

# A new generalized soil thermal conductivity model for sand–kaolin clay mixtures using thermo-time domain reflectometry probe test

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**Abstract** As the demand of exploitation and utilization of geothermal energy increases, more geothermal-related earth structures occur recently. The design of the structures depends upon an accurate prediction of soil thermal conductivity. The existing soil thermal conductivity models were mostly developed by empirical fits to datasets of soil thermal conductivity measurements. Due to the gaps in measured thermal conductivities between any two tested natural soils, the models may not provide accurate prediction for other soils, and the predicted thermal conductivity might not be continuous over the entire range of soil type. In this research, a generalized soil thermal conductivity model was proposed based on a series of laboratory experiments on sand, kaolin clay and sand–kaolin clay mixtures using a newly designed thermo-time domain reflectometry probe. The model was then validated with respect to  $k_{\text{dry}}-n$  (thermal conductivity of dry soils and porosity) and  $k_r-S_r$  (normalized thermal conductivity and degree of saturation) relationships by comparing with previous experimental studies. The predicted thermal conductivities were found to be in a good agreement with the experimental data collected from both this study and the other literatures with at least 85% confidence interval. It is concluded that the proposed model accounts for the effects of both environmental factors (i.e., moisture content and dry density) and compositional factors (i.e., quartz content and soil type) on soil thermal conductivity, and it

has a great potential in predicting soil thermal conductivity more accurately for geothermal applications.

**Keywords** Sand–kaolin clay mixtures · Thermal conductivity model · Thermo-TDR probe

## 1 Introduction

With a rapidly increasing demand of resource utilization of geothermal energy, and the great importance of soil thermal property to the design of various energy geostructures, such as geothermal energy piles (GEP), ground source heat pumps (GSHP) and borehole thermal energy storage (BTES), the study on soil thermal property becomes even more significant recently. Some work had been reported regarding the thermal conductivity of soils, rock and construction materials since it is the most important thermal property compared to thermal diffusivity and heat capacity [6, 12, 17, 26, 28]. Brandl [7] presented the importance of soil thermal property in the behaviors of thermo-active ground structures in his Rankine lecture. Amatya et al. [3] conducted a series of theoretical and experimental studies on thermo-mechanical behaviors of energy piles. In above cases, geothermal energy can be extracted during heat exchange process between soils and embedded circulating fluid for heating purpose in winter. Reversely, it can be injected into the ground for cooling purpose in summer. Soil moisture migration coupled with heat transfer process may happen in the surrounding areas of energy piles due to the thermal gradient induced by temperature difference between soils and circulating fluid. Then, the soil–structure interaction and mechanical properties of piles might be affected in a long operation period. Thermal conductivity is

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a fundamental thermal property that will influence the heat conduction in materials. Therefore, studying soil thermal conductivity is not only crucial to understand heat conduction process in soils, but also valuable to the design of geothermal-related earth structures.

As reported in literature [11, 13, 16, 18, 23, 24, 30, 46], many researchers developed soil thermal conductivity prediction models which are primarily based on the empirical fits to experimental data of natural soils. For example, Kersten [24] performed a series of laboratory experiments to measure the thermal conductivity of 19 different types of soils and proposed an empirical relationship between thermal conductivity and moisture content and dry density. Johansen [23] proposed the normalized thermal conductivity ( $k_r$ ) concept which has been used to study the effects of soil type, porosity ( $n$ ), degree of saturation ( $S_r$ ) and mineral component on soil thermal conductivity in a unique way through  $k_r$ - $S_r$  relationship. He also presented some different  $k_r$ - $S_r$  relationships for different soil types and developed a new thermal conductivity model based on this concept. Cote and Konrad [13] further studied the thermal conductivity of soils and construction materials and established a new  $k_r$ - $S_r$  relationship incorporating variable  $\kappa$  accounting for the soil type effect. In addition, accounting for the particle shape effect, variables  $\chi$  and  $\eta$  were also incorporated to calculate the thermal conductivity of dry soils. Lu et al. [30] conducted the thermo-time domain reflectometry (TDR) tests on 12 different natural soils ranging from sand, silts to loam or clay loam, and proposed a thermal conductivity model through a new  $k_r$ - $S_r$  relationship across a wide range of soil moisture condition.

There are also some other empirical soil thermal conductivity models, e.g., Chen [11] developed a thermal conductivity model for sands with extremely high quartz content; Zhang et al. [46] improved Cote and Konrad [13] model to predict thermal conductivity of pure quartz sands; Balland [5] further modified Johansen's [23] model for considering the effect of organic matter on thermal conductivity of solid; Donazzi [16] and Gangadhara Rao [18] established the relationships between thermal resistivity and porosity ( $n$ ), degree of saturation ( $S_r$ ), dry density ( $\rho_d$ ) and moisture content ( $w$ ) based on laboratory experiments; then, thermal conductivity model was derived according to the reciprocal correlation between conductivity and resistivity.

However, due to the gaps existing in measured thermal conductivities between any two tested natural soils in previous experimental studies, the predicted soil thermal conductivity may not agree with the actual values very well for some other soils. Moreover, the predicted thermal conductivity might not be continuous over the entire range of soil type. Another shortcoming in the existing models is

that sand content was usually assumed to be identical to quartz content, and it was used to calculate the thermal conductivity of solid, resulting in an overestimation of soil thermal conductivity [8, 30]. Thus, a lack of data for quartz content is another critical issue, hindering the successful application of the existing models [37].

On the other hand, some theoretical thermal conductivity models are summarized as follows: Gemant [19] attempted to derive an analytical solution of finite-element analyses method in determining thermal conductivity of lattice, but this does not appear to have been widely used. The weighted average thermal conductivity of each constituent in soil matrix was introduced in De Vries's [15] model which was developed based on Maxwell equation. The disadvantage of this model is the uncertainty of soil field capacity which is considered as a limit moisture content in determining whether water or air is the continuous media in soils. Gori [20] proposed a soil cubic model considering the effects of water films and bridges around soil particles on heat transfer process, but the complexity of the formula is the main limitation for future applications. Tong et al. [38] studied the heat transfer process in porous media and proposed a more generalized thermal conductivity model. But the dependence of parameters  $\eta_1$  and  $\eta_2$  on soil types was not clarified in his study. Haigh [21] presented a thermal conductivity model for sands based on a three-phase soil contact element. The model showed higher prediction accuracy for sands compared with other models, but its applicability on other soil types still needs further study.

The objective of this research is to develop a generalized soil thermal conductivity model considering the effects of porosity ( $n$ ), degree of saturation ( $S_r$ ), quartz content and soil type. Laboratory experiments were performed on pure sand (>99% quartz content), kaolin clay and sand-kaolin clay mixtures in different proportions at different porosities and degree of saturations using a newly designed thermo-time domain reflectometry (TDR) probe. Then, the model was developed based on normalized thermal conductivity ( $k_r$ ) concept proposed by Johansen [23], followed by the model validation for  $k_{dry}$ - $n$  (thermal conductivity of dry soils and porosity) and  $k_r$ - $S_r$  (normalized thermal conductivity and degree of saturation) relationships. The model performance was also evaluated through the comparison of predicted thermal conductivity with the experimental data from both this study and the previous literatures.

