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SHORT COMMUNICATION

Optimal location of piles in slope stabilization by limit analysis

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Abstract Many studies have been conducted to establish the optimal location of a row of piles to reinforce and stabilize slopes. However, the results obtained are very different, and in some cases even inconsistent and contradictory. The factor of safety of piled slopes is determined by the magnitude of resistive forces exerted by the piles on the slope. At the same time, the maximum retaining forces provided by the piles are also affected by the pile position. In this paper, the problem of the optimal location of piles used to stabilize slopes is analyzed using a combination of limit slope stability analysis and the theory of Ito and Matsui (Soils Found 15:43–59, 8) to calculate limit lateral loads on piles. Using an illustrative example slope, some of the issues including the most effective position, the most suitable position, and the position with the largest safety factor are discussed. The results show that the most effective pile position, the most suitable pile position, and pile position where the factor of safety can take maximum value are different from each other for a given slope.

Keywords Limit analysis · Optimal location · Pile · Slope stabilization

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1 Introduction

The stabilization of slopes by installation of piles is one of the innovative slope reinforcement techniques that have been introduced in recent years. Many successful cases have been reported [6, 8–11, 18], and numerous methods have been developed for the analysis of piled slopes [1, 2, 7, 9–11, 12, 18, 20]. One of the main mechanisms by which drilled shafts and driven piles can enhance the stability of the soil slope is through soil arching, in which the interslice forces transmitted to the soil slice behind the shafts are reduced [15]. Several studies were conducted to consider the arching mechanism of piles in slope stabilization using different methods [14, 15, 21, 22]. A major design issue is to determine the most suitable location of piles within the slope [18]. Many studies have been conducted in order to establish the optimal location of the piles within a slope. However, the results obtained are rather different, and in some cases even inconsistent and contradictory.

The limit equilibrium method is used by many researchers to study this issue. Poulos [18] pointed out that the row of piles should be located in the vicinity of the center of the critical failure wedge to avoid merely relocating the failure surface behind or in front of the piles. Lee et al. [12], using the simplified Bishop's slip circle approach, found out that the most effective pile positions are at the toe and crest of the slope for homogeneous cohesive soil slopes, and between the middle and the crest of the slope for a two-layered soil slope where the upper soft layer is underlain by a stiff layer. The extended friction circle method and Ito's approach were used by Hassiotis et al. [7] to analyze slopes reinforced with one row of piles. According to their studies, the piles should be located close to the top of the slope to achieve the maximum safety factor, especially when the slope is steep.

Numerical methods are also very popular in this issue. Cai and Ugai [2], using the three-dimensional finite element method, showed that the pile should be located in the middle of the slope to achieve the maximum safety factor for the slope. Won et al. [20], using FLAC3D [5], draw the conclusion that the piles should be installed in the middle of slope where the pressure acting on the piles is the largest. Wei and Cheng [19], also using FLAC3D, considered the problem of slope reinforced with one row of piles. Their numerical results showed that the optimal pile position lies between the middle of slope and the middle of the critical slip surface of the slope with no pile and is very close to the middle of the slope.

Ausilio et al. [1] developed a methodology for the stability of slopes reinforced with piles using the kinematic approach of limit analysis. According to their studies, the optimal location of the piles within the slope is near the toe of the slope where the stabilizing force needed to increase the safety factor to the desired value takes a minimum value. They also found out that piles also appear to be very effective when they are installed in the region from the middle to the toe of the slope. Nian et al. [17], using limit analysis, concluded that the optimal location of the piles is near the toe of the slope where the force needed to increase the slope stability to the design safety factor has the lowest value.

From the above review, the results of all numerical analyses show that the optimal location of piles is very close to the middle of the slope, while by using combined limit equilibrium analysis and Ito-Matsui's equation, it is close to the top of the slope, and for the limit analysis method, it is near the toe of the slope. The divergences are due to the fact that the force provided by the piles was considered in different ways. It is evident that different conclusions will be drawn depending on whether the soil-pile interaction is considered.

