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RESEARCH PAPER

Interference effect of two nearby strip footings on layered soil: theory of elasticity approach

Priyanka Ghosh · Anirudh Sharma

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Abstract In recent times, rapid urbanisation coupled with scarcity of land forces several structures to come up ever closer to each other, which may sometime cause severe damage to the structures from both strength and serviceability point of view, and therefore, a need is felt to devise simplified methods to capture the effect of footing interference. In the present study, an attempt has been made to model the settlement behaviour of two strip footings placed in close spacing on layered soil deposit consisting of a strong top layer underlying a weak bottom layer. Theory of elasticity is employed to derive the governing differential equations and subsequently solved by the finite difference method. The perfectly rough strip footings are considered to be resting on the surface of two-layer soil system, and the soil is assumed to behave as linear elastic material under a range of static foundation load. The effect of various parameters such as the elastic moduli and thickness of two layers, clear spacing between the footings and footing load on the settlement behaviour of closely spaced footings has been determined. The variation of vertical normal stress at the interface of two different soil layers as well as at the base of the failure domain also forms an important part of this study. The results are presented in terms of settlement ratio (ξ_δ), and their variation is obtained with the change in clear spacing between two footings. The present theoretical investigation indicates that the settlement of closely spaced footings is found to be higher than that of single isolated footing, which further reduces with increase in the spacing between the footings.

P. Ghosh (✉) · A. Sharma
Department of Civil Engineering, Indian Institute of Technology, Kanpur, Kanpur 208 016, India
e-mail: priyog@iitk.ac.in

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List of symbols

B	Width of footing
E	Elastic modulus
E_1, E_2	Elastic moduli for top and bottom soil layers, respectively
E_m	Elastic modulus for middle soil layer in case of three-layer deposit
G	Shear modulus of soil
G_1, G_2	Shear moduli for top and bottom soil layers, respectively
H	Total thickness of soil stratum
H_1, H_2	Thicknesses of top and bottom soil layers, respectively
H_m	Thicknesses of middle soil layer in case of three-layer deposit
S	Clear spacing between two footings
i	Subscript representing space domain along x-direction
j	Subscript representing space domain along z-direction
q	Applied load intensity on footing
u	Displacement along x-direction
w	Displacement along z-direction
x	Horizontal co-ordinate axis
z	Vertical co-ordinate axis
δ_v	Vertical settlement of single isolated footing
v	Poisson's ratio
σ_{xx}	Normal stress in x-direction
σ_{zz}	Normal stress in z-direction
τ_{xz}	Shear stress on xz plane
ξ_δ	Settlement ratio

1 Introduction

The problem of interaction between adjacent footings is of paramount practical significance, as, in many cases, foundations encountered in practise are not isolated and they often interfere with each other on account of their close spacing. The interference effect of two nearby strip footings on their ultimate bearing capacity was studied theoretically by Stuart [29] on the basis of limit equilibrium method. In this analysis, a partial non-plastic trapped wedge was considered below the footing base, in comparison with the available theories for an isolated rough footing. In this approach, the shape of failure surface was chosen as a combination of logarithmic spiral and straight line. In order to examine the effect of interference, the zone below the two interfering rough footings was assumed to comprise of partly a small non-plastic wedge and partly a plastic shear zone. Later, using the failure mechanism similar to that earlier used by Stuart [29], West and Stuart [30] employed the method of stress characteristics to obtain the solution for the interference of two closely spaced strip footings; the solution was, however, numerically, obtained only for $\phi = 35°$, where ϕ is the angle of internal friction of soil. The values of the efficiency factors obtained by West and Stuart [30], on the basis of method of characteristics for $\phi = 35°$, were shown to be smaller than those obtained earlier by Stuart [29] using the limit equilibrium method; the efficiency factor is defined as the ratio of the load carried by the single footing in presence of the other footing to that of the single isolated footing of the same size.

For the problem of three closely spaced strip footings on sand, Graham et al. [10] and Kouzer and Kumar [14] obtained the solution for determining the interfering effect on the central footing (not the two outer footings). This theory was meant for obtaining the interference effect due to a number of strip footings. The failure mechanism in this approach was chosen symmetrical about the centre line for any footing. It was demonstrated that the failure load increases considerably with decrease in the spacing of two footings. However, this theory cannot be employed to obtain the failure load of just two interfering footings. Later on, in order to study the interference effect of two nearby shallow foundations on single layer soil deposit, a number of investigations were performed theoretically as well as experimentally by various researchers [4, 7, 8, 11, 15–19, 22, 26]. In the field, the soil deposit is generally found to be in layers. Chow [2], Davies and Banerjee [6], Gazetas [9], Jommi and Novati [13], Maier and Novati [21], Shou [27], and Small and Booker [28] applied different numerical procedures such as finite element method, boundary element method and integral transform technique to predict the behaviour of footings placed on non-homogeneous layered soil. However, the study on the interference effect of nearby footings on layered soil system has not drawn much attention from the researchers. Therefore, the importance of study on the interfering footings on layered soil bed cannot be ignored. Very few research works are available on the study of footing interference on layered soil. Hanna [12] and Das et al. [5] studied experimentally the effect of interference on the bearing capacity of closely spaced footings resting on layered foundation bed. Hanna [12] investigated the ultimate bearing capacity of closely spaced footings resting on subsoils consisting of a weak sand layer overlying a strong deposit. Das et al. [5] performed laboratory model tests to determine the ultimate bearing capacity of two closely spaced surface footings supported by a layer of dense sand of limited depth, underlain by a soft clay layer extending to a great depth. The present investigation deals with the settlement characteristics of two closely spaced perfectly rough strip footings resting on two-layer soil medium formed with a strong top layer underlying a weak bottom layer. Most of the previous research works dealt with the enhancement of bearing capacity of closely spaced footings at the ultimate failure. However, the study on the settlement behaviour of closely spaced footings on layered soil system has not been explored much in the literature though the settlement characteristics of adjacent footings are expected to be more critical. In this paper, the settlement of single footing in presence of other footing is compared with that of single isolated footing. The analysis has been performed using the elasticity approach proposed by Maheswari and Viladkar [20]; where the solution is obtained by numerically solving the equilibrium equations under plane strain condition using the finite difference technique. The effect of various parameters, such as elastic moduli of different layers, depth of layers and load intensity on each footing, on the settlement behaviour of closely spaced footings has been explored in this paper.

