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Preparation of multi-atom specially entangled W-class state and splitting quantum information

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We give a protocol to prepare specially entangled W-class state of multi-atom which can be used to exactly teleport an arbitrarily unknown two-level two-atom state. During the process, the quantum information is split into *n* parts and the original quantum information can be sent to anyone of the *n* recipients with the other *n*–1 recipients' collaboration. In addition, we will give a suggestion to realize this scheme via QED cavity.

quantum teleportation, W-class state, QED cavity

We know that quantum entanglement plays an important role in quantum communication and computation[1]. Teleportation[2–4], one of the quantum communication, has drawn much attention because of its novel feature and latent applied prospects. And most theoretical schemes for teleportation have been considered with different entangled states, such as composing of the Einstein-Podolsky-Rosen (EPR) pair as well as combination of EPR pairs[5–14] or GHZ state[15–17]. In 1998, Karlsson and Bourennane[18] firstly proposed a protocol for controlled teleportation with a tripartite GHZ state[19] as quantum channel. In their work, they showed that an arbitrary unknown state of a qubit could be teleported to either of two receivers conditioned on the measurement outcome of the other. After that, the researchers[20–23] studied more about the scheme. Then people began to think how to use W state[24] in quantum information. Shi and Tomita[25] first proposed a scheme for teleporting a single qubit by a tripartite W state, and Dai et al.[26,27] suggested another protocols to teleport multi-particle state, but their schemes are just feasible for probabilistically teleporting a single qubit or two-qubit entangled state, such as $\alpha|00\rangle+\beta|11\rangle$ or $\alpha|01\rangle+\beta|10\rangle$, and cannot be used to determinately teleport an arbitrary two-atom state. Recently, Zheng[28] described a procedure for faithfully teleporting a qubit state using W-class entangled states as the quantum channel. The scheme split quantum information into two or more parts so that if and only if the other agents cooperate, the original qubit state can be restored. However, the scheme in ref. [28] was just feasible for teleporting one qubit; in addition, the author did not tell us how to prepare partly entangled W-class state of multi-qubit.

In this article, we first present a protocol for preparation of a specially entangled W-class state of multi-atom, which is different from that of ref. [28]. Then we give a scheme for exactly teleporting an arbitrarily unknown two-level two-atom state using two multi-atom W-class states as quantum channels, and we will see that this kind of entangled state makes the scheme not only more robust against decoherence but also more efficient than those in refs. [18,20–23]. We split the quantum information into more parts, thus the original quantum state can be sent to any one with the other parties' cooperation, that is to say, this process is controlled by one or more parties.

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1 Preparation of multi-atom specially entangled W-class state

At first, we consider two-level atoms 1 and 2 simultaneously interacting with a single-mode cavity field. In the large detuning limit, there is no exchange between the atoms and cavity, and thus the effective Hamiltonian can be written as[29]

$$H_{\mathrm{e}} = H_0 + H_i, \tag{1}$$

with

$$H_0 = \lambda\left[\sum_{j=1}^{2}(|1\rangle_j\langle 1|aa^+ - |0\rangle_j\langle 0|a^\dagger a)\right], \tag{2}$$

and

$$H_i = \lambda(S_1^+S_2^- + S_1^-S_2^+), \tag{3}$$

where H_0 is the photon-number dependent Stark shifts, H_i is the dipole coupling between atoms induced by the cavity, and $\lambda=g^2/\delta$ is Rabi frequency, g is the atom-cavity coupling strength, $\delta=\omega_0-\omega$ is the detuning between atomic transition frequency ω_0 and cavity frequency ω, $S_j^+=|1\rangle\langle 0|$ and $S_j^-=|0\rangle\langle 1|$ are pseudo-spin operators of the jth atom, with $|0\rangle$ and $|1\rangle$ being the ground and excited states of atom, $a^\dagger$ and a are the creation and annihilation operators for the cavity mode. Since H_0 commutes with H_i, the evolution operator of the system can be described by

$$U(t) = \mathrm{e}^{-iH_0t}\mathrm{e}^{-iH_it}, \tag{4}$$

which will result in the following evolutions, respectively:

$$|1_10_2\rangle \to \mathrm{e}^{-i\lambda t}(\cos\lambda t\,|1_10_2\rangle - i\sin\lambda t\,|0_11_2\rangle), \tag{5a}$$

