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# Analysis of resistance factors for LFRD of soil nail walls against external stability failures

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**Abstract** This paper presents reliability analyses of soil nail walls against two external ultimate limit states, global and sliding stabilities, which are related to the external stability failures of soil nail walls. Reliability analyses are conducted using Monte Carlo simulation technique. Soil nailing is a popular retaining system in highway construction and slope stabilization, and its current design practice is still based on the working stress design. There remains a need to establish a more rational design framework-load and resistance factor design-based on the concept of limit state design and reliability analysis for soil nail walls. The development of load and resistance factor design approach must consider multiple ultimate limit states, associated with external, internal, and facing failures. The analyses of resistance factors against two external failures are conducted in this study considering various influencing factors, including statistical parameters of soil friction angle, ultimate bond strength between soil and nails, soil type, wall geometry (wall height, back slope angle, and face batter angle), and nail configurations (nail inclination angle, drillhole diameter, and nail spacing). In the end, a series of resistance factors are proposed for potential application of load and resistance factor design approach against external failures for soil nail walls according to different design codes.

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<sup>1</sup> Department of Civil Engineering, Ryerson University, Toronto, ON M5B 2K3, Canada **Keywords** External stability · Load and resistance factor design · Reliability analysis · Resistance factor · Soil nail wall · Ultimate limit state

# **1** Introduction

Soil nailing is an important earth retention technique that has been widely applied in stabilizing existing slopes and embankments or supporting cuttings for railway and highway construction [7, 27, 39–41, 50]. It has gained popularity worldwide because of its cost-effectiveness and fast construction.

A soil nail wall (SNW) must be designed to satisfy required safety checks against all potential failure modes, including external, internal, and facing failure modes. External failures refer to failures in global stability, sliding stability, and bearing capacity. Global stability refers to the loss of overall stability of the reinforced soil mass, which may occur when the total loads outweigh the resistances provided by soil along the critical failure surface and by the nails extending through it. Sliding stability refers to the horizontal movement of the entire nailed soil mass along its base. Bearing capacity failure can only occur when an SNW is built over very soft fine-grained soils. Since soft finegrained soil is normally not among the favorable grounds for soil nails [30], the bearing capacity failure is very rare and will be excluded in this study, while the other two external failure modes will be investigated in this study.

Currently, its design is based on the working stress design (WSD) method [19, 23, 41], where factors of safety (FS) are designated to control the safety levels of a structure against various potential failure modes. The FS is defined as the ratio of the overall resistance of a system to the total loads that the system has to carry. Frequently, a single FS is inadequate to

guarantee a consistent and uniform safety level for a structure as it may involve widely varying degrees of uncertainties [16]. The load and resistance factor design (LRFD) approach, which was initiated by structural engineers for steel and concrete structural design in the 1970s, considers variability in the design equations by defining load and resistance factors (LRFs) [34, 38]. The direct benefit of using LRFD over WSD is more consistent safety levels under different design scenarios.

In structural engineering area, most design codes have completed the transition from WSD to LRFD. While in geotechnical engineering field, a great deal of efforts has been devoted to facilitate the transition. The LRFD design framework has been adopted for some geotechnical structures, like shallow and deep foundations [6, 22]. In retaining structures, a systematical study has been carried out for mechanically stabilized earth (MSE) walls [10, 28, 33], while there have been limited researches on LRFD of SNWs [4, 30]. Based on reliability analyses of an SNW with specific wall geometry and nail parameters, Babu and Singh [4] proposed a set of partial load and resistance factors. Indeed, there are many design variables, including soil type, wall geometry (e.g., wall height, face batter angle, and back slope angle), soil nail pattern (e.g., drillhole diameter, nail inclination, and nail spacing), which may influence the analysis results. Hence, it is important to investigate the influence of these variables and develop a set of resistance factors to cover these design scenarios in practice. Lazarte [30] developed a database for nail pullout resistance and then calibrated the corresponding resistance factor using the method proposed by Allen et al. [1]. In addition, a set of resistance factors were proposed in his study by fitting to the FS in WSD codes. Although these factors are in consistent with the design format of LRFD, they are not genuinely reliability-based and hence there still remains a need to perform reliability analysis and develop reliability-based resistance factors for SNWs.

The AASHTO [2], CHBDC [12], and Eurocode 7 [18] are three widely used design codes in North America and Europe. In these codes, LRFD specifications are incorporated for a variety of retaining structures excluding for SNWs. The main objectives of this study are to: (1) inspect the effects of different design variables on the resistance factors for SNWs against external failures, including soil type, soil shear strength, wall geometry, and nail configurations and (2) propose a set of reliability-based resistance factors that can be used in conjunction with these three codes.

### **2** Performance functions

The main task of this part is to establish the performance functions (PFs) for global and sliding stability checks. These two potential failure modes are typically shown in Fig. 1. Usually, conventional limit equilibrium analyses can be used to define the performance functions for SNWs. This section introduces the PFs for global and sliding stability based on the limit equilibrium analyses provided in FHWA [19].

