

Overcharging a Reissner-Nordström Taub-NUT regular black hole

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The destruction of a regular black hole event horizon might provide us the possibility to access regions inside black hole event horizon. This paper investigates the possibility of overcharging a charged Taub-NUT regular black hole via the scattering of a charged field and the absorption of a charged particle. For the charged scalar field scattering, both the near-extremal and extremal charged Taub-NUT regular black holes cannot be overcharged. For the test charged particle absorption, the result shows that the event horizon of the extremal charged Taub-NUT regular black hole still exists while the event horizon of the near-extremal one can be destroyed. However, if the charge and energy cross the event horizon in a continuous path, the near-extremal charged Taub-NUT regular black hole might not be overcharged.

weak cosmic censorship conjecture, black hole thermodynamics, Taub-NUT black hole

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1 Introduction

The discovery of gravitational waves [1-3], generated by black hole or neutron star binaries, opened a new window towards highly relativistic systems, such as cosmology [4] and black holes [5]. Recently, gravitational wave has become a powerful tool to help us to distinguish general relativity from other gravity theories [6-14]. Furthermore, gravitational waves aroused a lot of interest to study properties of black holes, such as no hair theory [15], quasi-topological properties [16], and thermodynamics [17-22]. However, the existence of spacetime singularities might indicate the failure of predictability of gravity theory. To protect the predictability of gravity theory, Penrose [23] proposed the weak cosmic censorship conjecture, which states that

spacetime singularities are always hidden inside the event horizons of black holes and cannot be seen by distant observers. Although the conjecture is still unproved, it has become one of the foundations of black hole physics. The existence of event horizons protects the predictability outside the event horizons. However, it also makes it hard to access the black hole interior by experiment. Thus, the destruction of black hole event horizons would provide the possibility of observing regions inside black holes and relevant new physics to build a consistent theory of quantum gravity.

There are many ways to consider the violation of the weak cosmic censorship conjecture [24-29]. The seminal work that tried to destroy the event horizon of an extremal black hole in order to violate the weak cosmic censorship conjecture was first proposed by Wald [30], where a test particle with large charge or large angular momentum was thrown into an extremal Kerr-Newman black hole. The result suggests that

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particles causing the destruction of an event horizon would not be captured by the black hole. The systematical studies of Rocha and Cardoso [31] for the BTZ black hole, higher-dimensional Myers-Perry family of rotating black holes and a large class of five-dimensional black rings [32, 33] support the result that extremal black holes cannot be destroyed in the test particle approximation. While, Hubeny [34] extended the research to near-extremal black holes and found that a near-extremal charged black hole can “jump over” the extremal limit and become a naked singularity. Further research of Jacobson and Sotiriou [35] suggests that a near-extremal Kerr black hole can be overspun. Recently, Li and Bambi [36] considered the destruction of the event horizons of regular black holes, such as Bardeen and Hayward black holes. They demonstrated that the event horizons of these black holes can be destroyed by test particles, and claimed that the destruction of such event horizons might provide us the possibility to access regions inside black hole event horizons. However, using the new version of gedanken experiment proposed by Sorce and Wald [37], when the second-order approximation of the perturbation that comes from the matter fields was taken into account, Jiang and Gao [38] showed that a static charged regular black hole coupled to nonlinear electromagnetic field cannot be overcharged, and much work has been done to destroy the event horizons of black holes using this new version of gedanken experiment [39-42].

Another intriguing method of destroying the event horizon of a black hole is by using the scattering of a test classical or quantum field. The scattering of a classical field to destroy the event horizon was first proposed by Semiz [43] and the result shows that the event horizon of an extremal dyonic Kerr-Newman black hole cannot be destroyed. This method was further developed by others [44-46]. Recently, Gwak [47] considered that the change of a Kerr-(anti) de Sitter black hole is infinitesimal since the scattering process happens during the infinitesimal time interval, and found that the black hole cannot be overspun. More studies following this line can be found in refs. [48-50]. This consideration of the scattering process in an infinitesimal time interval may imply that the time interval for particles across the event horizon may play an important role in destroying the event horizon [51-54]. Furthermore, when quantum mechanics is taken into account, near-extremal black holes may indeed be destroyed by the scattering of quantized fields [55-59].