$$|0_11_2\rangle \to \mathrm{e}^{-i\lambda t}(\cos\lambda t\,|0_11_2\rangle - i\sin\lambda t\,|1_10_2\rangle), \tag{5b}$$

$$|0_10_2\rangle \to |0_10_2\rangle). \tag{5c}$$

Suppose we have $n-1$ identical single-mode cavity fields in empty states and $n-1$ microwave fields $M_j\,(j=1,2,\cdots n-1)$ which can accomplish the unitary transformation $|0\rangle\to i|0\rangle$ on atom j. Let us now consider n identical two-level atoms which are initially in the state $|0_1\cdots 0_{n-1}0_n\rangle$ (see Figure 1). We first let atoms 1 and 2 simultaneously interact with the first cavity field described above. With the choice of $\lambda t=\frac{\pi}{4}$, the untangled two-atom state will undergo a maximally entangled state

$$|1_10_2\rangle \to \frac{1}{\sqrt{2}}(|1_10_2\rangle - i\,|0_11_2\rangle), \tag{6}$$

with the cavity still in its original state. After that, atom 1 will fly through M_1, and we have

$$|1_10_2\rangle \to \frac{1}{\sqrt{2}}(|1_10_2\rangle + |0_11_2\rangle). \tag{7}$$

Then, atoms 2 and 3 pass through the second cavity. If we do the same things on the two atoms as that on atoms 1 and 2 discussed above, the two atoms will undergo the following evolutions:

$$\begin{aligned}&|1_20_3\rangle \to \frac{1}{\sqrt{2}}(|1_20_3\rangle + |0_21_3\rangle),\\ &|0_20_3\rangle \to |0_20_3\rangle.\end{aligned} \tag{8}$$

Thus a tripartite W-class state is obtained:

$$\frac{1}{\sqrt{2}}\left[|1_10_20_3\rangle + \frac{1}{\sqrt{2}}(|1_11_20_3\rangle + |1_10_21_3\rangle)\right], \tag{9}$$

where we discard some phase factors appearing in eq. (9). As we all know, these phase factors can be changed by local unitary operations, so they have no effect on our results.

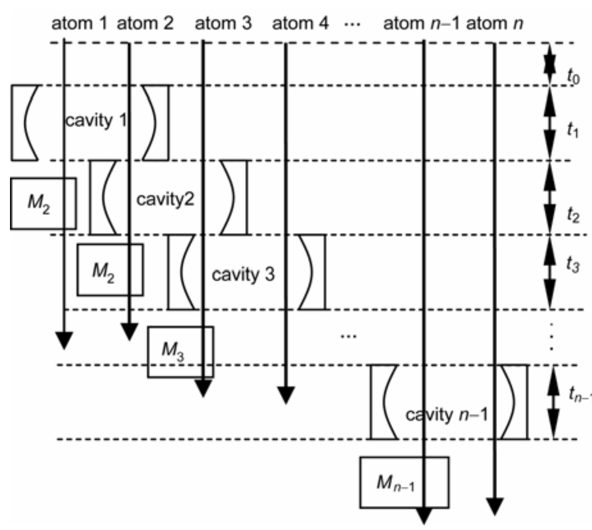

Figure 1 Schematic diagram of the interaction between the n atoms and the n–1 cavities. t_j denotes the interacting time between atoms and the jth cavity field and the arrows show the direction of atomic motion.

Similarly let the jth (j=4, 5, n−1) atom with the (j+1)th atom pass through the jth cavity field sequentially, and still let the interaction time satisfy $\lambda t_j=\pi/4$ and apply the operation $|0\rangle\to i|0\rangle$ on the jth atom with the aid of microwave field M_j, we will obtain a special W-class state of n-atom:

$$|\phi\rangle_{\{j\}}=\frac{1}{\sqrt{2}}\{|1_1 0_2 0_3\cdots 0_n\rangle+\frac{1}{\sqrt{2}}\Big[|0_1 1_2 0_3\cdots 0_n\rangle+\cdots$$
$$+\frac{1}{\sqrt{2}}(|0_1 0_2 0_3\cdots 1_{n-1}0_n\rangle+|0_1 0_2 0_3\cdots 0_{n-1}1_n\rangle)\Big]\},\quad (10)$$

where $\{j\}=\{1,2,\ldots n\}$ is a number sequence.