In this study, a method that combines the kinematic approach of limit analysis and Ito-Matsui's theory is used to evaluate the optimal location of piles in slope stabilization. The proposed method rigorously considers the effects of the pile position on the required reinforcing force, and the force that piles can provide to stabilize slopes. The most effective pile position, the most suitable pile position, and the pile position with the largest safety factor are analyzed.

2 Method of analysis

2.1 Shear strength reduction method

Slope stability analysis is generally formulated in terms of the factor of safety F against failure, which can be defined with respect to soil shear strength parameters as [1]

$$F = \frac{c}{c_{\mathrm{m}}} = \frac{\tan \varphi}{\tan \varphi_{\mathrm{m}}} \tag{1}$$

where c and φ are the cohesion and internal friction angle of the soil; c_{m} and φ_{m} are the mobilized cohesion and internal friction angle for the slope required to attain the state of critical stability. This definition of F is exactly the same as that used in limit equilibrium methods and has been adopted in many other studies [1, 4, 7, 16, 20]. Herein, F can also be understood as the factor by which the soil shear strength parameters are reduced to give rise to incipient failure. According to the kinematic theory of limit analysis, the factor of safety determined by equating the rate of external work to the rate of internal energy dissipation for any kinematically admissible velocity field is no less than the true solution of slope stability analysis. Thus, the safety factor can be calculated by minimizing F with all kinematically admissible failure mechanisms.

When a row of piles is inserted in a slope, the additional resistance that each pile can provide depends on the soil strength. It is suggested that the retaining force be calculated with the reduced values of c and φ to get conservative results in the design of piled slopes.

2.2 Limit analysis of piled slopes

The kinematic approach of limit analysis is used herein to analyze the stability of slopes reinforced with one row of piles. The upper bound limit analysis is based on the assumption that soil will be deformed according to the associated flow rule and the convexity of the soil yield condition [16]. For simplicity, the slope is assumed to be composed of homogeneous, isotropic, and dry soil. Of the various potential failure mechanisms of a slope, rotational failure has been found to be the most adverse for earth slopes. Thus, the rotational log-spiral collapse mechanism, which was earlier examined by Chen [3] and many other researchers, is adopted herein. The geometry of the failure surface (Fig. 1) is described by the log-spiral equation

$$r = r_0 \exp\left[(\theta - \theta_0)\frac{\tan \varphi}{F}\right] \tag{2}$$

where r and θ are the radius and corresponding angle of the log spiral, r_0 is the radius of the log spiral with respect to

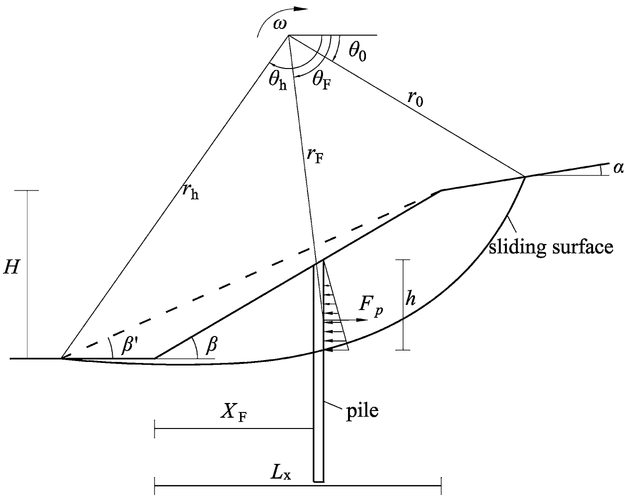

Fig. 1 Rigid rotation collapse mechanism for a slope reinforced with a row of piles

angle θ_0. The failing soil mass rotates as a rigid body about the point O with angular velocity ω. The slope geometry is defined by height H, and angles α and β, which are also indicated in Fig. 1.

The kinematic approach of limit analysis states that a slope will collapse if the rate of work done by external loads as well as the body forces exceeds the energy dissipation rate for any assumed kinematically admissible failure mechanism. The rate of external work due to the soil weight takes the form

$$\dot{W}_1 = \gamma r_0^3 \omega (f_1 - f_2 - f_3 - f_4) \tag{3}$$

where γ is the soil unit weight, and the functions f_1–f_4 depend on the angles θ_0, θ_h, φ, β and β'. Expressions for f_1–f_4 can be found in several works [1, 3, 13]. For completeness, these expressions are included in the "Appendix" of this paper.