# 2.1 Global stability

#### 2.1.1 Critical failure surface

The FHWA [19] provides a simplified method for analysis of global stability, in which the critical failure surface is simply assumed planar and extends from the toe of the wall to the top with an inclination angle of  $\psi$  from the horizontal. Similar to an analysis for slope stability,  $\psi$  can be taken as  $(45^\circ + \phi'/2 - \alpha/2)$ , where  $\phi'$  is the internal friction angle of soil and  $\alpha$  is the face batter angle. This assumption of failure plane is appropriate when the wall is



Fig. 1 Cross-sectional view of a general SNW: **a** global stability; **b** sliding stability

near vertical or the reinforced soils are cohesionless [32, 47], while for cohesive soils, the critical failure surface tends to be more curved. In another words, the assumption of a planar failure surface will create additional model uncertainties. However, due to the lack of system failure tests for SNWs, this issue cannot be readily resolved based on the current state of knowledge. For simplification, this planar failure surface is assumed in this study to be applicable for both cohesionless and cohesive soils.

#### 2.1.2 Performance function

An SNW may be subjected to different loads, including dead load and live load. Normally, the dead load in global stability,  $DL_G$ , is the vertical earth pressure due to the weight of an unstable block,  $W_{GL}$ , while the live load,  $LL_G$ , can be attributed to the surcharge load,  $q_s$ , on the top of the nailed slope. The  $DL_G$  and  $LL_G$  imposing on the potentially unstable block can be, respectively, expressed as follows:

$$DL_G = W_{GL} \sin \psi \tag{1}$$

$$LL_G = Q_s \sin \psi \tag{2}$$

where  $Q_s$  is the total surcharge load depending on the wall geometry, and  $\phi'$ . Both the expressions of  $W_{GL}$  and  $Q_s$  are given in "Appendix." The expressions of other quantities in the remainder of this paper are also given in "Appendix."

The total resistance against global failure comes from two parts: shear strength of the soil and stabilization effects of the nails. The shear strength of the soil along the critical failure surface, per the FHWA [19], can be written as

$$R_{soil} = cL_f + (W_{GL} + Q_s)\cos\psi\tan\phi'$$
(3)

where *c* is the soil cohesion;  $L_f$  is the length of the failure plane.

Stabilization effects provided by the nails can be quantified as

$$R_{nail} = T_{nail}[\sin(\psi + i)\tan\phi' + \cos(\psi + i)]$$
(4)

where  $T_{\text{nail}}$  is the total pullout resistance of the nails relating to the ultimate bond strength of nails,  $q_{\text{u}}$ , effective nail length beyond failure plane,  $L_{\text{e}}$ , drillhole diameter,  $D_{\text{DH}}$ , and vertical and horizontal nail spacing,  $S_{\text{v}}$  and  $S_{\text{h}}$ ; *i* is the nail inclination as defined in Fig. 1.

The PF for global stability can be achieved as follows:

$$g_G = R_{soil} + R_{nail} - DL_G - LL_G \tag{5}$$

### 2.2 Sliding stability

#### 2.2.1 Critical failure surface

As shown in Fig. 1b for sliding stability, the whole nailed soil mass is assumed to be a rigid block like the cases for

gravity retaining walls or MSE walls [28]. The horizontal movement of this nailed soil block would be initiated if the lateral thrust acting on it exceeds the shear resistance developed along its base. Therefore, the critical failure surface of sliding stability lies along the base of the nailed soil block with a length of  $B_{\rm L}$ .

#### 2.2.2 Performance function

The lateral thrust comprises two sources: One is the horizontal active earth pressure caused by the soil behind the potential sliding surface; the other is the lateral force due to surcharge load,  $q_s$ . The former is conceived as a dead load,  $DL_s$ , while the latter a live load,  $LL_s$ . They are, respectively, expressed as

$$DL_{S} = \frac{1}{2} \gamma H_{eff}^{2} K_{a} \cos \theta \tag{6}$$

$$LL_S = K_a H_{eff} q_s \cos\theta \tag{7}$$

where  $K_a$  denotes the coefficient of active lateral earth pressure;  $H_{eff}$  represents the effective height over which the earth pressure acts;  $\theta$  is the angle of back slope as defined in Fig. 1;  $\gamma$  is the unit weight of soil. The active lateral earth pressure coefficient based on Coulomb's theory is used in this study.

The shear resistance is developed through mobilizing soil shear strength along the critical failure surface, which can be determined as follows:

$$R_{SL} = cB_L + [W_{SL} + (DL_S + LL_S)\tan\theta]\tan\phi'$$
(8)

where  $W_{\rm SL}$  is the weight of the nailed soil block.

For those nails extending below the base of the nailed soil block, they also contribute to the resistance part. To consider the worst case, this part of contribution is neglected in this paper. Hence, the PF for sliding stability can be obtained as

$$g_S = R_S - DL_S - LL_S \tag{9}$$

One of the most popular LRFD approaches is to lump all the uncertainties in the resistance side into a single resistance factor while assigning different load factors to load components. The general expression of this LRFD approach can be written as [43]

$$\phi R_n \ge \gamma_D D L_n + \gamma_L L L_n \tag{10}$$

where  $R_n$ ,  $DL_n$ , and  $LL_n$  are the nominal resistance, dead load, and live load, respectively;  $\phi$ ,  $\gamma_D$ , and  $\gamma_L$  are the corresponding resistance and load factors.

