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RESEARCH PAPER

Development of a coupled pile-to-pile interaction model for the dynamic analysis of pile groups subjected to vertical loads

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Abstract

This paper presents a new analytical model for calculating the dynamic performance of pile groups subjected to vertical loads. The derived solution allows considering the robust pile-to-pile coupled interaction on impedance of the pile group, by accounting for the secondary waves generated by the vibration of the receiver pile. For that, we introduce a dynamic coupling factor to modify the classical pile-to-pile interaction factor to illustrating the coupling effect of the source and receiver pile. Numerical results obtained for typical problem parameters indicate that this coupling effect can be important for closely spaced pile group, pile groups with large slenderness ratio or pile groups with large group members. The proposed solution is capable of determining the frequency-dependent impedance of large pile groups comprising arbitrary numbers of piles, which numerical modelling can be cumbersome.

Keywords Analytical methods · Dynamic analysis · Pile groups · Soil-structure interaction

1 Introduction

Piles are always designed in a group to transfer dynamic loads to competent soil layers, which covers the issue of how each pile interacts with its surrounding soil (pile-to-soil interaction) [1, 3, 4, 11, 19, 20], and of interaction between piles in the same group (pile-to-pile interaction) [7, 10, 15, 18, 21]. The effects of pile-to-soil and pile-to-pile interaction phenomena on load transfer mechanisms are coupled sophisticatedly, rendering a number of numerical methods such as the finite element method (FEM) [13, 16] and the boundary element method (BEM) [2, 9, 14]. The complex geometry of pile groups with large group members and dense mesh discretization of the domain and its boundaries, especially for dynamic problems involving wave propagation, lead to large systems of equations and therefore to computationally expensive effort. This is not surprising that some approximate wave propagation methods [5, 6, 8, 12, 15, 17] are still in use to reveal load transfer mechanisms of pile-to-soil and pile-to-pile interaction.

Principle of superposition pioneered by Poulos [15] offers a method to calculate settlement of pile groups with arbitrary number of piles, by considering the interaction between two floating piles subjected to static vertical loads. The interaction factor is defined as a ratio of the additional displacement of a pile due to the presence of a source pile, over the displacement of the same pile under its own load. Later, Dobry and Gazetas [5] extended the static interaction factor of two piles to dynamic problems to serve as a basis for further studies on vertical, lateral and rocking vibrations of pile groups. These solutions were proposed based on a simple three step to calculate the dynamic interaction factor, while assuming that cylindrical waves are emanated from the source pile and spread radially to strike an adjacent pile, and that a receiver pile follows exactly the soil attenuation function. These assumptions

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suggest that secondary waves generated by the receiver pile vibration are ignored when the radially spreading incident waves strike the adjacent pile, and that the interaction between the receiver pile and its surrounding soil is neglected. Mylonakis and Gazetas [10] refined the pile-to-pile interaction model of Dobry and Gazetas [5] to account for the diffraction effect of the arriving wave by simulating the pile as a Winkler beam supported by springs and dashpots, while they did not consider the generated secondary waves.

In all the above-mentioned studies, only incident waves are considered when evaluating the pile-soil-pile interaction of pile groups subjected to vertical dynamic loads. In reality, secondary waves which are generated by the vibration of the receiver pile also exist in the soil layer after the incident waves striking the receiver pile, which tends to interfere with the incident waves form the source pile. Intuitively, the contribution of the secondary wave to the responses of the pile group will depend on the pile spacing S and the number of piles in the same group. To the authors' knowledge, this mechanism is ignored in all existing analytical solutions used to evaluate dynamic responses of a pile group. In the following, we address a four-step process for the dynamic analysis of pile-soil-pile systems that considers both the incident waves and the secondary waves, and use the new analytical model to define a "coupling factor" that accounts for the coupling effect of the source and receiver pile vibrations. Accordingly, we employ this coupling factor to obtain a new expression for the pile-to-pile interaction factor, which is used for dynamic responses of a pile group subjected to vertical loads. Finally, we discuss the effect of the refinement introduced in this paper on the popular interaction factor of two identical piles and the impedance of pile groups via comparison with existing solutions of Mylonakis and Gazetas [10].

2 A new proposed model for mutual interaction of piles in a group

The problem addressed in this paper is illustrated in Fig. 1: a group of m vertical circular piles with length H, diameter d ($d = 2r_0$), cross-sectional area A_p, modulus of elasticity E_p and mass density ρ_p are embedded in soil layers and subjected to a vertical harmonic load $P_G e^{i\omega t}$. ω is the excited frequency; $i = \sqrt{-1}$. The soil layer is modelled as linear viscoelastic material, with damping ratio β_s, complex Lame constants $\lambda = \lambda^*(1 + 2i\beta_s)$ and $G = G^*(1 + 2i\beta_s)$ and mass density ρ_s. The pile members spaced with S are connected through a rigid weightless pile cap which is not in contact with the soil, rendering this model competent for