For the rigid-block mechanism considered, the energy dissipation takes place both alone the sliding surface and surrounding the piles. The rate of energy dissipation caused by soil cohesion of the sliding surface is

$$\dot{D}_1 = c r_0^2 \omega f_5 \tag{4}$$

where c = soil cohesion, and f_5 is given in "Appendix". To account for the presence of the piles, a lateral force is assumed to be applied at the failing soil mass. So the rate of energy dissipation by the piles can be calculated as

$$\dot{D}_2 = F_p \sin \theta_F r_F \omega \tag{5}$$

where F_p is the force exerted on unit width of sliding mass by the piles, r_F is the radius of F_p about the rotation center, and the angle θ_F specifies the position of the retaining piles.

The total rate of internal energy dissipation is given by the sum of D_1 and D_2. Therefore, equating the rate of external work to the rate of energy dissipation leads to the following expression for F_p:

$$F_p = \frac{\gamma r_0^3 (f_1 - f_2 - f_3 - f_4) - c r_0^2 f_5}{\sin \theta_F r_F} \tag{6}$$

Equation (6) gives the force per unit width of soil that must be provided by a row of piles to achieve the desired value of the safety factor of the slope. In Eq. (6), f_1–f_5 are functions of F, the soil properties and the slope geometry (see the "Appendix"). For any known failure surface, the only unknowns are F_p and F. If F_p is known, the safety factor can be obtained. By considering all possible failure surfaces, a minimum safety factor of the slope can be found.

In this paper, the value of F_p is estimated using the plasticity theory developed by Ito and Matsui. As the value of F_p is related with the length of piles between the failure surface and the ground surface, every time a new slip surface is selected, the magnitude of the force F_p is calculated according to the newly selected failure surface. This force is then used in Eq. (6) to determine a new safety factor.

2.3 Forces on piles undergoing lateral soil movement

To determine the magnitude of force F_p, a theory, which was developed earlier by Ito and Matsui [8] to calculate earth pressures on a row of passive piles, is chosen in the present work. The soil around the piles is assumed to be in plastic equilibrium, satisfying the Mohr–Coulomb yield criterion. Then, the lateral load acting on the piles can be calculated regardless of the state of equilibrium of the slope. Based on these assumptions, the lateral force per unit thickness of soil layer acting on the piles, p, is estimated by the following equation [8–10]:

$$\begin{aligned} p(z) = {} & cA\left(\frac{1}{N_\varphi \tan\varphi}\left\{\exp\left[\frac{D_1 - D_2}{D_2} N_\varphi \tan\varphi \tan\left(\frac{\pi}{8} + \frac{\varphi}{4}\right)\right]\right.\right. \\ & \left.\left. -2N_\varphi^{(1/2)} \tan\varphi - 1\right\} \frac{2\tan\varphi + 2N_\varphi^{(1/2)} + N_\varphi^{-(1/2)}}{N_\varphi^{(1/2)} \tan\varphi + N_\varphi - 1}\right) \\ & - c\left(D_1 \frac{2\tan\varphi + 2N_\varphi^{(1/2)} + N_\varphi^{-(1/2)}}{N_\varphi^{(1/2)} \tan\varphi + N_\varphi - 1} - 2D_2 N_\varphi^{-(1/2)}\right) \\ & + \frac{\gamma z}{N_\varphi}\left\{A \exp\left[\frac{D_1 - D_2}{D_2} N_\varphi \tan\varphi \tan\left(\frac{\pi}{8} + \frac{\varphi}{4}\right)\right] - D_2\right\} \end{aligned} \tag{7}$$

where D_1 = center-to-center spacing between piles, D_2 = opening between piles, $D_1 - D_2$ = pile diameter,

γ = unit weight of soil, z = depth of soil layer from ground surface, $N_\varphi = \tan^2(\pi/4 + \varphi/2)$, and $A = D_1 (D_1/D_2)^{(N_\varphi^{1/2}\tan\varphi + N_\varphi - 1)}$. The total lateral force acting on a pile due to the plastically deforming soil layer around the pile, F_t, can be obtained by integrating Eq. (7) along the depth of the pile in the failing wedge. Then, the stabilizing force per unit width of soil provided by the pile, F_p, can be calculated by dividing F_t with the center-to-center distance between the piles, D_1 (i.e., $F_p = F_t/D_1$). The validity of this theory in the design of stabilization of slopes with piles was examined by Hassiotis et al. [7].