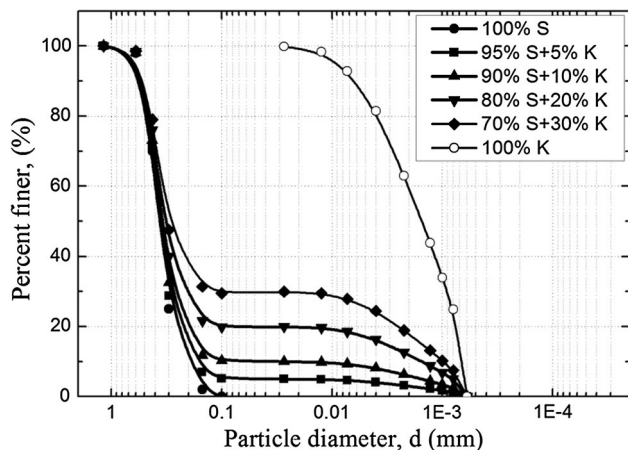
## 2 Materials and methods

### 2.1 Materials

Laboratory experiments were performed on pure sand, kaolin clay and sand-kaolin clay mixtures. The sand is Ottawa-type silica sands with white color and round to sub-

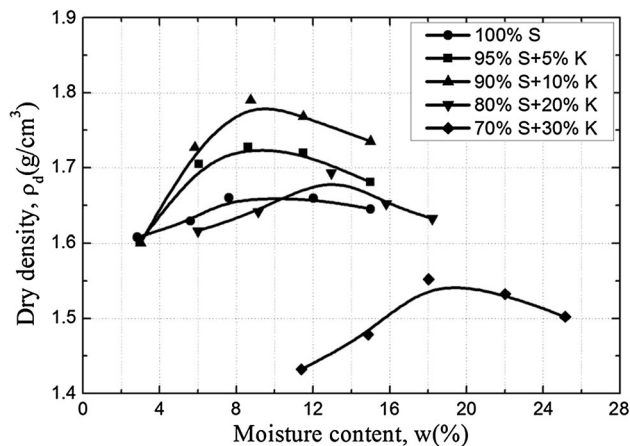
**Table 1** Experiment program comprising of sand, kaolin clay and sand–kaolin clay mixtures

	No	1	2	3	4	5	6	$G_s$
Content by dry weight, (%)	Sand	100	95	90	80	70	0	2.65
	Kaolin	0	5	10	20	30	100	2.58

**Fig. 1** Gradation curves of sand (*S*), kaolin clay (*K*) and sand–kaolin clay mixtures

rounded shape, complying with the requirement of ASTM C778-13 [1] manufactured by US Silica. It contains over 99% quartz which has the highest thermal conductivity of  $7.7\text{--}8.4\text{ W m}^{-1}\text{K}^{-1}$  among all the soil minerals [15]. Therefore, sand content is reasonably assumed to be the same as quartz content. The kaolin clay is mainly composed of anhydrous aluminum silicate, and its thermal conductivity is around  $2.9\text{ W m}^{-1}\text{K}^{-1}$  [27]. The sand was mixed with kaolin clay in different proportions with clay content ranging from 5, 10, 20 to 30% by dry weight, as experiment program presented in Table 1. Pure sand and kaolin clay were also tested in the experiments to determine the upper and lower bound of thermal conductivity of sand–kaolin clay mixtures. The gradation curves of sand (*S*), kaolin clay (*K*) and sand–kaolin clay mixtures are shown in Fig. 1.

Soil samples were prepared at different porosities and degree of saturations according to the corresponding compaction curves (shown in Fig. 2) obtained from standard compaction tests [2]. In Fig. 2, it is found that the maximum dry density was increased with an increase in clay content from 0 to 10%, and it was decreased when clay content was greater than 10%. For 70% S + 30% K mixture, the maximum dry density reached the lowest level and the corresponding optimum moisture content reached the highest level compared with other mixtures. Accordingly, the target dry density and degree of saturation of soil samples are presented in Table 2. Since the excessive side resistance will cause probe deformation and affect test

**Fig. 2** Compaction curves of sand (*S*) and sand–kaolin mixtures (ASTM D698 [2])

results when inserting the thermo-TDR probe into kaolin clay with relatively high dry density, the target dry density of clay samples was much lower than sand and sand–kaolin clay mixtures. Test methods and sample preparation procedures will be presented in the following section.

## 2.2 Test methods

Time domain reflectometry (TDR) technique has been widely used to monitor soil moisture content and dry density in situ and in laboratory [4, 22, 31, 39, 40, 45]. Moreover, a thermo-TDR probe was successfully designed and applied in measuring both soil thermal property and moisture content and dry density [42–44]. The performance of this probe was also evaluated by Zhang et al. [46].

As shown in Fig. 3, this thermo-TDR probe has a center probe where a resistance wire of total length of 162 mm is installed and connected to a D.C. current source (i.e., BK Precision 17850B). There are two identical outer probes located on two sides of the center probe with same distance of 6 mm. Three thermocouples are also embedded at mid-height in each probe and connected to a temperature readout unit (i.e., TC-08) to record the temperature variation for thermal conductivity analyses. Basically, the center probe and outer probes are considered as a line heat source and two temperature response recorders, respectively.

The thermo-TDR probe is also connected to a signal generator (i.e., Campbell Scientific TDR 100) by a RG 58 coaxial cable of length of 1.7 m and a PMTDR data

acquisition system which is used to record reflected TDR waveforms for predicting moisture content and dry density. This study only focuses on the thermal conductivity property of sand, kaolin clay and sand–kaolin clay mixtures. The photograph of experimental setup is shown in Fig. 4. The sample preparation and test procedures are described as follows:

1. Dry sand, kaolin clay and deionized water with predetermined weight were mixed thoroughly and then poured into a PVC (i.e., polyvinyl chloride) mold of inner diameter of 67.7 mm and height of 47.5 mm;
2. The mixture was compressed in the mold using triaxial test setup to obtain the target dry density and degree of saturation (shown in Table 2);
3. The soil sample was then wrapped by plastic cover to avoid moisture loss, and left for 24 h under room temperature (22–23 °C) to obtain uniformly distributed moisture content;

**Table 2** Target dry density and degree of saturation of sand, kaolin clay and sand–kaolin clay mixtures

Name	Sand	Sand–kaolin clay mixtures				Clay
		2	3	4	5	
No	1	2	3	4	5	6
$\rho_d$ (g/cm <sup>3</sup> )	1.55	1.55	1.55	1.55	1.40	0.65
	1.60	1.60	1.60	1.60	1.45	0.68
	1.65	1.65	1.65	1.65	1.50	0.71
		1.70	1.75			
$S_r$	0	0	0	0	0	0
	0.1	0.1	0.1	0.1	0.3	0.3
	0.2	0.2	0.2	0.25	0.4	0.4
	0.25	0.25	0.3	0.5	0.5	0.6
	1	1	1	1	1	1

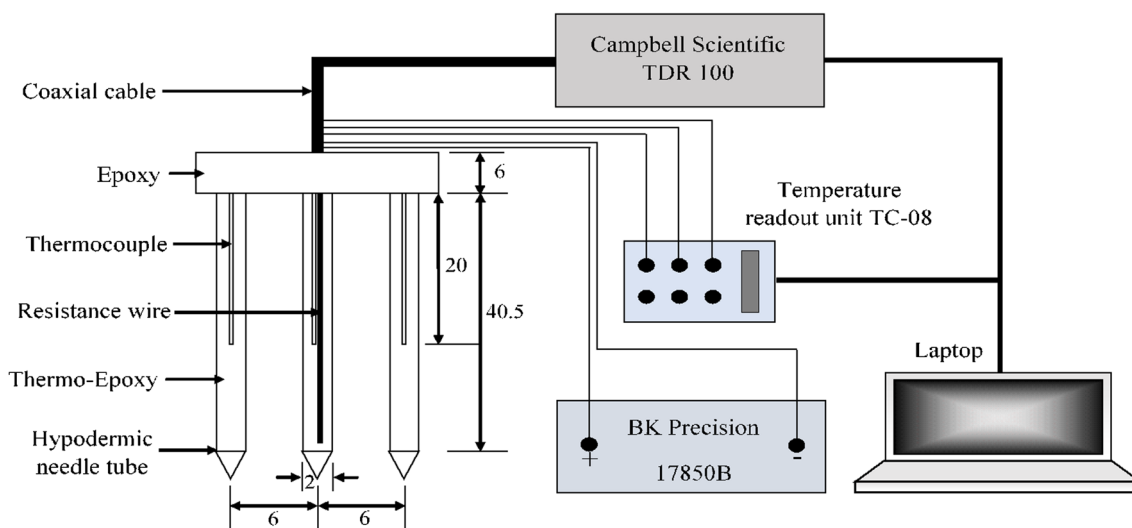
4. The thermo-TDR probe was vertically inserted into soil samples at the center from top surface after removing the plastic cover;
5. Current with a magnitude of 0.15 amps was applied for 15 s to heat the samples; the temperatures at center probe and two outer probes were recorded every second during this heating and following cooling processes;
6. A few soil samples around the thermo-TDR probe were dug out of the mold and tested by oven-drying method at 105 °C for actual moisture content measurement;
7. Thermal conductivity of soil samples was analyzed and calculated from the recorded temperature variations.