This kind of entangled n-atom W-class state has two peculiarities. On one hand, the state is composed of n–1 layers and every layer is normalized. For example, the last two layers in eq. (10) can be expressed as

$$\frac{1}{\sqrt{2}}\begin{bmatrix}|0_1 0_2 0_3\cdots 1_{n-2}0_{n-1}0_n\rangle\\ +\frac{1}{\sqrt{2}}|0_1 0_2 0_3\cdots 0_{n-2}\rangle(|1_{n-1}0_n\rangle+|0_{n-1}1_n\rangle)\end{bmatrix},\quad (11a)$$

$$\frac{1}{\sqrt{2}}|0_1 0_2 0_3\cdots 0_{n-2}\rangle\big(|1_{n-1}0_n\rangle+|0_{n-1}1_n\rangle\big),\quad (11b)$$

which are all normalized. On the other hand, if we perform a Von Neumann measurement on the first atom on the basis {0,1}, the others will be left into one of the following states with a equal probability:

$$|0_2 0_3\cdots 1_{n-2}0_{n-1}0_n\rangle \quad (12a)$$

or

$$\frac{1}{\sqrt{2}}\{|1_2 0_3 0_4\cdots 0_n\rangle+\frac{1}{\sqrt{2}}[|0_2 1_3 0_4\cdots 0_n\rangle+\cdots$$
$$+\frac{1}{\sqrt{2}}(|0_2 0_3 0_4\cdots 1_{n-1}0_n\rangle+|0_2 0_3 0_4\cdots 0_{n-1}1_n\rangle)]\}\quad (12b)$$

with the measurement result $|1\rangle_1$ or $|0\rangle_1$, respectively. It is the two particularities that make the teleportation deterministic rather than probabilistic[25–27].

2 Splitting quantum information via W-class state and multi-controlled teleportation

Suppose atoms 1 and 2 to be teleported initially in an unknown state

$$|\psi\rangle_{12}=(a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle_{12},\quad (13)$$

where $|a|^2+|b|^2+|c|^2+|d|^2=1$. Consider that Alice wants to send the information encoded in eq. (13) to either one of two recipients Bob and Charlie. Suppose they share two quantum channels expressed as

$$|\psi\rangle_{345}=(\frac{1}{2}|001\rangle+\frac{1}{2}|010\rangle+\frac{1}{\sqrt{2}}|100\rangle)_{345},\quad (14a)$$

$$|\psi\rangle_{678}=(\frac{1}{2}|001\rangle+\frac{1}{2}|010\rangle+\frac{1}{\sqrt{2}}|100\rangle)_{678},\quad (14b)$$

with the atom pair (3,6) belonging to the sender Alice and the rest two pairs (4,5), (7,8) belonging to Bob and Charlie, respectively. Initially the quantum state of the whole system, consisting of 8 atoms, reads

$$|\psi\rangle_{\text{tot}}=(a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle)_{12}$$
$$\otimes(\frac{1}{2}|001\rangle+\frac{1}{2}|010\rangle+\frac{1}{\sqrt{2}}|100\rangle)_{345}$$
$$\otimes(\frac{1}{2}|001\rangle+\frac{1}{2}|010\rangle+\frac{1}{\sqrt{2}}|100\rangle)_{678}.\quad (15)$$

Rewriting eq. (15) in terms of the four Bell states $|\phi^{\pm}\rangle=\frac{1}{\sqrt{2}}\big(|00\rangle\pm|11\rangle\big)$ and $|\psi^{\pm}\rangle=\frac{1}{\sqrt{2}}\big(|01\rangle\pm|10\rangle\big)$, we get