Hence, the resistance factor can be determined as

$$\phi \ge \frac{\gamma_D D L_n + \gamma_L L L_n}{R_n} \tag{11}$$

# **3** Random Variables and Deterministic Parameters

There is a certain degree of inherent uncertainties for each parameter appeared in Eqs. (5) and (9). However, it is unrealistic and unpractical to consider all these parameters as random variables. An alternative strategy is to classify all the parameters based on their degrees of uncertainties into two groups: (1) random variables and (2) deterministic parameters. For example, the ultimate bond strength,  $q_u$ , is expected to vary widely due to both the complex interaction between soil and nails and uncertainties that might also arise in the process of nail installation. Consequently,  $q_u$  is deemed as a random variable. On the other hand, the nail spacing can be much better controlled and thus it is feasible to consider it as a deterministic parameter. The following two sections address the random variables and deterministic parameters in this study.

#### 3.1 Random variables

The basic random variables that are related to external failures include: (1) soil unit weight,  $\gamma$ ; (2) friction angle,  $\phi'$ ; (3) soil cohesion, *c*; (4) ultimate bond strength,  $q_u$ ; and (5) live uniform surcharge load,  $q_s$ . Substantial studies have been conducted by various researchers to investigate the statistical parameters for these random variables. Table 1 summarizes the values of the coefficient of variation (COV) reported by researchers. This study adopts COVs comparable to values used by other researchers, as shown in Table 2.

The most favorable soil conditions for soil nailing include stiff to hard fine-grained soils and dense to very dense granular soils with some apparent cohesion. Two types of normally consolidated soils, dense sand and stiff clay, are adopted for the reliability analysis of SNWs. Since the majority of SNWs are constructed above the groundwater table, the parameters for shear strength in drained condition are used for these soils. The mean values of  $\gamma$ ,  $\phi'$ , c for both sand and clay are determined from Holtz et al. [26], and their bias factors are all assigned as 1.0. A bias factor is defined as the ratio of mean to nominal value. Due to complex geology processes involved during the formation of natural soils, the engineering behaviors of soils vary significantly in both horizontal and vertical directions. Many important soil features, including layered conditions, soil stress history, soil cementation, are not considered in this study to simplify this problem.

The statistical parameters of  $q_u$  are determined based on the database compiled by Lazarte [30]. The mean of  $q_u$  is estimated to be 123 kPa for sand and 66 kPa for clay. The

 
 Table 1
 Summary of the coefficients of variation (COVs) for random variables

Random variable	Reported COV	References		
γ	0.04–0.08	[51]		
	0.02-0.2	[42]		
	0.03-0.07	[4, 16]		
	0.05-0.1	[5]		
	0.03	[21]		
	0.1	[28]		
	0.0235	[32]		
	0.05	[8]		
$\phi'$	0.05-0.15 for sand;	[31]		
	0.12-0.56 for clay			
	0.05-0.15 for sand;	[13]		
	0.10-0.50 for clay			
	0.025 for sand	[49]		
	0.008-0.017 for sand	[35]		
	0.02-0.05 for sand	[29]		
	0.025 for sand	[28]		
	0.05-0.15	[42]		
	0.02-0.13	[16]		
	0.0833, 0.10	[33]		
	0.02-0.10	[4]		
С	0.1-0.55	[5, 29, 42]		
	0.13-0.40	[16]		
$q_{\mathrm{u}}$	0.293 for clay <sup>a</sup> ;	[30]		
	0.23 for sand <sup>a</sup>			
$q_{\rm s}$	0.205	[28]		
	0.0-0.30	[9, 36]		

Note: <sup>a</sup> The authors obtain these two values based on the database developed by Lazarte [30]

design value of  $q_u$  is taken as 125 kPa for sand and 50 kPa for clay from FHWA [19] in consideration of soil types and construction methods. Correspondingly, the bias factors of  $q_u$  are 0.98 for sand and 1.32 for clay.

The value of  $q_s$  is normally specified in the design codes, although its value varies from one code to another. Unfortunately, the guidelines on how to determine  $q_s$  for SNWs are not explicitly specified in the three codes under consideration. Nevertheless,  $q_s$  for other similar retaining structures can be used, e.g., for MSE wall. This is considered to be reasonable since for a structural system,  $q_s$  is usually not dependent on the specific structural elements but on the external elements such as vehicular load. Per AASHTO [2],  $q_s$  acting on the top of an SNW due to the vehicular load can be converted to an equivalent height of the soil sitting below the loading. The equivalent height is equal to 0.6 m when the height H of a retaining wall equals or exceeds 6.0 m. For H = 3.0 m, the equivalent height is

Table 2 Statistical parameters for the random variables

Random variable <sup>a</sup>	Soil <sup>b</sup>	Design value		Bias factor	COV		
		Baseline case	Range for study		Baseline case	Range for study	
$\gamma$ (kN/m <sup>3</sup> )	NC sand	19.0	19.0	1.0	0.05	0.05	
	NC clay	17.0	17.0		0.05	0.05	
$\phi'$ (°)	NC sand	37	30-40	1.0	0.08	0.05-0.15	
	NC clay	30	24–34		0.15	0.10-0.20	
<i>c</i> ' (kPa)	NC sand	0	0	N/A	N/A <sup>c</sup>	N/A	
	NC clay	0	0		N/A	N/A	
$q_{\rm u}$ (kPa)	NC sand	125	100-180	0.98	0.23	0.20-0.30	
	NC clay	50	40-80	1.32	0.293	0.20-0.40	
$q_{\rm s}$ (kN/m)		AASHTO: 0.8y for A	AASHTO: $0.8\gamma$ for $H = 4$ m; $0.6\gamma$ for $H \ge 6$ m CHBDC: $0.8\gamma$		0.205	0.205	
		CHBDC: 0.8y					
		Eurocode: 10					

<sup>a</sup> All the random variables are assumed log-normally distributed

<sup>b</sup> NC normally consolidated

<sup>c</sup> N/A not applicable

set 0.9 m. For *H* between 3.0 and 6.0 m, the equivalent height can be estimated by a linear interpolation. Details can be referred to the AASHTO [2]. The evaluation of  $q_s$  in the CHBDC [12] is to take it as an equivalent additional fill height of 0.80 m, regardless of the wall height.