3 Illustrative example

The approach outlined above is illustrated considering the slope shown in Fig. 2 as an example. The slope has a height of 13.7 m, a slope angle of 30°, and is made of a homogeneous soil with cohesion of 23.94 kPa, friction angle of 10°, and unit weight of 19.63 kN/m^3. The water table is not considered. This case was previously examined by Ausilio et al. [1] and Michalowski [16] using limit analysis. It was found that the safety factor of the slope (without the pile reinforcement) was about 1.11. Since a safety factor of 1.11 is considered inadequate, the slope may be reinforced by installing a row of piles to increase the safety factor to a desired value. In the present example, the piles are assumed to be 0.9 m in diameter with a center-to-center distance of 2.25 m (to satisfy a D_2/D_1 ratio of 0.6). The pile is assumed to be rigid. For the convenience of analysis, large diameter piles are used in this example, but it should be noted that large diameter piles driven in the slope are rarely used in practice to increase the factor of safety as such action may invoke slope instability during the pile driving operation.

4 Results and discussions

As pointed out by Poulos [18], the design of piles to reinforce slopes involves not only evaluating the force needed to increase the safety factor to the desired value but also the maximum force that each pile can provide to resist sliding. In the present study, the optimal location of the piles was determined by considering both the stabilizing force needed to increase the safety factor to the desired value and the maximum force that each pile can provide according to Ito and Matsui's theory. The pile position is denoted by the dimensionless abscissa X_F/L_x, where $L_x = H/\tan\beta$ (Fig. 1). It is assumed that X_F/L_x varies between 0.1 and 0.9. These limits correspond to the pile positions near the toe and the crest of the slope, respectively. The reason for this assumption is that when the value of X_F/L_x below 0.1 or over 0.9, the failure surfaces do not pass through the piles are considerably large.

4.1 The most effective pile location

The most effective position of the piles within the slope is where the stabilizing force needed to increase the safety factor to the desired value takes the minimum value. Figure 3 gives the results of needed force to improve the slope stability to required safety factors for different pile positions. As can be expected, F_p increases with increasing required safety factor. In all the cases examined, the most effective location of the piles is near the toe of the slope, where the force need to be provided by the piles to achieve the selected value of safety factor takes the lowest value. This result is corroborated by Ausilio et al. [1]. As pointed out by these authors, this is due to the shape of the sliding surface that is a log-spiral curve having a radius that increases as the surface develops from the top to the base of the slope. For a rotational failure mechanism as shown in Fig. 1, the required stabilizing moment due to F_p, with respect to the rotation center, has an arm that increases as the location of the piles approaches to the slope toe, and consequently force F_p decreases.

The effects of pile location on the required F_p show that if the retaining force that a row of piles can provide is large enough, the piles should be installed near the toe of the slope where the stabilizing force can produce maximum stabilization results. It also indicates that the most economic location for piles in slope stabilization is near the toe of the slope.

4.2 The most suitable pile location

The most effective pile position may be different from the most suitable pile position, which is discussed below. Soil is a plastic material. As a result, according to Ito and Matsui's theory, the stabilizing force that a row of piles can provide is proportional to the depth of soil