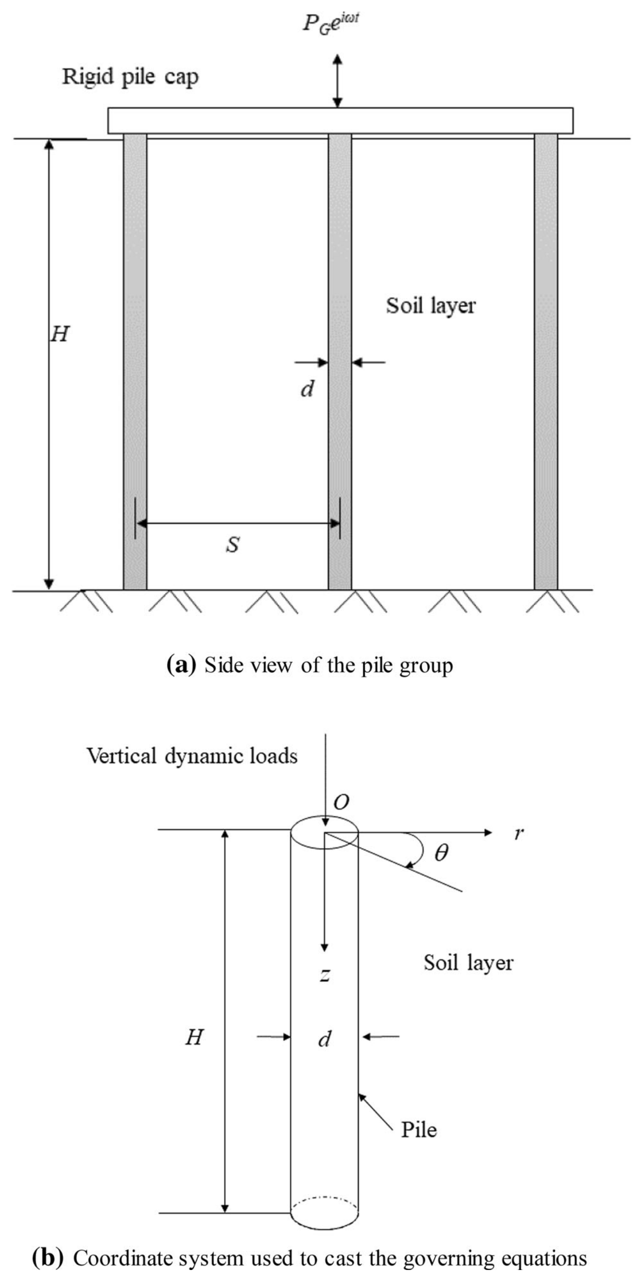


Fig. 1 Schematic of a pile group subjected to a vertical dynamic load: **a** side view of the pile group and **b** coordinate system used to cast the governing equations

high-rise pile groups. A right-handed cylindrical ($r - \theta - z$) coordinate system, shown in Fig. 1b, is used to cast the governing equations of the problem. The origin of the system lies at the centre of the pile head, and the z-axis coincides with the pile axis which points downwards, with clockwise polar angles θ taken as positive. In addition, we introduce the following assumption that the pile members are simulated as Winkler beams; no slippage takes place at the pile-soil interface and that the pile group is rested on the rigid bedrock.

As discussed above, secondary waves also exist in the soil after the incident waves striking receiver piles, which tends to amplify or de-amplify the free soil displacement at location of the source pile. In order to depict this mechanism mathematically, the development of pile-to-pile coupled interaction is decomposed into the following four steps with reference to Fig. 2, which follows from the model proposed by Dobry and Gazetas [5] and Mylonakis and Gazetas [10]:

Step 1: The response (vertical displacement $W_{11}(z)$) of a solitary source pile subjected to a time-harmonic vertical load is determined using an analytical method.
Step 2: Cylindrical waves are generated along the source pile and spread radially with the vertical soil displacement $U_{21}(S, z)$ given by:

$$U_{21}(S,z) = \psi(S)W_{11}(z) \tag{1}$$

in which $\psi(S)$ is the soil attenuation function:

$$\psi(S) = \frac{U_{21}(S,z)}{U_{21}\left(\frac{d}{2},z\right)} = \frac{H_0^{(2)}\left(\frac{S}{d}\frac{a_0}{\sqrt{1+2i\beta_s}}\right)}{H_0^{(2)}\left(\frac{1}{2}\frac{a_0}{\sqrt{1+2i\beta_s}}\right)} \tag{2}$$

where $a_0 = \omega d/V_s$, V_s is shear wave velocity of the soil layer, and $H_0^2()$ is the Hankel function of zero order and the second kind.
Step 3: The unloaded receiver pile is modelled as a Winkler beam to account for diffraction effect of the arriving wave. Notice that the mechanics of step 3 is in a sense the reverse to that in step 1: in step 1 the source pile induces displacements on soil, whereas in step 3, the incident wave field induces displacements on the receiver pile.
Step 4: The soil reaction at location of the source pile arising from the receiver pile interferes with the incident free displacement, rendering responses of the source pile different. To account in a simple realistic way for this interaction, the source pile is modelled as a Winkler beam, and the soil displacement $U_{12}(S, z)$ induced by the receiver pile is applied at the distributed soil springs and dashpots.