Recently, black holes with NUT parameters have been investigated in many aspects, such as gravitational lensing [60], particle acceleration [61], black hole complexity [62], and holography [63]. The Taub-NUT solutions were proposed as black hole candidates with Misner string singularities on the axes. However, the physical interpretation of the NUT parameter still remains controversial [64-69]. Fortunately,

recent researches show that the Misner string singularities are much less defective than that previously expected and the Taub-NUT solutions may actually be physically relevant [70-72]. On the other hand, black hole thermodynamics plays an important role in the study of the weak cosmic censorship conjecture. Even though black hole solutions with a NUT parameter have been found out for more than half a century, there are no convincing results for their thermodynamics [18, 73-81]. Recently, the thermodynamics of black holes with a NUT parameter seems to have been reasonably formulated with the existence of Misner strings [17, 20, 21].

In this paper, we try to destroy the event horizon of the Reissner-Nordström Taub-NUT regular black hole by adopting the latest results of the black hole thermodynamics. This black hole has no spacetime singularity, so it is not protected by the weak cosmic censorship conjecture. The destruction of its event horizon might provide us the possibility to access regions inside the black hole and give us useful information to build a consistent theory of quantum gravity.

The rest of this paper is organized as follows. In sect. 2, we review the charged Taub-NUT black hole and its thermodynamics. In sect. 3, the scattering of a charged scalar field is explored in the charged Taub-NUT black hole background. In sect. 4, we study the conserved charges for the charged scalar field in the scattering process. We try to destroy the event horizon of the charged Taub-NUT black hole with a test scalar field and a test particle in sects. 5 and 6, respectively. The last section summarizes our results.

2 The thermodynamics of the charged Taub-NUT black hole

The charged Taub-NUT black hole is a solution of the Einstein-Maxwell theory. The metric of the charged Taub-NUT spacetime reads

$$ds^2 = -\frac{f(r)}{r^2 + n^2}(dt + 2n \cos \theta d\phi)^2 + \frac{r^2 + n^2}{f(r)}dr^2 + (r^2 + n^2)(d\theta^2 + \sin^2 \theta d\phi^2), \quad (1)$$

where

$$f(r) = r^2 - 2Mr - n^2 + e^2, \quad (2)$$

and the electromagnetic potential is

$$\mathbf{A} = \frac{-er}{r^2 + n^2}(dt + 2n \cos \theta d\phi), \quad (3)$$

where M , n , and e denote the mass, the NUT parameter, and the electric parameter of the black hole, respectively. For the charged Taub-NUT black hole, there exist Misner string singularities located at $\theta = 0$ and $\theta = \pi$, which can be seen from

Figure 1 [82]. The Reissner-Nordström solution will be recovered from eq. (1) with the NUT parameter n vanishing. For the charged Taub-NUT black hole, its scalar curvature vanishes. To check if the black hole is regular, we obtain the expression of the Kretschmann scalar of this black hole as:

$$\begin{aligned}
 K &= R^{\mu\nu\rho\tau} R_{\mu\nu\rho\tau} \\
 &= \frac{8}{(n^2 + r^2)^6} \left[e^4 (7n^4 - 34n^2 r^2 + 7r^4) \right. \\
 &\quad - 12e^2 (n^6 - 10n^5 r^2 + 5n^2 r^4) \\
 &\quad - 12e^2 M r (5n^4 - 10n^2 r^2 + r^4) \\
 &\quad + 6(n^2 - M^2)(n^2 - r^2)(n^2 - 4nr + r^2)(n^2 + r^2) \\
 &\quad \left. \times (n^2 + 4nr + r^2) + 24Mn^2 r (n^2 - 3r^2)(3n^2 - r^2) \right]. \quad (4)
 \end{aligned}$$

The plot of the Kretschmann scalar is shown in Figure 2. It shows that the Kretschmann scalar is regular everywhere, which means that the charged Taub-NUT solution (1) has no spacetime singularity. So this black hole is regular and can be considered as a regular solution of the Reissner-Nordström

