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Zero-difference and single-difference precise orbit determination for LEO using GPS

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Various methods for precise orbit determination (POD) of low earth orbiters (LEO) are briefly introduced in this paper. Based on the software named SHORD-III developed by our institute, single-difference (SD) and zero-difference (ZD) dynamic POD based on LEO carrying an on-board GPS receiver is mainly discussed. The approaches are tested using real GRACE data (November 5—25, 2002) and independently validated with Satellite Laser Ranging (SLR) measurements over the same 21 days. Comparisons with the scientific orbits provided by GFZ indicate that the SD POD RMS accuracy can achieve 5, 10 and 6 cm in radial, along and cross the track, and the ZD POD RMS accuracy can achieve 4, 8 and 4 cm in radial, along and cross the track. SLR validation shows that SD POD accuracy is better than 8 cm in distance, and ZD POD accuracy is better than 6 cm.

low earth orbiters (LEO), precise orbit determination (POD), single-difference (SD), zero-difference (ZD)

Along with the rapid development of space technology and Global Positioning System (GPS), on-board GPS has become one of the main approaches of precise orbit determination (POD) for Low Earth Orbiter (LEO) satellites. Although POD with GPS has been demonstrated with great success by various satellites such as Topex/ Poseidon, GPS/MET, etc., there are still many open issues concerning the optimum way to determine LEO satellite orbits with GPS. On one hand, the quality of on-board GPS receivers has been considerably improved; on the other hand, researches and applications inquire more and more strictly accurate LEO orbit[1].

However, there is a variety of orbit determination methods for LEO using GPS-based data. According to the dynamic information which will be used or not, LEO POD approaches may be divided into three classes: 1) kinematical POD, which is independent of satellite dynamics, only requiring the geometric information contained in the GPS observations (at least, four GPS satellites) to determine the LEO satellite position directly; 2) completely dynamic POD, namely the traditional POD, which relies on physically accurate force models and adjusting a relatively small number of force model parameters as part of the orbit solution process. In such a way, the resulting orbit represents all observations best in a least squares sense, the orbit is completely determined by the dynamic model implemented in the equations of motion[2−4]; 3) reduced-dynamic POD, which balances the contributions from the force models and the geometric information, estimating the pseudo-stochastic pulses (in general, one degree Gauss-Markov process noise) acting on spacecraft to compensate for the dynamic force model errors[1,3,4].

According to the way of obtaining GPS satellite orbits and clock corrections, LEO POD approaches may be divided into two classes: one is single-step method, and the other is two-step method. In the single-step POD, taking LEO as a flying station, orbits of LEO and GPS, ground station coordinates, Earth orientation parameter, troposphere refraction parameter, etc. are estimated together simultaneously. However, so numerous are

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parameters that, not only procedure design is complicated, but also the calculation workload is huge, posing rather high demand to computer CPU[5]. In the two-step POD, GPS orbits and clocks should be computed in a separate computation or taken from external source such as the IGS firstly, and then solving LEO orbits and other parameters. Generally speaking, the two approaches lead to similar results.

In this article, we mainly discuss the dynamic orbit determination for LEO, but with GPS orbits and clock corrections fixed. Different from common zero-difference (ZD) and double-difference approaches[1−3,6,7], we use zero- and single-difference (SD) methods to determine LEO orbit. ZD method, which is based on the ionosphere-free combination of dual-frequency P-code and carrier phase measurements, requires the estimation of phase ambiguities and of epoch-wise clock corrections for the receiver, while SD method can pre-eliminate LEO clock errors completely by forming difference between two GPS satellites. Both SD approach and ZD approaches were implemented in the POD software named SHORD-III which was developed by our own institute. In addition, they were tested using real Gravity Recovery And Climate Experiment (GRACE) satellite data and independently validated with Satellite Laser Ranging (SLR) measurements. The algorithm and results will be presented in the subsequent part of this article.

