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A new method for detecting line spectrum of ship-radiated noise using Duffing oscillator

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A detection scheme for line spectrum of ship-radiated noise is proposed using Duffing oscillator. The chaotic trajectory of Duffing oscillator is analyzed and the state equation of the system is improved to detect weak periodic signals in different frequencies. According to the simulation results, the phase transforms of Duffing oscillator are sensitive to periodic signals and immune to the random noise and the periodic interference signals which have larger angular frequency difference from the referential signal. By employing Lyapunov exponents in the field of detection as the criteria for chaos, the phase transforms of dynamic behaviors in quantity are successfully determined. Meanwhile, the threshold value in critical state has been evaluated more accurately. Based on the phase transforms of Duffing oscillator, a new method for detecting line spectrum of ship-radiated noise is given. Three types of ship-radiated noise signals are analyzed and the values of line spectrum are acquired successfully by this method. The experimental results show that this method has high sensitivity and high resolution.

Duffing oscillator, Lyapunov exponents, criteria for chaos, ship-radiated noise, line spectrum

Ship-radiated noise plays an important role in the ship performances including type, speed, tonnage and so on. Investigating the characteristics of ship-radiated noise is significant for national defense to design acoustic fuse of underwater weapon as well as to identify and attack enemy target effectively[1−3]. Ship-radiated noise is produced by various vibrations and sound sources from ship body, mainly coming from the driving system, airscrew, donkey engine, effect of hydrodynamic force or movement of ship, etc. The major part of the single-frequency noise (just line spectrum) is generated by airscrew. It is different in the frequency of line spectrum among ships that have different types or the same types but with different speeds. According to line spectrum of low frequency (lower than 100 Hz), which is more stable and powerful than continuous spectrum, abundant information indicates that it is appropriate for passive detection, identification as well as tracking of underwater targets because of its less loss and longer distance in underwater transmission. In addition, there are some uncertain elements of line spectrum caused by such items as the spectrum expansion, frequency shift and the fluctuation of amplitude and phase, which are induced by the influence of navigation status and the underwater transmission channels.

There are some limitations to the traditional signal processing of target detection based on the spectrum analysis and the random theory, especially to the farther targets or the weak signals. Over the past ten years, chaos theory was extensively utilized in different fields[4], such as medicine, bionomics, secret communication and electronic countermeasure. Particularly, the applications of chaotic oscillator in the weak signal detection, which

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have been developed quickly in recent years, become one of the hot issues in the contemporary nonlinear science[5−7]. In this paper, utilizing the to-be-detected signal as a perturbation of chaotic system, the state is forced to change essentially in the light of the sensitivity depending on the parameters of system. Then, the weak periodic signal under the background of strong noise can be detected by identifying the phase space trajectory. With the results from simulations and theoretical calculation, it is proved that there is a huge difference in the phase space trajectories between the chaotic state and the periodic state on a large scale, and this difference can be used as the evidence in the chaotic system for the detection of weak signal based on Duffing oscillator. Meanwhile, Lyapunov exponents are adopted as criteria for chaos in the field of detection, the method to determine the phase transform of dynamic behaviors in quantity is proposed and the threshold value in chaotic critical state (chaos, but on the verge of changing to the periodic state) is evaluated more accurately. From the above, a new method based on Duffing oscillator is put forward to detect line spectrum of ship-radiated noise. Results of analyzing real ship signals show that this method has high sensitivity and high resolution to ship-radiated noise.

1 Theory and methods

1.1 The establishment of the chaotic detection system using Duffing oscillator

The chaos state is one kind of specific states in some nonlinear dynamic systems. Although the chaotic movements are stochastic, the equations for their description are determinate, such as Duffing, Lorenz and Vandpul. The Duffing equation is one of the widely studied classic nonlinear systems consequently, here the system based on it is chosen.