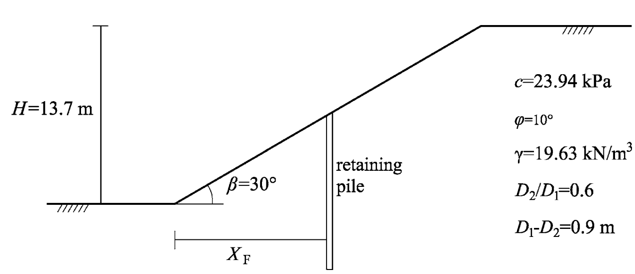

Fig. 2 Illustrative example of a slope reinforced with piles

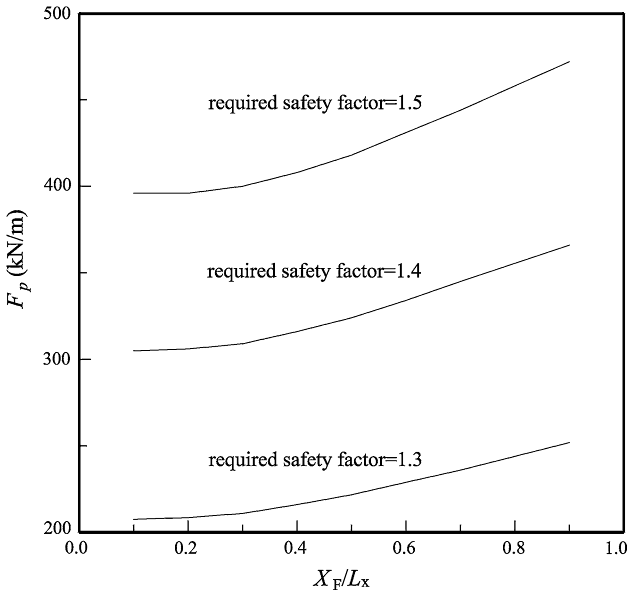

Fig. 3 Pile location versus F_p for required safety factor = 1.3, 1.4, and 1.5

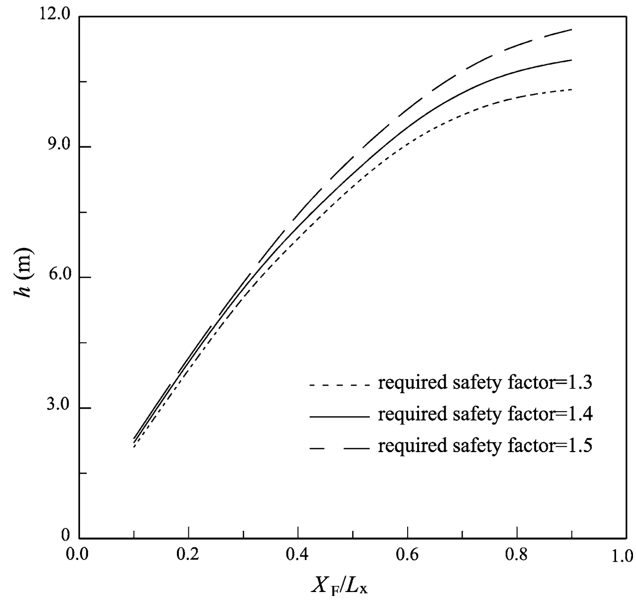

Fig. 4 Pile location versus h for required safety factor = 1.3, 1.4, and 1.5

layer from ground surface. The length of the portion of the pile between the sliding surface and the ground surface is denoted by h and plotted against the dimensionless abscissa X_F/L_x in Fig. 4. It can be seen that the magnitude of h is greatly affected by the pile location especially when the piles are placed in the lower-middle part of the slope. As a result, it is important to determine whether the piles can provide enough stabilizing force when they are located close to the toe of the slope. At this location, the value of h is very small due to the smaller height of the slope. Figure 5 plots the required and provided forces to achieve the required safety factors against pile locations. In all the cases examined, there is a point of intersection for the two lines, which means the force provided by the piles is below the needed force when the piles are located between the slope toe and the intersection point. Consequently, the piles must be placed in the upper part of the intersection point to obtain the required safety factor.

In the design of piled slopes, the pile length should be carefully considered. According to Poulos [18], the total length of the piles may be preliminarily assumed as $L_p \approx 2\,h$. For both safe and economical considerations, it can be concluded from Figs. 4 and 5 that the point of intersection is the most suitable pile position because the pile length is the shortest at this point.

It can also be seen from Fig. 5 that the most suitable pile positions are different for a given slope when the required safety factors are different. The larger the required safety factor, the higher in the slope should the piles be installed. It should be noted that these results are obtained under the condition that the pile diameter and spacing remain unchanged. The force that can be provided by the piles is determined by h, pile diameter, pile spacing, and soil properties. Actually, the diameter and spacing are also very important parameters in the design of piles to reinforce unstable slopes.