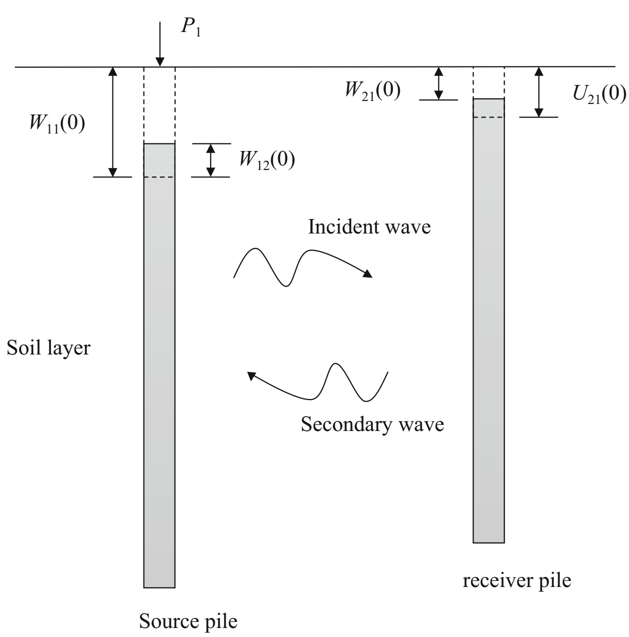


Fig. 2 Schematic illustration of the proposed model to account for pile-to-pile mutual interaction

For a harmonically vibrating source pile, the dynamic equilibrium of the infinitesimal pile segment gives the following governing equation:

$$E_pA_p\frac{d^2W_{11}(z)}{dz^2} - \left(k_z + i\omega c_z - m\omega^2\right)W_{11}(z) = 0 \tag{3}$$

where $W_{11}(z)$ denotes the axial displacement of the vibrating source pile, k_z and c_z are frequency-dependent springs and dashpots, respectively, $m = \rho_pA_p$. The general solution of Eq. (3) is:

$$W_{11}(z) = A_{11}e^{\lambda z} + B_{11}e^{-\lambda z} \tag{4}$$

where $\lambda^2 = \frac{k_z+i\omega c_z-m\omega^2}{E_pA_p}$, A_{11} and B_{11} are constants relating with boundary conditions of the source vibrating pile. For special end-bearing piles, we introduce the boundary conditions at the top and bottom of the pile as:

$$\left.\frac{dW_{11}(z)}{dz}\right|_{z=0} = -\frac{P_1}{E_pA_p} \tag{5}$$

$$W_{11}(z)|_{z=H} = 0 \tag{6}$$

where P_1 denotes the harmonic vertical load atop the source pile.

Substituting Eqs. (5) and (6) into Eq. (4) yields:

$$A_{11} = -\frac{P_1}{E_pA_p\lambda(1+e^{2H\lambda})} \tag{7}$$

$$B_{11} = \frac{e^{2H\lambda}P_1}{E_pA_p\lambda(1+e^{2H\lambda})} \tag{8}$$

Similarly, the dynamic equation of a receiver pile can be governed as:

$$\begin{aligned} &E_pA_p\frac{d^2W_{21}(z)}{dz^2} + m\omega^2W_{21}(z) \\ &- (k_z + i\omega c_z)[W_{21}(z) - U_{21}(S,z)] \\ &= 0 \end{aligned} \tag{9}$$

where

$$U_{21}(S,z) = \psi(S)W_{11}(z) = \psi(S)\left(A_{11}e^{\lambda z} + B_{11}e^{-\lambda z}\right)$$

$W_{21}(z)$ is the displacement of the receiver pile induced by vibration of the source pile.

The solution of Eq. (9) is:

$$W_{21}(z) = \frac{k_z + i\omega c_z}{2\lambda E_p A_p}\psi(S)z\left(-A_{11}e^{\lambda z} + B_{11}e^{-\lambda z}\right) + A_{21}e^{\lambda z} + B_{21}e^{-\lambda z} \tag{10}$$

in which A_{21} and B_{21} are integration constants. A_{21} and B_{21} are calculated considering zero force atop the receiver pile and fixed bottom of the receiver pile i.e.

$$\left.\frac{dW_{21}(z)}{dz}\right|_{z=0} = 0 \tag{11}$$

$$W_{21}(z)|_{z=H} = 0 \tag{12}$$

Accordingly,

$$A_{21} = \frac{(k_z + i\omega c_z)\psi(S)\left(A_{11} - B_{11} - B_{11}\lambda H + A_{11}\lambda H e^{2\lambda H}\right)}{2E_pA_p\lambda^2(1 + e^{2H\lambda})} \tag{13}$$

$$B_{21} = \frac{(k_z + i\omega c_z)\psi(S)\left(-A_{11}e^{2\lambda H} + B_{11}e^{2\lambda H} - B_{11}\lambda H + A_{11}\lambda H e^{2\lambda H}\right)}{2E_pA_p\lambda^2(1 + e^{2H\lambda})} \tag{14}$$

Considering the secondary waves striking the source pile, the governing equation for this step can be expressed as:

$$E_pA_p\frac{d^2W_{12}(z)}{dz^2} + m\omega^2W_{12}(z) - (k_z + i\omega c_z)[W_{12}(z) - U_{12}(S,z)] = 0 \tag{15}$$

In which $U_{12}(S,z) = \psi(S)W_{21}(z) = \psi(S)\left[\frac{k_z+i\omega c_z}{2\lambda E_pA_p}\psi(S) z\left(-A_{11}e^{\lambda z} + B_{11}e^{-\lambda z}\right) + A_{21}e^{\lambda z} + B_{21}e^{-\lambda z}\right]$; $W_{12}(z)$ is a displacement induced by the secondary wave, which points upwards.