1 ZD and SD LEO GPS observation equation

The observation equation for LEO zero-difference POD using P-code or carrier-phase measurements in units of distance for the frequency i between LEO receiver and GPS satellite s can be written as follows:

$$P_{\mathrm{LEO},i}^{s} = \rho_{\mathrm{LEO}}^{s} + c\delta t_{\mathrm{LEO}} - c\delta t^{s} + \delta\rho_{\mathrm{ion},i} + \delta\rho_{\mathrm{rel}} + \delta\rho_{\mathrm{pco}} + \delta\rho_{\mathrm{pco}}^{s} + \varepsilon_i, \quad (1)$$

where $P_{\mathrm{LEO},i}^{s}$ is LEO zero-difference pseudo-range measurement; ρ_{LEO}^{s} geometrical distance; c speed of light in vacuum; δt_{LEO} and δt^{s} LEO and GPS satellite clock corrections, respectively; $\delta\rho_{\mathrm{ion},i}$ ionosphere delay; $\delta\rho_{\mathrm{rel}}$ relativistic correction; $\delta\rho_{\mathrm{pco}}$ LEO phase center offset; $\delta\rho_{\mathrm{pco}}^{s}$ GPS satellite phase center offset; ε_i observation noise.

Since neutrosphere altitude is less than 80 km while LEO satellite orbit altitude is more than 200 km, there is no troposphere delay to be taken into account in LEO case. Moreover, in contrast to the conventional ground-based receiver, on-board GPS receiver is not influenced by solid earth tide and ocean tide, even when multi-path is minimized by elaborately designing antenna height, so the right side of eq. (1) does not contain troposphere delay and multi-path effects.

In order to eliminate ionospheric delays, the ionosphere-free linear combination (PC) can be formed between the LEO P-code measurements $P_{\mathrm{LEO},1}^{s}$ and $P_{\mathrm{LEO},2}^{s}$ on carrier frequencies f_1 and f_2, respectively

$$PC = \frac{f_1^2}{f_1^2 - f_2^2} P_{\mathrm{LEO},1}^{s} - \frac{f_2^2}{f_1^2 - f_2^2} P_{\mathrm{LEO},2}^{s}. \quad (2)$$

In the same way, carrier-phase measurement linear combination (LC) can be defined as:

$$LC = \frac{f_1^2}{f_1^2 - f_2^2} L_{\mathrm{LEO},1}^{s} - \frac{f_2^2}{f_1^2 - f_2^2} L_{\mathrm{LEO},2}^{s}. \quad (3)$$

Combining eq. (2) with eq. (3), a new observable b_{LC} can be generated:

$$b_{LC} = \left(\frac{f_1^2}{f_1^2 - f_2^2} P_{\mathrm{LEO},1}^{s} - \frac{f_2^2}{f_1^2 - f_2^2} P_{\mathrm{LEO},2}^{s} \right) - \left(\frac{f_1^2}{f_1^2 - f_2^2} L_{\mathrm{LEO},1}^{s} - \frac{f_2^2}{f_1^2 - f_2^2} L_{\mathrm{LEO},2}^{s} \right). \quad (4)$$

In this combination, the signal is largely dominated by pseudo-range noise and multi-path signal. In a given interval of time without any cycle slips, the above b_{LC} is nearly constant and b_{LC} can be estimated by directly averaging the right side of eq. (4) calculated at each epoch. In this case, the observation equation based on zero-difference ionosphere-free measurement for LEO can be written as follows:

$$\tilde{\rho} = \rho_{\mathrm{LEO}}^{s} + c\delta t_{\mathrm{LEO}} - c\delta t^{s} + \delta\rho_{\mathrm{rel}} + \delta\rho_{\mathrm{pco}} + \delta\rho_{\mathrm{pco}}^{s} + \varepsilon_3, \quad (5)$$

where $\tilde{\rho}$ denotes the ionospheric-free carried-phase smoothed P-code pseudo-range.