Eurocode 7 [18] does not specify a particular value for  $q_s$  or a method to calculate it. This paper sticks to the value of  $q_s$  commonly used in UK. In UK, the BD37/01 [25] recommends  $q_s$  a value of 10 kPa for HA loading and 20 kPa for 45 units of HB loading based on the uniform pressure approach. On the other hand, the newly completed PD6694-1 [11] proposes a new approach for determining  $q_s$ due to highway traffic loads in the design of retaining structures. The new approach emphasizes the concentrated effects of vehicle loading toward the top of retaining walls and is thus very different from the uniform pressure approach. The BD 37/01 [25] states that the type HA loading is the normal design loading for Great Britain where it adequately covers the effects of all permitted normal vehicles other than those used for the carriage of abnormal indivisible loads. In the calculation of resistance factors for Eurocode 7, this study presumably chooses a characteristic value of 10 kPa for  $q_s$  for reference.

The bias factor for live load in highway bridge LRFD is reported to vary from 0.6 to 1.2, depending on the span length and number of lanes of a bridge [37]. This paper conservatively uses a bias factor of 1.2 for  $q_s$ , which is consistent to the one used by Kim and Salgado [28]. In addition, all the random variables are assumed to be lognormally distributed so as to be consistent with other researchers, e.g., Fenton and Griffiths [20] and Kim and Salgado [28]. Statistical parameters for all the random variables are summarized in Table 2.

# 3.2 Deterministic parameters

The deterministic parameters specified in this study include the soil types (i.e., NC sand and clay), wall geometry (i.e., wall height, *H*, face batter angle,  $\alpha$ , and back slope angle,  $\theta$ ), and nail configurations (nail inclination angle, *i*, drillhole diameter,  $D_{\text{DH}}$ , and horizontal and vertical nail spacing,  $S_{\text{h}}$  and  $S_{\text{v}}$ ). As given in Table 3, these deterministic parameters are first chosen for the baseline case and then varied to investigate their effects on the calculation of resistance factors.

## **4** Target Reliability And Load Factors

Calibration of resistance factors to a specific design code requires the adoptions of target reliability index and load factors from that code during the calibration. In AASHTO [2], the design of overall stability for all bridge substructures (e.g., slope and foundation) is categorized into the Service I Limit State, leading to all the load factors equal to 1.00; the related resistance factors are calculated by fitting to FS in conventional WSD codes. The categorization of overall stability into Service Limit State is of great controversy, and this issue will not be resolved until additional information or studies are available [30]. Since the target reliability index,  $\beta_{\rm T}$ , for Service Limit States is currently unavailable in AASHTO [2], this study calibrates the resistance factors on the basis of considering overall stability Strength Limit State.

In the AASHTO LRFD Design Specifications, the Strength I Limit State has been calibrated for a  $\beta_{\rm T}$  of 3.5 during a 75-year design life of a bridge [2], which

	Deterministic parameter	Baseline case	Range of study
Slope	Soil type	NC sand and NC clay	NC sand and NC clay
	Wall height, H (m)	10	4–12
	Angle of face batter, $\alpha$ (°)	0	0–40
	Angle of back slope, $\theta$ (°)	0	0–20
Nail	Nail inclination, <i>i</i> (°)	15	0–30
	Drillhole diameter, $D_{\rm DH}$ (mm)	150	100–200
	Nail spacing, $S_{\rm h} \times S_{\rm v}  ({\rm m}^2)$	2.25	1.5-3.0

Table 3 Deterministic parameters for calculations in this study

corresponds to a probability of failure,  $P_{\rm f} = 2.0 \times 10^{-4}$ . The relationship between a probability of failure and a reliability index is plotted in Fig. 2 along with the  $\beta_{\rm T}$  specified in various design codes.

In terms of the Strength I Limit State in AASHTO [2], the load factors are selected as  $\gamma_s = 1.75$  for the live uniform surcharge load (live load in both global and sliding stabilities),  $\gamma_{EV} = 1.35$  for the vertical earth pressure (dead load in global stability), and  $\gamma_{EH} = 1.50$  for the horizontal active earth pressure (dead load in sliding stability). In Eurocode [17], load factors are selected based on design approaches. Design approaches and combinations relating to the implementations of partial resistance factors are not considered in this paper. The applications of load combinations in design of an SNW may also vary within European nations as specified by National Annex from each member country. The consideration of load combinations in this study sticks to the National Annex of UK. Table 4 shows the values of  $\beta_{\rm T}$  and load factors suggested in AASHTO [2], CHBDC [12], and Eurocode [17], which will be used to calculate the resistance factors for SNWs.