The Duffing equation is the second order differential equation containing cubic item, which can be motivated by exterior stimulations to engender oscillation movement and then generate chaotic movement and periodic movement. Duffing-Holmes equation is in the form:

$$x''(t)+kx'(t)-x(t)+x^3(t)=F\cos(t), \qquad (1)$$

where $F\cos(t)$ is the forced periodic term in the equation (the reference signal is an internal signal in the system), and k denotes the rate of damping, and the term $-x(t)+x^3(t)$ is the nonlinear recovery force term of the equation. When the forced periodic $F\cos(t)$ is invariable, the kinematical state of the system is mainly determined by the equation's recovery force. Comprehensively considering various factors of the lowest limit of detecting weak signal, the SNR of chaotic detection system and the demonstration of the chaotic criterion[8], we transform the recovery force term into $-x^3(t)+x^5(t)$, and then get a modified Duffing-Holmes equation:

$$x''(t)+kx'(t)-x^3(t)+x^5(t)=F\cos(t). \qquad (2)$$

However, there are some limitations in the system by eq. (2): (1) The frequency of to-be-detected periodic signal must be $\omega=1$ rad/s; (2) The waveform should be the same as that of the reference signal $\cos(t)$.

To apply eq. (2) to detecting weak signals with different frequencies, we must do some frequency transforms. From eq. (2), defining $t=\omega\tau$, we obtain

$$\frac{\mathrm{d}^2x}{\mathrm{d}\tau^2}+k\omega\frac{\mathrm{d}x}{\mathrm{d}\tau}-\omega^2x^3+\omega^2x^5=\omega^2F\cos(\omega\tau). \qquad (3)$$

Since eq. (3) is derived from eq. (2), the system is seen on another time scale, and the dynamic properties and bifurcation behavior are not changed. The differences lie in the running speed, because x and x' now are ω times as large as before.

Inserting the coefficients c_1, c_2, c_3 and c_4 into eq. (2), we obtain

$$x''(t)+kc_1x'(t)-c_2x^3(t)+c_3x^5(t)=c_4F\cos(\omega t), \qquad (4)$$

where $c_1=\omega$ and $c_2=c_3=c_4=\omega^2$. Then eq. (4) is the same as eq. (2). Using eq. (4), the sinusoidal signal with random frequency but no periodic signal with random waveform could be detected.

Furthermore, the chaos inhibition scheme can be used to construct the chaotic detection system[11]. Adding a weak periodic perturbation item to the coefficient of x^5 in Duffing equation, the equation turns out to be

$$x''+k\omega x'-\omega^2x^3+\omega^2[1+as(\omega t)]x^5=\omega^2F\cos(\omega t), \qquad (5)$$

where $as(\omega t)$ is the to-be-detected weak periodic signal and $F\cos(\omega t)$ is the internal reference signal in the system. When $a=0$, the system itself has no parameter perturbation item, and it will be selected in the chaotic critical state. Here, by adding weak periodic perturbation into the nonlinear coefficient of x^5, a weak periodic signal can be detected by the phase space trajectories via a transition from the chaotic state to the large-scale

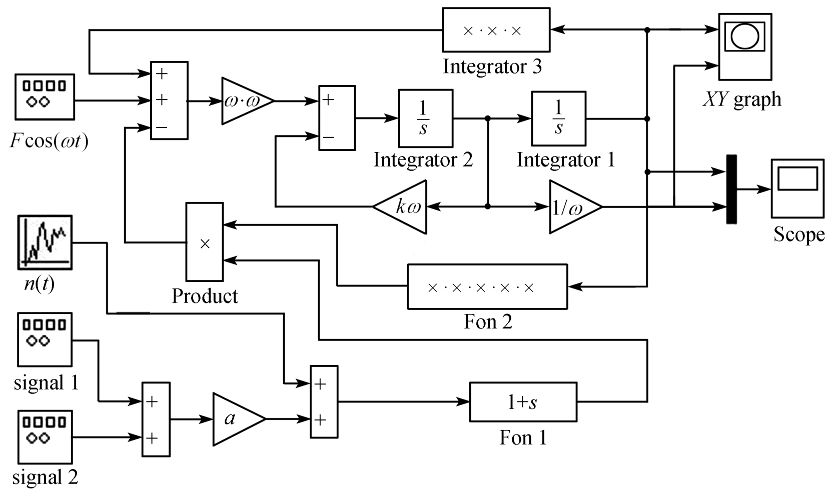

Figure 1 The system's simulation model.

periodic state. We rewrite eq. (5) in the dynamic form:

$$\begin{cases} x' = \omega v, \\ v' = \omega^2\{-kv + x^3 - [1 + as(\omega t)]x^5 + F\cos(\omega t)\}. \end{cases} \tag{6}$$

According to eq. (6), we can use MATLAB software to construct the system's simulation model based on Duffing oscillator, as shown in Figure 1.