### 2.3 Determination of thermal conductivity

In accordance with an infinite line heat source theory, temperature variations at the center probe and two outer probes were recorded and used to analyze soil thermal conductivity. In an infinite medium, the temperature change is a function of time and radial distance away from the heat source [14, 25]. The mathematic expression of the temperature response is as follows:

$$T(r, t) = \frac{Q}{4\pi\alpha} \left[ Ei\left(\frac{-r^2}{4\alpha(t-t_0)}\right) - Ei\left(\frac{-r^2}{4\alpha t}\right) \right] \quad (1)$$

where  $Q$  is the strength of heat source, defined as  $Q = q/\rho_c$ ,  $q$  is the quantity of line heat source, W/m;  $\rho_c$  is the volumetric heat capacity, J m<sup>-3</sup>K<sup>-1</sup>;  $T$  is the temperature change, °C;  $t$  is the time, s;  $t_0$  is the duration of heat pulse, s;  $\alpha$  is the thermal diffusivity, mm<sup>2</sup>/s;  $r$  is the radial distance, m;  $Ei(x)$  is the exponential integral.



**Fig. 3** Schematic of thermo-TDR probe and experimental setup (unit: mm) (Zhang et al. [46])

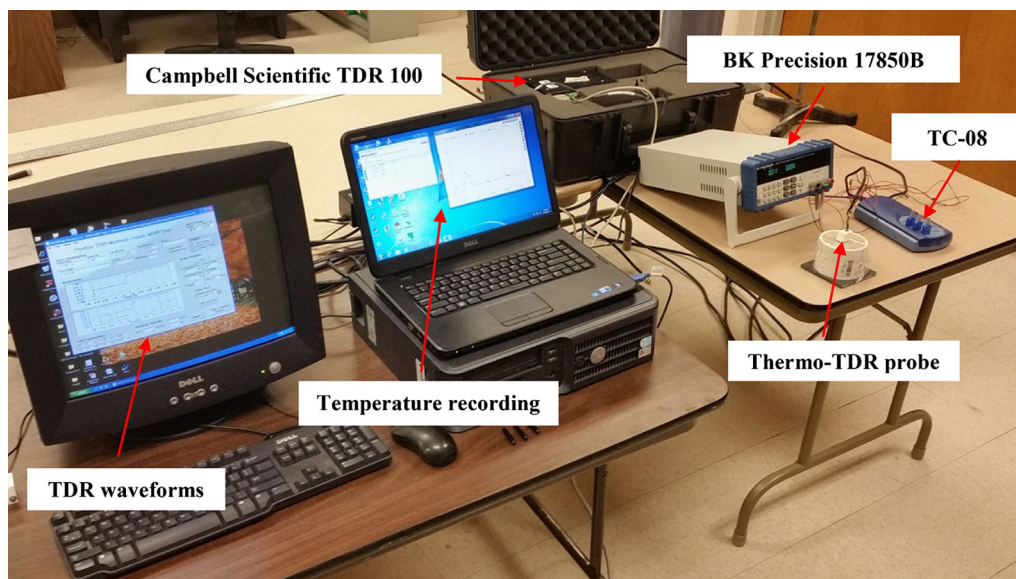


Fig. 4 Photograph of experimental setup

Nonlinear regression analysis method was adopted by Bristow et al. [10] and Welch et al. [41] to calculate soil thermal properties based on the recorded temperature variations. By taking the derivative of Eq (1) with respect to time and setting the result equals to zero, Bristow et al. [9] proposed the following equations to determine soil thermal properties,

$$\rho_c = \frac{q}{4\pi\alpha T_m} \left[ Ei\left(\frac{-r^2}{4\alpha(t_m - t_0)}\right) - Ei\left(\frac{-r^2}{4\alpha t_m}\right) \right] \quad (2)$$

$$\alpha = \frac{r^2}{4} \left[ \frac{1}{(t_m - t_0)} - \frac{1}{t_m} \right] / \ln \left[ \frac{t_m}{(t_m - t_0)} \right] \quad (3)$$

Based on the relationship among thermal conductivity ( $k$ ), thermal diffusivity ( $\alpha$ ) and volumetric heat capacity ( $\rho_c$ ),  $k = \rho_c \alpha$ , thermal conductivity is calculated by

$$k = \frac{q}{4\pi T_m} \left[ Ei\left(\frac{-\ln[t_m/(t_m - t_0)]}{t_0/t_m}\right) - Ei\left(\frac{-\ln[t_m/(t_m - t_0)]}{t_0/(t_m - t_0)}\right) \right] \quad (4)$$

where  $q$  is the heating power of center probe, W/m;  $t_0$  is the duration of heat pulse, i.e., 15 s;  $r$  is the radial distance between the center probe and outer probe, i.e., 6 mm (shown in Fig. 3);  $t_m$  is the time when the maximum temperature occurs at outer probe;  $T_m$  is the maximum temperature change at outer probe, °C.

In order to obtain accurate measurement of soil thermal conductivity property, heating power must be selected carefully to ensure that heat conduction is the main heat transfer mechanism in soils. With very low heating power, temperature rise is not obvious and the recorded temperature variation is greatly affected by electrical noise, while too much heating power will induce more moisture

migration and influence measured soil thermal conductivity. Consequently, the heating power  $q$  in Eq (4) is determined as follows:

$$q = I^2 R / L = 0.15^2 \times 22.23 / 0.0405 = 12.35 \text{ W/m} \quad (5)$$

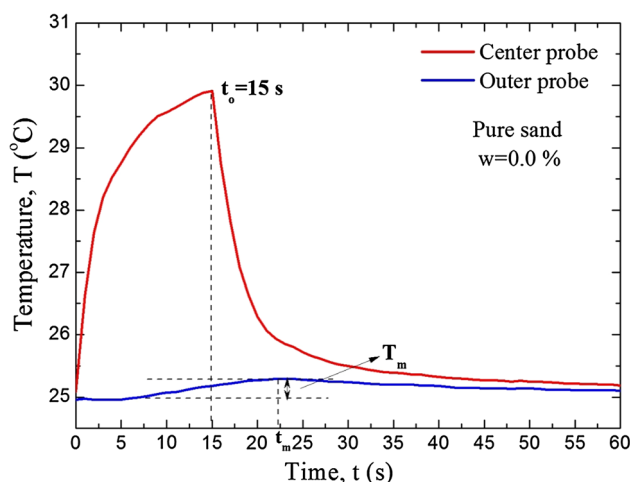
where  $I$  is the applied current, i.e., 0.15 amps;  $R$  is the resistance of the heating wire, i.e., 22.23  $\Omega$ ;  $L$  is the length of the center probe, i.e., 0.0405 m. According to Ren’s et al. [32] study, this heating power can minimize the convective heat flow and the effect of moisture movement on measured soil thermal conductivity property.