2 Definition of the problem

The soil deposit consists of two layers having thicknesses of top and bottom layers as H_1 and H_2, respectively, with an underlying rigid base (Fig. 1). The total depth of the stratum is H. The strength parameters of the soil layers are represented by their elastic moduli E_1 and E_2, and Poisson's ratio v; where E_1 and E_2 are the elastic moduli of top and bottom layers, respectively. For the sake of simplicity, the Poisson's ratio for both the layers is assumed as same [20]. Two closely spaced perfectly rough rigid strip footings of width B are placed on the surface of top layer. Each footing carries a load intensity of q per metre run, which causes only the elastic deformation of the foundation bed. It is worth mentioning here that the soil is assumed to be

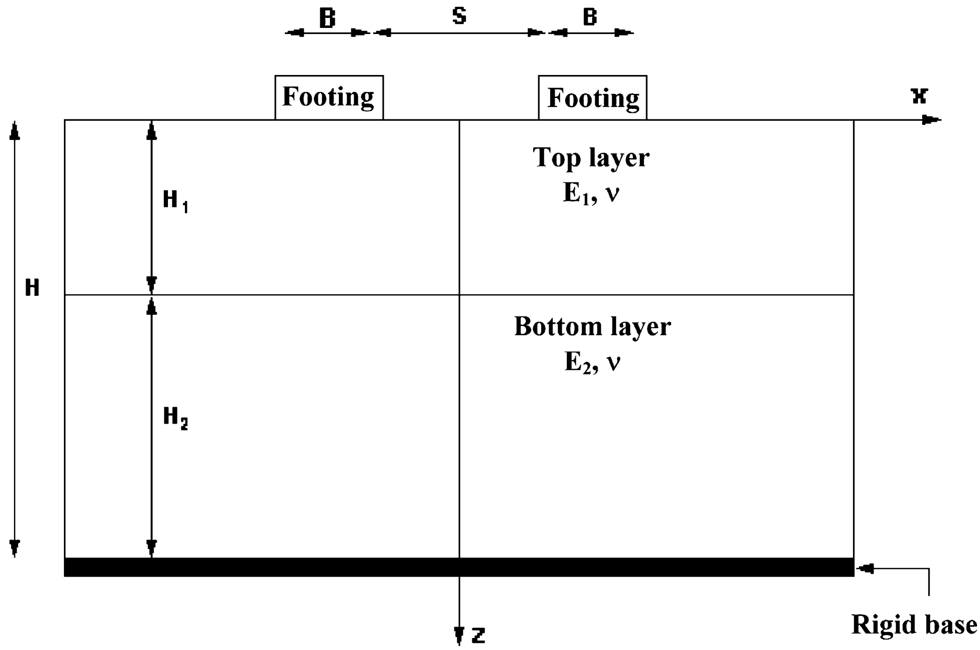

Fig. 1 Failure domain and different parameters

linearly elastic and the applied loads too are assumed to be such that the soil does not reach the plastic state as proposed by Maheswari and Viladkar [20]. In Fig. 1, S represents the clear spacing between the footings.

3 Analysis

The theory of elasticity is adopted to derive the governing differential equations under plane strain condition. The equilibrium conditions under plane strain case are given by the following equations:

$$\frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{xz}}{\partial z} = 0 \tag{1a}$$

$$\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \sigma_{zz}}{\partial z} = 0 \tag{1b}$$

The stress–displacement relations can be established from the generalised Hooke's law and are expressed as follows:

$$\sigma_{xx} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\frac{\partial u}{\partial x} + \nu\frac{\partial w}{\partial z}\right] \tag{2a}$$

$$\sigma_{zz} = \frac{E}{(1+\nu)(1-2\nu)}\left[\nu\frac{\partial u}{\partial x} + (1-\nu)\frac{\partial w}{\partial z}\right] \tag{2b}$$

$$\tau_{xz} = G\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right] \tag{2c}$$

where u and w are the displacements along x and z directions, respectively. Combining Eqs. 1 and 2, the governing differential equations are obtained as follows:

$$G\nabla^2 w + \frac{G}{(1-2\nu)}\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right] = 0 \tag{3a}$$

$$G\nabla^2 u + \frac{G}{(1-2\nu)}\frac{\partial}{\partial x}\left[\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}\right] = 0 \tag{3b}$$

It is worthy to mention here that the different values of Poisson's ratio for different layers could be easily incorporated just by modifying the governing equations. The discretised failure domain along with the boundary and continuity conditions is shown in Fig. 2. Due to the symmetry along the centre line of domain, only half of the domain is considered in the analysis. For the single isolated footing, half of the footing is considered where the centre line of the footing coincides with that of the full domain. However, in case of interference, the full width of the footing is included as shown in Fig. 2. The stresses and displacements approach zero as we move away from the footing, i.e., along the extreme boundaries (ST and UT).