$$|\psi\rangle_{\text{tot}}=|\phi^{\pm}\rangle_{13}|\phi^{\pm}\rangle_{26}\Bigg[\frac{a}{2}(|01\rangle+|10\rangle)_{45}(|01\rangle+|10\rangle)_{78}+\gamma_{26}\frac{b}{\sqrt{2}}(|01\rangle+|10\rangle)_{45}|00\rangle_{78}$$
$$+\gamma_{13}\frac{c}{\sqrt{2}}|00\rangle_{45}(|01\rangle+|10\rangle)_{78}+\gamma_{13}\gamma_{26}d|0000\rangle_{4578}\Bigg]$$
$$+|\psi\pm\rangle_{13}|\psi\pm\rangle_{26}\Bigg[a|0000\rangle_{4578}+\gamma_{26}\frac{b}{\sqrt{2}}|00\rangle_{45}(|01\rangle+|10\rangle)_{78}$$
$$+\gamma_{13}\frac{c}{\sqrt{2}}(|01\rangle+|10\rangle)_{45}|00\rangle_{78}+\gamma_{13}\gamma_{26}\frac{d}{2}(|01\rangle+|10\rangle)_{45}(|01\rangle+|10\rangle)_{78}\Bigg]$$
$$+|\phi^{\pm}\rangle_{13}|\psi^{\pm}\rangle_{26}\Bigg[\frac{a}{\sqrt{2}}(|01\rangle+|10\rangle)_{45}|00\rangle_{78}+\gamma_{26}\frac{b}{2}(|01\rangle+|10\rangle)_{45}(|01\rangle+|10\rangle)_{78}$$
$$+\gamma_{13}c|0000\rangle_{4578}+\gamma_{13}\gamma_{26}\frac{d}{\sqrt{2}}|00\rangle_{45}(|01\rangle+|10\rangle)_{78}\Bigg]$$
$$+|\phi^{\pm}\rangle_{13}|\phi^{\pm}\rangle_{26}\Bigg[\frac{a}{\sqrt{2}}|00\rangle_{45}(|01\rangle+|10\rangle)_{78}+\gamma_{26}b|0000\rangle_{4578}$$
$$+\gamma_{13}\frac{c}{2}(|01\rangle+|10\rangle)_{45}(|01\rangle+|10\rangle)_{78}+\gamma_{13}\gamma_{26}\frac{d}{\sqrt{2}}(|01\rangle+|10\rangle)_{45}|00\rangle_{78}\Bigg],\quad (16)$$

where γ_{ij} denotes "±" corresponding to the Bell state's superscript φ_{ij}^{+} and φ_{ij}^{-} (or ψ_{ij}^{+} and ψ_{ij}^{-}). Alice performs a Bell state measurement on atom pairs (1,3) and (2,6), respectively. Assuming the measurement result is $|\varphi^{-}\rangle_{13}|\varphi^{+}\rangle_{26}$, the state of atoms 4, 5, 7 and 8 will collapse into the following state:

$$|\psi\rangle_{4578}=\frac{a}{2}(|01\rangle+|10\rangle)_{45}(|01\rangle+|10\rangle)_{78}+\frac{b}{\sqrt{2}}(|01\rangle+|10\rangle)_{45}|00\rangle_{78}$$
$$-\frac{c}{\sqrt{2}}|00\rangle_{45}(|01\rangle+|10\rangle)_{78}-d|0000\rangle_{4578}. \quad (17)$$

Obviously, the original two-qubit quantum information has been split into two parts carried by the atom pairs (4,5) and (7,8). In order to reconstruct the original two-atom state, Bob and Charlie must perform transformations

$$\frac{1}{\sqrt{2}}(|01\rangle+|10\rangle\rightarrow|01\rangle,$$
$$|00\rangle\rightarrow|00\rangle, \quad (18)$$

which leads to

$$|\psi\rangle_{4578}\rightarrow|00\rangle_{47}(a|11\rangle+b|10\rangle-c|01\rangle-d|00\rangle)_{58}. \quad (19)$$

Obviously, the original quantum information encoded in eq. (13) has been split into two parts held by Bob and Charlie, respectively. Neither of the two receivers can reincarnate the original state only by local operations on their own atoms. Assume that Alice and Charlie agree to let Bob possess the final information, then Alice informs Bob of her measurement result via classical communication channel and Charlie transmits his atom 8 to Bob. With this four-bit information at hand, Bob applies two rotations $i\sigma_y=(|0\rangle\langle 1|-|1\rangle\langle 0|)$ on atom 5 and $\sigma_x=(|0\rangle\langle 1|+|1\rangle\langle 0|)$ on atom 8, respectively. Now the states of atoms 5 and 8 are perfectly reincarnated into the initial state that Alice intends to send to him. It is evident that Bob must operate relevant transformations against Alice's different measurement results. Table 1 gives all Alice's different results and Bob's relevant operations. We can see that, in the process, Charlie works as a controller. Without his help, teleportation between Alice and Bob cannot be realized. Similarly, Charlie can become the final recipient in the same way controlled by Bob.