The solution of Eq. (15) is:

$$\begin{aligned}W_{12}(z) = {} & z\left[\frac{(k_z + i\omega c_z)^2\psi^2}{8\lambda^2E_p^2A_p^2}A_{11}z - \frac{(k_z + i\omega c_z)\psi}{2E_pA_p\lambda}A_{21} - \frac{(k_z + i\omega c_z)^2\psi^2}{8\lambda^3E_p^2A_p^2}A_{11}\right]e^{\lambda z} \\ & + z\left[\frac{(k_z + i\omega c_z)^2\psi^2}{8\lambda^2E_p^2A_p^2}B_{11}z + \frac{(k_z + i\omega c_z)\psi}{2E_pA_p\lambda}B_{21} + \frac{(k_z + i\omega c_z)^2\psi^2}{8\lambda^3E_p^2A_p^2}B_{11}\right]e^{-\lambda z} \\ & + A_{12}e^{\lambda z} + B_{12}e^{-\lambda z}\end{aligned} \tag{16}$$

in which A_{12} and B_{12} are integration constants. A_{12} and B_{12} are calculated considering zero force atop the pile head and fixed bottom of the pile i.e.

$$\left.\frac{dW_{12}(z)}{dz}\right|_{z=0} = 0 \tag{17}$$

$$W_{12}(z)|_{z=H} = 0 \tag{18}$$

Therefore,

$$A_{12} = -\frac{M_1 + e^{H\lambda}M_2\lambda}{\lambda + \lambda e^{2H\lambda}} \tag{19}$$

$$B_{21} = \frac{e^{H\lambda}\left(e^{H\lambda}M_1 - M_2\lambda\right)}{\lambda + \lambda e^{2H\lambda}} \tag{20}$$

where

$$M_1 = -\frac{A_{11}\psi^2(k_z + i\omega c_z)^2}{8A_p^2E_p^2\lambda^3} - \frac{A_{21}\psi(k_z + i\omega c_z)}{2A_pE_p\lambda} + \frac{B_{11}\psi^2(k_z + i\omega c_z)^2}{8A_p^2E_p^2\lambda^3} + \frac{B_{21}\psi(k_z + i\omega c_z)}{2A_pE_p\lambda},$$

$$\begin{aligned}M_2 = {} & H\left[\frac{(k_z + i\omega c_z)^2\psi^2}{8\lambda^2E_p^2A_p^2}A_{11}H - \frac{(k_z + i\omega c_z)\psi}{2E_pA_p\lambda}A_{21} - \frac{(k_z + i\omega c_z)^2\psi^2}{8\lambda^3E_p^2A_p^2}A_{11}\right]e^{\lambda H} \\ & + H\left[\frac{(k_z + i\omega c_z)^2\psi^2}{8\lambda^2E_p^2A_p^2}B_{11}H + \frac{(k_z + i\omega c_z)\psi}{2E_pA_p\lambda}B_{21} + \frac{(k_z + i\omega c_z)^2\psi^2}{8\lambda^3E_p^2A_p^2}B_{11}\right]e^{-\lambda H}.\end{aligned}$$

Since Mylonakis and Gazetas [10] only considered the incident wave emanating from the source pile, the ratio of $\alpha_1 = W_{21}(0)/W_{11}(0)$ was defined as the popular interaction factor of two piles. However, to account for the secondary waves due to vibrations of the receiver pile, we need to introduce a coupling factor κ, which is equal to the source pile head displacement due to vibration of the receiver pile, over the pile head displacement of the source pile:

$$\kappa = \frac{W_{12}(0)}{W_{11}(0)} \tag{21}$$

For m identical piles, pile displacement can be expressed mathematically in matrix as:

$$[D] = [\mathrm{A}][P] \tag{24}$$

where

$$[D] = \begin{bmatrix} D_1 \\ D_2 \\ . \\ . \\ . \\ D_m \end{bmatrix}, \quad [\mathrm{A}] = \frac{1}{K}\begin{bmatrix} 1 - \sum\limits_{j=1, j\neq 1}^{m} \kappa_{1j} & \alpha_{1,12} & \alpha_{1,13} & . \;\; . \;\; . & \alpha_{1,1m} \\ \alpha_{1,21} & 1 - \sum\limits_{j=1, j\neq 2}^{m} \kappa_{2j} & \alpha_{1,23} & . \;\; . \;\; . & \alpha_{1,2m} \\ . & . & . & . & . \\ . & . & . & . & . \\ . & . & . & . & . \\ \alpha_{1,m1} & \alpha_{1,m2} & . & . \;\; . \;\; . & 1 - \sum\limits_{j=1, j\neq m}^{m} \kappa_{mj} \end{bmatrix}, r[P] = \begin{bmatrix} P_1 \\ P_2 \\ . \\ . \\ . \\ P_m \end{bmatrix}.$$

Considering the coupled interaction between the source and receiver pile, the value of $W_{11}(0) - W_{12}(0)$ is the finally desired displacement of the source pile. Therefore, the interaction factor which accounts for secondary waves propagating from the receiver pile can be expressed as:

$$\alpha = \frac{W_{21}(0)}{W_{11}(0) - W_{12}(0)} = \frac{W_{21}(0)}{(1-\kappa)W_{11}(0)} \tag{22}$$