When there is no signal input ($a=0$), k is fixed, and then, with the gradual accretion of the range of F, the system state is finally changed into the large-scale periodic state, experiencing the interims of chummage trajectory, the bifurcation trajectory, the chaotic state and the chaotic critical state[9]. By observing the phase trajectory, the complete difference of the phase trajectories between chaotic state and the large-scale periodic state can be considered as evidence in the signal detection.

However, it is not convincing to determine the transformation of the system's chaotic states only by observing the phase trajectory diagrams. Firstly, it is a man-made identification with a low efficiency and possible misjudgment especially when there is no enough simulation time. Secondly, the threshold value in the chaotic critical state is hard to acquire accurately. Therefore, an appropriate criterion is needed to judge the transformation of the system states.

1.2 Lyapunov exponents used in the criteria for chaos

Lyapunov exponents are used in the phase space to measure the time-varying degree of the attraction or disassociation of two phase trajectories in different initial conditions judged by index percent. The ratio of attraction or disassociation of these trajectories is called Lyapunov exponent. The dynamic properties of a certain system are reflected statistically by Lyapunov exponents which are described as follows[10].

To a two-dimensional map:

$$\begin{cases} x_{n+1} = X(x_n, y_n), \\ y_{n+1} = Y(x_n, y_n), \end{cases} \tag{7}$$

its Jacobian matrix is

$$J(x_n, y_n) = \begin{bmatrix} \dfrac{\partial X}{\partial x_n} & \dfrac{\partial X}{\partial y_n} \\ \dfrac{\partial Y}{\partial x_n} & \dfrac{\partial Y}{\partial y_n} \end{bmatrix}. \tag{8}$$

Supposing the points successive mapping from the initial point $P_0(x_0, y_0)$ as $P_1(x_1, y_1)$, $P_2(x_2, y_2)$,…, $P_n(x_n, y_n)$, the Jacobian matrix of the previous $(n-1)^{\text{th}}$ point is

$$J_0 = J(x_0, y_0),\ J_1 = J(x_1, y_1), \cdots,\ J_{n-1} = J(x_{n-1}, y_{n-1}). \tag{9}$$

Defining $J^{(n)} = J_{n-1}J_{n-2}\cdots J_1 J_0$, the modulo of eigenvalue for $J^{(n)}$ is $j_1^{(n)}$, $j_2^{(n)}$, and $j_1^{(n)} > j_2^{(n)}$, the Lyapunov exponents are defined as follows:

$$L_1 = \lim_{n\to\infty} \sqrt[n]{j_1^{(n)}},\quad L_2 = \lim_{n\to\infty} \sqrt[n]{j_2^{(n)}}.$$

In criteria for chaos, whether a system is in the chaotic state or not could be determined by the sign of the Lyapunov exponents[12]. If either of the two Lyapunov exponents is positive, the system is in the chaotic state; if both of the two Lyapunov exponents are negative, the system is in the large-scale periodic state; if either of the two Lyapunov exponents is zero or approximately equal to zero, the system is in the chaotic critical state transforming from the chaotic state to the large-scale periodic state. Therefore, through analyzing the Lyapunov exponents of chaotic detection system, the system's dynamic instant motion-state could be detected clearly, and then we can make sure whether the to-be-detected signal exists, and at the same time, the threshold value of the system in the chaotic critical state can be acquired accurately.

2 Simulations and discussion

2.1 Simulation I

The to-be-detected periodic signal $as(\omega t)$ that consisted of the sinuous s_1 and the square wave s_2 is taken as an example (Figure 2).