Figure 5 shows the temperature variations of pure dry sand at center probe and one outer probe. It also illustrates the way to determine  $t_m$  and  $T_m$  in Eq. (4). The temperature variation of the other outer probe is not shown in the figure because the two outer probes had almost the same temperature variation in the experiments. By using Eq. (4), soil thermal conductivity was eventually calculated by the average value of the two outer probes.

### 3 Experimental results

#### 3.1 Measurements of thermal conductivity and porosity

Figure 6 indicates the relationships between thermal conductivity and porosity for sand, kaolin clay and sand–kaolin clay mixtures in semilogarithmic scale. It is evident that the thermal conductivity increased with a decrease in porosity at each level of degree of saturation. Moreover, excellent linear trends between thermal conductivity and



**Fig. 5** Temperature variations of pure dry sand at center probe and outer probe

porosity were found for all the soil samples, which are also consistent with Chen's [11] study. Another feature is that the thermal conductivity of each soil sample converged to a unique value at vertical axis by linear regression analyses as porosity decreased to zero. But this value was decreasing along vertical axis as clay content increased as shown in Fig. 6. Thermal conductivity of soils is actually the thermal conductivity of solid when porosity equals to zero (i.e.,  $n = 0$ ), and it is mainly affected by the quartz content of soil samples due to the highest thermal conductivity of quartz among all the soil minerals. Hence, thermal conductivity of soil samples at  $n = 0$  was found to be decreased as sand content (i.e., quartz content) decreased.

### 3.2 Measurements of thermal conductivity and degree of saturation

Figure 7 indicates the relationships between thermal conductivity and degree of saturation for sand, kaolin clay and sand–kaolin clay mixtures. It is found that thermal conductivity increased with an increase in degree of saturation at each level of porosity. For pure sand, the increase in thermal conductivity at low degree of saturations was more considerable than sand–kaolin clay mixtures and pure clay. However, this increment reduced gradually as clay content increased. Furthermore, three approximate linear trends of thermal conductivity with degree of saturation were observed when clay content increased up to 30%.

This is mainly attributed to the effects of particle size and quartz content on soil thermal conductivity. In sands, water films and bridges are more easily formed at low degree of saturations among sand particles to facilitate heat transfer process; therefore, the thermal conductivity is improved significantly. The subsequent slight increase in thermal conductivity of sand is due to an increase in

thermal conductivity of pore fluid as more voids in sands are replaced by water. In contrast, the moisture in sand–clay mixtures is distributed more uniformly when clay is added. The effects of water films and bridges on thermal conductivity are not predominately over the entire range of degree of saturation. This is also because that hydration is the dominating water retention mode when clayey soils are under high matric suction or low moisture content conditions [29]. The hydrated water is mostly located on the soil particle surface or inside the mineral crystal structures surrounding the exchangeable cations [33].

For pure clay, the linear trends between thermal conductivity and degree of saturation were also very obvious as 70% S + 30% K mixture. But its thermal conductivity decreased dramatically because of the lower thermal conductivity of solid phase (i.e., kaolin clay) and the higher porosity compared with sand and sand–kaolin clay mixtures.

## 4 Model development

### 4.1 Framework of normalized thermal conductivity

The normalized thermal conductivity proposed by Johansen [23] is expressed as follows:

$$k_r = \frac{k - k_{\text{dry}}}{k_{\text{sat}} - k_{\text{dry}}} \quad (6)$$

where  $k_r$  is the normalized thermal conductivity,  $k_{\text{dry}}$  and  $k_{\text{sat}}$  are the thermal conductivities of soils at dry and fully saturated conditions, respectively,  $\text{W m}^{-1}\text{K}^{-1}$ . Then, establishing a  $k_r$ – $S_r$  relationship can study the effects of porosity, moisture content and soil mineral component on thermal conductivity in a unique way because of the unique relationship for a given type of soil [13]. The lower and upper limit conditions of Eq. (6) are given as follows:

Lower limit condition :  $S_r = 0 \rightarrow k_r = 0$

Upper limit condition :  $S_r = 1 \rightarrow k_r = 1$

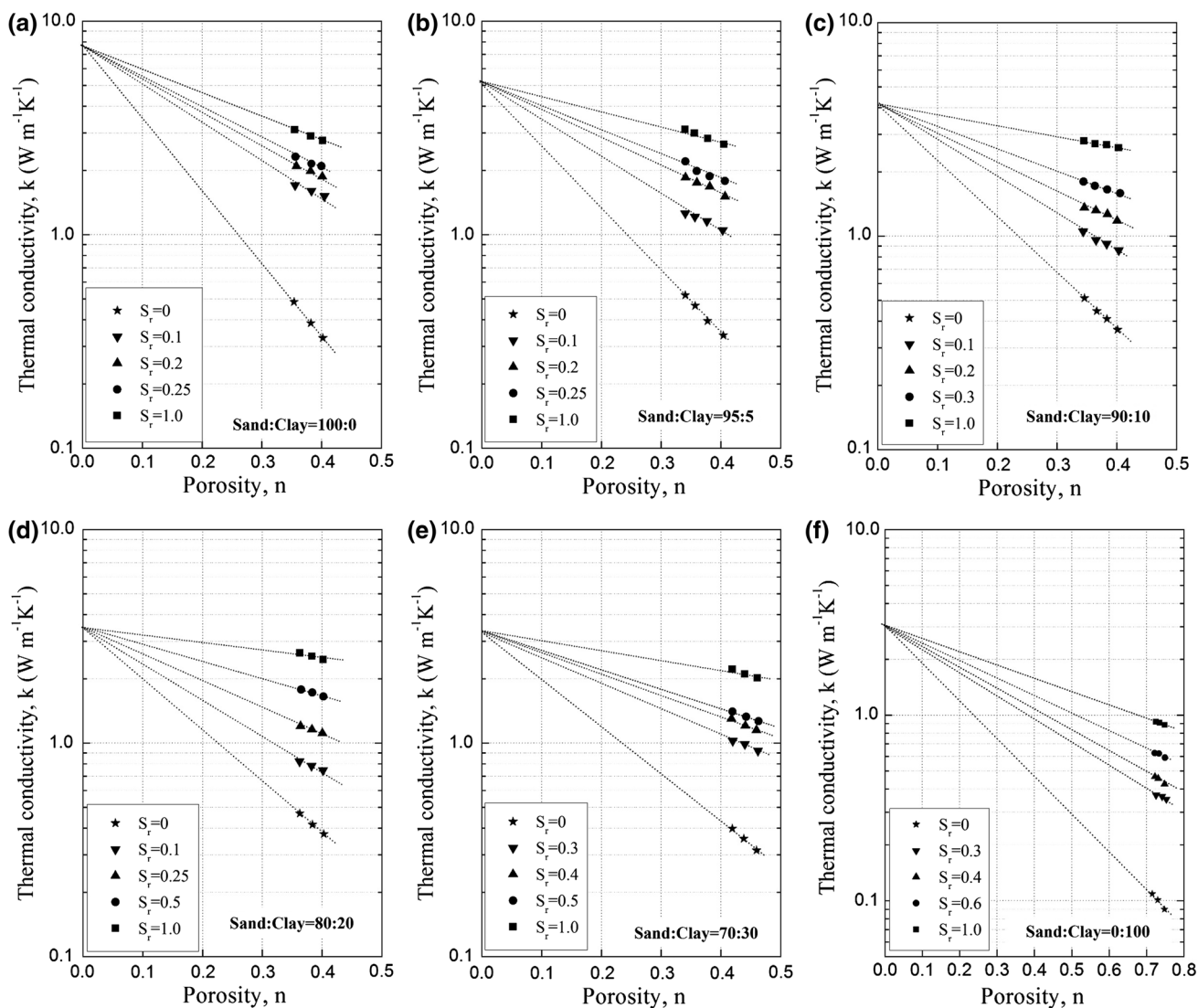
Thus, soil thermal conductivity can be calculated from Eq. (6) as given below

$$k = k_r \times (k_{\text{sat}} - k_{\text{dry}}) + k_{\text{dry}} \quad (7)$$

The simplest expression for calculating the thermal conductivity of saturated soils has been widely used as follows [34]:

$$k_{\text{sat}} = k_{\text{water}}^n k_{\text{solid}}^{1-n} \quad (8)$$

Cote and Konrad [13] proposed a generalized thermal conductivity model for soils and construction materials in a wide range of soil types, porosity and degree of saturation.