Let us now investigate the scheme for teleporting an arbitrary two-atom state to any of n recipients controlled by other n–1 recipients.

We use two W-class states prepared in Section 1 as quantum channels

$$|\chi\rangle_{\{2j+1\}}=\frac{1}{\sqrt{2}}\Big(|1\rangle_3|\varphi\rangle_{\{2k+3\}}+|0\rangle_3|\chi\rangle_{\{2k+3\}}\Big), \quad (20a)$$

Table 1 Alice's measurement results and Bob's corresponding operations. σ_j^i is an operation on atom i, I^i denotes the unit operation on the ith atom.

Alice's result	$\|\psi\rangle_{58}$	Bob's operation
$\|0_1 0_2 0_3 0_4\rangle$	$a\|11\rangle-b\|10\rangle+c\|01\rangle-d\|00\rangle$	$\sigma_y^5\sigma_y^8$
$\|0_1 0_2 0_3 1_4\rangle$	$a\|10\rangle-b\|11\rangle+c\|00\rangle-d\|01\rangle$	$\sigma_y^5 I^8$
$\|0_1 0_2 1_3 0_4\rangle$	$a\|01\rangle-b\|00\rangle+c\|11\rangle-d\|10\rangle$	$I^5\sigma_y^8$
$\|0_1 0_2 1_3 1_4\rangle$	$a\|00\rangle-b\|01\rangle+c\|10\rangle-d\|11\rangle$	$I^5 I^8$
$\|0_1 1_2 0_3 0_4\rangle$	$a\|10\rangle-b\|11\rangle+c\|00\rangle-d\|01\rangle$	$\sigma_y^5\sigma_z^8$
$\|0_1 1_2 0_3 1_4\rangle$	$a\|11\rangle-b\|10\rangle+c\|01\rangle-d\|00\rangle$	$\sigma_y^5\sigma_x^8$
$\|0_1 1_2 1_3 0_4\rangle$	$a\|00\rangle-b\|01\rangle+c\|10\rangle-d\|11\rangle$	$I^5\sigma_z^8$
$\|0_1 1_2 1_3 1_4\rangle$	$a\|01\rangle-b\|00\rangle+c\|11\rangle-d\|10\rangle$	$I^5\sigma_x^8$
$\|1_1 0_2 0_3 0_4\rangle$	$a\|01\rangle-b\|00\rangle+c\|11\rangle-d\|10\rangle$	$\sigma_z^5\sigma_y^8$
$\|1_1 0_2 0_3 1_4\rangle$	$a\|00\rangle-b\|01\rangle+c\|10\rangle-d\|11\rangle$	$\sigma_z^5 I^8$
$\|1_1 0_2 1_3 0_4\rangle$	$a\|11\rangle-b\|10\rangle+c\|01\rangle-d\|00\rangle$	$\sigma_x^5\sigma_y^8$
$\|1_1 0_2 1_3 1_4\rangle$	$a\|10\rangle-b\|11\rangle+c\|00\rangle-d\|01\rangle$	$\sigma_x^5 I^8$
$\|1_1 1_2 0_3 0_4\rangle$	$a\|00\rangle-b\|01\rangle+c\|10\rangle-d\|11\rangle$	$\sigma_z^5\sigma_z^8$
$\|1_1 1_2 0_3 1_4\rangle$	$a\|01\rangle-b\|00\rangle+c\|11\rangle-d\|10\rangle$	$\sigma_z^5\sigma_x^8$
$\|1_1 1_2 1_3 0_4\rangle$	$a\|10\rangle-b\|11\rangle+c\|00\rangle-d\|01\rangle$	$\sigma_x^5\sigma_z^8$
$\|1_1 1_2 1_3 1_4\rangle$	$a\|11\rangle-b\|10\rangle+c\|01\rangle-d\|00\rangle$	$\sigma_x^5\sigma_x^8$

$$|\chi\rangle_{\{2j+2\}}=\frac{1}{\sqrt{2}}\Big(|1\rangle_4|\varphi\rangle_{\{2k+4\}}+|0\rangle_4|\chi\rangle_{\{2k+4\}}\Big), \quad (20b)$$