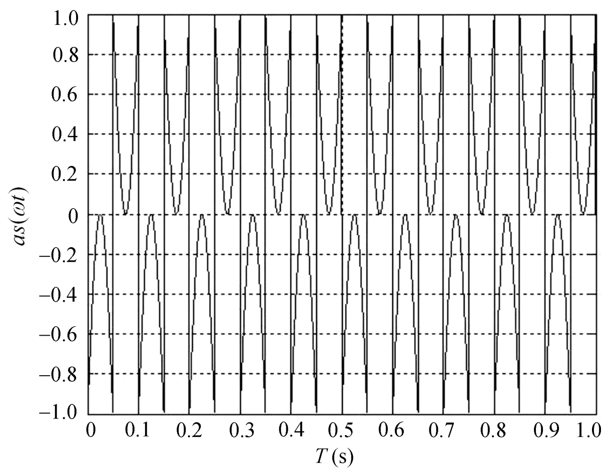

Figure 2 The waveform of the to-be-detected signal $as(\omega t)$.

Fixing the frequency of the reference signal at 10 Hz and the damping ratio at $k = 0.5$, we can primarily judge by the phase projection that the system movement changes into the chaotic state when $F \approx 0.5$. With about 100 points in the region $F = [0.5, 1]$ chosen to calculate the Lyapunov exponents (shown in L_1, L_2), 30 typical values are recorded in the Table 1, and the Lyapunov exponent curves are plotted as given in Figure 3.

In Figure 3, the symbol "*" remarked on abscissa indicates one Lyapunov exponent value ($L_1 = -0.0069$) which is closest to zero, so the threshold value of chaotic critical state $F_d = 0.7395$ can be obtained. Therefore, when $F<F_d$, there are more than one positive Lyapunov exponent, the system is in the chaotic state; when $F>F_d$, both of the two Lyapunov exponents are negative, and the system is in the large-scale periodic state.

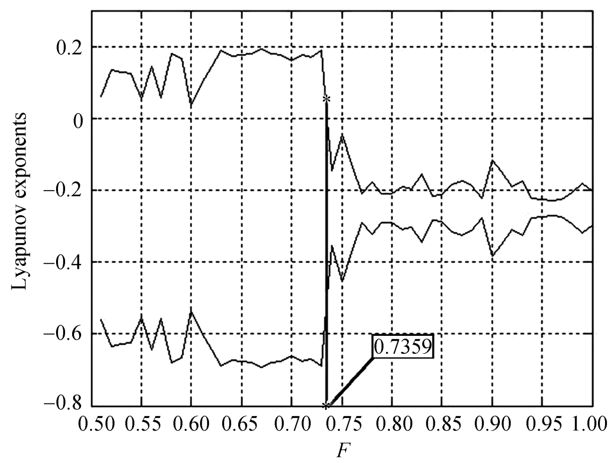

Figure 3 The relational curves of Lyapunov exponents and the amplitude of reference signal F.

2.2 Simulation II

(1) Adjust the amplitude of the reference signal to $F=F_d= 0.7359$, and then set the system in the chaotic critical state. With the white-noise signal $n(t)$ merged into the system and then gradually $n(t)$ increased, the system still keeps in the chaotic state (see in Figure 4). However, adding the to-be-detected periodic signal and the white-noise signal at the same time $as(20\pi t)+n(t)$, we can find that the phase plane orbits of the system will be immediately changed from the chaotic critical state to