Thus, by forming difference between two GPS satellites, the observation equation based on single-difference measurement for LEO can be written as follows:

$$\Delta\tilde{\rho}^{s_1 s_2} = \Delta\rho^{s_1 s_2} - c(\delta t^{s_1} - \delta t^{s_2}) + \Delta\rho_{\mathrm{rel}}^{s_1 s_2} + \Delta\rho_{\mathrm{pco,LEO}}^{s_1 s_2} + \Delta\rho_{\mathrm{pco}}^{s_1 s_2} + \varepsilon, \quad (6)$$

where $\Delta\tilde{\rho}^{s_1s_2}$ is the differences of ionosphere-free linear combination measurement between s_1 and s_2 satellite; $\Delta\rho^{s_1s_2}$ the geometrical distance difference; $\Delta\rho_{\mathrm{rel}}^{s_1s_2}$ the relativistic correction difference; $\Delta\rho_{\mathrm{pco,LEO}}^{s_1s_2}$ the LEO phase center offset difference; $\Delta\rho_{\mathrm{pco}}^{s_1s_2}$ the GPS satellite phase center offset difference; ε the observation noise difference.

Our SD algorithm is based on the combination principle for two GPS satellites. First, choose one GPS satellite which is continuously tracked in a rather long time period as the reference satellite, then form difference between the reference satellite and other tracked GPS satellite. It is very important to note that LEO satellite clock corrections are eliminated while forming single-difference between two GPS satellites; the huge load of solving epoch-wise LEO clock corrections does not matter any more, so SD method is simple and efficient as well.

2 LEO dynamic models and POD scheme

In the dynamic POD, there is no denying that the quality of dynamical models and parameterization are the two main factors that influence the POD precision. According to the feature, we may divide the forces $\boldsymbol{A}_{\mathrm{total}}$ acting on satellites into three groups: central gravitation from Earth to satellite $\boldsymbol{A}_{\mathrm{two\text{-}body}}$, conservative acceleration $\boldsymbol{A}_{\mathrm{grav}}$ and non-conservative acceleration $\boldsymbol{A}_{\mathrm{non\text{-}grav}}$.

$$\boldsymbol{A}_{\mathrm{total}} = \boldsymbol{A}_{\mathrm{two\text{-}body}} + \boldsymbol{A}_{\mathrm{grav}} + \boldsymbol{A}_{\mathrm{non\text{-}grav}},$$

$$\boldsymbol{A}_{\mathrm{two\text{-}body}} = -\frac{GM_e}{r^3}\boldsymbol{r},$$

$$\boldsymbol{A}_{\mathrm{grav}} = \boldsymbol{A}_{\mathrm{N}} + \boldsymbol{A}_{\mathrm{NS}} + \boldsymbol{A}_{\mathrm{Tides}} + \boldsymbol{A}_{\mathrm{RO}} + \boldsymbol{A}_{\mathrm{REL}},$$

$$\boldsymbol{A}_{\mathrm{non\text{-}grav}} = \boldsymbol{A}_{\mathrm{drag}} + \boldsymbol{A}_{\mathrm{solar}} + \boldsymbol{A}_{\mathrm{earth}} + \boldsymbol{A}_{\mathrm{rad}} + \boldsymbol{A}_{\mathrm{tangent}} + \boldsymbol{A}_{\mathrm{RTN}},$$

where GM_e is gravity constant times mass of the Earth; $\boldsymbol{r}$ LEO geocentric position (J2000); $\boldsymbol{A}_{\mathrm{N}}$ acceleration due to N-body perturbations; $\boldsymbol{A}_{\mathrm{NS}}$ non-spherical perturbations; $\boldsymbol{A}_{\mathrm{Tides}}$ tides perturbations, including solid earth tide, ocean tide and atmospheric tide; $\boldsymbol{A}_{\mathrm{RO}}$ rotational deformation due to polar motion; $\boldsymbol{A}_{\mathrm{REL}}$ relativistic perturbation; $\boldsymbol{A}_{\mathrm{drag}}$ drag perturbation; $\boldsymbol{A}_{\mathrm{solar}}$ solar radiation pressure perturbation; $\boldsymbol{A}_{\mathrm{earth}}$ Earth radiation pressure perturbation; $\boldsymbol{A}_{\mathrm{rad}}$ satellite thermal radiator perturbation; $\boldsymbol{A}_{\mathrm{RTN}}$ empirical RTN perturbations.