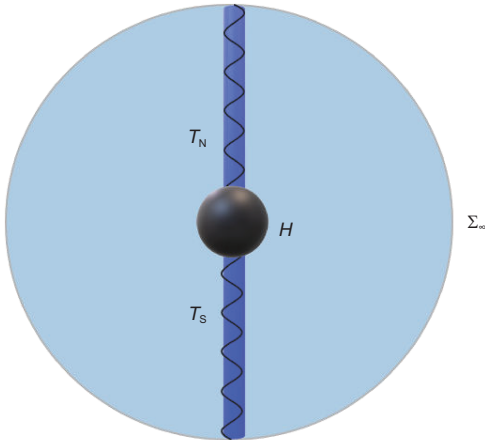


Figure 1 (Color online) Charged Taub-NUT boundaries [82]: Misner tubes. The charged Taub-NUT space-time not only has the standard boundaries: the event horizon H and spatial infinity Σ_∞ , but also includes two Misner tubes T_S and T_N located at the south and north axes, respectively.

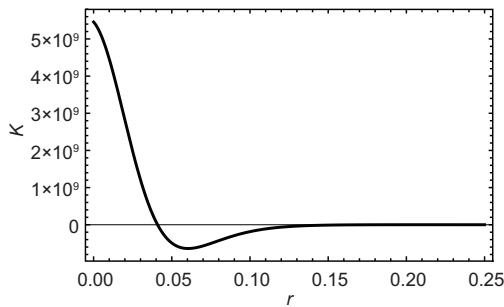


Figure 2 Plot of the Kretschmann scalar K for a charged NUT black hole with the mass $M = 1$, the charge parameter $e = 1$, and the NUT parameter $n = 0.1$.

black hole. Furthermore, this black hole is geodesically complete and according to the classification in ref. [83], it can be regarded as a one-way hidden wormhole. Therefore, after the destruction of the event horizon, there is no naked singularity and it is possible for us to explore regions inside the black hole and to find new physical phenomenon.

The horizons of the charged Taub-NUT black hole are determined by the equation

$$f(r) = r^2 - 2Mr - n^2 + e^2 = 0. \quad (5)$$

The outer and inner horizons can be obtained easily as:

$$r_{\pm} = M \pm \sqrt{M^2 + n^2 - e^2}, \quad (6)$$

where the “+” sign corresponds the event horizon of the black hole and we denote it by r_h in the following sections. When $M^2 + n^2 = e^2$, the two horizons coincide with each other and the black hole becomes an extremal one. The horizons of the black hole disappear for $M^2 + n^2 < e^2$.

The Hawking temperature T and the entropy S of the black hole are

$$T = \frac{1}{4\pi r_h} \left(1 - \frac{e^2}{r_h^2 + n^2} \right), \quad (7)$$

and

$$S = \pi(r_h^2 + n^2). \quad (8)$$

The electric potential is

$$\Phi_h = \frac{er_h}{r_h^2 + n^2}. \quad (9)$$

The first law of thermodynamics for the charged Taub-NUT black hole has been investigated extensively, and its expression is given by [20]

$$dM = TdS + \Phi_h dQ + \Psi dN, \quad (10)$$

where Q is the electric charge surrounded by the event horizon

$$Q = \frac{e(r_h^2 - n^2)}{r_h^2 + n^2}, \quad (11)$$

and the Misner charge N is associated with the NUT parameter

$$N = -\frac{4\pi n^3}{r_h} \left[1 - \frac{e^2(n^2 + 3r_h^2)}{(n^2 + r_h^2)^2} \right], \quad (12)$$

with its conjugate quantity Ψ

$$\Psi = \frac{1}{8\pi n}. \quad (13)$$

Here we want to point out that the conjugate quantity Ψ diverges as the NUT parameter $n \rightarrow 0$. This peculiar feature is similar to the thermodynamics of accelerated black holes [84].

