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SHORT COMMUNICATION

Kinematics of shear bands

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Abstract Shear bands appear at limit states of soil bodies. They are analysed as thin zones of localised deformation that takes place as simple (i.e. dilatant) shear. It can be observed, however, that shear bands are discontinuous and also may be "reflected" at rigid boundaries. These phenomena appear as incompatible with the assumed shear deformation. The analysis in this paper reveals the kinematics of such "incompatibilities" in terms of continuous deformation fields.

Keywords Incompatible deformation · Riedel shear · Shear bands

1 Introduction

In soil mechanics, the most common limit states in terms of kinematics are represented by so-called rigid body failure mechanisms, i.e., a continuous body (soil, underground) is considered as being disintegrated into several parts that are rigid bodies or blocks. These blocks move relatively to each other along plane or circular–cylindrical surfaces. Coulomb's earth pressure theory and the conventional slip circle analysis are realisations of this concept. Thus, slip surfaces and slip lines are principal features of limit states (or, commonly, failures) and have since long been the subject of research in soil mechanics. They are closely related to a peculiar property of solids, i.e. the tendency of deformation to get localised into thin bands. Localisation and shear band formation constitute deviations from the initially homogeneous deformation of test specimens in the laboratory. Considering kinematics, the question arises whether slip surfaces are strong or weak discontinuities. The first one implies a jump of velocity, whereas the second one implies a jump of the velocity gradient, whereas the velocity field is continuous. It proves that this question is a matter of scale: what at first glance appears as a strong discontinuity proves in detail to be a weak discontinuity. Of course, strong discontinuities are still conceivable and possible in geomaterials. The interpretation of slip lines as weak discontinuities occurs on the basis of shear bands, i.e. 'thin' zones with simple shear (i.e. dilatant shear) in their interior. The thickness of shear bands is in laboratory experiments small, say ca 10 mean grain diameters for sand. In geological scales it can amount to several meters or more. Whether a shear band is discernible with naked eyes or not [4], it proves to be a useful concept, e.g. for the statical analysis of shear band formation, which is now also an important check to prove the quality of constitutive models. A proper mechanical analysis requires consistent kinematical concepts. Such concepts are based on observation but need not to be 100% realistic. However, they need to be 100% consistent and, therefore, they need to be simple.

2 Objectives

The analysis of shear bands as weak discontinuities considers a continuous velocity (or displacement) field, thus the equations of compatibility (i.e. continuity of displacement) are not violated. However, some apparently incompatible patterns of displacement may be observed: (1) shear bands are not straight but can be 'reflected' at rigid

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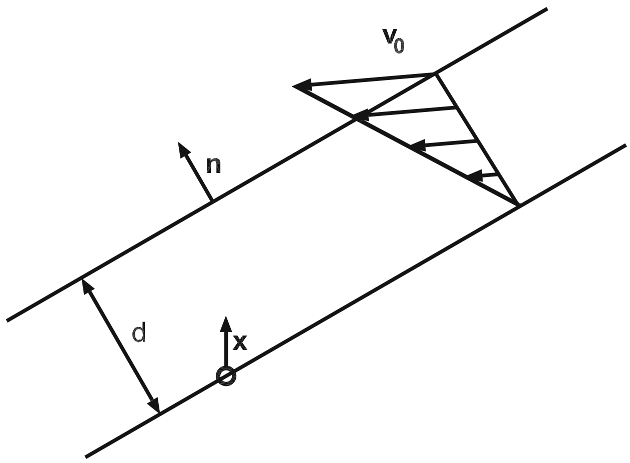

Fig. 1 Simple shear within a shear band of thickness d. ν is the angle of dilatancy