3.1 Boundary and continuity conditions

In the present analysis, the following boundary conditions are employed (Fig. 2) along different boundaries.

Along ground surface on the left of footing (PQ) [$x < 0.5S$ and $z = 0$]: Along PQ, which is away from the loaded area, no normal or shear stresses act.

$$\sigma_{zz} = 0;\ \tau_{xz} = 0 \tag{4}$$

Along footing base (QR) [$0.5S \le x \le (0.5S + B)$ and $z = 0$]: Along QR, which lies below the loaded area, the

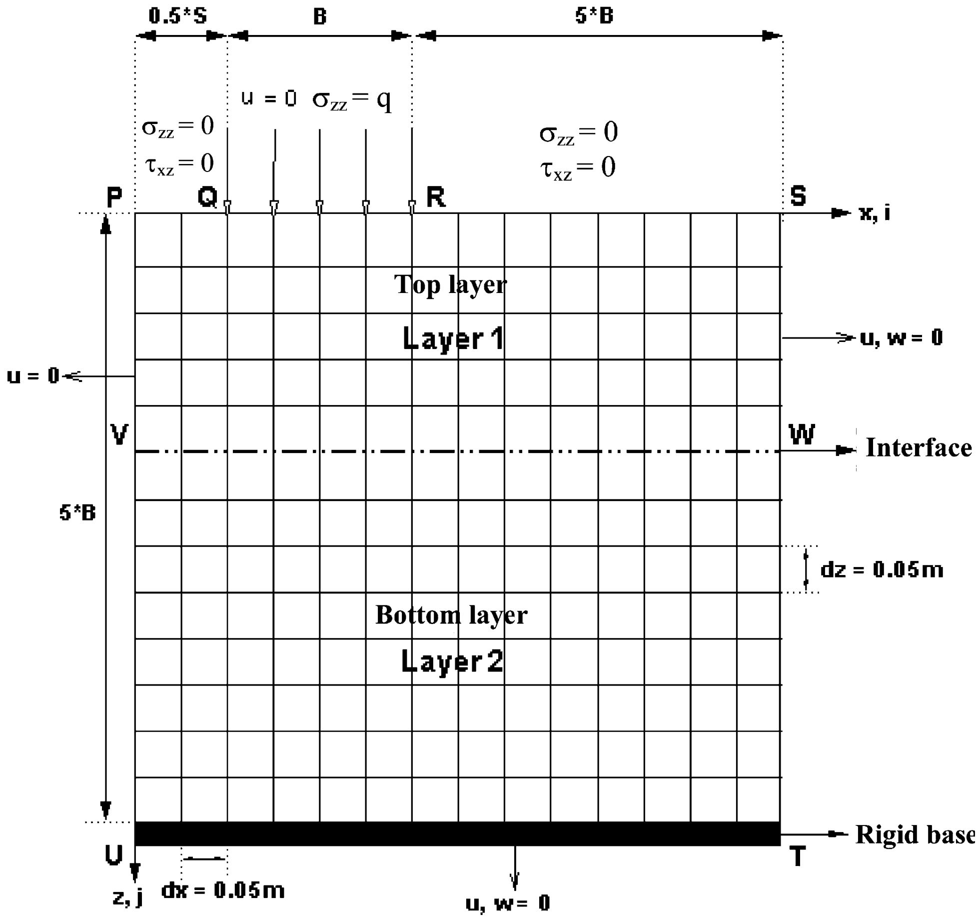

Fig. 2 Discretised failure domain with boundary and continuity conditions

normal stress is equal to the applied loading intensity. In this region, the lateral displacements of the soil particles are restricted to simulate the perfectly rough condition below the footing.

$$\sigma_{zz} = q; \quad u = 0 \tag{5}$$

Along ground surface on the right of footing (RS) [x > (0.5S + B) and z = 0]: The loading condition along RS is essentially the same as that along PQ as no surcharge is considered along the ground surface. Thus, the boundary conditions specified in Eq. 4 are applicable to this region as well.

Along the right extreme boundary of domain (ST) [x = (0.5S + B + 5B) and for all z]: Along the right extreme boundary ST, the displacements along both x and z directions are considered to be equal to zero since it is far away from the loaded area, and therefore, it is assumed to have no effect due to the application of load on the foundation.

$$u = 0; \quad w = 0 \tag{6}$$

Along the bottom boundary of domain (UT) [for all x and z = H]: The bottom most boundary UT represents the fixed base prevented from movement in all directions; hence, the displacements along boundary UT are also zero in both directions as given in Eq. 6.

Along the centre line of domain (PU) [x = 0 and for all z]: Along the centre line of failure domain (PU), only the vertical displacements are allowed and hence

$$u = 0 \tag{7}$$

The continuity of normal as well as shear stresses is satisfied at the interface of two layers (VW) and is given by

$$\sigma_{zz}^{(1,2)} = \sigma_{zz}^{(2,1)} \quad \text{and} \quad \tau_{xz}^{(1,2)} = \tau_{xz}^{(2,1)} \tag{8}$$

In Eq. 8, the expression implies that the normal stress σ_{zz} and shear stress τ_{xz} along the interface VW have the same values irrespective of it is approached either from top to bottom $\left(\sigma_{zz}^{(1,2)}, \tau_{xz}^{(1,2)}\right)$or from bottom to top $\left(\sigma_{zz}^{(2,1)}, \tau_{xz}^{(2,1)}\right)$, where 1 and 2 stand for top and bottom layers, respectively.