3 Massive complex scalar field in charged Taub-NUT spacetime

In this section, we consider the scattering of a massive complex scalar field φ minimally coupled to gravity in the charged Taub-NUT spacetime. The dynamics of the complex scalar field satisfies the Klein-Gordon equation

$$\frac{1}{\sqrt{-g}}(\partial_\mu - iqA_\mu) \left[\sqrt{-g}g^{\mu\nu}(\partial_\nu - iqA_\nu)\varphi \right] - \mu_s^2\varphi = 0, \quad (14)$$

where μ_s is the mass of the complex scalar field φ . Without loss of generality, the complex scalar field can be decomposed as [85]:

$$\varphi(t, r, \theta, \phi) = e^{-i\omega t} R(r)\Theta(\theta)e^{im\phi}, \quad (15)$$

where $\Theta(\theta)$ denotes a generalized spheroidal function. Inserting the metric (1) and the decomposition (15) into the equation of motion (14), we get the radial part of the equation

$$\frac{d}{dr} \left(f(r) \frac{dR(r)}{dr} \right) + \left(\frac{L^2}{f(r)} - \mu_s^2(r^2 + n^2) - \rho \right) R(r) = 0, \quad (16)$$

and the angular part [86]

$$\frac{1}{\sin\theta} \frac{d}{d\theta} \left(\sin\theta \frac{d\Theta}{d\theta} \right) - \left[\left(2\omega n \cot\theta - \frac{m}{\sin\theta} \right)^2 - \rho \right] \Theta = 0, \quad (17)$$

where $\rho = l(l+1) + O(l^2)$ is the separation constant and

$$L = eqr + \omega(n^2 + r^2). \quad (18)$$

Using the normalization condition [47], the angular part $\Theta(\theta)$ can be reduced to unity in the charge and energy fluxes, so the exact expression of $\Theta(\theta)$ is inessential in this paper. Thus we will focus on the radial part function $R(r)$. To simplify the radial part eq. (16), we introduce the tortoise coordinate

$$\frac{dr}{dr_*} = \frac{f(r)}{n^2 + r^2}. \quad (19)$$

Then eq. (16) becomes

$$\frac{d^2 R(r)}{dr_*^2} + \frac{2rf(r)}{(r^2 + n^2)^2} \frac{dR(r)}{dr_*} + \left[\left(\omega - \frac{eqr}{n^2 + r^2} \right)^2 - \frac{(\mu_s^2 n^2 + \mu_s^2 r^2 + \rho)f(r)}{(r^2 + n^2)^2} \right] R(r) = 0. \quad (20)$$

Near the event horizon, $f(r) \rightarrow 0$ and eq. (20) can be reduced to

$$\frac{d^2 R(r)}{dr_*^2} + (\omega - q\Phi_h)^2 R(r) = 0. \quad (21)$$

The solution of the radial part near the horizon can be solved as:

$$R(r) = e^{\pm(\omega - q\Phi_h)r_*}. \quad (22)$$

There are two branches of the solution. First branch with positive sign corresponds to an outgoing wave, and the second one with minus sign corresponds to an ingoing wave which is chosen as a physically acceptable solution in this paper. Therefore, the wave function near the horizon is

$$\varphi(t, r, \theta, \phi) = e^{-i\omega t} e^{-(\omega - q\Phi_h)r_*} \Theta(\theta) e^{im\phi}. \quad (23)$$

With this function, we can study changes of the black hole parameters after the ingoing wave scattering at the event horizon.

4 Conserved charges under scattering of the scalar field

In this paper, we neglect the self-force effect and other interactions, which means that the energy and electric charge carried by the wave must be small enough. The energy change of the black hole is related to the energy flux, which is determined by the energy-momentum tensor of the massive scalar field

$$T_\nu^\mu = \frac{1}{2} D^\mu \varphi \partial_\nu \varphi^* + \frac{1}{2} D^{*\mu} \varphi^* \partial_\nu \varphi - \delta_\nu^\mu \left(\frac{1}{2} g^{\alpha\beta} D_\alpha \varphi D_\beta^* \varphi^* + \mu_s^2 \varphi^* \varphi \right). \quad (24)$$

With the energy-momentum tensor (24) and the ingoing wave function (23), the energy flux through the event horizon is obtained by

$$\frac{dE}{dt} = \int_H T_t^r \sqrt{-g} d\theta d\phi = \omega(\omega - q\Phi_h)(r_h^2 + n^2), \quad (25)$$

and the charge flux is

$$\frac{dQ}{dt} = - \int_H j^r \sqrt{-g} d\theta d\phi = q(\omega - q\Phi_h)(r_h^2 + n^2), \quad (26)$$

where the electric current j^μ is obtained by

$$j^\mu = -\frac{1}{2} iq[\varphi^*(\partial^\mu + iqA^\mu)\varphi - \varphi(\partial^\mu - iqA^\mu)\varphi^*]. \quad (27)$$