As for SHORED-III, neither atmospheric tide perturbation nor satellite thermal radiator perturbation is taken into account. The empirical RTN perturbation is defined as eq. (5), in addition, the force models and parameter estimation schemes for zero- and single-difference POD with real GRACE data over one day are given in Table 1.

$$\boldsymbol{A}_{RTN} = \begin{bmatrix} A_R \\ A_T \\ A_N \end{bmatrix} = \begin{bmatrix} C_R \cos u + S_R \sin u \\ C_T \cos u + S_T \sin u \\ C_N \cos u + S_N \sin u \end{bmatrix}, \tag{7}$$

where u is satellite latitude, others are the estimated parameters.

Table 1 Models and parameters used in SHORED-III

Forces/model	Description	Remarks
Gravity field model	GGM02C	150×150
Atmospheric drag	DTM94	c_d estimated every three hours
Solar radiation pressure	Box-Wing	not estimated
Solid earth tides	IERS96	not estimated
Ocean tides	CSR4.0	not estimated
Empirical rtn perturbations	equation(5)	c_T, s_T, c_N, s_N estimated per 1.5 h
EOP	IERS Bulletin B	not estimated
Initial State vector	3-D position and velocity	estimated per day
Ephemeris	JPL DE/LE 200	not estimated
Ambiguity	float ambiguity	estimated per pass
LEO clock correction*	ZD parameter	estimated epoch-wise

The most important error source in modeling the orbit of a very low flying satellite like GRACE is gravity field model errors, so choosing a reasonable gravity filed model is crucial to improve POD precision. Thanks to the success of GRACE and CHAMP satellite missions, now, there are several high-precision gravity field models available such as GGM02C[8], GGM02S, GGM01C, GGM01S and EIGEN3P. GGM02C (http://www.csr.utexas.edu/grace/gravity/) model is produced by combining GGM02S[8,9] model which is derived using 363 days of data spanning from April 2002 to December 2003 from the GRACE satellite with terrestrial gravity information, while GGM01C model (http://www.csr.utexas.edu/grace/gravity/ggm01/) is produced by combining terrestrial gravity information with GGM01S model which is derived using 111 days of data from GRACE satellite. Otherwise, EIGEN3P model is derived using three years of data from CHAMP (http://www.gfz-potsdam.de/champ/results/grav/008_eigen-3p.html). The five gravity field models described above have been tested through orbit determination using real GRACE data. These are dynamic orbit solutions in which tracking data (GPS), estimated parameters and models are kept identical but not the gravity field. The actual results show that the five models provide orbits of similar quality. Nevertheless, all the results presented hereafter are based on GGM02C gravity field model.

3 Experiment results and analysis

3.1 Compared with the PSO

In order to assess the accuracy of SD and ZD POD, we computed GRACE satellites' orbits over a period of 21 days (November 5—25, days 309—329, 2002)[10,11] and compared these two orbit types with the so-called Precise Science Orbits (PSO) computed by GeoForschungsZentrum (GFZ). Those comparisons will be presented in the following. What is more, the precise GPS orbits and clock corrections sampled at 30 s interval used in SHORD-III were provided by GFZ[10,11].

Let us first see the differences between our single- and zero-difference POD solutions and PSO over the 21 days: 1) the histogram of the root-mean-squared (RMS) in the three-dimensional (3D) position for GRACE-A and GRACE-B each day is displayed in Figure 1. The horizontal line in the plot indicates the days of year, and the vertical line indicates the three-dimensional accuracy in centemeter. 2) Time series of radial (*R*) differences, along-track (*T*) differences and cross-track (*N*) differences of GRACE-A are plotted in Figure 2, while differences of GRACE-B are displayed in Figure 3. The vertical line indicates the RMS at *R*, *T* or *N* direction in centimeter.

Figure 1 clearly shows that the agreement between our single-difference dynamic orbits and PSO is good with an RMS well below 15 cm while the agreement between our zero-difference dynamic orbits and PSO is good with an RMS well below 10 cm. Assuming PSO has a quality of 5—6 cm, then the accuracy of single-difference dynamic orbits and zero-difference dynamic orbits is at a level of 14 cm and 8 cm, respectively.