Table 1 Lyapunov exponents in Duffing detection system

No.	F	L_1	L_2	No.	F	L_1	L_2	No.	F	L_1	L_2
1	0.5200	0.1355	−0.6355	11	0.7100	0.1770	−0.6770	21	0.8200	−0.1959	−0.3041
2	0.5400	0.1241	−0.6241	12	0.7200	0.1702	−0.6702	22	0.8400	−0.2165	−0.2835
3	0.5600	0.1426	−0.6426	13	0.7300	0.1877	−0.6877	23	0.8600	−0.1829	−0.3171
4	0.5800	0.1780	−0.6780	14	0.7359	−0.0069	−0.4931	24	0.8800	−0.1884	−0.3116
5	0.6000	0.0344	−0.5344	15	0.7400	−0.1470	−0.3529	25	0.9000	−0.1152	−0.3848
6	0.6200	0.1451	−0.6451	16	0.7500	−0.0448	−0.4552	26	0.9200	−0.1901	−0.3526
7	0.6400	0.1734	−0.6734	17	0.7600	−0.1259	−0.3740	27	0.9400	−0.2217	−0.2783
8	0.6600	0.1800	−0.6800	18	0.7700	−0.2094	−0.2906	28	0.9600	−0.2280	−0.2719
9	0.6800	0.1785	−0.6785	19	0.7800	−0.1782	−0.3218	29	0.9800	−0.2064	−0.2937
10	0.7000	0.1594	−0.6594	20	0.8000	−0.2110	−0.2889	30	1.0000	−0.2049	−0.2951

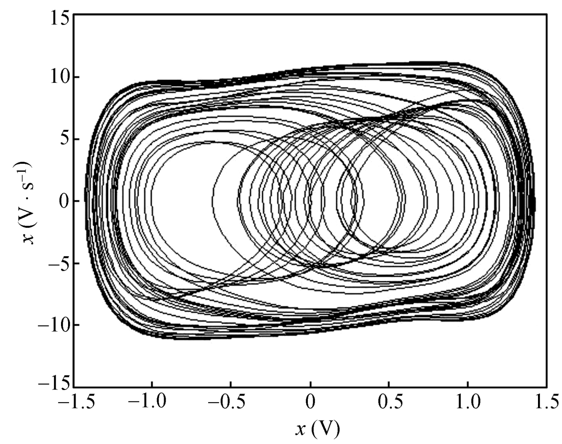

Figure 4 The phase plane orbits in the chaotic critical state.

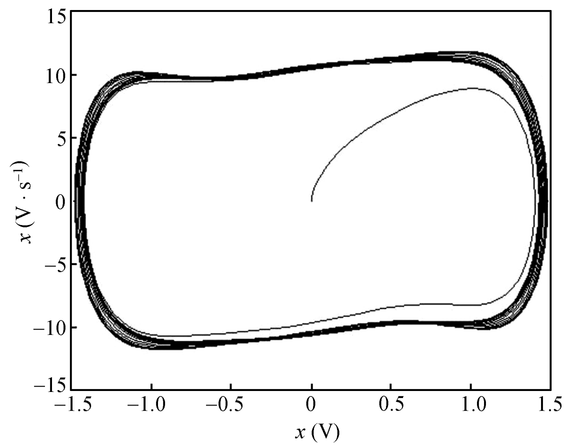

Figure 5 The phase plane orbits in the large-scale periodic state.

the large-scale periodic state (Figure 5). This indicates that the system has great sensitivity to the certain periodic signal with the same frequency of the reference signal, and also has certain immunity to white-noise signal.

(2) Adjust the amplitude of the reference signal to $F = F_d$, and then set the system in the chaotic critical state. Merge the mixed periodic signal with two different frequencies into the system, that is, $a(t) = a_1 s(10\pi t) + a_2 s(20\pi t)$. When $a_1 = 0$ and $a_2 \neq 0$, $a(t) = a_2 s(20\pi t)$, the system will be changed from the critical state to the large-scale periodic state; when $a_1 \neq 0$ and $a_2 = 0$, $a(t) = a_1 s(10\pi t)$, the system remains in the chaotic state; when $a_1 \neq 0$ and $a_2 \neq 0$, $a(t) = a_1 s(10\pi t)+a_2 s(20\pi t)$, the system will also be changed from the critical state to the large-scale periodic state. It is indicated that the phase transforms of Duffing oscillators are only sensitive to the periodic signal with the same frequency of the reference signal, and are immune to the periodic signals of other frequencies that are actually viewed as noises to the system. The power ratio of input signals $a_2 s(20\pi t)$ to $a_1 s(10\pi t)$ is about

$$\frac{S_i}{N_i} = 10\lg\frac{a_2^2}{a_1^2} = 10\log\frac{(8\times 10^{-7})^2}{(10^{-3})^2} = -71 \text{ dB}.$$

