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Entransy dissipation number and its application to heat exchanger performance evaluation

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Based on the concept of the entransy which characterizes heat transfer ability, a new heat exchanger performance evaluation criterion termed the entransy dissipation number is established. Our analysis shows that the decrease of the entransy dissipation number always increases the heat exchanger effectiveness for fixed heat capacity rate ratio. Therefore, the smaller the entransy dissipation number, the better the heat exchanger performance is. The entransy dissipation number in terms of the number of exchanger heat transfer units or heat capacity rate ratio correctly exhibits the global performance of the counter-, cross- and parallel-flow heat exchangers. In comparison with the heat exchanger performance evaluation criteria based on entropy generation, the entransy dissipation number demonstrates some distinct advantages. Furthermore, the entransy dissipation number reflects the degree of irreversibility caused by flow imbalance.

entransy, entransy dissipation number, entropy generation, heat exchanger effectiveness

With the sky-rocketing prices of petroleum and coal, how to use energy sources efficiently is one of the most effective ways to reduce energy demand. Heat exchanger as a device of energy utilization is widely applied to chemical process, petroleum, power, refrigeration, food processing, etc. Hence, it is of great importance to decrease the unnecessary energy dissipation and improve the performance of heat exchanger devices.

The performance evaluation criteria for heat exchanger are generally classified into two groups: the first is based on the first law of thermodynamics; the second is based on the second law of thermodynamics. In recent decades, the study of the second group has attracted a lot of attention[1]. However, the heat transfer occurring in heat exchangers usually involves turbulence, the heat conduction under finite temperature difference, the fluid friction under finite pressure drop and fluid mixing. These processes are characterized as irreversible non-equilibrium thermodynamic processes. Unfortunately, the irreversible non-equilibrium thermodynamics is still in its infancy. Therefore, there are still lots of unsolved problems in the investigation of the performance evaluation criteria and optimization designs of heat exchangers.

Based on the minimum entropy production principle advanced by Prigogine[2], Bejan[3,4] developed the entropy generation minimization approach to the heat exchanger optimization design. However, among all the variational principles in thermodynamics, Prigogine's minimum entropy generation principle is still the most debated one[5]. Accordingly the entropy generation minimization approach, widely applied to modeling and optimization of thermal systems that owe their thermodynamic imperfection to heat transfer, mass transfer, and fluid flow irreversibilities, demonstrates some inconsistencies and paradoxes in the applications of heat exchanger designs[6]. This is because the focus of the entropy generation minimization approach is on the heat-to-power conversion processes, while in heat ex-

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changer designs the rate and efficiency of heat transfer are more concerned. By analogy with the electrical conduction, Guo et al. defined a new physical concept, entransy, which describes the heat transfer ability[7,8]. Based on the entransy, heat transfer efficiency can be defined and the optimization design of heat exchanger can be discussed. It is found that in the irreversible processes the entransy is dissipated and the heat transport capability is reduced[9,10]. The more dissipation of the entransy implies the higher degree of irreversibility in heat transfer process. Thus the entransy dissipation may serve as a figure of merit for assessing the irreversibility of heat transfer processes.

When the entransy dissipation is applied to the performance evaluation and optimization design of the heat exchanger, it must be appropriately non-dimensionalised. Here, it is worth recalling that the non-dimensionalisation of the entropy generation was not trivial, in contrast it has played a pivotal role in the applications of the minimum entropy generation approach to heat exchanger designs[1,6]. In the present work, a dimensionless method of the entransy dissipation of the heat exchanger is established. Subsequently, a quantity called the entransy dissipation number of heat exchanger is defined and its physical meaning is discussed. The validity of the entransy dissipation number as the performance evaluation criterion of heat exchanger is examined. The comparison between the entransy dissipation number and entropy generation number is made.