From Figures 2 and 3, we can see that: 1) the difference between our solution and PSO at *T* direction is the biggest one among *R*, *T* and *N* directions, which corresponds to the nature of orbit dynamics; 2) there is no obvious systematic bias in *R*, *T* and *N* directions, those biases were caused, primarily, by the periodic errors of

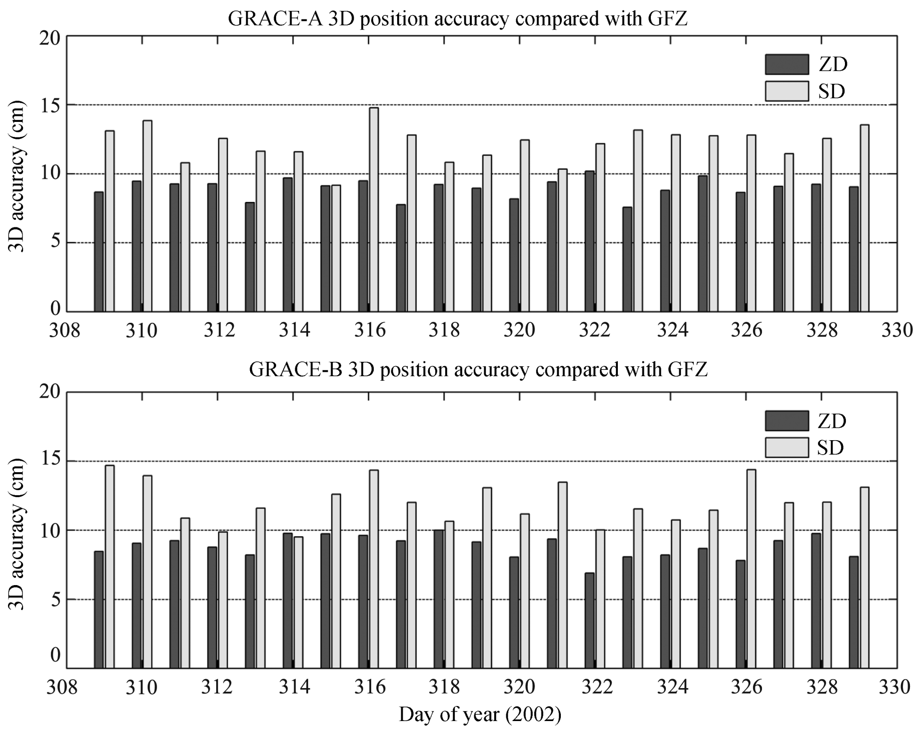

Figure 1 Histogram of RMS in 3D position: GRACE-A and GRACE-B.

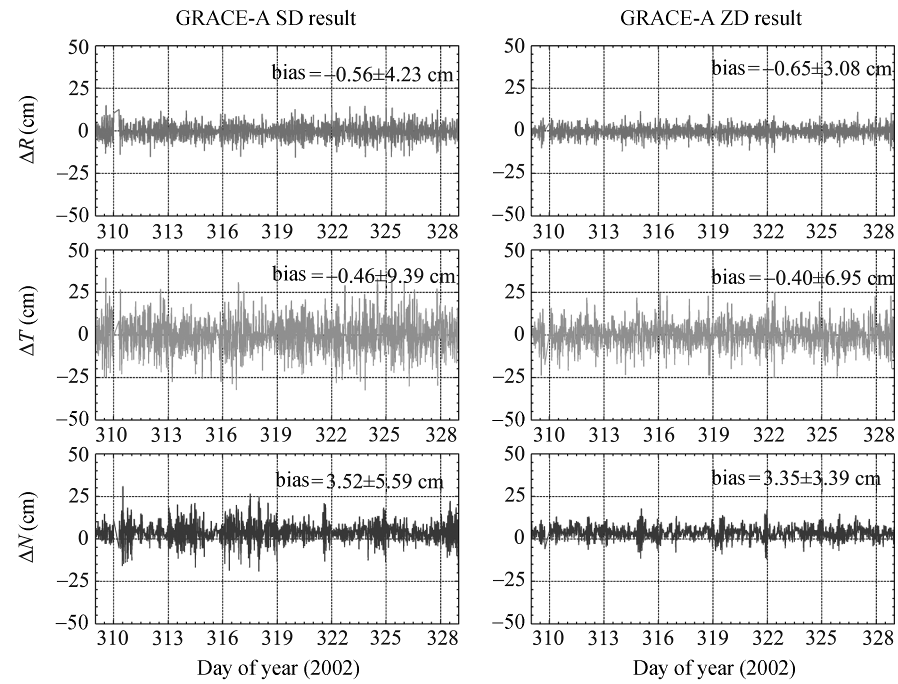

Figure 2 Differences between our solution and PSO for GRACE-A, days 309—329, 2002.