3 Results of detecting line spectrum

In this section, three analyzed ships are specified as Ship A, Ship B and Ship C. The sampling frequency of all ship-radiated noise signals is 48 kHz. Sample data are selected at random from signals of each ship-type. The length of each sample data is 48000 points within one second. First, we do FFT to the three types of sample data. The distributions of frequency spectra from DC to 100 Hz are acquired primarily, as shown in Figure 6.

From Figure 6(d), the distribution of line spectra to Ship A can be observed clearly ranging from 20 to 30 Hz, temporarily ignoring the line spectra in 50 and 100 Hz which are caused by disturbance of the electric network nearby.

Merge the data of Ship A as the to-be-detected signal into the detection system using Duffing oscillator (Figure 1) for further simulations, in the following steps:

(1) Suppose the frequency of the reference to be 20 Hz. Adjust the amplitude of the reference signal F to set the system into the chaotic critical state. Then, the threshold value of the critical state F_d has been ascertained through observing the phase plane orbits and calculating Lyapunov exponents.

(2) When the ship data are fed into the system, by increasing gradually the amplitude a, the phase plane orbits of the system will be changed from the critical state into the large-scale periodic state. Immediately, the value of a is recorded down.

(3) The Lyapunov exponents are calculated. If both of the two Lyapunov exponents are negative, it can be concluded quantitatively that the system has certainly changed from the critical state into the large-scale periodic state. Otherwise, we should increase the value a to force the system to be in the periodic state completely.

(4) By changing the frequency of the reference signal every one Hertz from 20 to 30 Hz, the ship signal is further detected according to above three steps. The value a recorded every time can be use to determine the specific distribution of the characteristic line spectrum.