1 Entransy dissipation theory

Based on the analogies between thermal and electrical conductions Guo et al.[7,8] defined the entransy as half of the product of heat capacity and temperature:

$$E_{\mathrm{h}} = \frac{1}{2} Q_{\mathrm{vh}} T = \frac{1}{2} UT \, , \tag{1}$$

where T is the temperature, Q_{vh} is the heat capacity at constant volume in general and the internal energy U for ideal gases in particular. For a heat exchanger with one-dimensional steady flows and under the usual assumptions such as no heat exchanging with environment, ignoring the change of kinetic and potential energy, and neglecting the longitudinal heat conduction, the energy balance equation of the hot fluid can be written as

$$C_{\mathrm{h}} \frac{dT_{\mathrm{h}}(x)}{dx} = -Q(x) \, , \tag{2}$$

where C_{h} is the heat capacity rate, $Q(x)$ is the heat transfer rate at x, $T_{\mathrm{h}}(x)$ is the temperature of the hot fluid at x. Multiplying both sides of eq. (2) by the hot fluid temperature $T_{\mathrm{h}}(x)$ and integrating with respect to the spatial variable x, yield the following entransy balance equation of the heat exchanger:

$$\frac{1}{2} C_{\mathrm{h}} T_{\mathrm{h,i}}^2 - \frac{1}{2} C_{\mathrm{h}} T_{\mathrm{h,o}}^2 = \int Q(x) T_{\mathrm{h}}(x) d\mathrm{x} \, , \tag{3}$$

where the subscript h refers to the hot fluid, the subscript i and o refer to the inlet and outlet of heat exchanger, respectively. Eq. (3) indicates the entransy transferred to the cold fluid from the hot fluid equals to the difference between the entransies at the inlet and outlet of the heat exchanger. Similarly, the entransy balance equation of the cold fluid is written as

$$\frac{1}{2} C_{\mathrm{c}} T_{\mathrm{c,i}}^2 + \int Q(x) T_{\mathrm{c}}(x) d\mathrm{x} = \frac{1}{2} C_{\mathrm{c}} T_{\mathrm{c,o}}^2 \, , \tag{4}$$

where the subscript c refers to the cold fluid. Eq. (4) indicates that the entransy of the cold fluid at the outlet of the heat exchanger equals to the sum of the entransy of the cold fluid at the inlet of the heat exchanger and the entransy absorbed from the hot fluid. Adding eq. (3) to eq. (4) and rearranging the resulting equation, we obtain

$$\begin{aligned} \Delta E = E_{\mathrm{inlet}} - E_{\mathrm{outlet}} &= \left[\frac{1}{2} C_{\mathrm{h}} T_{\mathrm{h,i}}^2 + \frac{1}{2} C_{\mathrm{c}} T_{\mathrm{c,i}}^2 \right] \\ &- \left[\frac{1}{2} C_{\mathrm{h}} T_{\mathrm{h,o}}^2 + \frac{1}{2} C_{\mathrm{c}} T_{\mathrm{c,o}}^2 \right] \\ &= \int Q(x)(T_{\mathrm{h}}(x) - T_{\mathrm{c}}(x)) d\mathrm{x} \, . \end{aligned} \tag{5}$$

For a one-dimensional steady heat conduction of the infinite plate with thickness d, if some heat Q is transferred from the end at $x=d$ with the high temperature T_{h} to the end at $x=0$ with the low temperature T_{c}, the entransy dissipation for this heat conduction process is given as follows[8]:

$$\int_0^d \phi_{\mathrm{h}} d\mathrm{x} = -\int_0^d Q \frac{dT}{d\mathrm{x}} d\mathrm{x} = Q \int_{T_{\mathrm{c}}}^{T_{\mathrm{h}}} dT = Q(T_{\mathrm{h}} - T_{\mathrm{c}}) \, , \tag{6}$$

where ϕ_{h} is the dissipation function. Inspired by this example, one may define the maximum entransy dissipation in a heat exchanger as $Q(T_{\mathrm{h,i}}-T_{\mathrm{c,i}})$, where Q is the actual heat transfer rate, $(T_{\mathrm{h,i}}-T_{\mathrm{c,i}})$ is the maximum temperature difference in the heat exchanger. By analogy with the definition of heat exchanger effectiveness ε, the entransy dissipation in heat exchanger can be non-dimensionalised by the maximum entransy dissipation,

$$\Delta E^* = \frac{\Delta E}{Q(T_{\rm h,i} - T_{\rm c,i})} = \frac{\left[\frac{1}{2}C_{\rm h}T_{\rm h,i}^2 + \frac{1}{2}C_{\rm c}T_{\rm c,i}^2\right] - \left[\frac{1}{2}C_{\rm h}T_{\rm h,o}^2 + \frac{1}{2}C_{\rm c}T_{\rm c,o}^2\right]}{\varepsilon C_{\min}(T_{\rm h,i} - T_{\rm c,i})^2}, \quad (7)$$