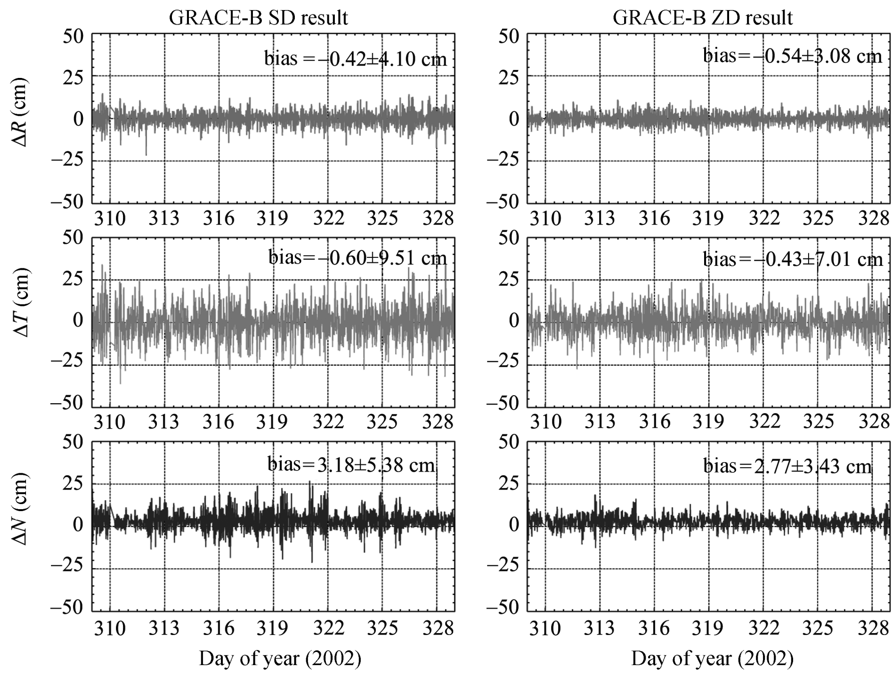

Figure 3 Differences between our solution and PSO for GRACE-B, days 309—329, 2002.

satellite orbit; 3) the accuracy of zero-difference dynamic orbits is better than that of single-difference dynamic orbits.

3.2 SLR validation

GRACE-A and GRACE-B satellites were not only equipped with a GPS receiver, but also with an array of SLR retro-reflectors. Since 1-cm accuracy of SLR observation has been claimed[4], we can use high accurate SLR measurements to validate the orbits computed with GPS data. The SLR residuals were computed as the difference between the SLR measurements minus the distance between the SLR station and the GPS-derived orbit position; hence, the corrections to SLR measurements such as troposphere delay should be corrected first[6,12].

The two orbit types were independently validated with SLR over the same 21 days (days 309－329, 2002). SLR data which have been compressed in 5 s interval normal point were obtained from the quick look CDDIS data repository. During the processing, tides correction, station offset, satellite center of mass correction, tropospheric delay and relativistic correction are applied in SLR measurements. Since GRACE satellites fly fast at very low altitudes, it is difficult to track for the ground station. Altogether 1858 SLR residuals (81 passes) were obtained in this way using data from 14 SLR stations for GRACE-A, while 1664 SLR residuals (73 passes) were obtained from 13 SLR stations for GRACE-B.

SLR residuals of GRACE-A and GRACE-B are summarized in Tables 4 and 5, respectively. From Tables 2 and 3, we immediately notice that the assessment of these single-difference dynamic orbits and zero-difference dynamic orbits with SLR measurements results in an RMS of 7.1 cm and 5.0 cm, respectively. What's more, no significant systematic bias could be detected in the SLR residuals both in single- and zero-difference approach. For example, for zero-difference approach of GRACE-B, the bias is –0.05±5.40 cm. These results further show that there is no significant bias between our solutions and PSO; meanwhile, they are in accord with the GPS-based solutions as well.