3 Mutual interaction of pile group

Every single pile in a pile group acts both as a source and receiver pile, simultaneously. On basis of principle of superposition, displacement of every solitary pile comprises two components: (1) The source pile displacement induced by load applied on its own head i.e. $\left(1 - \sum_{j=1, j\neq i}^{m} \kappa_{ij}\right) W_i$; (2) The receiver pile displacement induced by neighbouring piles i.e. $\sum_{j=1, j\neq i}^{m} \alpha_{1,ij} W_j$, where κ_{ij} is a factor accounting for the coupling effect of pile j on pile i; $\alpha_{1,ij}$ denotes the traditional interaction factor between pile j and pile i as defined by Mylonakis and Gazetas [10]; W_i and W_j are the displacement of pile i and j, respectively, due to the atop vertical loads P_i and P_j. Therefore, the resultant displacement D_i of each pile i in the pile group can be cast as:

$$D_i = \left(1 - \sum_{j=1, j\neq i}^{m} \kappa_{ij}\right) W_i + \sum_{j=1, j\neq i}^{m} \alpha_{1,ij} W_j \tag{23}$$

Vertical loads atop the pile can be expressed as:

$$[P] = [\mathrm{A}]^{-1}[D] \tag{25}$$

Pile members are connected through a rigid pile cap, which yields:

$$P_G e^{i\omega t} = \sum_{i=1}^{m} P_i \tag{26}$$

and

$$D_1 = D_2 = \cdots = D_m = D_G \tag{27}$$

The pile group impedance can be defined as:

$$K_G = \frac{P_G e^{i\omega t}}{D_G} \tag{28}$$

which can be recast in the summation of a real component and an imaginary component:

$$K_G = E_p d(k_G + ic_G) \tag{29}$$

4 Results and discussion

The aim of this section is twofold: first, we present examples to illustrate secondary waves generated by the vibration of the receiver pile on dynamic responses of the source pile through the coupling factor. Next, we use the introduced coupling factor to combine with the two-pile interaction factor to shed some light on how the refinement of the receiver pile affects dynamic responses of a pile group via selected arithmetic examples. Unless otherwise stated, the parameters considered in the analysis are

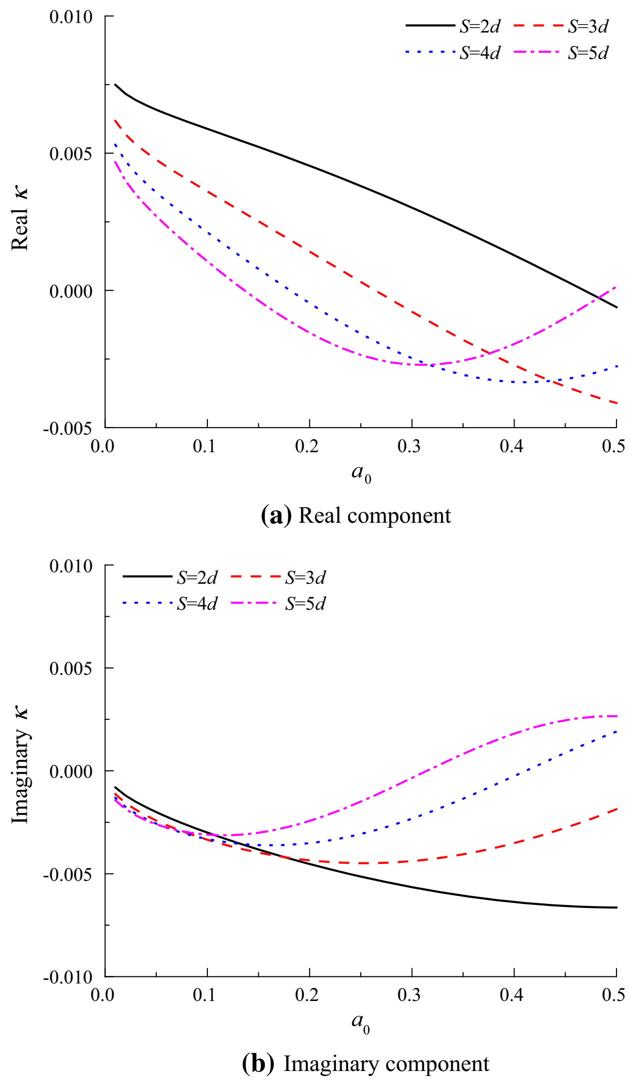


Fig. 3 Effects of pile spacing on the coupling factor **a** real component and **b** imaginary component. The results for H/d = 20