where ΔE^* is called the entransy dissipation number of heat exchanger and represents the ratio of actual entransy dissipation to maximum entransy dissipation in heat exchanger. When $C_{\rm h} < C_{\rm c}$, the application of the definition of the heat exchanger effectiveness leads to

$$\Delta E^* = \frac{(T_{\rm h,i} + T_{\rm h,o})(T_{\rm h,i} - T_{\rm h,o}) + (T_{\rm c,i} + T_{\rm c,o})(T_{\rm c,i} - T_{\rm c,o})/C^*}{2\varepsilon(T_{\rm h,i} - T_{\rm c,i})^2} = \frac{\varepsilon(T_{\rm h,i} - T_{\rm c,i})[2(T_{\rm h,i} - T_{\rm c,i}) - \varepsilon(T_{\rm h,i} - T_{\rm c,i})(1 + C^*)]}{2\varepsilon(T_{\rm h,i} - T_{\rm c,i})^2}, \quad (8)$$

where C^* is the heat capacity rate ratio. From Eq.(8) the entransy dissipation number finally arrives at

$$\Delta E^* = \frac{2 - \varepsilon(1 + C^*)}{2}. \quad (9)$$

When $C_{\rm h} \geqslant C_{\rm c}$, similar derivation leads to the same expression as eq. (9). Therefore, eq. (9) is the general expression of the entransy dissipation number for heat exchangers with two streams.

2 Entransy dissipation number and entropy generation number

2.1 Counter flow heat exchanger

For the counter flow arrangement, the heat exchanger effectiveness is[11,12]

$$\varepsilon = \frac{1 - \exp[-Ntu(1 - C^*)]}{1 - C^* \exp[-Ntu(1 - C^*)]}, \quad (10)$$

where *Ntu* is the number of exchanger heat transfer units. Substituting eq. (10) into eq. (9) yields the expression of entransy dissipation number for counter flow heat exchanger.

The variation of the entransy dissipation number with the number of exchanger heat transfer units and exchanger effectiveness for various heat capacity rate ratios are depicted in Figure 1 (a) and Figure 1 (b), respectively. Figure 1 (a) illustrates that the entransy dissipation number monotonously decreases with the increase of the number of exchanger heat transfer units for the fixed heat capacity rate ratio. With the increasing of the heat exchanger effectiveness the entransy dissipation number approximately linearly decays as shown in Figure 1(b). But when the effectiveness reaches its maximum value 1, the entransy dissipation number does not vanish. Therefore, the entransy dissipation number is not equal to zero unless C^*=1. The exceptional case corresponds to the balanced flow. For the imbalanced flow although the heat transfer area tends to the infinity, the flow imbalance still causes irreversibility in the heat exchanger. This kind of irreversibility is called the 'irreversibility due to flow imbalance' discussed by Bejan[4]. Figure 1 shows that the larger the heat capacity rate ratio, the smaller the entransy dissipation number is. Therefore to some extent the entransy dissipation number quantifies the irreversibility caused by flow imbalance.

The entropy generation number was defined as the ratio of the entropy generation to the larger heat capacity rate[11,12]. When the hot fluid heat capacity rate is smaller, the entropy generation number for the counter flow heat exchanger can be written as follows[6]:

$$N_{\rm s} = C^* \ln\left[1 - \varepsilon\left(1 - \frac{T_{{\rm c},i}}{T_{{\rm h},i}}\right)\right] + \ln\left[1 + C^*\varepsilon\left(\frac{T_{{\rm h},i}}{T_{{\rm c},i}} - 1\right)\right]. \quad (11)$$

From Eq. (11), it is evident that the entropy generation number is not only related to the heat exchanger effectiveness and heat capacity rate ratio, but also related to the inlet temperatures of both fluids. The variations of entropy generation number with the exchanger effectiveness and number of exchanger heat transfer units are displayed in Figure 2(a) and Figure 2(b), respectively.