The governing differential equations in Eq. 3 can be derived in terms of displacements as it is rather difficult to solve them in terms of stresses. Since the interest lies in the settlement behaviour of footings under interference, this formulation is found to be ideal as the solution of equations directly gives the displacements at various points in the domain. This also shows the necessity of expressing the boundary and continuity conditions, as given in Eqs. 4–8 in terms of displacements to ensure the compatibility. Therefore, the boundary conditions can be expressed as

Along PQ (0 < x < 0.5S, z = 0)

$$\sigma_{zz} = \frac{E_1}{(1+v)(1-2v)}\left[v\frac{\partial u}{\partial x} + (1-v)\frac{\partial w}{\partial z}\right] = 0 \quad (9a)$$

$$\tau_{xz} = G_1\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right] = 0 \quad (9b)$$

Along QR (0.5S ≤ x ≤ (0.5S + B), z = 0)

$$\sigma_{zz} = \frac{E_1}{(1+v)(1-2v)}\left[v\frac{\partial u}{\partial x} + (1-v)\frac{\partial w}{\partial z}\right] = q \quad (10a)$$

$$\tau_{xz} = G_1\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right] = 0 \quad (10b)$$

Along RS (x > (0.5S + B), z = 0)

$$\sigma_{zz} = \frac{E_1}{(1+v)(1-2v)}\left[v\frac{\partial u}{\partial x} + (1-v)\frac{\partial w}{\partial z}\right] = 0 \quad (11a)$$

$$\tau_{xz} = G_1\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right] = 0 \quad (11b)$$

Similarly, at the layer interface VW, the continuity conditions in terms of displacements can be written as follows:

$$\frac{E_1}{(1+v)(1-2v)}\left[v\frac{\partial u}{\partial x} + (1-v)\frac{\partial w}{\partial z}\right] = \frac{E_2}{(1+v)(1-2v)}\left[v\frac{\partial u}{\partial x} + (1-v)\frac{\partial w}{\partial z}\right] \quad (12)$$

$$G_1\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right] = G_2\left[\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right] \quad (13)$$

3.2 Finite difference formulation

The governing differential equations given in Eq. 3 are coupled partial differential equations, and therefore, it is rather difficult to solve them analytically. The finite difference formulation is therefore chosen to solve these equations numerically. The governing differential equations as well as the boundary and continuity conditions can be expressed in the finite difference form [20]. The central difference scheme is adopted in the present analysis since it approximates the derivative on both sides of the point under consideration and is consequently found to be more accurate. Considering a general node i, j in the failure domain, the governing differential equation for displacement in the x-direction can be expressed as follows:

$$\left[\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\mathrm{d}x)^2} + \frac{u_{i,j+1} - 2u_{i,j} + u_{i,j-1}}{(\mathrm{d}z)^2}\right] + \frac{1}{(1-2v)} \times \left[\frac{u_{i+1,j} - 2u_{i,j} + u_{i-1,j}}{(\mathrm{d}x)^2} + \frac{w_{i+1,j+1} - w_{i+1,j-1} - w_{i-1,j+1} + w_{i-1,j-1}}{8\mathrm{d}x\mathrm{d}z}\right] = 0 \quad (14)$$

The expression given in Eq. 14 can be simplified by assuming $\mathrm{d}x = \mathrm{d}z$, i.e., the finite difference nodes are equally spaced in both x and z directions. Based on this simplification and after re-arranging the terms in Eq. 14, the following expression for the displacement along x-direction can be obtained, which is applicable at each node in the domain.

$$u_{i,j} = \frac{(1-v)}{(3-4v)}\left(u_{i-1,j} + u_{i+1,j}\right) + \frac{(1-2v)}{2(3-4v)}\left(u_{i,j-1} + u_{i,j+1}\right) + \frac{1}{8(3-4v)}\left(w_{i+1,j+1} - w_{i+1,j-1} - w_{i-1,j+1} + w_{i-1,j-1}\right) \quad (15)$$

Similarly, the displacement along z-direction can be derived as follows:

$$w_{i,j} = \frac{(1-v)}{(3-4v)}\left(w_{i,j-1} + w_{i,j+1}\right) + \frac{(1-2v)}{2(3-4v)}\left(w_{i-1,j} + w_{i+1,j}\right) + \frac{1}{8(3-4v)}\left(u_{i+1,j+1} - u_{i+1,j-1} - u_{i-1,j+1} + u_{i-1,j-1}\right) \quad (16)$$

The boundary conditions can also be obtained in the finite difference form and are expressed for the following cases [25].