The analyses of the most suitable pile location show that the maximum retaining force that a row of piles can provide should be considered in the design, and that the most suitable pile position change with the required safety factor.

4.3 Pile position with maximum safety factor

Assuming that the piles are long enough, the safety factors are calculated considering the force that can be provided by the piles. This relationship is given in Fig. 6. It can be seen that the safety factor increases with X_F until the piles are placed close to the top of the slope. The safety factor takes its maximum value when X_F/L_x is about 0.75. This result is very close with that of Hassiotis et al. [7] for the same slope. The height of the portion of the pile above the sliding surface h and the corresponding stabilizing force provided by piles F_p are both plotted against X_F/L_x in Fig. 7. The length of pile above the failure surface and the stabilizing force provided by the piles take the maximum

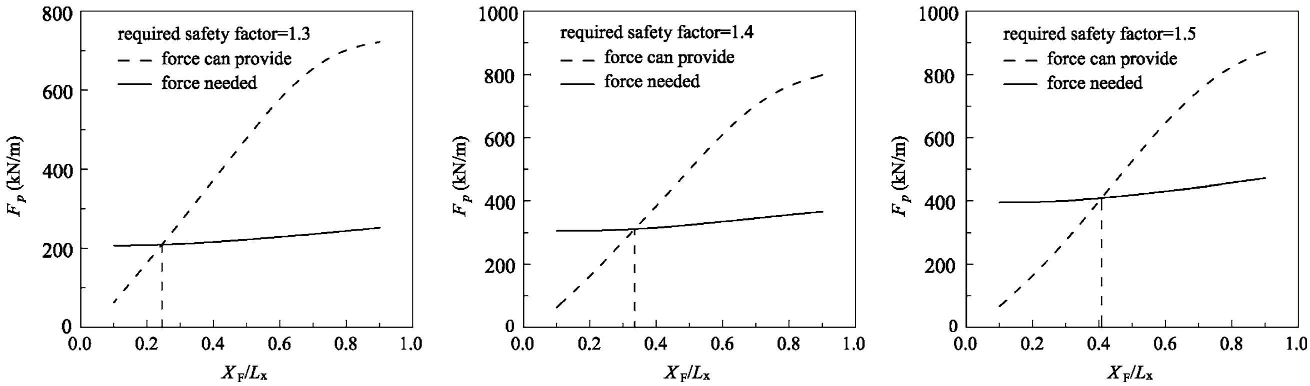

Fig. 5 Pile location versus needed and provided F_p to obtain the required safety factor of 1.3, 1.4, and 1.5