From Figure 2(a), one can see that with the increasing of the exchanger effectiveness, the entropy generation number first increases, then decreases. The dependence of the entropy generation number on the number of heat transfer units demonstrates the similar behavior, as illustrated in Figure 2(b). Therefore, there exists a value of the exchanger effectiveness denoted as ε^*, at this point the entropy generation number achieves the maximum. From eq. (11), we obtain

$$\varepsilon^* = \frac{1}{1 + C^*}. \quad (12)$$

It can be proved that eq. (12) is still valid for $C_{\rm h}>C_{\rm c}$. From eq. (12), one can see that ε^* is only dependent on the heat capacity rate ratio. When C^*=1, namely the heat capacity rates of the hot and cold flow are the same, and then according to eq. (12), ε^*=0.5. In this case the de-

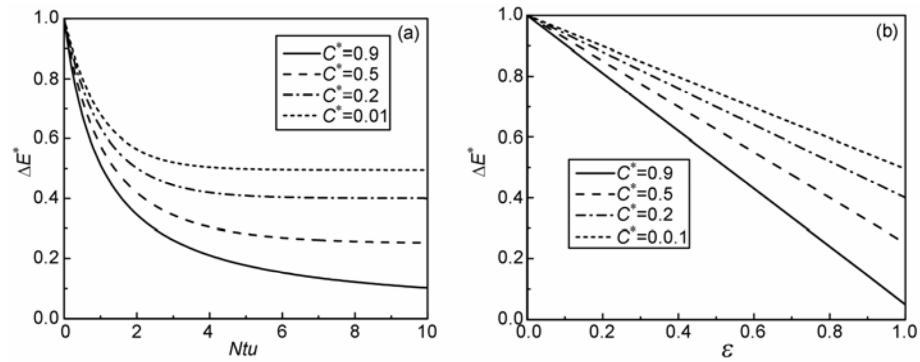

Figure 1 The entransy dissipation number for the counter flow heat exchanger. (a) Entransy dissipation number versus *Ntu*; (b) Entransy dissipation number versus ε.

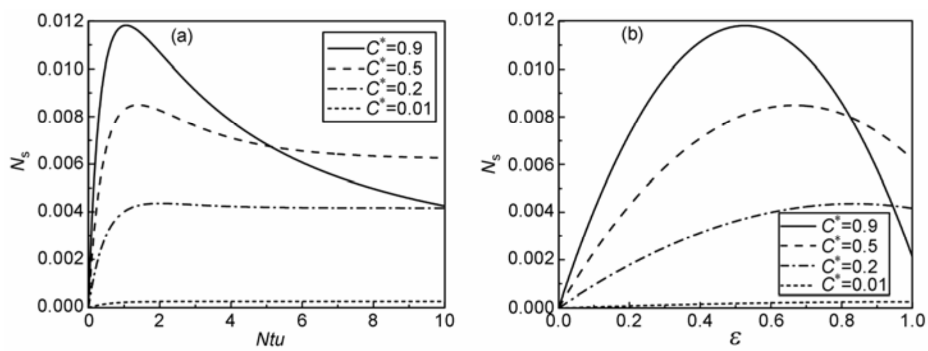

Figure 2 The entropy generation number for counter flow heat exchanger. (a) The entropy generation number versus ε ($T_{h,i}/T_{c,i}$=1.25); (b) the entropy generation number versus *Ntu* ($T_{h,i}/T_{c,i}$=1.25).

pendence of the entropy generation number on the exchanger effectiveness is illustrated in Figure 3.

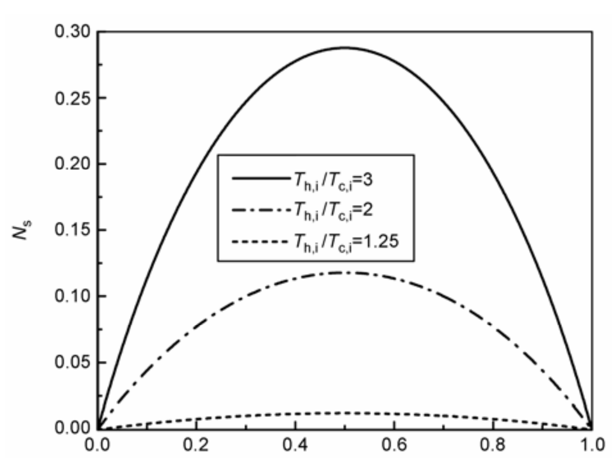

Figure 3 The dependence of the entropy generation number on the exchanger effectiveness for the balanced counter flow.