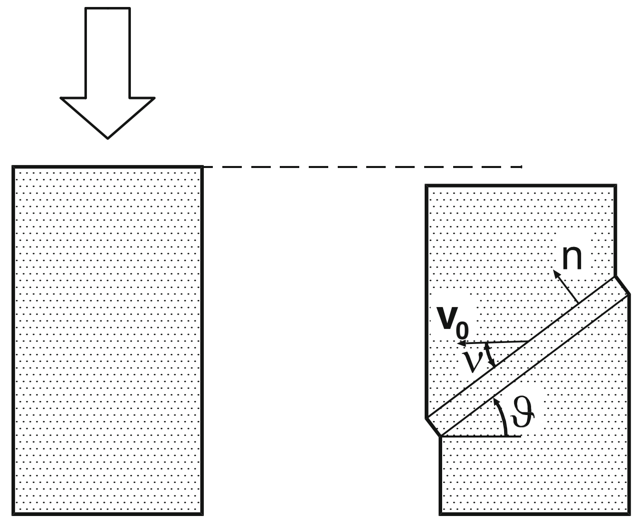

Fig. 2 Soil sample before and after shear-banding in plane (biaxial) deformation

boundaries and (2) shear bands are discontinuous, i.e. they end somewhere and continue at another place. Shear bands that end within the sample are similar (in case of strong discontinuities) to edge dislocations, which constitute a standard pattern of incompatible deformation. This paper aims to show that the aforementioned incompatibilities are only apparent (in terms of weak discontinuities), since they can be described in terms of continuous velocity fields.

3 Shear bands

Simple shear can be described by the stretching **D** given by the velocity $\mathbf{v}_0$, the shear band thickness d and the unit normal **n** (Fig. 1):

$$\begin{aligned} \mathbf{D} &= \frac{1}{2d}(\mathbf{v}_0 \otimes \mathbf{n} + \mathbf{n} \otimes \mathbf{v}_0) \\ D_{ij} &= \frac{1}{2d}\left(v_{0i}n_j + v_{0j}n_i\right) = \frac{1}{d}v_{0(i}n_{j)}. \end{aligned} \tag{1}$$

In the expression $v_{0(i}n_{j)}$ the lower parentheses indicate the symmetric part of the dyadic product $v_{0i}n_j$.

For the particular case of a biaxial compression test (Fig. 2), the author assumed the validity of Eq. 1 inside the shear band and a rigid body motion outside this band. This assumption is realistic [2, 11]. Using this assumption and the static condition $\dot{\mathbf{T}}\mathbf{n} = \mathbf{0}$ at the boundary of the shear band and the hypoplastic constitutive equation, he obtained a 'class A' prediction of the shear band formation (i.e. the strain and stress at which a shear band appears and its inclination) in sand [6, 7].[1] This prediction was realistic as compared with experimental results by Vardoulakis [12, 13]. However, in the present paper only the kinematics of shear bands, i.e. the pertinent velocity (or displacement) fields, will be further considered.

In Fig. 2, the thickness of the shear band is exaggerated, as in nature it amounts only to, say, ten times the mean grain diameter.

Considering the aforementioned kinematics of shear band, we observe that shear bands may 'hit' rigid boundaries of the considered soil specimen [9]. It can be observed that a so-called 'reflected' shear band emanates from the point of incidence on the boundary. Another observation is that shear bands may end somewhere in the specimen and continue at other places. Such configurations can be observed in the photographs shown in Figs. 3 and 4. These photographs show biaxial (i.e. plane strain) compression of reconstituted samples of kaolin clay. The test apparatus designed by the author, belongs to the type shown in Fig. 5b. To preserve visibility, the clay specimen is not placed into a rubber membrane. In direction 3 (see Fig. 5) a transparent perspex wall allows visual inspection of the sample, which is provided with coloured vertical lines that make shear bands discernible. The lateral cell pressure is applied by glycerine, which does not enter into the pores of the sample.

Reflexions and interruptions of shear bands constitute apparent violations of compatibility, i.e. of continuity of displacement. It should be noted that there is no general principle requiring the fulfilment of the compatibility condition, violations of which have been accepted since long, e.g. under the name 'dislocations'. Continuously distributed dislocations have been analysed as realisations of non-euclidean geometry of the deformation of continua, and applications to rock mechanics have been considered on this basis [8]. However, this paper deals with single shear bands and related dislocations.

[1] $\dot{\mathbf{T}}$ denotes the objective time rate of the Cauchy stress **T**.

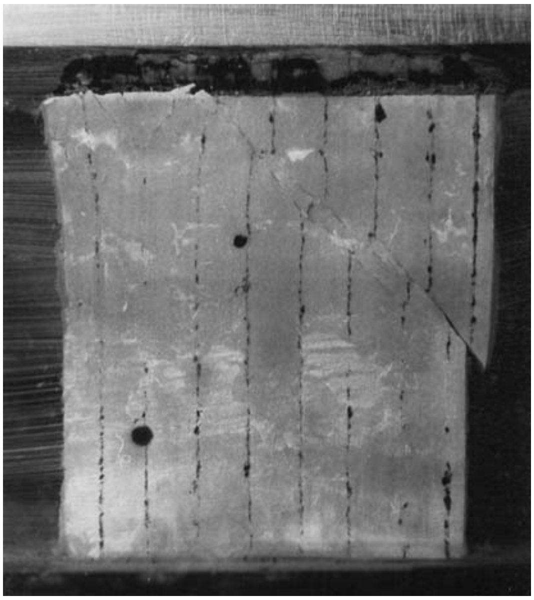

Fig. 3 Discontinuous shear bands in a clay sample compressed in plane strain [5]