where

$$|\chi\rangle_{\{2k+3\}}=\frac{1}{\sqrt{2}}\Big\{|1_5 0_7\cdots 0_{2n+3}\rangle+\frac{1}{\sqrt{2}}\big[|0_5 1_7\cdots 0_{2n+3}\rangle+\cdots+\frac{1}{\sqrt{2}}(|0_5\cdots 1_{2n+1}0_{2n+3}\rangle+|0_5\cdots 0_{2n+1}1_{2n+3}\rangle)\big]\Big\}, \quad (21a)$$

$$|\chi\rangle_{\{2k+4\}}=\frac{1}{\sqrt{2}}\Big\{|1_6 0_8\cdots 0_{2n+4}\rangle+\frac{1}{\sqrt{2}}\big[|0_6 1_8\cdots 0_{2n+4}\rangle+\cdots+\frac{1}{\sqrt{2}}(|0_6\cdots 1_{2n+2}0_{2n+4}\rangle+|0_6\cdots 0_{2n+2}1_{2n+4}\rangle)\big]\Big\}, \quad (21b)$$

$$|\varphi\rangle_{2k+3}=|0_5 0_7\cdots 0_{2n+3}\rangle, \quad (21c)$$

$$|\varphi\rangle_{2k+4}=|0_6 0_8\cdots 0_{2n+4}\rangle, \quad (21d)$$

with j, k=1, 2,…n, {2k+3}={5, 7,…2n+3} and {2k+4}={6, 8,…2n+4}. It is obvious that each state in eq. (21) is normalized, which is important to ensure that the teleportation is deterministic and faithful. We let the atoms 1, 2, 3 and 4 belong to the sender Alice, and the (2k+3)th and (2k+4)th atoms belong to the kth recipient. Initially the quantum state of the whole system, consisting of 2n+4 atoms, reads

$$|\psi\rangle_{\text{tot}}=|\psi\rangle_{12}\otimes|\chi\rangle_{\{2j+1\}}\otimes|\chi\rangle_{\{2j+2\}}. \quad (22)$$

Rewriting eq. (22) in terms of the four Bell states $|\varphi^{\pm}\rangle=\frac{1}{\sqrt{2}}(|00\rangle\pm|11\rangle)$ and $|\psi^{\pm}\rangle=\frac{1}{\sqrt{2}}(|01\rangle\pm|10\rangle)$, we get

$$\begin{aligned}|\psi\rangle_{\text{tot}}=\frac{1}{4}\Big[&|\phi^{\pm}\rangle_{13}|\phi^{\pm}\rangle_{24}(\alpha|\chi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}}\\&+\gamma_{24}b|\chi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}}\\&+\gamma_{13}c|\varphi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}}+\gamma_{13}\gamma_{24}d|\varphi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}})\\&+|\psi^{\pm}\rangle_{1,3}|\psi\pm\rangle_{2,4}(a|\varphi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}}\\&+\gamma_{24}b|\varphi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}}\\&+\gamma_{13}c|\chi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}}+\gamma_{24}b|\varphi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}})\\&+|\phi^{\pm}\rangle_{1,3}|\psi^{\pm}\rangle_{2,4}(a|\chi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}}\\&+\gamma_{24}b|\chi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}}\\&+\gamma_{13}c|\varphi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}}+\gamma_{13}\gamma_{24}d|\varphi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}})\\&+|\phi^{\pm}\rangle_{13}|\phi^{\pm}\rangle_{24}(\alpha|\varphi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}}\\&+\gamma_{24}b|\varphi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}}\\&+\gamma_{13}c|\chi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}}\\&+\gamma_{13}\gamma_{24}d|\chi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}})\Big],\end{aligned} \quad (23)$$

where γ_{ij} corresponds to the Bell state's superscript "±" of atom i and j.

As we all know that the joint Bell state measurement (BSM) is still a bottleneck problem, with the aid of QED cavity, the four Bell states can be converted into four unentangled states[28,29]

$$|\phi\pm\rangle\rightarrow\begin{cases}|11\rangle,\\ i|00\rangle,\end{cases}\qquad |\psi\pm\rangle\rightarrow\begin{cases}|01\rangle,\\ -i|10\rangle.\end{cases} \quad (24)$$