The thermodynamic analysis indicates that less entropy generation means less thermodynamic irreversibility and better performance of the heat exchanger. However, from Figsure 2 (a) and 3 one can see that the entropy generation number increases with the increasing of the heat exchanger effectiveness when the effectiveness is less than ε^*. This phenomenon is called the 'entropy generation paradox'[4]. As opposed to the entropy generation number the entransy dissipation number demonstrates the reasonable relationship with the exchanger effectiveness as shown in Figure 1 (b).

2.2 Parallel flow heat exchanger

For the parallel flow heat exchanger, the heat exchanger effectiveness is[11,12],

$$\varepsilon = \frac{1-\exp[-Ntu(1+C^*)]}{1+C^*}. \tag{13}$$

Then, substituting eq. (13) into eq. (9) gives rise to the expression of entransy dissipation number for parallel flow heat exchanger. The variation of the entransy dissipation number with the exchanger effectiveness is depicted in Figure 4(a) for various heat capacity rate ratios. Note that although the parallel and counter flow heat exchangers share the same formula for the entransy dissipation number, their achievable ranges of the heat ex-

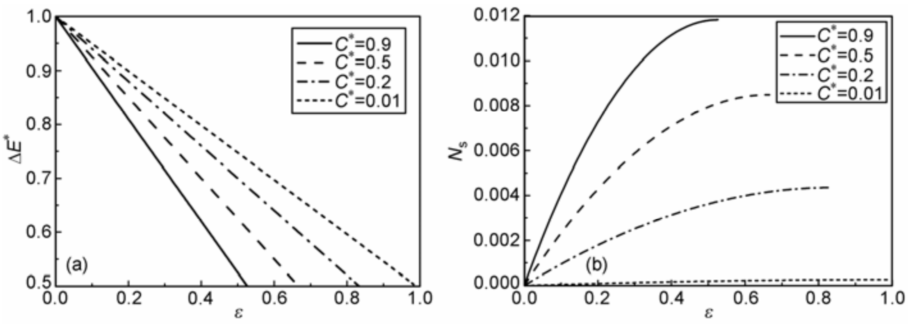

Figure 4 The entransy dissipation number and entropy generation number with the heat exchanger effectiveness for parallel flow heat exchanger ($T_{h,i}/T_{c,i}$=1.25). (a) The entransy dissipation number versus the effectiveness; (b) the entropy generation number versus the effectiveness.

changer effectiveness are different for the fixed heat capacity rate, as shown in Figsure 1(b) and 4(a). Figure 4(b) illustrates the variation of the entropy generation number with the exchanger effectiveness for various heat capacity rate ratios. From this figure one can see that the entropy generation number always increases with the increasing of the heat exchanger effectiveness. Thus the 'entropy generation paradox' is more prominent than the counter flow heat exchanger. Actually, in parallel flow heat exchanger the highest possible outlet temperature of the cold fluid is the same as the outlet temperature of the hot fluid in the ideal case. It is in this ideal case that the entropy generation number reaches its maximum value. For the counter flow heat exchanger, when the heat exchanger effectiveness increases and reaches certain value, the outlet temperature of the cold flow equals to the outlet temperature of the hot flow, and the entropy generation number achieves its maximum value. If the heat exchanger effectiveness increases further, the outlet temperature of the cold flow will exceed the outlet temperature of the hot flow, and the entropy generation number begins to decay.

3 Comparison between the modified entropy generation number and entransy dissipation number

As discussed above, the entropy generation number defined by Bejan suffers from the 'entropy generation paradox'. In an effort to resolve this paradox, a number of ways for non-dimensionalising the entropy generation are proposed, such as by Q/T_a (T_a is environmental temperature)[13] and $Q/T_{c,i}$[6]. In the present work, the latter is employed, namely, the entropy generation is non-dimensionalised as follows:

$$N_{s1} = \frac{\dot{S}_{gen} T_{c,i}}{Q}, \tag{14}$$

where N_{s1} is called the modified entropy generation number in the following discussion. The modified entropy generation number avoids the 'entropy generation paradox' and behaves in a more intuitively reasonable way[6]. But it is necessary to point out that the physical meaning of the modified entropy generation number is not definite. The modified entropy number N_{s1} was interpreted as the entropy generation under the unit heat transfer capacity[13]. However, the characteristic temperature in N_{s1} adds some uncertainty. Moreover, the inlet temperatures of fluids have more effect on N_{s1}. Thus when it is employed to assess the performance of different flow arrangements, some problems occur, as illustrated in Figure 5. Three curves in Figure 5 represent the modified entropy generation numbers of the counter flow arrangement with $T_{h,i}/T_{c,i}$=1.6, cross flow arrangement with $T_{h,i}/T_{c,i}$=1.5 and parallel flow ar-