Due to the presence of the Misner strings, the integrations turn to be more subtle. As shown in Figure 1, the event horizon H does not contain the string singularities, which are located at $\theta = 0$ and $\theta = \pi$, respectively. In eqs. (25) and (26), the normalization condition

$$\begin{aligned} \lim_{\varepsilon \rightarrow 0} \int_\varepsilon^{\pi-\varepsilon} \Theta^2(\theta) \sin\theta d\theta \int_0^{2\pi} d\phi \\ = \int_0^\pi \Theta^2(\theta) \sin\theta d\theta \int_0^{2\pi} d\phi = 1 \end{aligned} \quad (28)$$

has been used. Here we have set the radii of both Misner tubes to be the same and to equal ε ($\varepsilon \ll 1$).

With the energy flux (25) and the charge flux (26), we can get the changed energy and charge within a given infinitesimal time interval dt as:

$$dM = dE = \omega(\omega - q\Phi_h)(r_h^2 + n^2) dt, \quad (29)$$

$$dQ = q(\omega - q\Phi_h)(r_h^2 + n^2) dt. \quad (30)$$

From the above equations, we can see that the relation between ω and $q\Phi_h$ determines the directions of the energy flux and the charge flux. When $\omega > q\Phi_h$, the energy and charge of the black hole increase. They remain unchanged when $\omega = q\Phi_h$ while decrease when $\omega < q\Phi_h$. The latter means that the energy and charge are extracted out by the scattering field, which is so called superradiance [87].

There are three parameters in the charged Taub-NUT black hole: the mass M , the electric parameter e , and the NUT parameter n . For a black hole far from extremal, its final state is still a black hole after exchanging the energy and charge. So we can use laws of black hole thermodynamics to investigate the changes of the three parameters during the scattering process as the authors did for Kerr-Taub-NUT black hole [51].

If we assume that the test field only changes the energy and charge of the black hole, and the NUT parameter n stays fixed in this process, through the first law of black hole thermodynamics (10), we get

$$\begin{aligned} dS &= \frac{1}{T}(dM - \Phi_h dQ - \Psi dN) \\ &= \frac{q^2 r_h^2}{T} \left(\frac{\omega}{q} - \frac{e}{r_h} \right) \left(\frac{\omega}{q} - \frac{er_h}{r_h^2 + n^2} \right) dt. \end{aligned} \quad (31)$$

Since the NUT parameter n is non-vanishing, it is clear that

$$\frac{e}{r_h} > \frac{er_h}{r_h^2 + n^2}. \quad (32)$$

For the wave modes with ω/q satisfying

$$\frac{e}{r_h} > \frac{\omega}{q} > \frac{er_h}{r_h^2 + n^2}, \quad (33)$$

the entropy S of the black hole decreases in the scattering process and this goes against the second law of black hole thermodynamics [88, 89]. Therefore, the NUT parameter n should change in the scattering process.

Now we consider the case that the Misner charge N is unchanged in the scattering process. The change of the entropy is

$$dS = \frac{1}{T}(dM - \Phi_h dQ - \Psi dN) = \frac{1}{T}(\omega - q\Phi_h)^2 dt \geq 0. \quad (34)$$

The result shows that the entropy never decreases and this satisfies the second law of thermodynamics of the black hole. So we will consider that the Misner charge N is conserved in the following discussion.

5 Destroying the event horizon with test scalar field

In this section, we try to destroy the black hole event horizon by scattering a classical complex scalar field into an extremal and a near-extremal charged Taub-NUT black hole, respectively. The energy and charge of the black hole will change after scattering the test scalar field. If a black hole is overcharged, its event horizon will disappear and its inner structure of the black hole might be seen.