Table 2 SLR residuals for GRACE-A

Station ID station name	Mean (cm)		RMS (cm)		No. passes	No. npt
	SD	ZD	SD	ZD		
1884(Riga, Latvia)	1.2	2.3	5.5	4.3	4	94
7080(McDonald Observatory, Texas, USA)	2.8	2.4	0.4	1.2	1	5
7090(Yarragadee, Australia)	0.9	3.3	6.1	5.1	17	238
7105(Greenbelt, Maryland, USA)	–1.2	–1.2	7.3	2.5	6	160
7110(Monument Peak, California, USA)	–0.5	–0.9	4.6	5.3	7	224
7210(Haleakala, Hawaii)	–1.2	1.5	7.2	3.0	7	159
7237(Changchun, China)	–4.0	–1.5	6.1	3.8	7	133
7811(Borowiec, Poland)	0.3	–3.7	6.0	4.5	2	20
7835(Grasse, France)	–2.8	–2.5	4.3	4.1	3	60
7836(Potsdam, Germany)	3.2	–1.0	5.5	4.0	4	178
7837(Shanghai, China)	–1.4	–0.8	7.8	1.5	4	43
7838(Simosato, Japan)	–2.6	–0.5	10.3	4.9	3	109
7839(Graz, Austria)	0.6	–1.2	6.2	4.1	13	368
7840(Herstmonceux, United Kingdom)	5.7	–2.9	4.3	4.0	3	67
All	–0.00	–0.59	6.72	4.64	81	1858

Table 3 SLR residuals for GRACE-B

Station ID station name	Mean (cm)		RMS (cm)		No. passes	No. npt
	SD	ZD	SD	ZD		
1884(Riga, Latvia)	3.8	0.4	7.5	2.7	4	103
7090(Yarragadee, Australia)	–0.2	1.6	6.9	5.3	21	295
7105(Greenbelt, Maryland, USA)	–3.2	–1.0	2.9	4.5	4	70
7110(Monument Peak, California, USA)	–2.7	–0.0	8.8	5.8	9	265
7210(Haleakala, Hawaii, USA)	0.3	–3.8	3.6	1.8	2	50
7237(Changchun, China)	–0.2	–1.6	8.4	6.4	9	187
7811(Borowiec, Poland)	3.2	3.8	3.1	1.6	2	27
7835(Grasse, France)	6.2	4.2	1.1	0.9	1	20
7836(Potsdam, Germany)	–4.1	0.1	5.5	4.0	4	112
7837(Shanghai, China)	4.8	5.3	2.4	3.6	1	7
7838(Simosato, Japan)	–0.2	–2.3	9.1	6.6	3	145
7839(Graz, Austria)	–1.1	–1.2	5.7	5.1	10	300
7840(Herstmonceux, United Kingdom)	0.2	–1.6	5.8	4.5	3	83
All	–0.66	–0.04	7.42	5.40	73	1664

4 Conclusion

We have developed two kinds of algorithm for completely dynamic orbit determination for LEO based on GPS ionosphere-free single- and zero-difference observations. The algorithm is implemented in the computer program SHORD-III. Based on various tests (external comparison and SLR residuals), we can draw the following conclusions:

1) an accuracy of about 15 cm in 3D position, 5, 10 and 6 cm in radial, along and cross track, respectively, has been achieved for our single-difference dynamic orbits;

2) an accuracy of about 10 cm in 3D position, 4, 8 and 4 cm in radial, along and cross track, respectively, has been achieved for our zero-difference dynamic orbits;

3) according to the SLR residuals, an accuracy of about 8 cm and 6 cm in distance has been achieved for our single- and zero-difference dynamic orbits, respectively.

We have shown that both single- and zero-difference POD approaches can achieve rather high accuracy, nevertheless, single-difference approach is simple and efficient while zero-difference approach is a little better in accuracy. Single-difference approach is just our test. There is still potential for improving orbit accuracy with further experiments with the optimal parameter estimation schemes and carrier phase ambiguity resolution. Moreover, the dynamic orbit modeling needs further improvement to get satisfactory results.

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