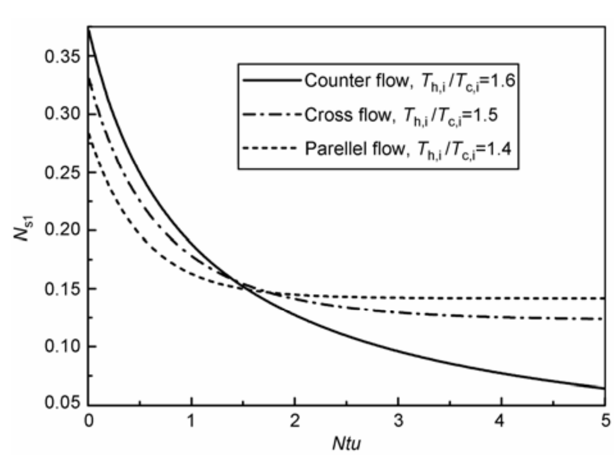

Figure 5 The modified entropy generation number versus the number of exchanger heat transfer units for different flow arrangements (C^*=0.9).

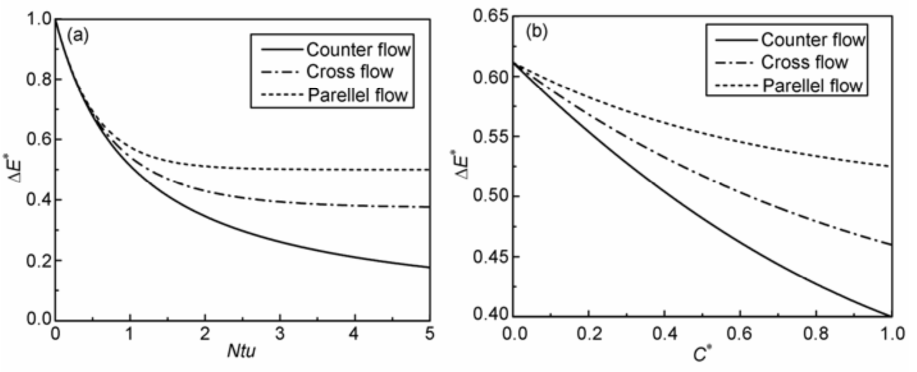

Figure 6 The entransy dissipation number for different flow arrangements. (a) The entransy dissipation number versus the number of exchanger transfer units (C^*=0.9); (b) the entransy dissipation number versus the heat capacity rate ratio (*Ntu*=1.5).

rangement with $T_{h,i}/T_{c,i}$=1.4 in terms of the number of exchanger heat transfer units. Among these three flow arrangements it is well known that the counter flow arrangement exhibits the best performance. However, this is only consistent with the last parts of the three curves where the number of exchanger heat transfer units is large; otherwise the counter flow arrangement is the worst one in terms of the modified entropy generation number. This is due to the sensitive dependence of the modified entropy generation number on the ratio of the inlet temperature of the hot fluid to the counterpart of the cold fluid. Therefore, even the modified entropy generation number is incapable of correctly evaluating the global performance of heat exchangers.

The variations of the entransy dissipation number with the number of exchanger heat transfer units and the heat capacity rate ratio are illustrated in Figure 6(a) and Figure 6(b), respectively. For both cases, one can clearly see that the counter flow arrangement is the best, the parallel flow arrangement is the worst and the cross flow arrangement is in between. Therefore, the entransy dissipation number correctly reflects the global performance of the heat exchangers and is competent for evaluating the heat exchanger performance.

4 Concluding remarks

In summary, the entransy dissipation number is introduced by non-dimensionalising the entransy dissipation of heat exchanger. It is found that when the entransy dissipation number decreases, the heat exchanger effectiveness always increases. In this respect, it demonstrates an obvious advantage in comparison with the entropy generation number which suffers from the so-called ‘entropy generation paradox’. For the parallel-, cross- and counter-flow heat exchangers, our analysis shows that the entransy dissipation number clearly and correctly demonstrates their global performances. Therefore, the entransy dissipation number can serve as a thermodynamic figure of merit for assessing heat exchanger performance.

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