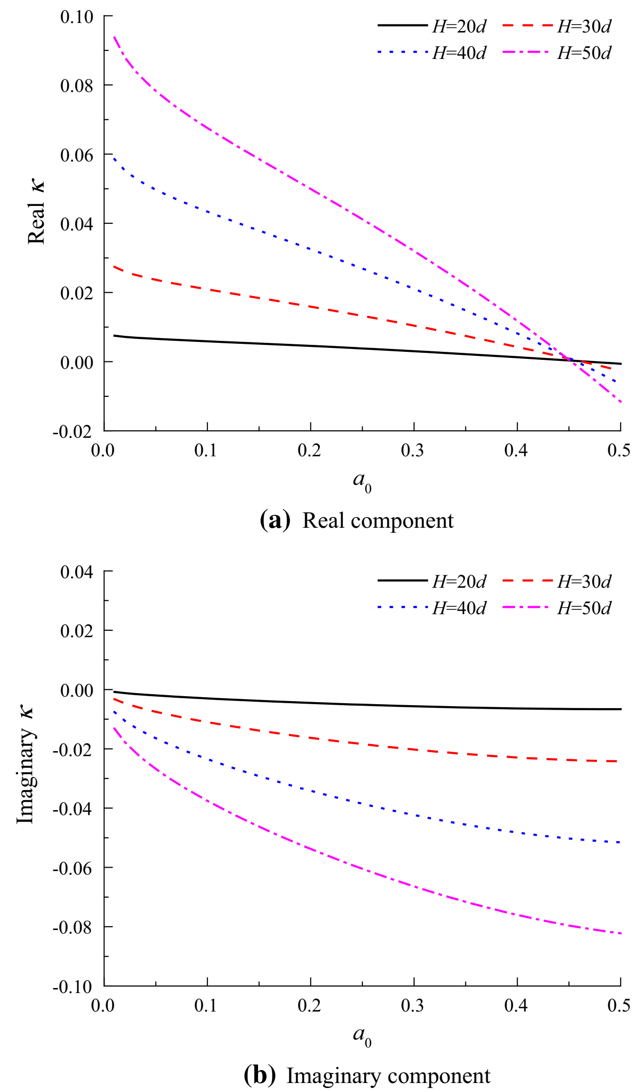
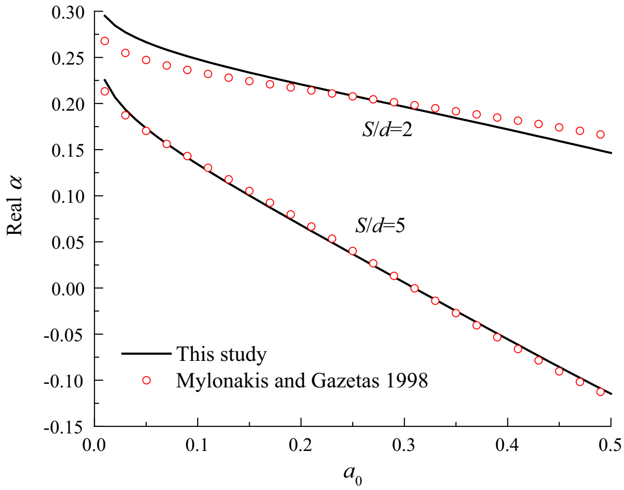


Fig. 4 Effects of slenderness ratio of the pile on the coupling factor **a** real component and **b** imaginary component. The results for S/d = 2

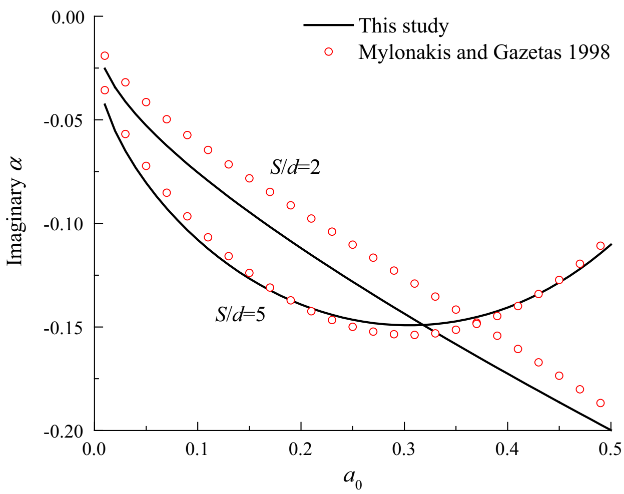
presented as follows: elasticity modulus of pile E_p= 25GPa, mass density of pile ρ_p= 2500 kg/m^3, mass density of soil ρ_s= 2000 kg/m^3, shear modulus of soil G_s= 10 MPa, damping ratio of soil β_s= 0.02 and Poisson's ratio of soil v_s= 0.4. In addition, the results are presented in terms of the normalized frequency $a_0 = \omega d/V_s$, where $V_s = \sqrt{G_s/\rho_s}$.

4.1 Effect of receiver pile vibration on the source pile and the interaction factor

First, variations of the real and imaginary components of the coupling factor κ with the dimensionless frequency a_0 for different pile spacing and different pile slenderness ratio are, respectively, illustrated in Figs. 3 and 4. Notice in Fig. 3 that resonance amplitude of the coupling factor (both the real component and the imaginary component) tends to decrease with the increasing pile spacing. The variation of the coupling factor with pile spacing is not trivial, suggesting that the effect of secondary waves on responses of the source pile is important, especially for closely spaced pile groups. Also notice in Fig. 4 that the coupling factor is, as expected, dependent on the pile slenderness: large slenderness ratio results in significant coupling effect. This is owing to the fact that the interaction between the receiver pile and its surrounding soil becomes obvious with the increasing pile length, leading to increased secondary waves. This coupling effect between the source pile and the receiver pile cannot be captured by the conventional model, as it does not consider the secondary waves caused by the existence of the receiver pile.