For nodes that are just below the footing base (QR)

$$w_{i,j-1} = \left[\frac{2q(1+v)(1-2v)\mathrm{d}x}{E_1(1-v)}\right] + \frac{v}{(1-v)}\left(u_{i+1,j} - u_{i-1,j}\right) + w_{i,j+1} \quad (17)$$

For nodes along the ground surface (PQ and RS), which are outside the footing base

$$w_{i,j-1} = \frac{v}{(1-v)}\left(u_{i+1,j} - u_{i-1,j}\right) + w_{i,j+1} \quad (18)$$

For all nodes, except where lateral displacement is explicitly specified

$$u_{i,j-1} = \left(w_{i+1,j} - w_{i-1,j}\right) + u_{i,j+1} \quad (19)$$

At the interface of two layers, the continuity of normal stresses gives [3]

$$E_1\left(\frac{w_{i,j-1}-w_{i,j}}{dz}\right)=E_2\left(\frac{w_{i,j}-w_{i,j+1}}{dz}\right)$$

or,

$$E_2\left(\frac{w_{i,j}-w_{i,j+1}}{dz}\right)=E_1\left(\frac{w_{i,j}-w'_{i,j+1}}{dz}\right)$$

or,

$$w'_{i,j+1}=w_{i,j}-\frac{E_2}{E_1}(w_{i,j}-w_{i,j+1}) \tag{20}$$

Similarly, the continuity of shear stresses at the interface gives

$$E_2\left(\frac{u_{i,j}-u_{i,j+1}}{dz}+\frac{w_{i+1,j}-w_{i,j}}{dx}\right)$$
$$=E_1\left(\frac{u_{i,j}-u'_{i,j+1}}{dz}+\frac{w_{i+1,j}-w_{i,j}}{dx}\right)$$

or,

$$u'_{i,j+1}=u_{i,j}-\frac{E_2}{E_1}(u_{i,j}-u_{i,j+1})+\left(1-\frac{E_2}{E_1}\right)(w_{i+1,j}-w_{i,j}) \tag{21}$$

where, $u'_{i,j+1}$ and $w'_{i,j+1}$ are the equivalent horizontal and vertical displacements, respectively. The equivalent displacements are the displacements of nodes lying at the interface. Equations 15 and 16 can be solved along with Eqs. 17–21 using the finite difference technique, and the displacements can be obtained at each node of the finite difference mesh [1].

3.3 Sensitivity analysis for optimum mesh and domain size

As mentioned earlier, the governing differential Eq. 3 can be solved using the finite difference method, where the final results are mostly dependent on the number of grid points in the mesh and eventually the size of mesh and domain. A detailed study has been carried out to check the sensitivity of the obtained results to the change in these parameters. The optimum value of various geometric parameters required to define the finite difference mesh can be obtained by trial and error such that, beyond which there is no significant variation in the results. It is important to note that the sensitivity analysis has been conducted exclusively for each specific case to obtain the optimum value of different geometric parameters of the failure domain. However, the optimum values of these geometric parameters are found to be almost same for all the cases. For the sake of illustration, a few of those sensitivity analyses are reported here. In Fig. 3, the vertical settlement under the centre of the footing is plotted by varying the mesh size from 0.5 to 0.025 m for $q = 0.25$ MN/m, $B = 1$ m, $E_2/E_1 = 3.0$, $H_1/H = 0.8$ and $S/B = 1.0$. At mesh size smaller than 0.05 m, no significant variation in the results can be observed, and hence, a mesh size of 0.05 m is considered as the optimum mesh size in the present analysis. In Fig. 4, the domain width is varied from 4 to 8 times the footing width (B), and the vertical settlement below the centre of the footing is obtained. It can be seen that there is practically no change in the results beyond the domain width of $5B$ and, therefore, the optimum domain width is adopted as $5B$. In Fig. 5, the domain depth is varied from $4B$ to $7B$ and the vertical settlement below the centre of the footing is determined. It can be noted that there is a little change in the magnitude of settlement below the centre of the

