Fig. 5 Effect of permeability b^* on the dynamic impedance

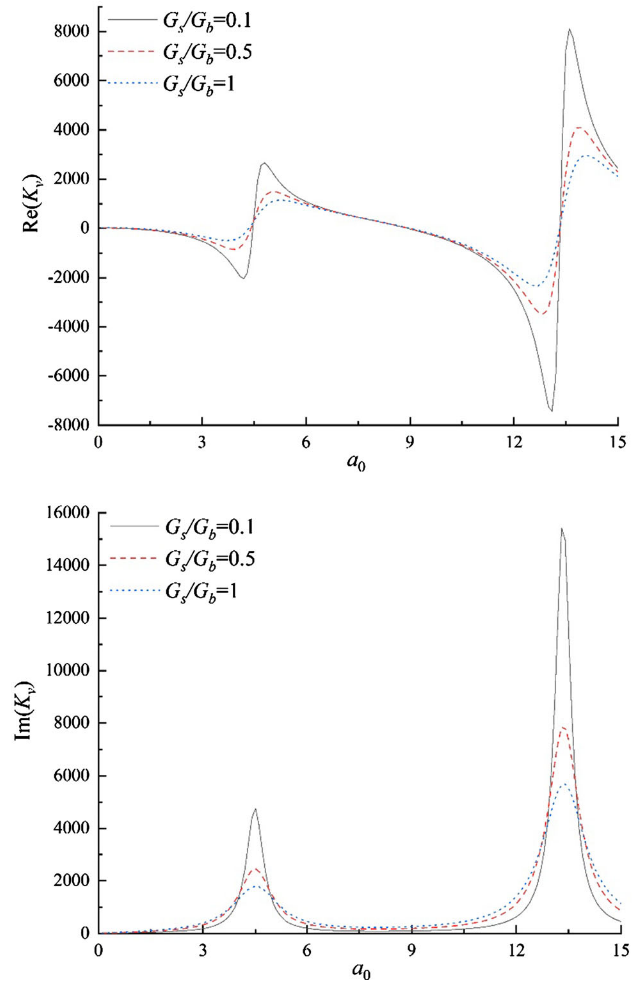

Fig. 6 Effect of relative stiffness of soil G_s/G_b on dynamic impedance

Figure 7 depicts the effect of the relative stiffness of the pile E_p^* on the dynamic impedance K_v. It is clear that both the real and imaginary components of the impedance are sensitive to the pile stiffness and exciting frequencies. The amplitudes of the impedance K_v increase obviously with the increase of the relative stiffness E_p^*. The results also indicate that considering a pile as an infinitely rigid foundation may lead to substantial errors in certain cases.

4 Conclusions

An analytical solution for an elastic pile embedded in poroelastic soil overlying rigid bedrock was presented in this paper. The derived solution agrees well with the existing solutions, and a parametric analysis was developed to highlight the influence of the excitation frequency, the substratum thickness, the poroelastic parameter and the relative stiffness of the soil and the pile. Results suggest that the stiffness of both the soil and the pile has a pronounced effect on the impedance, and considering the stratification of soil and elasticity of pile is more realistic to the dynamic response. The effect of the substratum thickness and poroelastic parameter is less prominent, but also cannot be ignored if an accurate estimation is required. The solution is convenient to code and provides a quick estimation of the vertical dynamic response of an embedded pile.

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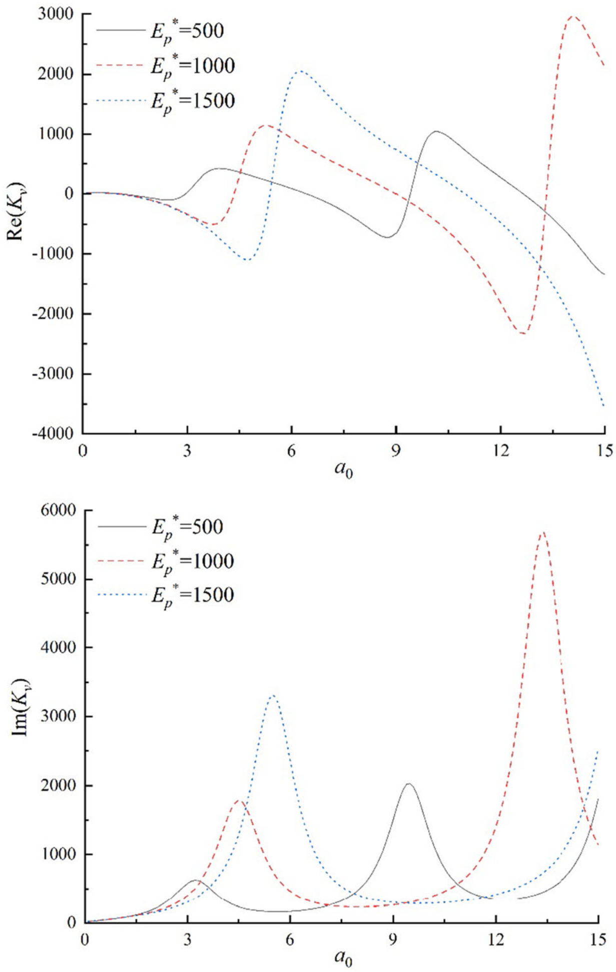

Fig. 7 Effect of relative stiffness of pile E_p^* on dynamic impedance

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