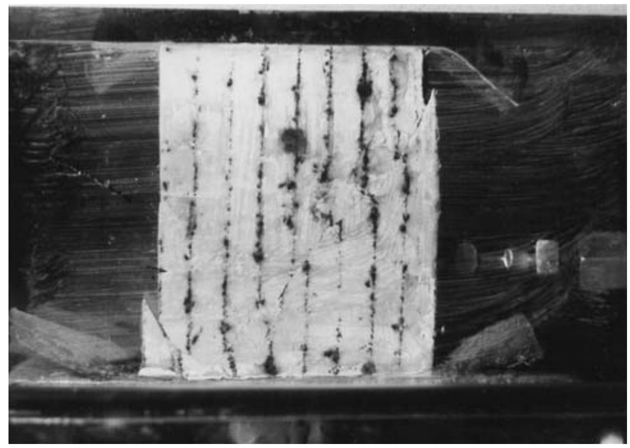

Fig. 4 Discontinuous shear bands in another clay sample compressed in plane strain [5]

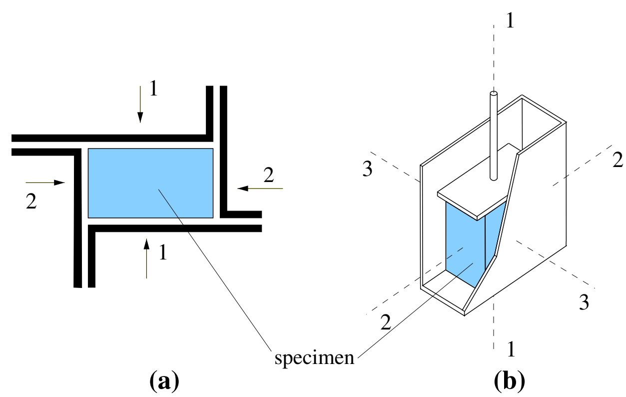

Fig. 5 Principles of biaxial deformation. *Left* Hambly-type. *Right* Cell with liquid pressure in direction 2

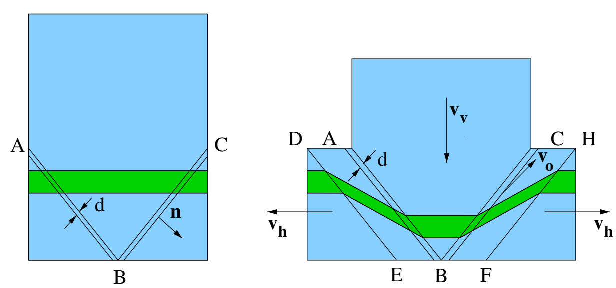

Fig. 6 Shear bands adjacent to rigid wall. The coloured strip indicates a material strip and is added just for illustration of the motion. Being initially horizontal (*left*), it obtains with deformation the form shown in the *right* part

4 Reflexion at rigid boundaries

It can be observed that a shear band 'hits' a rigid boundary at a location from where a somehow reflected shear band emanates (cf. Fig. 6; [3]). The underlying kinematics can be described with reference to Fig. 6. The boundaries of the shear bands AB and BC are not material surfaces (i.e. they do not permanently consist of the same material particles) but are stationary weak discontinuities. During deformation they are crossed by particles (material points). The shear bands have the thickness d and the velocity gradient $1/d\ \mathbf{v}_0 \otimes \mathbf{n}$ with $\mathbf{v}_0 = \mathbf{v}_h - \mathbf{v}_v$. Outside the shear bands, rigid motion takes place. The resulting motion appears paradoxical: the upper part of the sample has an apex at the lower rigid boundary and moves downwards with a constant velocity $\mathbf{v}_v$. Despite this motion, its apex at B remains permanently sharp! The explanation is that the apex B is not a material point. As a further consequence of the assumed motion, the thickness of the zones ABED and CBFH permanently increases.