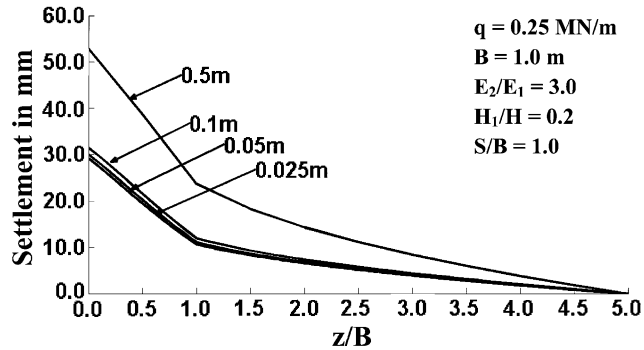

Fig. 3 Sensitivity analysis for optimum mesh size

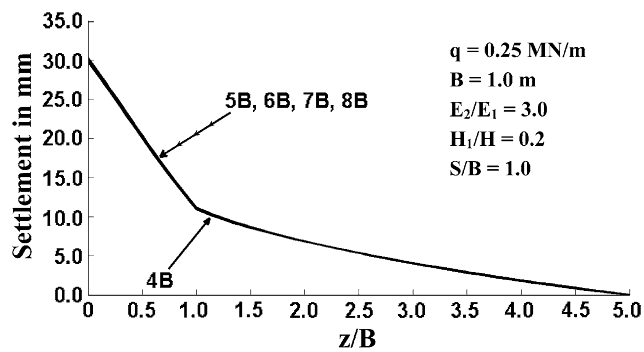

Fig. 4 Sensitivity analysis for optimum domain width

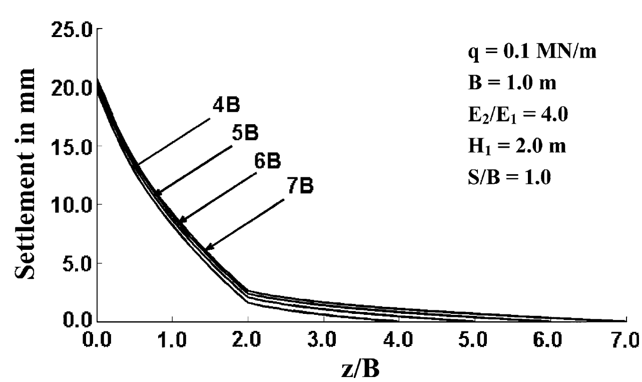

Fig. 5 Sensitivity analysis for optimum domain depth

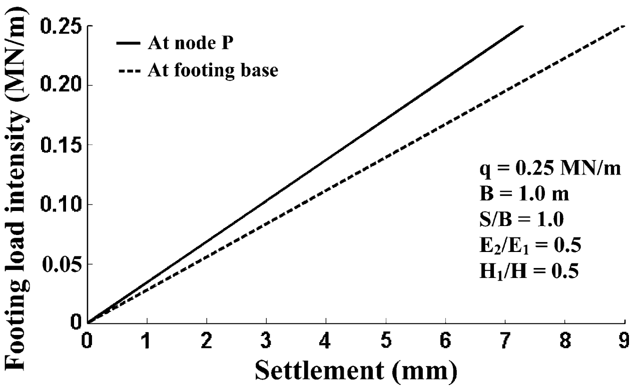

Fig. 6 Variation of load-settlement curves for $q = 0.25$ MN/m, $B = 1.0$ m, $S/B = 1.0$, $E_2/E_1 = 0.5$ and $H_1/H = 0.5$

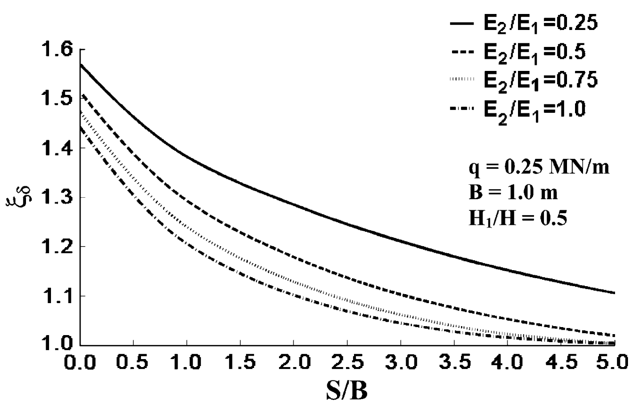

Fig. 7 Variation of ξ_δ with S/B for different E_2/E_1 with $q = 0.25$ MN/m, $B = 1.0$ m and $H_1/H = 0.5$

footing beyond domain depth of 5B. Thus, the optimum domain depth is chosen as 5B. All the cases presented in this paper have been, therefore, analysed with the domain size of 5B along both x and z directions. Figure 2 shows the discretised soil system along with the domain size and mesh size that are obtained from the sensitivity analysis.

4 Results and discussion

The parameters chosen to study the variation of settlement ratio (ξ_δ) are the relative elastic moduli and relative thicknesses of two soil layers, and footing load; where the settlement ratio (ξ_δ) is defined as the ratio of the vertical settlement of the single footing in presence of the other footing to that of the single isolated footing of the same size. The settlement within the failure domain occurs under the linear elastic mode for different combinations of the elastic modulus of soil and the applied load intensity on the footing. Figure 6 shows the variation of settlement with the applied load intensity at node P (Fig. 2) and just below the footing for $q = 0.25$ MN/m, $B = 1.0$ m, $S/B = 1.0$, $E_2/E_1 = 0.5$ and $H_1/H = 0.5$. It can be observed that the settlement at the node P is higher than that at the base of the footing, which indicates that the interference of footings causes higher settlement in the bed than that happens for the single isolated footing. The response of closely spaced footings on the single layer soil deposit is also explored and presented in this paper.

Figure 7 shows the variation of settlement ratio with S/B in two-layer system for different magnitudes of E_2/E_1 with $q = 0.25$ MN/m, $B = 1.0$ m and $H_1/H = 0.5$. In Fig. 7, the elastic modulus of the top layer is kept constant as 80 MPa and that of the bottom layer is varied from 20 to 80 MPa to obtain different magnitude of E_2/E_1 varying from 0.25 to 1.0. It can be noticed that ξ_δ decreases with increase in S/B and E_2/E_1. However, the actual settlement of interfering footings can be obtained by multiplying ξ_δ with the settlement of single isolated footing (δ_v) as given in Table 1. It is worth mentioning here that the magnitude of different parameters has been taken from the data reported by Maheswari and Viladkar [20] so as to obtain the elastic settlement of closely spaced footings. The variation of ξ_δ with S/B for different magnitudes of H_1/H with $B = 1.0$ m is shown in Fig. 8. In Fig. 8, the ratio of elastic modulus of the bottom layer to that of top layer (E_2/E_1) is kept as 0.5, and the footing load (q) is considered as 0.25 MN/m. The thickness of the top stronger layer is varied from 2 to 4 m to obtain different magnitudes of H_1/H (0.2, 0.5, 0.6 and 0.8). It can be seen that the magnitude of ξ_δ decreases with the increase in S/B and H_1/H. The variation of settlement ratio with S/B for different values of q with $B = 1.0$ m, $E_2/E_1 = 0.5$ and $H_1/H = 0.5$ is shown in Fig. 9. The footing load is varied from 0.15 to