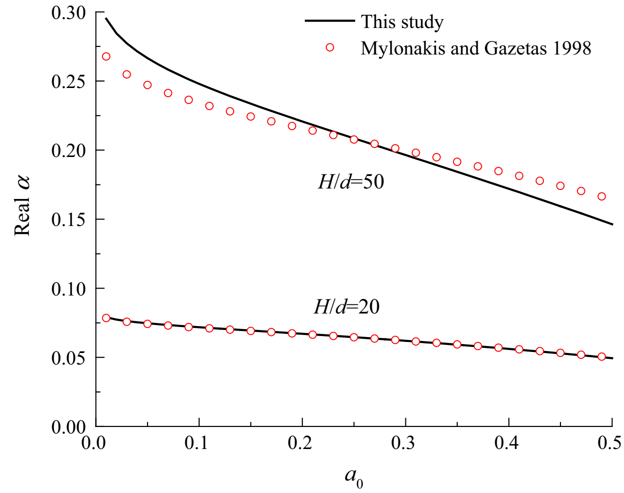


(a) Real component

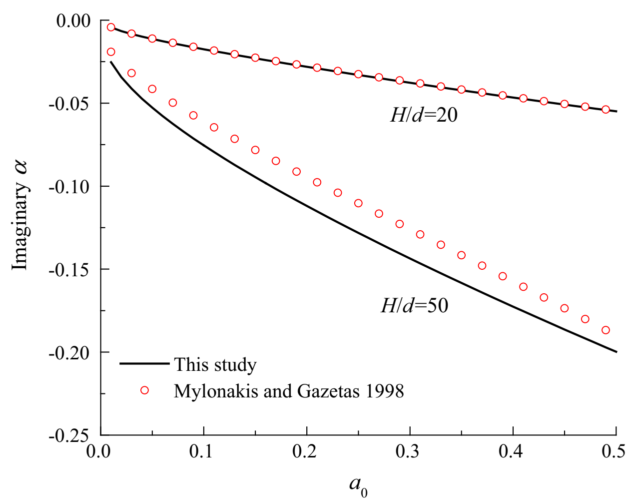


(b) Imaginary component

Fig. 5 Comparison of the interaction factor computed using the proposed model and the solution of Mylonakis and Gazetas [10] for the case of $S/d = 2$ and $S/d = 5$: **a** real component **b** imaginary component. The results for $H/d = 50$



(a) Real component



(b) Imaginary component

Fig. 6 Comparison of the interaction factor computed using the proposed model and the solution of Mylonakis and Gazetas [10] for the case of $H/d = 20$ and $H/d = 50$: **a** real component **b** imaginary component. The results for $S/d = 2$

Next, we will investigate the effect of secondary waves generated by the vibration of the receiver pile on its calculated interaction factor, by comparing results of the presented solution against those obtained by Mylonakis and Gazetas [10]. Interaction factors calculated through the two solutions, which are both expressed via α uniformly here, are illustrated in Figs. 5 and 6 for different pile spacing and slenderness ratio, respectively. Comparatively noticeable discrepancies are observed for the case of $S/d = 2$ in Fig. 5 and $H/d = 50$ in Fig. 6, while as expected, the two solutions converge as the pile spacing increases and the pile slenderness ratio decreases (see $S/d = 5$ in Fig. 5 and $H/d = 20$ in Fig. 6). Therefore, (1) secondary waves emanating from the receiver pile become important for pile groups with close pile spacing or with large slenderness ratio, and (2) the presented solution agrees well with the established solutions proposed by Mylonakis and Gazetas [10] when the pile spacing is sufficiently large enough or the pile slenderness ratio is comparatively small. It is noteworthiness that the secondary waves of the receiver pile on the interaction factor may not significant e.g. cases of $S/d = 5$ and $H/d = 20$, as only two piles are used to obtain the interaction factor. However, this coupling effect tends to be noticeable when calculating impedance of the pile group with increased group members, and we will demonstrate this in the following part through a selected arithmetic example.

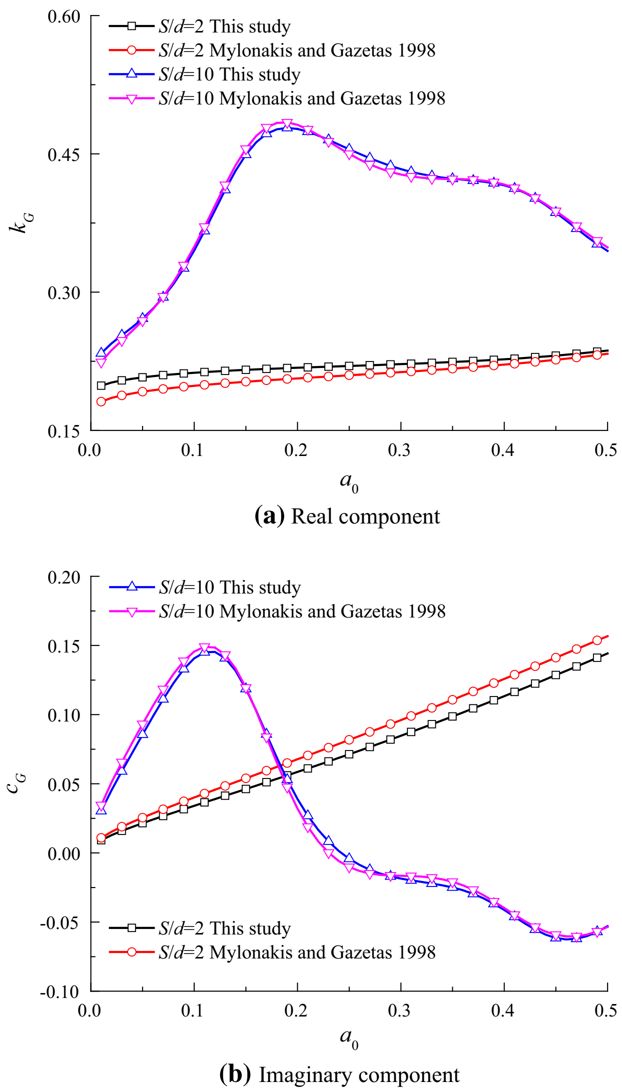


Fig. 7 Comparison of impedance of a symmetrical nine-pile group computed using the proposed solution with that obtained by Mylonakis and Gazetas [10] for the case of **a** *S*/*d* = 2, **b** *S*/*d* = 2. The results for *H*/*d* = 30