Thus Alice can achieve the joint BSM just by performing a Von Neumann measurement on her atoms 1, 2, 3 and 4, separately. Alice's measurement result will be one of the 16 basis {|0000}, |0001⟩,…|1111⟩ } with a certain probability of 1/16. Assume Alice's outcome is $|0_1 1_2 0_3 1_4\rangle$, then the rest atoms will collapse into the following state:

$$\begin{aligned}|\psi\rangle_{5,6,\cdots 2n+4}=\;&a|\chi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}}+b|\chi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}}\\&-c|\varphi\rangle_{\{2k+3\}}|\chi\rangle_{\{2k+4\}}-d|\varphi\rangle_{\{2k+3\}}|\varphi\rangle_{\{2k+4\}}.\end{aligned} \quad (25)$$

We can see that the original quantum information encoded in eq. (13) has been split into n parts held by all the recipients, respectively, and neither of them can reconstruct the original two-atom state by local operations on their own atoms. Assume Bob is the first recipient and the others agree to let him possess the final information, then the sender tells Bob the Bell state measurement result via classical communication channel and all the other recipients transmit their atoms to Bob. Now, Bob possesses all the atoms which carry the original information. After that, Bob applies the following unitary transformations on all his atoms:

$$|\chi\rangle_{\{2k+3\}}\rightarrow|1_5 0_7\cdots 0_{2n+3}\rangle, \quad (26a)$$

$$|\chi\rangle_{\{2k+4\}}\rightarrow|1_6 0_8\cdots 0_{2n+4}\rangle, \quad (26b)$$

$$|\varphi\rangle_{\{2k+3\}}\rightarrow|0_5 0_7\cdots 0_{2n+3}\rangle, \quad (26c)$$

$$|\varphi\rangle_{\{2k+4\}}\rightarrow|0_6 0_8\cdots 0_{2n+4}\rangle. \quad (26d)$$

As a result, the state of the Bob's whole atoms will be converted into

$$|\psi\rangle_{5,6,\cdots 2n+4}\rightarrow(a|1_5 1_6\rangle+b|1_5 0_6\rangle-c|0_5 1_6\rangle-d|0_5 0_6\rangle)|0_7 0_8\cdots 0_{2n+3}0_{2n+4}\rangle. \quad (27)$$

According to Alice's measurement result, Bob performs two relevant single-qubit unitary operations, $i\sigma_y = (|0\rangle\langle 1| - |1\rangle\langle 0|)$ on atom 5 and $\sigma_\chi = (|0\rangle\langle 1| + |1\rangle\langle 0|)$ on atom 6, respectively, thus the states of atoms 5 and 6 are perfectly reincarnated into the initial state that Alice intends to send to him. We can see that, in the process, the other $n-1$ recipients work as controllers. Without the collaboration from others, Bob can't successfully reconstruct the original quantum state that Alice intends to send. Similarly, any other recipient can become the final recipient in the same way.

It is evident that Bob must operate relevant transformations against Alice's different measurement results. Table 1 gives Alice's all different results and Bob's relevant operations.

3 Illustration of Bob's transformations

Let us now investigate on how to realize the transformations expressed in eq. (26) via cavity QED. We take eqs. (26a) and (26c) for example. To be convenient, we first discuss the case of n=3. Eqs. (21a) and (22c) will become

$$\chi\rangle_{\{2k+3\}} \rightarrow |\chi\rangle_{5,7,9} = \frac{1}{\sqrt{2}}\Big[|1_5 0_7 0_9\rangle + \frac{1}{\sqrt{2}}|0_5\rangle)(|1_7 0_9\rangle + |0_7 1_9\rangle)\Big], \quad (28a)$$

$$|\varphi\rangle_{\{2k+3\}} \rightarrow |\varphi\rangle_{5,7,9} = |0_5 0_7 0_9\rangle. \quad (28b)$$

After Alice's measurement on her atoms 3 and 4, to begin with, Bob applies an operation $|1\rangle \rightarrow i|1\rangle$ on the 9th atom, leading to

$$|\chi\rangle_{5,7,9} \rightarrow \frac{1}{\sqrt{2}}\Big[|1_5 0_7 0_9\rangle + \frac{1}{\sqrt{2}}|0_5\rangle(|1_7 0_9\rangle + i|0_7 1_9\rangle)\Big]. \quad (29)$$

Next, sending the last two atoms into the a single mode cavity described in Section 1 and letting the interaction time satisfy $\lambda t = \frac{\pi}{4}$, Bob will obtain the following evolutions:

$$\frac{1}{\sqrt{2}}(|1_7 0_9\rangle + i|0_7 1_9\rangle) \rightarrow |1_7 0_9\rangle), \quad (30a)$$

$$|0_7 0_9\rangle \rightarrow |0_7 0_9\rangle. \quad (30b)$$

Thus eq. (28a) will become

$$|\chi\rangle_{5,7,9} \rightarrow \frac{1}{\sqrt{2}}(|1_5 0_7\rangle + |0_5 1_7\rangle)|0_9\rangle, \quad (31)$$

with eq. (22b) having no change.