**Fig. 6** Relationships between thermal conductivity and porosity for sand, kaolin clay and sand–kaolin clay mixtures

The generalized  $k_r$ – $S_r$  relationship they provided is as follows:

$$k_r = \frac{\kappa S_r}{1 + (\kappa - 1)S_r} \tag{9}$$

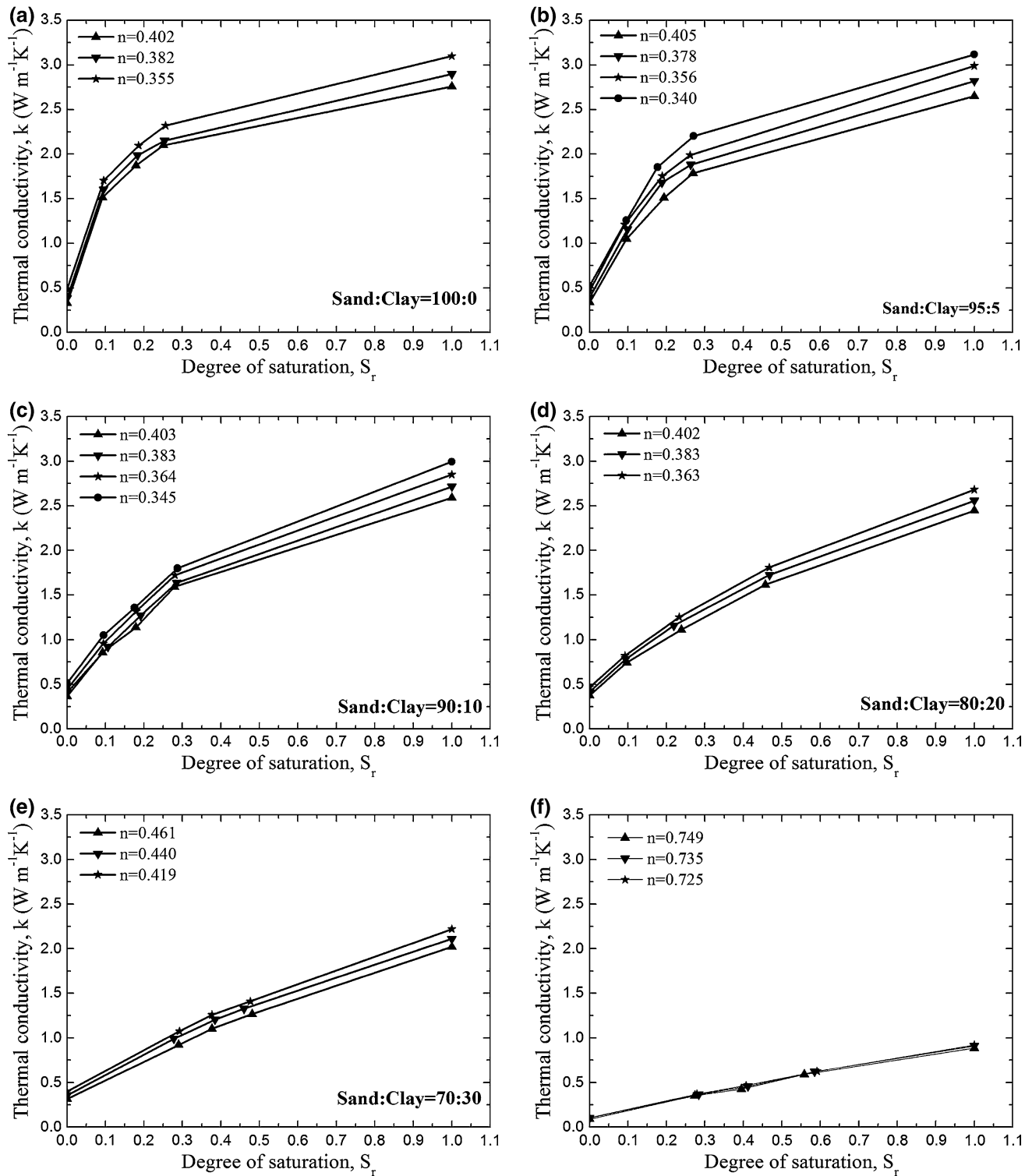
where  $\kappa$  is an empirical parameter accounting for the soil type effect under unfrozen and frozen states. The values of  $\kappa$  are 4.7 and 1.8 for the unfrozen and the frozen state, respectively. The relationship between thermal conductivity of dry soils and porosity in Cote and Konrad’s [13] study is given as follows:

$$k_{dry} = \chi 10^{-\eta n} \tag{10}$$

where  $\chi$  and  $\eta$  are material parameters that account for the particle shape effect and  $n$  is the porosity of dry soils.

### 4.2 A new generalized soil thermal conductivity model

Figure 8 shows the normalized thermal conductivity and degree of saturation for sand, kaolin clay and sand–kaolin clay mixtures. According to Eq. (9), different  $\kappa$  values were obtained for different quartz (sand) contents by nonlinear regression fits to the experimental data. It is found that  $\kappa$  was equal to 8 for pure sand defining an upper limit, and it was equal to 1.2 for kaolin clay defining a lower limit. In addition, the value of  $\kappa$  lies in between above two values for sand–kaolin clay mixtures as it increased as quartz (sand) content increased. According to Eqs. (7) and (9), higher  $\kappa$  value leads to higher soil thermal conductivity with  $k_{dry}$ ,  $k_{sat}$  and  $S_r$  remaining constant for a given type of soil. Thus, soil thermal conductivity



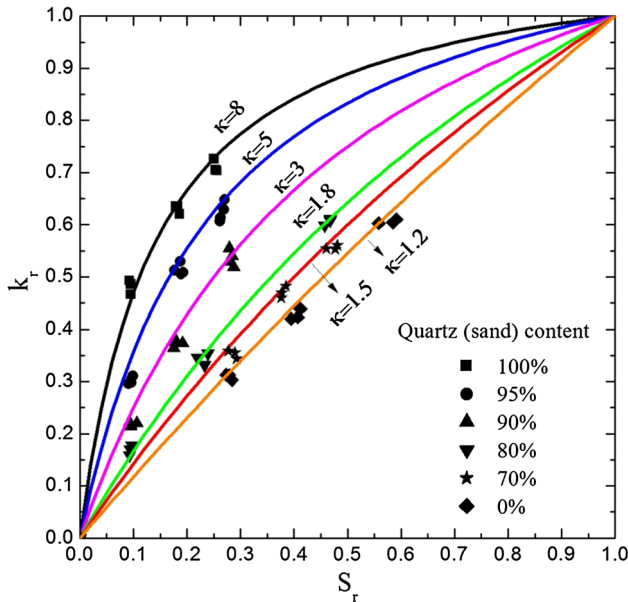
**Fig. 7** Relationships between thermal conductivity and degree of saturation for sand, kaolin clay and sand–kaolin clay mixtures

increased with increasing quartz (sand) content, which also conformed to the experimental results as shown in Figs. 6 and 7. Considering the dependence of  $\kappa$  on quartz (sand) content, an exponential function was adopted to describe the relation between  $\kappa$  and quartz (sand) content which is

shown in Fig. 9. Substituting this relation into Eq. (9), a new  $k_r$ – $S_r$  relationship is proposed as follows:

$$k_r = \frac{(2.168 \times 10^{-5} \times \exp(x/7.903) + 1.252)S_r}{1 + (2.168 \times 10^{-5} \times \exp(x/7.903) + 0.252)S_r} \quad (11)$$