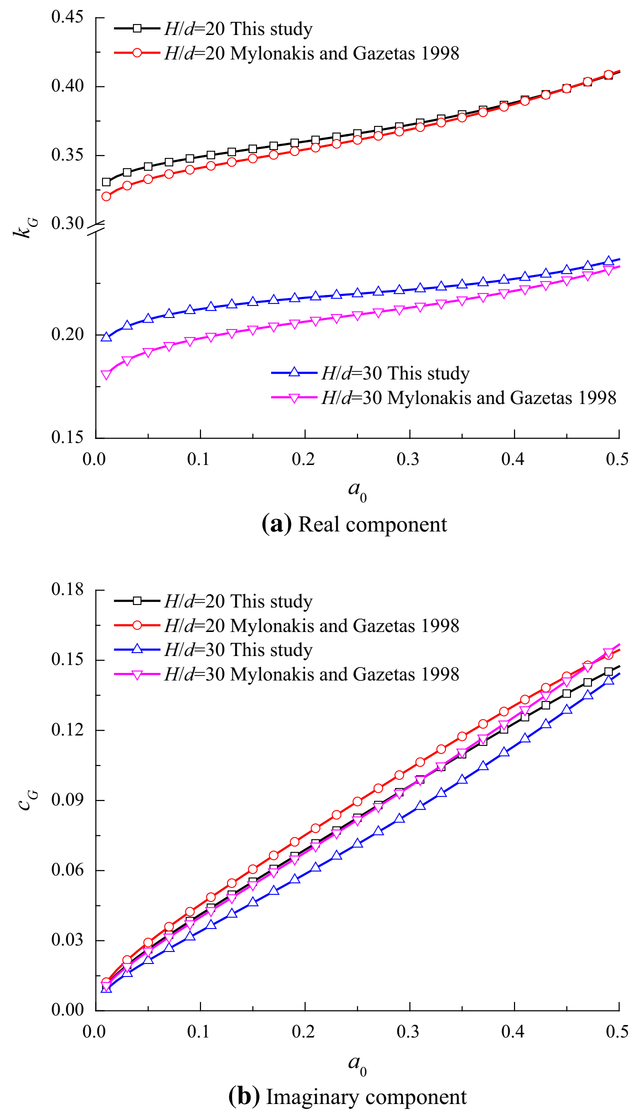


Fig. 8 Comparison of impedance of a symmetrical nine-pile group computed using the proposed solution with that obtained by Mylonakis and Gazetas [10] for the case of **a** *H*/*d* = 20, **b** *H*/*d* = 30. The results for *S*/*d* = 2

4.2 Effect of receiver pile vibration on pile group impedance

Effects of the receiver pile vibration on the impedance of a symmetrical nine-pile group are illustrated in Figs. 7 and 8, via making comparisons between the proposed solutions and the results obtained from Mylonakis and Gazetas [10]. Arithmetic results are presented for different pile spacing in Fig. 7 and different pile slenderness ratio in Fig. 8. Notice that the solution at hand agrees well with the pile group impedance resulting from that neglecting effects of the receiver pile, for the case of large-spaced pile group e.g. *S*/*d* = 10 in Fig. 7. However, notable discrepancies are observed with the decreased pile spacing e.g. *S*/*d* = 2, which confirm the important role of the receiver pile vibration. In addition, comparatively significant discrepancies, as expected, are observed for the case of *H*/*d* = 20 and *H*/*d* = 30 in Fig. 8, and this difference becomes more prominent for the case of *H*/*d* = 30. Notice in Fig. 6 that effect of the receiver pile vibration on the interaction factor is negligible small for the case of *H*/*d* = 20. However, discrepancies between the proposed solution and that from Mylonakis and Gazetas [10] in evaluating the impedance of a nine-pile group are still not trivial for the case of *H*/*d* = 20 in Fig. 8. Therefore, effects of the receiver pile vibration in a pile group with large group members become important for pile slenderness of practical interests.

5 Concluding remarks

We presented a simple physical model for describing the dynamic response of end-bearing pile groups subjected to harmonic vertical loads. The model is based on a pile-to-pile coupling interaction which allows considering the radially spreading waves emanated from the source pile and considering the secondary waves generated by the receiver pile vibration. The model allows calculating the coupling effect of the source and receiver pile through an introduced coupling factor, and permits the impedance of pile groups with arbitrary number of cylindrical piles to be obtained analytically. Apart from illustrating coupling factor between the source and receiver pile, which allows us to gain insight into the physical mechanics of the problem, we also presented arithmetic results to verify validation of the method and to quantify effects of the receiver pile vibration on dynamic responses of the pile group. Unquestionably, limitations of the model stem from certain assumptions introduced in the solution, such as the rigid weightless cap which is not in contact with soil and the perfect bonding at pile-soil interaction. Nevertheless, the main advantage of this model is that it is relatively easy to program, and is suitable for dealing with large closely spaced pile groups or pile groups with large members.

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