### 5 Resistance factor calculation

The resistance factors for an SNW against global stability failure are determined separately with the ones against sliding stability failure. The global stability failure mode is analyzed through following steps:

- 1. Formulate the PF for global stability failure mode, Eq. (5).
- 2. Evaluate the statistical parameters, distribution type, and bias factor for each random variable in the PF, as the baseline case shown in Table 2.
- 3. Choose the deterministic design parameters of the baseline case from Table 3.
- 4. Select a targeted  $\beta_{\rm T}$  from a design code, as summarized in Table 4.
- 5. Perform reliability analysis using Monte Carlo simulation (MCS) technique. The analyses are iterated to



Fig. 2 Reliability index and probability of failure

decide the nail length *L* under the conditions of Steps 2 and 3 until  $\beta_{\rm T}$  is reached.

- 6. Select load factors from the targeted code in Table 4 and apply them in Eq. (11) to calculate the resistance factor for global stability failure mode,  $\phi_{G}$ .
- 7. Change the statistical parameters for a certain random variable in Step 2 and repeat Steps 2 to 6 to investigate how the statistical parameters of a random variable influence the determination of  $\phi_{\rm G}$ . Table 2 shows the parameter ranges studied in this paper. Similarly, the influences of the deterministic parameters can also be investigated and the ranges of interest are shown in Table 3.

A similar procedure can be implemented to calculate the resistance factor for sliding stability failure mode,  $\phi_{\rm S}$ , and study the corresponding influencing factors.

# 6 Results and discussion

The influences of various factors on  $\phi_{\rm G}$  and  $\phi_{\rm S}$  are investigated using the  $\beta_{\rm T}$  and load factors from AASHTO [2] as an example. Figure 3 provides a general idea of how

Code Load factor  $\beta_T$ Design life (year)  $\gamma_{\rm EV}$  (global)  $\gamma_{\rm EH}$  (sliding)  $\gamma_{S}$  (both) AASHTO [2] 1.35 1.50 1.75 3.5 75 CHBDC [12] 3.75 75 1.25 1.25 1.70 50 Eurocode [17] 1.35 1.35 1.50 3.8

Table 4 Load factors and target reliability indices defined in various codes

one of the most important design parameters for SNWs, L/H (ratio of nail length to wall height), influences the reliability levels of global and sliding stability modes. For a 10-m-high SNW built in sand a relatively small increment of L/H, i.e., from 4.3 to 6.8 m, is adequate to rise the reliability index for global stability,  $\beta_G$ , from 2.0 to 5.0, while for the wall built in clay, the L/H must be at least doubled to have the same amount of increment in reliability level. This is mainly due to the fact that the value of  $q_u$  used in clay is approximately half of the value in sand. Similar trends can also be observed for the sliding case. It can also be found that the global stability controls the nail length for SNWs if the same level of reliability index is required for both ultimate limit states.

### 6.1 Influences from random variables

For global stability,  $\phi_{\rm G}$  is barely dependent on the means of both  $\phi'$  and  $q_{\rm u}$ . Nevertheless, their COVs are of critical importance, as shown in Figs. 4 and 5. As  ${\rm E}[\phi']$ , the mean value of  $\phi'$ , increases from 30° to 40° for sand, the corresponding  $\phi_{\rm G}$  decreases only approximately 5 % from 0.66 to 0.63. On the contrast,  $\phi_{\rm G}$  drops from 0.70 to 0.55 if the value of COV( $\phi'$ ) increases from 0.05 to 0.15. Similar trends for  $\phi_{\rm G}$  subjected to variations of  $q_{\rm u}$  can be found from Fig. 5.

Unlike global stability case, it is found that the resistance factor for sliding stability,  $\phi_S$ , is noticeably dependent on both the mean value and COV of  $\phi'$  though the influence from the COV is higher. The larger the  $E[\phi']$  or  $COV(\phi')$ , the lower the  $\phi_S$ . For example, for the sand case,  $\phi_S$  reduces from 0.80 to 0.67 as  $E[\phi']$  increases from 30° to 40°. The reduction due to a change in  $COV(\phi')$  is much more significant.  $\phi_S$  drops from 0.92 to 0.42 as  $COV(\phi')$ increases from 0.05 to 0.15. These findings imply that a thorough site investigation to reduce both  $COV(\phi')$  and  $COV(q_u)$  can result in a more economical design of SNWs.

#### 6.2 Influences from deterministic parameters

A total of seven deterministic design parameters are inspected to check their influences on  $\phi_{\rm G}$  and  $\phi_{\rm S}$ , including the soil type, slope geometry (*H*,  $\theta$ , and  $\alpha$ ), and nail design parameters (*i*,  $D_{\rm DH}$ , and  $S_{\rm h} \times S_{\nu}$ ). The statistical



Fig. 3 Relationships between  $\beta$  and L/H for the baseline design case in Tables 2 and 3

parameters needed during the calculations are as the baseline case shown in Table 2.

Their influences are shown in Fig. 6. There is no apparent dependency found between  $\phi_{\rm G}$  and  $\phi_{\rm S}$  and the slope geometry and nail design parameters, although a larger facing angle  $\alpha$  appears to result in a lower  $\phi_{\rm G}$ . For example,  $\phi_{\rm G}$  is approximately 0.65 for an SNW with a vertical facing and reduces to be 0.60 when facing is sloped to  $\alpha = 40^{\circ}$ .