**Fig. 8** Normalized thermal conductivity ( $k_r$ ) and degree of saturation ( $S_r$ )

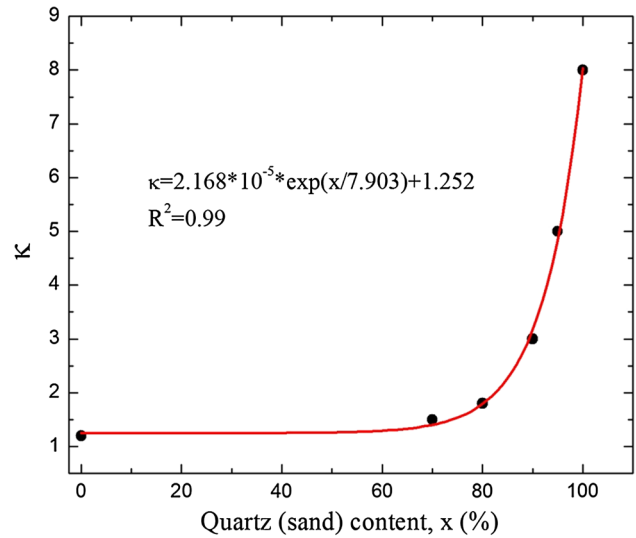
where  $x$  is the quartz (sand) content, %.

Collecting the experimental data of soil samples from Fig. 6, the thermal conductivity and porosity of sand, kaolin clay and sand–kaolin clay mixtures under dry condition are shown in Fig. 10. It is found that the slope ( $k$ ) and intercept ( $c$ ) of the fitted linear trends between thermal conductivity and porosity in semilogarithmic scale were changed with quartz (sand) content. Pure sand showed the highest intercept and the lowest slope, whereas pure clay showed the lowest intercept and the highest slope. Furthermore, all other thermal conductivity measurements of sand–kaolin clay mixtures were located in between above two fitted lines.

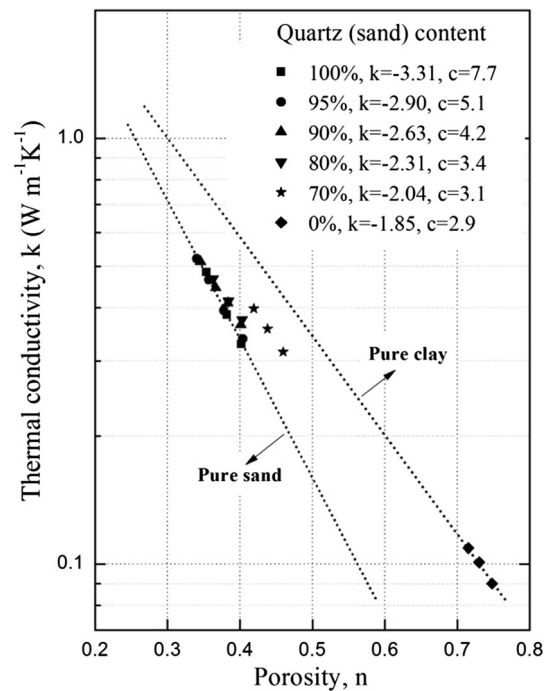
Figure 11 shows the slope ( $k$ ), intercept ( $c$ ) and quartz (sand) content for sand, kaolin clay and sand–kaolin clay mixtures. Two identical exponential functions were used to describe the relationships between  $k$ ,  $c$  and  $x$  with both  $R^2$  values very close to one. Comparing Eq. (10) with experimental results shown in Fig. 10, it is indicated that the absolute value of  $k$  equals to  $\eta$ , and  $c$  equals to  $\chi$ . Therefore, thermal conductivity of dry soils can be formulated according to Eq. (10), which is expressed as follows:

$$k_{dry} = (1.216 \times 10^{-6} \times \exp(x/6.599) + 3.034) \times 10^{(-0.003 \times \exp(x/16.452) - 1.840) \times n} \quad (12)$$

According to Eq. (8), thermal conductivity of saturated soils can be calculated by



**Fig. 9** Relationship between  $\kappa$  and quartz (sand) content

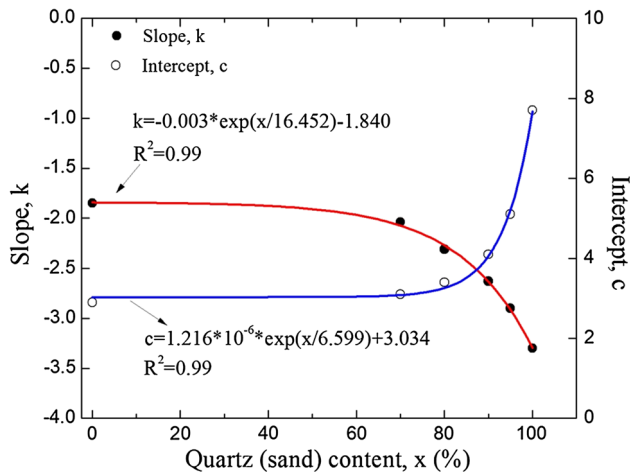


**Fig. 10** Thermal conductivity and porosity of sand, kaolin clay and sand–kaolin clay mixtures under dry condition

$$k_{sat} = k_{water}^n \left( k_{quartz}^{x/100} k_{kaolin}^{1-x/100} \right)^{1-n} \quad (13)$$

where  $k_{quartz}$  is assumed as  $7.5 \text{ W m}^{-1}\text{K}^{-1}$  [11] and  $k_{kaolin}$  is assumed as  $2.9 \text{ W m}^{-1}\text{K}^{-1}$  [27].

Substituting Eqs. (11), (12) and (13) into Eq. (7), a continuum thermal conductivity model was proposed as follows:



**Fig. 11** Relationships between slope ( $k$ ), intercept ( $c$ ) and quartz (sand) content

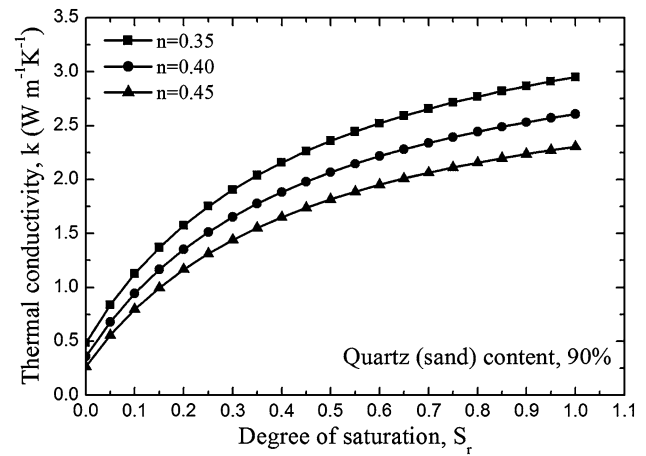
$$k = \frac{(2.168 \times 10^{-5} \times \exp(x/7.903) + 1.252)S_r}{1 + (2.168 \times 10^{-5} \times \exp(x/7.903) + 0.252)S_r} \times \left[ k_{\text{water}}^n \left( \frac{k_{\text{quartz}}^x k_{\text{kaolin}}^{1-x}}{k_{\text{quartz}}^{x/100} k_{\text{kaolin}}^{1-x/100}} \right)^{1-n} - (1.216 \times 10^{-6} \times \exp(x/6.599) + 3.034) \times 10^{(-0.003 \times \exp(x/16.452) - 1.840) \times n} \right] + (1.216 \times 10^{-6} \times \exp(x/6.599) + 3.034) \times 10^{(-0.003 \times \exp(x/16.452) - 1.840) \times n} \quad (14)$$

Comparing with the existing soil thermal conductivity models, the proposed model not only considers the influence of degree of saturation and porosity on soil thermal conductivity, but also introduces a new variable “ $x$ ” to account for the quartz (sand) content and soil type effects. Model prediction and validation will be presented in the following sections.