Then Bob does the same things on atoms 5 and 7 as those on atoms 5 and 9, and eq. (28) lastly evolves into

$$\frac{1}{\sqrt{2}}\Big[|1_5 0_7 0_9\rangle + \frac{1}{\sqrt{2}}|0_5\rangle(|1_7 0_9\rangle + |0_7 1_9\rangle)\Big] \rightarrow |1_5 0_7 0_9\rangle, \quad (32a)$$

$$|0_5 0_7 0_9\rangle \rightarrow |0_5 0_7 0_9\rangle. \quad (32b)$$

Similarly, if n is an arbitrarily positive integral, i.e. there are n recipients, we can still obtain the following evolutions:

$$\frac{1}{\sqrt{2}}\Big\{|1_5 0_7 \cdots 0_{2n+3}\rangle + \frac{1}{\sqrt{2}}\Big[|0_5 1_7 \cdots 0_{2n+3}\rangle + \cdots + |0_5 0_7 \cdots 0_{2n-1}\rangle + \frac{1}{\sqrt{2}}(|1_{2n+1} 0_{2n+3}\rangle |0_{2n+1} 1_{2n+3}\rangle)\Big]\Big\} \rightarrow |1_5 0_7 \cdots 0_{2n+3}\rangle, \quad (33a)$$

$$|0_5 0_7 \cdots 0_{2n+3}\rangle \rightarrow |0_5 0_7 \cdots 0_{2n+3}\rangle. \quad (33b)$$

In this way, the transformations expressed in eq. (20) can be realized.

4 Conclusions

We proposed a protocol to prepare a specially entangled W-class state of multi-atom which is used to exactly teleport an arbitrarily unknown two-level two-atom state. The distinct advantages of this kind of W-class states are obvious. For one thing, the W-class state is robust against decoherence, if one particle is traced out, there remains a large degree of entanglement which is still to be used for teleportation. For another, the multi-atom W-class state can not only make the teleportation process exact rather than probabilistic[25,27] but also implement the teleportation under the control of multi-party. Moreover, the successful probability of teleportation in the protocol presented here is 1.0 as well as that in the standard teleportation protocol[2]. In ref. [2], one EPR pair is used as the quantum channel and only one party can receive the unknown quantum state. And the original protocol involves the joint Bell-state-measurement (BSM) which is difficult to be realized experimentally[30]. On the contrary, the theoretical protocol presented here is free from direct BSM with the help of QED cavity and is feasible experimentally using current technologies. We now give a brief discussion on the experimental matters. For the Rydberg atoms with principal quantum numbers 49, 50 and 51, the radiative time is about $t_r = 3\times10^{-2}$ s, and the coupling constant is $g=2\pi \times 24$ kHz[31]. With the choice $\delta=10g$, the required atom-cavity-field interaction time is on the order of $\delta/g^2= 6.6\times10^{-5}$ s. If n=4, then the time needed to complete the whole procedure is 3.5×10^{-4} s, much shorter than $t_r=3\times10^{-2}$ s. A cavity with a quality factor Q=10^8 is experimentally achievable[31]. In the present case the cavity field frequency is about 50 GHz. The corresponding photon lifetime is $t_c=Q/2\pi v\simeq3.0\times10^{-4}$ s. In the present scheme, the cavity only has a small probability, about 0.01, of being excited during the passage of the atoms through the cavity. Thus the efficient decay time of the cavity is about 3.0×10^{-2} s, on the order of the atomic radiative time. Therefore, based on cavity QED techniques, the scheme proposed in this article might be realizable.

In addition, in this article, the specially entangled multi-atom W-class states are used as quantum channel instead of EPR pairs, which results in that the original quantum information encoded in unknown two-atom state is split into n parts and perfectly restored by any of the n recipients with the collaboration from the others.

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