It is interesting to find that  $\phi_G$  depends little on the soil type, while  $\phi_S$  relates closely to the soil type where the toe of an SNW lies. For the baseline case,  $\phi_S$  is found to be around 0.70 for the NC sand case and around 0.50 for the NC clay case. This is because the COV of  $\phi'$  for clay varies in a wider range than that for sand (as given in Table 2), which results in higher COVs for both resistance and load in Eq. (9) and leads to lower values of  $\phi_S$  for the clay case.

In summary, the COVs of random variables are significant to both  $\phi_{\rm G}$  and  $\phi_{\rm S}$ . In addition, the mean of  $\phi'$  also has considerable impact on  $\phi_{\rm S}$ . While for deterministic design parameters, i.e., soil type, slope geometry (H,  $\theta$ , and  $\alpha$ ), and nail design parameters (i,  $D_{\rm DH}$ , and  $S_{\rm h} \times S_{\rm v}$ ), neither the slope geometry nor the nail design considerations are found to be critical in determining both  $\phi_{\rm G}$  and  $\phi_{\rm S}$  in



Fig. 4 Influences of  $\phi'$  on  $\phi_G$  and  $\phi_S$ : a  $E[\phi']$ ; b  $COV(\phi')$ 

this study. However,  $\phi_S$  is greatly influenced by the soil where the toe of an SNW lies. The significance of each factor to the determinations of  $\phi_G$  and  $\phi_S$  is summarized in Table 5.

#### 6.3 Proposed resistance factors

Based on the findings listed in Table 5, different resistance factors have to be adopted for each design scenario in order to achieve a uniform reliability level across the whole design space, depending on the soil type, and means and COVs of both  $\phi'$  and  $q_u$ . Since there are an extraordinarily large number of design cases to consider, using a different pair of resistance factors for each case becomes unrealistic. Moreover, the recommendation of  $\phi_G$  and  $\phi_S$  should be simple and convenient to apply on condition that safety is



**Fig. 5** Influences of  $q_u$  on  $\phi_G$ : **a**  $E[q_u]$ ; **b**  $COV(q_u)$ 

well guaranteed. Therefore, to select the most favorable  $\phi_{\rm G}$  and  $\phi_{\rm S}$  for all design cases would be challenging.

Admittedly, safety should be the first priority in proposing these factors. In the meanwhile, being cost-effective is another issue of interest. The cost of an SNW design using any proposed  $\phi_G$  and  $\phi_S$  should be at a feasible level. From this point of view, this study recognizes the  $\phi_G$  and  $\phi_S$  for the baseline case defined in Tables 2 and 3 reasonable and representative. However, since the effect of COVs is of paramount importance, resistance factors corresponding to different levels of COV are also given for reference.

**Fig. 6** Influences from deterministic parameters on  $\phi_G$  and  $\phi_S$ : **a** H; **b**  $\theta$ ; **c**  $\alpha$ ; **d** i; **e**  $D_{DH}$ ; and **f**  $S_h \times S_v$  (statistical parameters involved in the calculations are as the baseline case shown in Table 2)



Group	up Influencing factor		
Random variable	Soil friction angle, $\phi'$		
	Mean	$\phi_{\rm S}$ only	
	COV	$\phi_{\rm G}, \phi_{\rm S}$	
	Ultimate bond strength, $q_u$		
	Mean	Neither	
	COV	$\phi_{ m G}$ only	
Deterministic param	eter		
Slope	Soil type	$\phi_{\rm S}$ only	
	Wall height, H	Neither	
	Angle of face batter, $\alpha$	Neither	
	Angle of back slope, $\theta$	Neither	
Nail	Nail inclination, <i>i</i>	Neither	
	Drillhole diameter, D <sub>DH</sub>	Neither	
	Nail spacing, $S_{\rm h} \times S_{\rm v}$	Neither	

 Table 5 Significance of various factors during the calculation of resistance factors

To propose resistance factors in terms of soil types is advantageous because (1) it produces more consistent reliability levels within various design scenarios; (2) SNW designs will be more cost-effective in comparison with those based on  $\phi_{\rm G}$  and  $\phi_{\rm S}$  without considering soil types; and (3) it does not essentially ruin the simplicity and convenience for application.

The values of  $\phi_G$  and  $\phi_S$  are rounded to the nearest 0.05 and then recommended for the potential application of the reliability-based LRFD of SNWs. The above study has targeted at the AASHTO [2]. Similar studies can be conducted to determine both  $\phi_G$  and  $\phi_S$  in conjunction with other two codes, CHBDC [12] and Eurocode [17]. Table 6 lists the recommended values that correspond to the load factors and target reliability indices prescribed in Table 4 for these three codes. LRFD design specifications for SNWs are not yet provided in these three codes. Nevertheless, resistance factors for global stability and sliding stability have been proposed for other types of retaining structures, e.g., MSE walls. Those resistance factors are also listed in Table 6 for comparison. In general, the values of currently adopted  $\phi_G$  and  $\phi_S$  for other types of retaining structures are higher than the recommended values for SNWs in this paper but basically comparable to those calculated based on the low COVs.