Table 1 Settlement of single isolated footing on two-layer soil deposit

Parameters	δ_v (mm)			
$q = 0.25$ MN/m, $B = 1.0$ m, $H_1/H = 0.5$	$E_2/E_1 = 0.25$	$E_2/E_1 = 0.50$	$E_2/E_1 = 0.75$	$E_2/E_1 = 1.00$
	7.83	6.86	6.23	5.78
$q = 0.25$ MN/m, $B = 1.0$ m, $E_2/E_1 = 0.5$	$H_1/H = 0.40$	$H_1/H = 0.50$	$H_1/H = 0.60$	$H_1/H = 0.80$
	7.23	6.86	6.55	6.03
$B = 1.0$ m, $E_2/E_1 = 0.5$, $H_1/H = 0.5$	$q = 0.15$ MN/m	$q = 0.20$ MN/m	$q = 0.25$ MN/m	$q = 0.30$ MN/m
	4.12	5.49	6.86	8.24

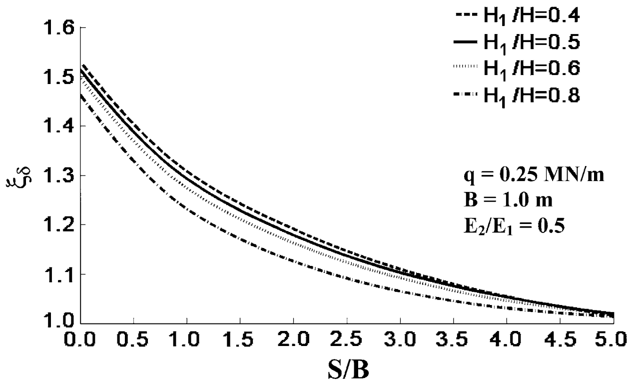

Fig. 8 Variation of ξ_δ with *S/B* for different H_1/H with $q = 0.25$ MN/m, $B = 1.0$ m, $E_2/E_1 = 0.5$

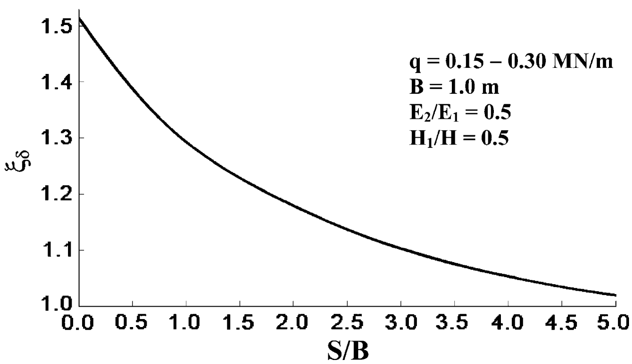

Fig. 9 Variation of ξ_δ with *S/B* for different *q* with $B = 1.0$ m, $E_2/E_1 = 0.5$ and $H_1/H = 0.5$

0.3 MN/m. It can be observed that there is no difference in the magnitude of ξ_δ for different values of *q* though a significant difference in the magnitude of settlement of closely spaced footings is noticed with the variation in *q*. The variation of ξ_δ with *S/B* for single layer system with different values elastic modulus (*E*) and load intensity (*q*) is shown in Fig. 10. The single layer system is considered as a special case of two-layer system, wherein both the layers have identical properties. It can be observed that although the settlement of footing varies for different values of *E* and *q* (Table 2), the value of ξ_δ remains constant for different *S/B*. The variation of normalised vertical normal stress (σ_{zz}/q) below the interface as well as at the base of the domain for different values of *S/B* is shown in Fig. 11. It can be noted, as the spacing between the footings increases, the vertical normal stress level reduces both at the interface and base. It can also be seen that the number of peaks in the vertical normal stress distribution becomes two from a single peak as the spacing between the footings increases.

The results presented in this paper predict the settlement behaviour of closely spaced footings on strong soil layer underlying a weak layer. Figure 7 shows that the settlement characteristics of interfering footing can be improved by

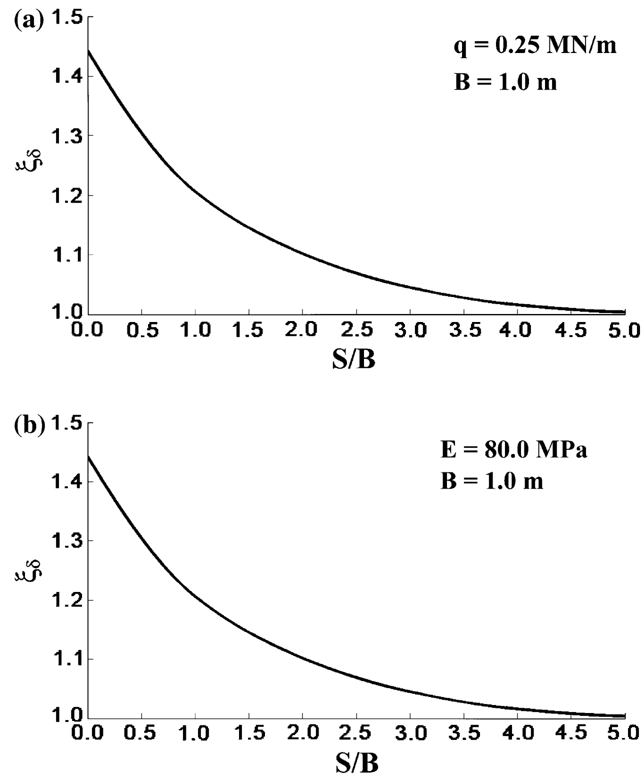

Fig. 10 Variation of ξ_δ with *S/B* for single layer with **a** different *E*, **b** different *q*

Table 2 Settlement of single isolated footing on single layer soil deposit

Parameters	δ_v (mm)	
$q = 0.25$ MN/m, $B = 1.0$ m	$E = 30$ MPa	$E = 120$ MPa
	15.42	3.86
$E = 80$ MPa, $B = 1.0$ m	$q = 0.15$ MN/m	$q = 0.20$ MN/m
	3.47	4.63

improving the properties of weak bottom layer. However, as the spacing between the footings increases, the vertical settlement of the strip footing approaches that of the single isolated footing and eventually, the value of ξ_δ becomes 1.0 at greater spacing. It is worth noting here that the vertical settlement of single isolated footing on two-layer bed is found to be exactly same as those reported by Maheswari and Viladkar [20]. From Figs. 7, 8, 9, 10 and 11, it can be clearly observed that the settlement as well as the vertical normal stress at different locations of the failure domain reduces with increase in the spacing between the footings.