### 4.3 Model prediction

Figure 12 shows the predicted thermal conductivity with degree of saturation for different porosities at quartz (sand) content of 90%. The model predictions matched the experimental results (shown in Fig. 7) reasonably well. Lower porosity led to higher thermal conductivity at each degree of saturation because heat transfer process was facilitated by an increase in number of inter-particle physical contact points as porosity reduced.

Figure 13 shows the predicted thermal conductivity with degree of saturation for different quartz (sand) contents at porosity of 0.4. It is observed that the thermal conductivity increased as quartz (sand) content decreased under dry condition, which also conformed to the experimental results shown in Fig. 10. This can be explained by the fact that more initial voids in soils are occupied by fine clay



**Fig. 12** Predicted thermal conductivity with degree of saturation for different porosities at  $x = 90\%$

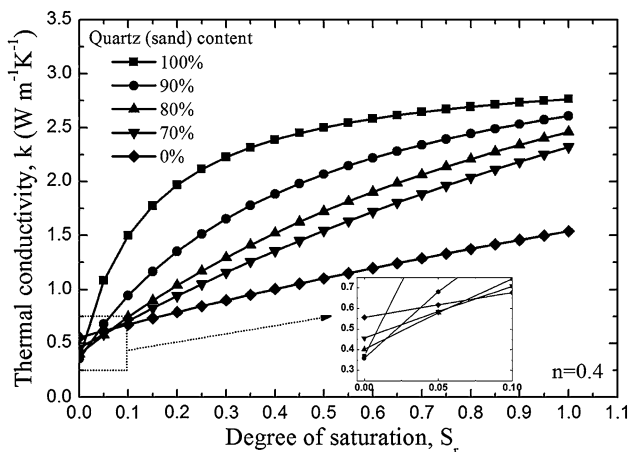
particles to form more inter-particle contacts as clay content increases. Although the thermal conductivity of clay particles is smaller than sand grains, it is still greater than that of air. Such newly formed inter-particle contacts at original interfaces between soil grains and air will reduce the thermal resistance somewhat and improve the overall soil thermal conductivity. On the contrary, the thermal conductivity was increasing with quartz (sand) content under moist conditions because of the moisture and quartz content effects.

In Figs. 12 and 13, the model not only predicted thermal conductivity accurately through the comparison with the experimental results, but also excellently reproduced the characteristics of thermal conductivity variation with degree of saturation at different porosities and quartz (sand) contents over the entire range of soil moisture condition.

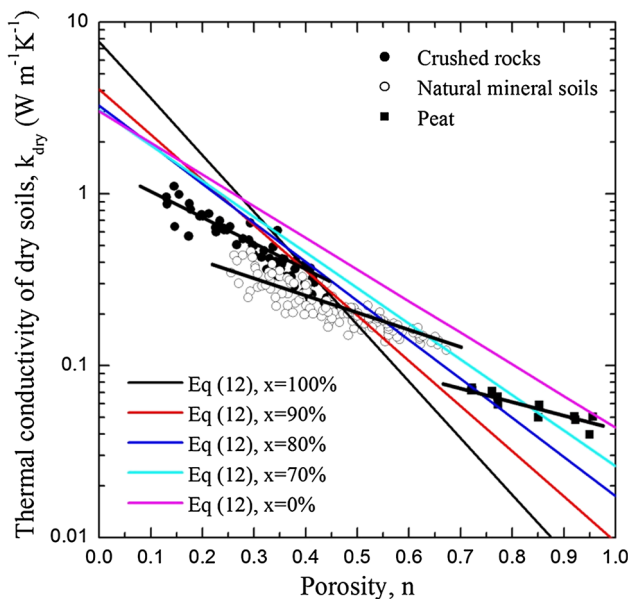
## 5 Model validation

### 5.1 Validation for $k_{\text{dry}}-n$ relationship

Figure 14 shows the comparison of predicted thermal conductivity of dry soils by Eq. (12) with experimental data from the previous literatures [13, 23, 24, 35, 36]. Three linear fitted lines of experimental data were also plotted in the figure to compare with the model prediction. It is observed that the predicted thermal conductivities were slightly higher than the measured values for crushed rocks and natural mineral soils, which is probably because of the extremely high quartz content of the test sand in this study. Assuming the quartz content of peat equals to zero, the predicted thermal conductivities were still greater than the measured values because of the difference in mineral components between kaolin clay and peat. Peat consists of



**Fig. 13** Predicted thermal conductivity with degree of saturation for different quartz (sand) contents at  $n = 0.4$



**Fig. 14** Comparison of predicted thermal conductivity of dry soils by Eq (12) with experimental data [13, 23, 24, 35, 36]

a large amount of organic matter that has lower thermal conductivity than anhydrous aluminum silicate which is the main component of kaolin clay. Since the exact quartz or sand contents of soil samples in the literature were not clarified, the model prediction did not agree with the experimental data very well, but it still exhibited a relatively high degree of prediction accuracy for thermal conductivity of dry soils.

**5.2 Validation for  $k_r-S_r$  relationship**

Figure 15a, b shows the comparison of  $k_r-S_r$  relationships between model prediction by Eq. (11) and Kersten’s [24] data of different soil types. In Fig. 15a,  $k_r-S_r$  relationship

was depicted according to Eq. (11) at quartz (sand) content of 100%. The model prediction showed very good agreement with experimental data of Ottawa sand and crushed quartz at low degree of saturation. But  $k_r$  was overestimated by the proposed model for other three sands (i.e., Northway fine sand, Norway sand and Lowell sand) at high degree of saturation. This is because that these sands have different quartz contents: Lowell sand has quartz content of only 43–72%; Northway sand and Northway fine sand contain only 10% quartz; whereas Ottawa sand and crushed quartz have the highest quartz content of 90–100%, which is more closed to the quartz content of 99% in the model prediction.

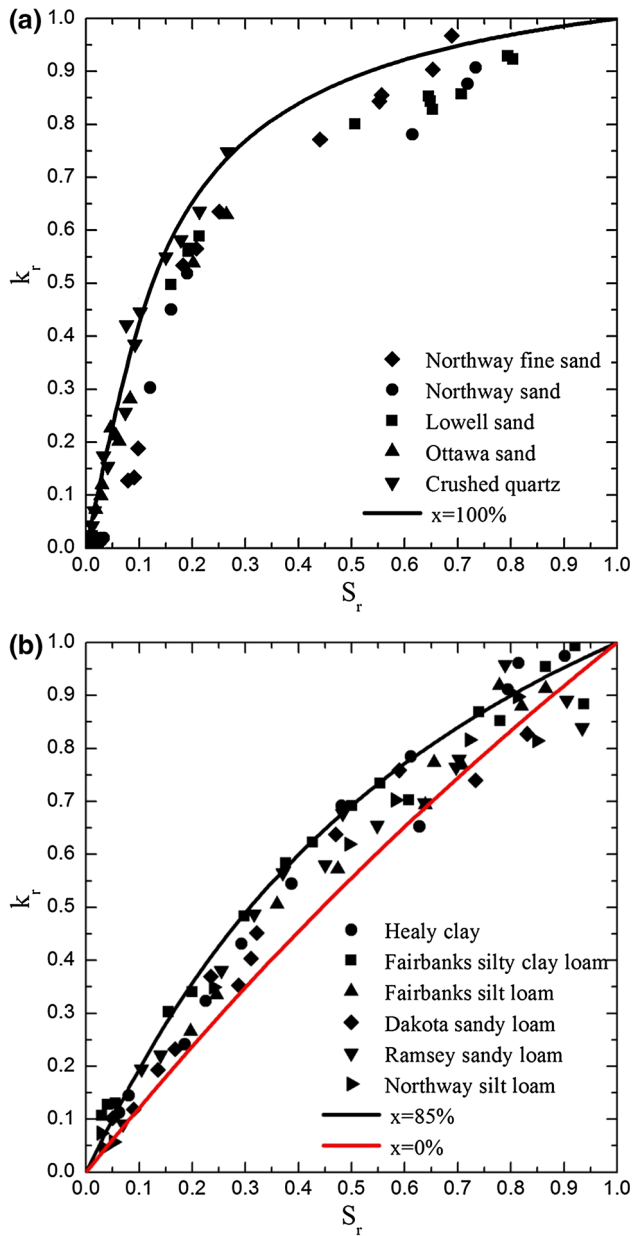
In Fig. 15b, two  $k_r-S_r$  relationships were depicted at quartz (sand) content of 85 and 0%, respectively. Although the exact quartz contents of silty soils, clayey soils, silts and clays in Kersten’s [23] study were not clarified,  $k_r-S_r$  relationship obtained from model prediction at  $x = 0%$  (i.e., pure clay) also defined a lower limit reasonably well through the comparison with the experimental data. Moreover, model prediction at  $x = 85%$  provided an upper limit for all the experimental data. Consequently, the proposed model is able to capture the characteristics of  $k_r-S_r$  relationship excellently in a wide range of soil type.