#### 6.4 Examples of SNW design

A design example given in FHWA [19] is adopted to illustrate the application of the resistance factors proposed in this study. The soil nail wall is 7.6 m in height with a vertical facing and a horizontal back slope. The soil is a medium dense, silty sand with an effective friction angle of  $32^{\circ}$ , soil cohesion of 4.8 kPa, and unit weight of 19.6 kN/m<sup>3</sup>. The nail spacing, nail inclination angle, and drill hole diameter will be 1.5 m horizontally and vertically,  $15^{\circ}$ , and 150 mm, respectively. The permanent load acting on the wall is mainly due to the weight of the soil behind the wall. The uniform live load on top of the wall is expected to be 7.2 kPa. There is no groundwater within the depth of concern. The nominal ultimate bond strength between soil and nail is 100 kPa. The main task is to determine the uniform nail length which enables the SNW to meet the targeted safety levels.

The normalized nail length is determined to be L/H = 0.70 for a FS of 1.51 for global stability based on a WSD-based design in FHWA [19]. The sliding stability is judged to be an unlikely scenario and is taken for granted from the external stability in the manual.

Based on the  $\phi_{\rm G}$  and  $\phi_{\rm S}$  values proposed by the present paper according to AASHTO [2], the *L/H* ratios are found to be 0.74 and 0.59 for global and sliding stability,

Table 6 Resistance factors proposed for SNWs

Design code	Soil type	Recommended in this study <sup>a</sup>		High COVs <sup>b</sup>		Low COVs <sup>c</sup>		Currently adopted in code for other types of retaining structures	
		$\phi_{G}$	$\phi_{S}$	$\phi_{\rm G}$	$\phi_S$	$\phi_{G}$	$\phi_S$	φ <sub>G</sub>	$\phi_{S}$
AASHTO [2]	NC sand	0.65	0.70	0.45	0.40	0.85	0.90	$0.75^{d}$ or $0.65^{d}$	0.90
	NC clay	0.65	0.50	0.45	0.40	0.85	0.70		
CHBDC [12]	NC sand	0.55	0.60	0.40	0.35	0.75	0.75	Not specified	0.8
	NC clay	0.55	0.40	0.40	0.30	0.75	0.55		
Eurocode 7 [18]	NC sand	0.60	0.60	0.40	0.35	0.80	0.80	0.91 <sup>e</sup>	0.91 <sup>e</sup>
	NC clay	0.60	0.40	0.40	0.30	0.80	0.60		

<sup>a</sup>  $\text{COV}(\phi') = 0.08$  and  $\text{COV}(q_u) = 0.23$  for NC sand, and  $\text{COV}(\phi') = 0.15$  and  $\text{COV}(q_u) = 0.293$  for NC clay

<sup>b</sup>  $\text{COV}(\phi') = 0.15$  and  $\text{COV}(q_u) = 0.30$  for NC sand, and  $\text{COV}(\phi') = 0.20$  and  $\text{COV}(q_u) = 0.40$  for NC clay

<sup>c</sup>  $\text{COV}(\phi') = 0.05$  and  $\text{COV}(q_u) = 0.15$  for NC sand, and  $\text{COV}(\phi') = 0.10$  and  $\text{COV}(q_u) = 0.20$  for NC clay

<sup>d</sup> The load factors are all equal to 1.00; details on which value to be used can be referred to AASHTO [2]

<sup>e</sup> This value corresponds to the design approach 2

Table 7 Results of the example of SNW design

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Design method	Code	Target safety	Normalized nail length, L/H			Design safety <sup>c</sup>	
			Global	Sliding	Adopted	Global	Sliding
WSD	FHWA [19]	FS = 1.50	0.70	_b	0.70	FS = 1.51	_b
LRFD with $\phi_G$ and $\phi_S$ recommended	AASHTO [2]	$\beta_T = 3.5$	0.74	0.59	0.74	$\beta_G = 3.8$	$\beta_s = 5.9$
in the present paper <sup>a</sup>	CHBDC [12]	$\beta_T = 3.75$	0.80	0.58	0.80	$\beta_G = 4.3$	$\beta_s = 6.5$
	Eurocode 7 [18]	$\beta_T = 3.8$	0.78	0.54	0.78	$\beta_G = 4.1$	$\beta_s = 6.3$

<sup>a</sup> The load and resistance factors for each code are as given in Tables 4 and 6

<sup>b</sup> Sliding stability was not checked in FHWA [19]

<sup>c</sup> Reliability indices were calculated using the COVs given in Table 2 for the baseline case

respectively. The final design ratio is taken as 0.74, which is comparable to 0.70 determined in FHWA [19]. With L/H = 0.74 and the COVs given in Table 2 for the baseline case, reliability analyses result in a  $\beta_{\rm G}$  of 3.8 for global stability and a  $\beta_{\rm S}$  of 5.9 for sliding stability. Both conditions are satisfied for the requirement of a minimum  $\beta_{\rm T} = 3.5$ . The higher  $\beta_{\rm S}$  confirms the assumption of an unlikely sliding stability failure in FHWA [19].

The example wall is also redesigned based on the proposed  $\phi_{\rm G}$  and  $\phi_{\rm S}$  values for CHBDC [12] and Eurocode 7 [18]. The normalized nail lengths are all similar to the case for AASHTO mentioned above. The results of this design example are summarized in Table 7 along with results from FHWA [19] for comparison.

# 7 Limitations and discussions

There are two main issues not addressed during the calculations of  $\phi_{\rm G}$  and  $\phi_{\rm S}$ : One is the spatial variability of soils, and the other is the model uncertainty in global stability failure mode.