5 Comparison

Schmertmann [23], and Schmertmann and Hartman [24] proposed the elastic settlement of flexible as well as rigid

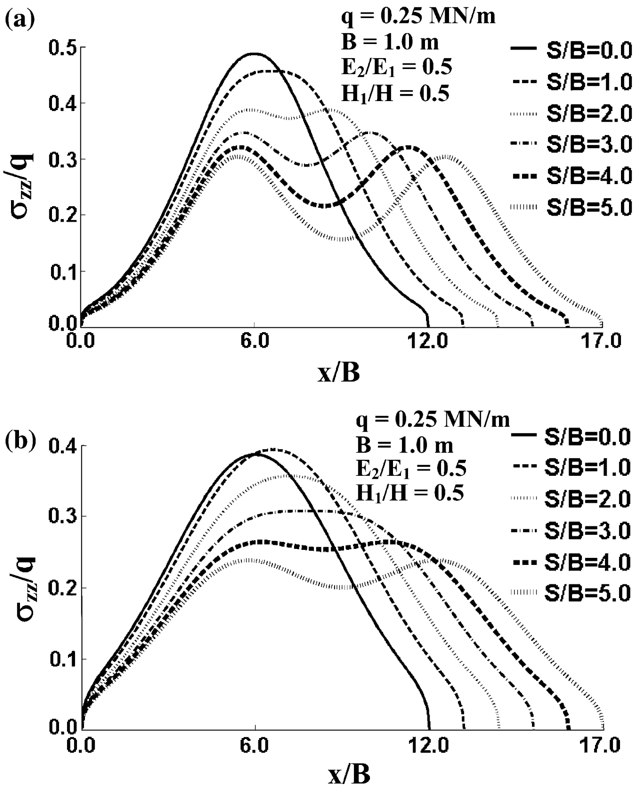

Fig. 11 Variation of normal stress for different *S*/*B* **a** below interface, **b** at base

isolated strip footing on granular soil using strain influence factor. For the comparison purpose, the settlement of the isolated footing on three-layer soil bed has been obtained as 24.12 mm using the present analysis for $q = 0.1$ MN/m, $B = 1.0$ m, $E_1 = 10$ MPa, $E_m = 40$ MPa, $E_2 = 10$ MPa, $H_1 = 1.5$ m, $H_m = 0.5$ m and $H_2 = 8$ m, whereas the settlement obtained from the theory reported by Schmertmann [23], and Schmertmann and Hartman [24] is found as 26.25 mm. In Fig. 12, the present values of settlement of single isolated footing on three-layer soil deposit are

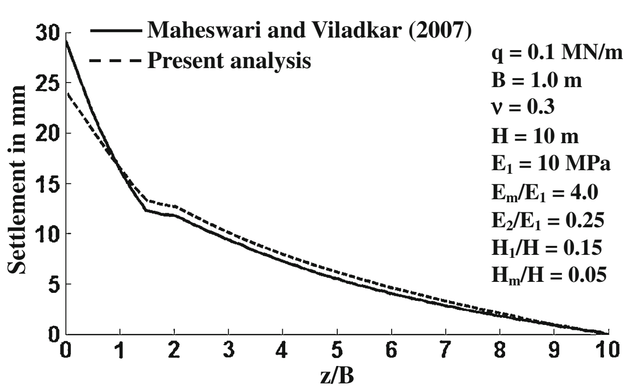

Fig. 12 Comparison of settlement of single isolated footing on three-layer system

compared with those reported by Maheswari and Viladkar [20] for $E_1 = 10$ MPa. It can be observed that there exists a marginal difference between the present values and the values given by Maheswari and Viladkar [20].

Very few research works on closely spaced strip footings on layered cohesionless soil bed are available in the literature [5, 12]. However, the present values of efficiency factor (ξ_δ) on layered system could not be compared with any of the available research works due to the lack of direct match in different soil parameters.

6 Conclusion

In the present study, a numerical finite difference analysis is performed to determine the settlement characteristics of an isolated as well as two nearby rough strip footings on two-layer bed with strong top layer underlying a weak bottom layer. The depth of top layer is varied by varying the value of H_1/H. It can be noted that the settlement of two nearby strip footings is greater than that of single isolated footing of the same size and placed on the soil having same properties. The settlement of closely spaced footings reduces continuously as the spacing between the footings increases and eventually becomes equal to that of a single isolated footing at larger spacing, which indicates the behaviour of single isolated footing free from any interference effect. The settlement of two interfering strip footings resting on layered soil can be reduced by improving the properties of the weak bottom layer. For closely spaced strip footings resting on layered soil, the vertical normal stress at the interface between two layers as well as at the base of the domain reduces as the spacing between the footings increases. From this investigation, it is quite clear that the closely spaced footings experience much higher settlement than that of single isolated footing, which may lead to catastrophic failure if not given due consideration.

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