**6 Model assessment**

Figure 16a–c shows the comparison of predicted thermal conductivity with measured values from the present study, Lu’s et al. [30] study and Chen’s [11] study. In Fig. 16a, the predicted thermal conductivities were found to be in a very good agreement with measured values from this study. The relative difference between them is observed to be less than 10%.

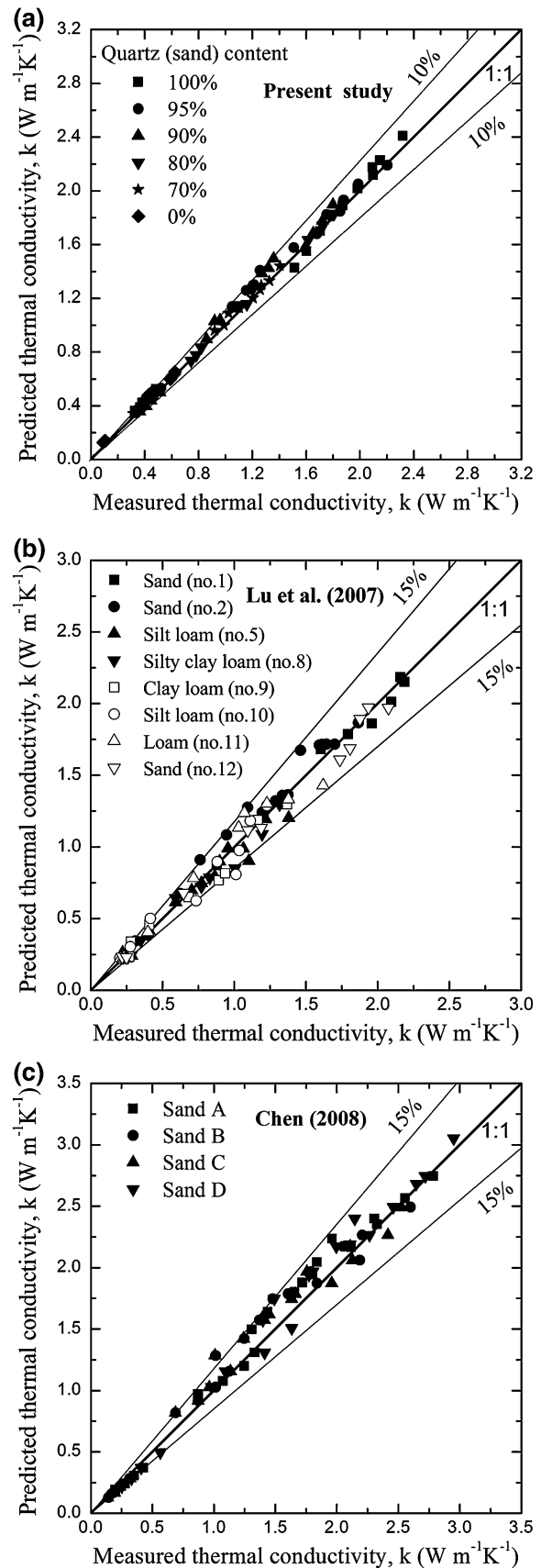
Lu et al. [30] performed a series of thermo-TDR tests on various types of soils ranging from sand, silt to loam and clay loam for measuring their thermal conductivities. He also provided the sand and clay content of test soils in his study. Hence, his experimental data were also collected to further evaluate the performance of the proposed model on different types of natural soils with varying sand or clay content. As shown in Fig. 16b, the predicted results did not match the measure values very well with deviation of around 15%. Sand content was assumed to be identical to quartz content in the model prediction, which may affect the prediction accuracy for the test soils in Lu’s et al. [30] study due to the uncertainty of quartz contents of his soil samples.

Chen [11] measured the thermal conductivity of four quartz sands using thermal probe method. All the test sands in his study contained quartz content of around 99% which was used in the model prediction in this study. As shown in



**Fig. 15** Comparison of  $k_r$ - $S_r$  relationships between model prediction and Kersten's [24] data: **a** Medium and fine sands; **b** silty soils, clayey soils, silts and clays

Fig. 16c, the model prediction agreed with Chen's [11] data well with deviation of approximate 15%. Although the quartz contents of soil samples were determined precisely for this case, the slight difference between the predicted and measured values is probably because of the effects of particle size distribution and particle shape on sand thermal conductivity. Based on the above analyses, the model can be used to predict soil thermal conductivity accurately with at least 85% confidence interval over a wide range of soil type.



◀**Fig. 16** Comparison of predicted thermal conductivity with measured values from: **a** present study; **b** Lu et al. [30]; **c** Chen [11]

## 7 Summary and conclusions

Soil thermal conductivity is a key soil property in the exploitation and utilization of geothermal energy as well as in design of energy geostructures such as geothermal energy piles (GEP), ground source heat pumps (GSHP) and borehole thermal energy storage (BTES). In this study, a new generalized soil thermal conductivity model was developed based on a series of laboratory experiments on sand, kaolin clay and sand–kaolin clay mixtures in different proportions using a newly designed thermo-TDR probe. The major conclusions resulted from this study can be drawn as follows:

1. Due to the considerable effect of quartz content on soil thermal conductivity, the fine graded sand with known quartz content is selected to perform laboratory experiments; therefore, such effect is studied quantitatively in this study.
2. The proposed model incorporates quartz (sand) content as a variable; therefore, it is capable of predicting thermal conductivity of solid more accurately than other existing models. The model can also further improve the prediction accuracy of soil thermal conductivity particularly when the quartz content of soils is known.
3. The effect of soil type on thermal conductivity is also taken into account by the change in sand (quartz) content in sand–kaolin clay mixtures. Since the sand content is assumed to be the same as quartz content reasonably in this study, there is no need to introduce other variables to account for such effect. However, the model may not provide very satisfied predicted results when quartz content is much less than sand content for a given type of soil.
4. The predicted soil thermal conductivity by the new model is continuous over the entire range of soil type, which fills the gaps of model prediction existing in between any two neighboring soil types.
5. It should be noted that the model assessment for soil thermal conductivity might not be adequate due to the lack of quartz content data of soil samples in the previous literatures, but the predicted thermal conductivities are still found to be in a good agreement with the experimental data with at least 85% confidence interval.

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