For spatial variability of soils, this study took the tacit assumptions that the random soil properties are perfectly correlated (correlation length equal to infinity) within a homogeneous layer. It is a logical fact that assuming a perfectly correlated random field will underestimate the reliability levels. This has been well investigated and confirmed by previous studies, e.g., Griffiths et al. [24] and Salgado and Kim [44]. This ignorance of spatial variability of soils accordingly leads to lower  $\phi_G$  and  $\phi_S$ . However, the underestimation of reliability in this study is intentional for two reasons: (1) It greatly simplifies the analysis; (2) we take the partial correlation as an extra line of safety margin.

Model uncertainty in global stability is another key component in reliability analysis and calculations of resistance factors. Christian et al. [14] considered two main sources of model uncertainty in slope stability analysis: the discrepancy between a 3-D analysis and a 2-D analysis (Azzouz et al. [3] found from 18 case studies that the ratio of the three-dimensional to the plane strain FS is 1.11 with a COV of 0.05), and the assumed failure surface, for which Christian et al. [14] used a bias factor of 0.95 and a COV of 0.05. These uncertainties contribute equally to the SNWs. Furthermore, the presence of soil nails complicates the investigation of model uncertainty of SNWs in twofolds: (1) The bond strength model of soil nails involves model uncertainty, and (2) the reinforcement of soil nails may shift the critical failure surface from that for a non-nailed slope and thus creates synergistic effects on the model uncertainty. Lazarte [30] developed a soil nail bond strength database. Nevertheless, there have been very few system failure tests (e.g., Stocker et al. [48], Schlosser [45], CLOUTERRE [15], and Sheahan [46]) for one to obtain meaningful statistics for the overall model uncertainty of the SNWs. Consequently, model error is not taken into account in this paper. Indeed, the issue of model error will remain unless abundant system failure tests are available.

The establishment of LRFD for SNWs is far from completed. For further study, instead of homogeneous soils, reliability analysis should be performed for SNWs in layered soils because it is much common the case in practice. More soil types and load combinations should also be taken into account. Substantial efforts must be devoted to quantify the model error as it is critical to determine the resistance factors. Developing the reliability approaches for performance-based design of SNWs also deserves more concerns and efforts.

In conclusion, the proposed  $\phi_G$  and  $\phi_S$  are deemed feasible at current stage. Definitely, as more information becomes available, both  $\phi_G$  and  $\phi_S$  should be updated.

#### 8 Summary and conclusions

Based on the concept of limit state design, reliability analyses of soil nail walls against external failures are performed using Monte Carlo simulation technique. With consideration of impacts from soil types, soil properties, wall geometry, and nail configurations, a series of resistance factors for the load and resistance factor design of soil nail walls are, respectively, recommended to potential applications according to three popular design codes: AASHTO [2], CHBDC [12], and Eurocode 7 [18]. The following are the main points of this paper:

- 1. For the random variables of soil friction angle,  $\phi'$ , and ultimate bond strength,  $q_u$ , their COVs are found to be critically important during the determination of  $\phi_G$ , while their mean values are of much less significance. As for  $\phi_S$ , it relates closely to both the mean and COV of  $\phi'$ .
- 2. For deterministic parameters, the soil types matter most for  $\phi_S$ . Influences from other factors are marginal on  $\phi_G$  and  $\phi_S$ , including wall geometry (i.e., wall height, back slope angle, and face batter angle), and nail parameters (i.e., nail inclination angle, drillhole diameter, and nail spacing).
- 3. A series of  $\phi_G$  and  $\phi_S$  are recommended for the load and resistance factor design of soil nail walls against external failures according to three popular design codes: AASHTO, CHBDC, and Eurocode 7.

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# Appendix

$$B_{L} = \frac{\cos(\alpha - i)L}{\cos \alpha};$$

$$H_{eff} = H + (B_{L} - H\tan\alpha)\tan\theta;$$

$$K_{a} = \frac{\sin^{2}(\omega + \phi')}{\sin^{2}\omega\sin(\omega - \delta')\left[1 + \sqrt{\frac{\sin(\phi' + \delta')\sin(\phi' - \theta)}{\sin(\omega - \delta')\sin(\omega + \theta)}}\right]^{2}}$$

where  $\omega = 90^{\circ} + \alpha$ ,  $\delta'$  is the angle of friction between the soil and the wall, in this paper, it is conservatively assumed as zero;  $L_e = L - \frac{\cos(\alpha + \psi)}{\sin(\psi + i)\cos\alpha}h_j$ , where  $h_j$  is the vertical distance from the *j*-th nail head to the base of the wall;  $L_f = \frac{\cos(\alpha + \theta)}{\sin(\psi - \theta)\cos\alpha}H$ ;

$$Q_s = \frac{\cos(\alpha + \psi)\cos\theta}{\cos(\alpha + \theta)} L_f q_s;$$

$$T_{nail} = \sum_{j=1}^{n} L_e \frac{\pi D_{DH} q_u}{S_h};$$

$$W_{GL} = \frac{\cos(\alpha + \psi)}{2\cos\alpha} HL_f \gamma; \text{ and}$$
$$W_{SL} = \frac{\gamma}{2} \left[ H_{eff}^2 \cot\theta + H^2(\tan\alpha - \cot\theta) \right].$$

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