5 Discontinuous shear bands

With respect to shear band kinematics, an apparent incompatibility is encountered at discontinuous or interrupted shear bands, as can be seen in the photographs of Figs. 3 and 4.

This incompatibility can be resolved if we consider a zone of rectilinear extension between two interrupted shear bands, as shown in Fig. 3. The interfaces of the shearing and stretching zones (sections AB and DC in Fig. 7) undergo a deformation that conforms to both motions. Within the zone of rectilinear extension, the stretching is

$$\mathbf{D} = \frac{v}{d} \sin \nu\, \mathbf{v}^0 \otimes \mathbf{v}^0,$$

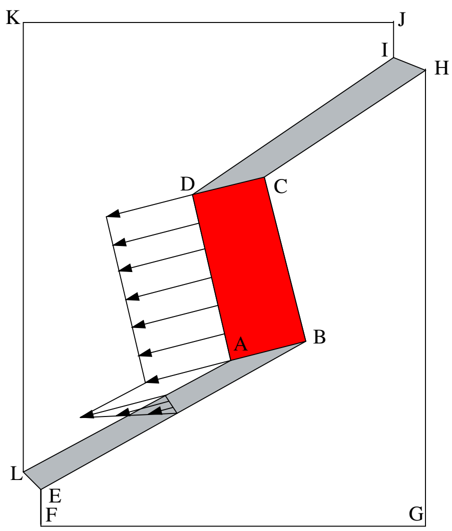

Fig. 7 Kinematics of interrupted shear band. The regions ALEB and DCHI are the two parts of an interrupted (or discontinuous) shear band. The parts KLADIJ and EFGHCB move as rigid bodies. In zone ABCD takes place a uniform extension. The bodies EFGH CB and ADIJKL do not deform. The stretching of the interfaces AB and DC conforms to simple shear *and* to rectilinear extension

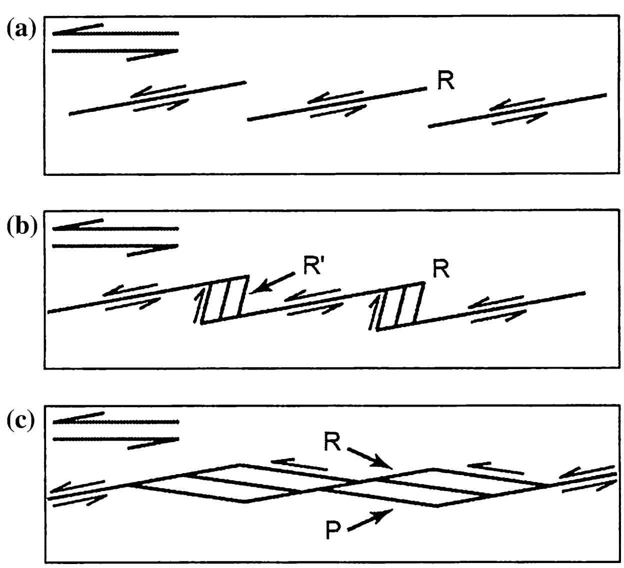

Fig. 8 Sequences of Riedel shear bands (from [1])

with ν being the angle of dilatancy within the shear band, $\mathbf{v}^0$ the unit vector in the direction of $\mathbf{v}$ and $v = |\mathbf{v}|$.

The length a of the stretched zone (i.e. the length of the sections AB and DC in Fig. 7) is given as

$$a = \frac{d}{\sin \nu}.$$

Clearly, $a = 0$ for $d = 0$ and $a \to \infty$ for $\nu \to 0$. In case of undrained clay shown in Figs. 3 and 4, we have $d \approx 0$ and $\nu \approx 0$. Hence, the length a is kinematically indeterminate. The width b (i.e. the length of AD and BC in Fig. 7) is also kinematically indeterminate and depends probably on the natural scatter of material properties. Note that the rectilinear extension of region ABCD (Fig. 7) is also prone to localisation of deformation and shear band formation. Such secondary shear bands do in fact appear and are known as Riedel bands. They have been first observed in clay cake models by Riedel and are also encountered in many other types of soil and rock. Skempton [10] reports on a tectonic shear zone in siltstone: "Over the greater part of its length the shear zone includes a pair of principal slip surfaces running parallel and less than 2-cm apart ...". The sequence of primary and secondary shear bands can be continued to produce a hierarchic system of scale-invariant and self similar patterns [1], see also Fig. 8.

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