The results of the first detection to Ship A in Table 2

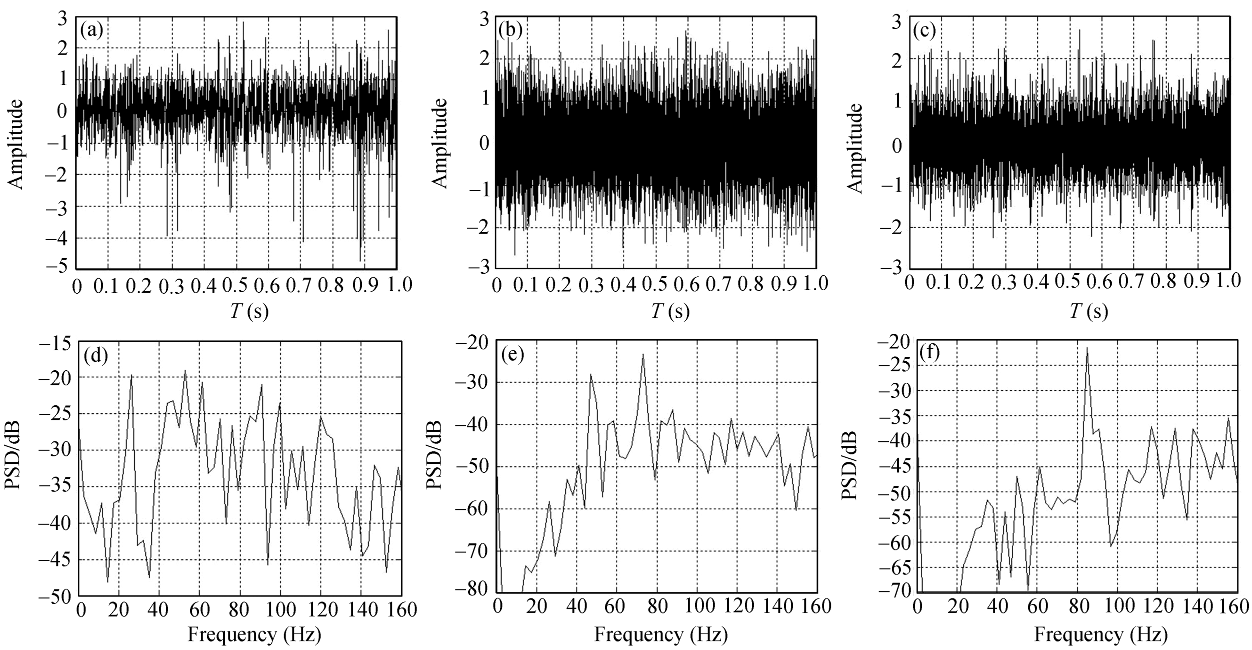

Figure 6 Three types of ship-radiated noise signal. (a) The waveform of Ship A; (b) the waveform of Ship B; (c) the waveform of Ship C; (d) the distribution of frequency spectrum to Ship A; (e) the distribution of frequency spectrum to Ship B; (f) the distribution of frequency spectrum to Ship C.

show that the distribution of the line spectrum is approximately from 26 to 27 Hz. After the second detection operated every 0.1 Hz from 26 to 27 Hz, we can determine the precise position to be 26.2 Hz.

Following this approach, from Figure 6(e) and (f), we can see that the distribution of the line spectrum is approximately from 65 to 75 Hz for Ship B, and from 75 to 85 Hz for Ship C. Furthermore, using the detection system, we can get the precise positions of line spectra of Ship B and C, which are 68.4 and 82.2 Hz, respectively (Table 2).

4 Conclusion

(1) The non-equilibrium phase transforms of Duffing oscillator are sensitive to weak periodic signals, and immune to the random noise and the periodic interference signals that have larger angular frequency differences of the reference signal.

(2) Using Lyapunov exponents in the field of detection as the criteria for chaos, which is easy to accomplish, we can evaluate the threshold value in criticalstate more accurately. Meanwhile, the way of determin ing the phase transforms of system dynamic behavior in

Table 2 Results of the distributions of line spectrum by chaotic detection[a)]

Ship A first time		Ship A second time		Ship B first time		Ship B second time		Ship C first time		Ship C second time	
Frequency (Hz)	a ($\times10^{-7}$)	Frequency (Hz)	a ($\times10^{-7}$)	Frequency (Hz)	a ($\times10^{-7}$)	Frequency (Hz)	a ($\times10^{-7}$)	Frequency (Hz)	a ($\times10^{-7}$)	Frequency (Hz)	a ($\times10^{-7}$)
20	–	26.0	0.30	65	–	68.0	0.20	75	–	81.5	0.57
21	–	26.1	0.23	66	12	68.1	0.20	76	–	81.6	0.53
22	–	26.2	0.21	67	1.3	68.2	0.20	77	40	81.7	0.56
23	20	26.3	0.29	68	0.2	68.3	0.18	78	3.0	81.8	0.51
24	4.2	26.4	0.39	69	0.26	68.4	0.15	79	4.1	81.9	0.47
25	3.4	26.5	0.45	70	4.1	68.5	0.22	80	2.5	82.0	0.40
26	0.3	26.6	0.48	71	6.3	68.6	0.23	81	0.6	82.1	0.35
27	0.6	26.7	0.51	72	17	68.7	0.24	82	0.4	82.2	0.33
28	32	26.8	0.56	73	–	68.8	0.24	83	0.7	82.3	0.49
29	–	26.9	0.63	74	–	68.9	0.24	84	2.0	82.4	0.57
30	–	27.0	0.60	75	–	69.0	0.26	85	4.8	82.5	0.62

a) "–" denotes that the frequency component here is weak comparatively, that is to say, the system cannot be changed from chaotic state into the large-scale periodic state by the to-be-detected signal in such frequency.

quantity has been developed.

(3) This method in detecting line spectrum of ship-radiated noise using Duffing oscillator, is more effective and sensitive than traditional methods based on power spectrum and the resolution is also higher. It is the first time to realize the detection of line spectrum of ship-radiated noise directly in time domain. Our method can simplify the complexity in engineering and will be helpful for further studying underwater target signals in both identification and classification.

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