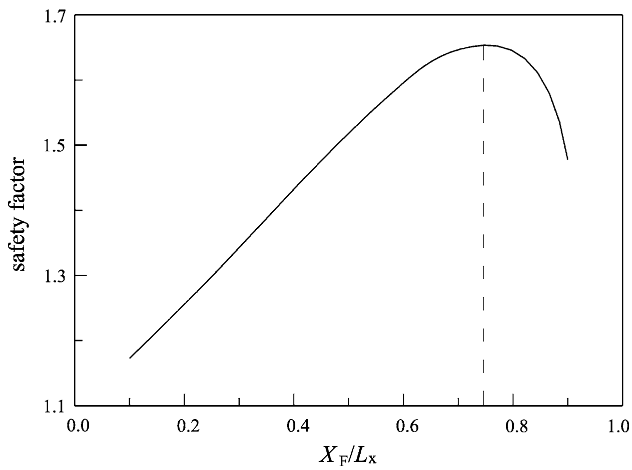

Fig. 6 Pile location versus safety factor of illustrative slope

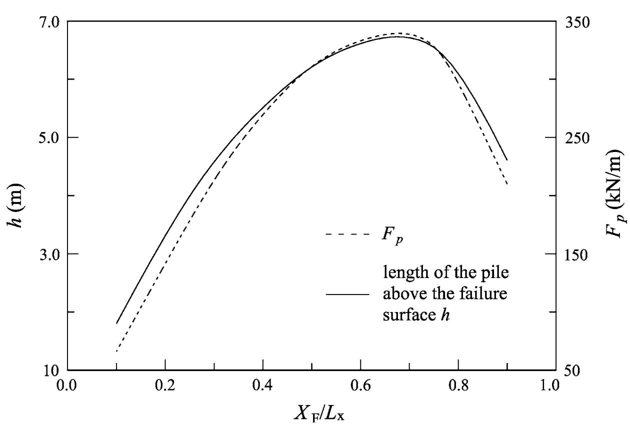

Fig. 7 Pile location versus length of the pile above the failure surface h (*solid line*) and F_p (*dashed line*)

value when the piles are placed in the upper-middle part of the slope. This can be the explanation to the influence of pile position on the safety factor. Evidently, the piles need to be placed close to upper-middle part of the slope to achieve a high value of safety factor.

It can be seen from above analyses that for sufficiently long piles, the position where the safety factor takes the maximum value lies in the upper-middle part of the slope where the resisting force is the greatest.

5 Conclusions

This paper deals with the optimal location of piles within slopes reinforced with a row of piles. The upper bound theorem of limit analysis was used in the slope stability analysis, and the plastic state theory developed by Ito and Matsui was employed to determine the stabilizing force provided by the piles. A simple example slope is given to illustrate the validity of the method. The influence of pile position on stabilizing effects was analyzed. Based on the numerical results, the following conclusions were reached:

1. If the maximum force that a row of piles can provide is large enough, the most effective pile positions are near the toe of the slope where the stabilizing force needed to increase the safety factor to the desired value takes the minimum value.
2. The most suitable place for stabilizing piles should be determined by both the force needed and provided. The most suitable pile position changes with the required safety factors.
3. For piles that are long enough, the position where safety factor takes maximum value lies in the upper-middle part of the slope where the force supply is greatest.

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Appendix

$$f_1 = \frac{(3\tan\varphi_t\cos\theta_h + \sin\theta_h)\exp[3(\theta_h-\theta_0)\tan\varphi_t]}{3(1+9\tan^2\varphi_t)} - \frac{3\tan\varphi_t\cos\theta_0 + \sin\theta_0}{3(1+9\tan^2\varphi_t)}$$

$$f_2 = \frac{1}{6}\frac{L}{r_0}\left(2\cos\theta_0 - \frac{L}{r_0}\cos\alpha\right)\sin(\theta_0+\alpha)$$

$$f_3 = \frac{\exp[(\theta_h-\theta_0)\tan\varphi_t]}{6}\left[\sin(\theta_h-\theta_0) - \frac{L}{r_0}\sin(\theta_h+\alpha)\right] \times \left\{\cos\theta_0 - \frac{L}{r_0}\cos\alpha + \cos\theta_h\cdot\exp[(\theta_h-\theta_0)\tan\varphi_t]\right\}$$

$$f_4 = \left(\frac{H}{r_0}\right)^2\frac{\sin(\beta-\beta')}{2\sin\beta\sin\beta'} \times \left(\cos\theta_0 - \frac{L}{r_0}\cos\alpha - \frac{H}{3r_0}[\cot\beta+\cot\beta']\right)$$

$$f_5 = \frac{1}{2\tan\varphi}\{\exp[2(\theta_h-\theta_0)\tan\varphi_t] - 1\}$$

where β is slope angle, $\tan\varphi_t = \tan\varphi/F$, and L is the distance between the failure surface at the top of the slope and the edge of the slope. It is given by

$$L = \frac{r_0\sin(\theta_h-\theta_0)}{\sin(\theta_h+\beta'')} - \frac{r_0\sin(\theta_h+\beta'')}{\sin(\theta_h+\beta'')\sin(\beta'-\beta'')} \{\sin(\theta_h+\beta'')\exp[(\theta_h-\theta_0)\tan\phi_t] - \sin(\theta_0+\beta'')\}$$

$$\frac{H}{r_0} = \frac{\sin\beta'}{\sin(\beta'-\beta'')} \times \{\sin(\theta_h+\beta'')\exp[(\theta_h-\theta_0)\tan\varphi_t] - \sin(\theta_0+\beta'')\}$$

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