To guarantee the existence of the event horizon, the minimum value of the metric function $f(r)$ must be non-positive. By using this condition, we can check whether the black hole event horizon is destroyed. For an extremal or a near-extremal charged Taub-NUT black hole, the minimum value of the metric function $f(r)$ is

$$f_{\min} = -M^2 + e^2 - n^2, \quad (35)$$

and the corresponding coordinate r is located at $r_0 = M$. The initial state of the black hole at the minimum value is represented by $f_{\min}(M, Q, N)$. After the scattering of the scalar field, the parameters of the final state become

$$\begin{aligned} M &\rightarrow M' = M + dM, \\ Q &\rightarrow Q' = Q + dQ, \\ N &\rightarrow N' = N, \end{aligned} \quad (36)$$

where we have supposed that the Misner charge N is unchanged during the scattering process. The minimum value of the final state metric function $f_{\min}(M + dM, Q + dQ, N)$ can be expressed in term of the initial state function $f_{\min}(M, Q, N)$,

$$\begin{aligned} f_{\min}^{\text{final}} &\equiv f_{\min}(M + dM, Q + dQ, N) \\ &= f_{\min}(M, Q, N) + \left(\frac{\partial f_{\min}}{\partial M} \right)_{Q,N} dM + \left(\frac{\partial f_{\min}}{\partial Q} \right)_{M,N} dQ, \end{aligned} \quad (37)$$

where

$$\begin{aligned} \left(\frac{\partial f_{\min}}{\partial M} \right)_{Q,N} &= \frac{2[n^4 - n^2 r_h^2 - M^2(n^2 - 9r_h^2) + 6Mr_h(n^2 - r_h^2)]}{-6n^2 r_h + 6r_h^3 + M(n^2 - 9r_h^2)}, \\ \left(\frac{\partial f_{\min}}{\partial Q} \right)_{M,N} &= \frac{2e[2r_h^3(n^2 - 3r_h^2) + M(n^4 + 2n^2 r_h^2 + 9r_h^4)]}{(n^2 + r_h^2)(-Mn^2 + 6n^2 r_h + 9Mr_h^2 - 6r_h^3)}. \end{aligned} \quad (38)$$

When the minimum value of the final state metric function f_{\min}^{final} becomes positive, the black hole is overcharged in the scattering process and its event horizon is destroyed.

For an extremal charged Taub-NUT black hole, the metric function $f(r)$ has only one intersection with the r -axis and

this means that the localization of the event horizon coincides with the one of the minimum value, i.e., $r_h = r_0 = M$. Then the minimum value of the final state metric function f_{\min}^{final} (37) can be obtained as:

$$f_{\min}^{\text{final}} = -\frac{2(n^4 + 4n^2r_h^2 + 3r_h^4)}{5r_hn^2 + 3r_h^3}(\omega - q\Phi_h)^2(n^2 + r_h^2)dt \leq 0. \quad (39)$$

Here the equal sign is taken only when $\omega = q\Phi_h$ and this means that the final state is still an extremal black hole. For $\omega \neq q\Phi_h$, the result shows that the extremal black hole will become a non-extremal one after absorbing the test scalar field. Thus, the event horizon of the extremal charged Taub-NUT black hole cannot be destroyed after the scattering of the test scalar field.

Now we consider the near-extremal charged Taub-NUT black hole case by using the above method. With the transferred energy (29) and the transferred charge (30) during the time interval dt , the minimum value of the final state metric function f_{\min}^{final} (37) is given by

$$f_{\min}^{\text{final}} = -M^2 + e^2 - n^2 + \Gamma(\omega - q\tilde{\Phi})(\omega - q\Phi_h)(n^2 + r_h^2)dt, \quad (40)$$

where

$$\Gamma = \frac{2[n^4 - n^2r_h^2 - M^2(n^2 - 9r_h^2) + 6Mr_h(n^2 - r_h^2)]}{-6n^2r_h + 6r_h^3 + M(n^2 - 9r_h^2)}, \quad (41)$$

and

$$\tilde{\Phi} = \Phi_h \frac{2r_h^3(n^2 - 3r_h^2) + M(n^4 + 2n^2r_h^2 + 9r_h^4)}{n^4r_h - n^2r_h^3 - M^2r_h(n^2 - 9r_h^2) + 6Mr_h^2(n^2 - r_h^2)}. \quad (42)$$

Since the distance between the minimum point r_0 and the event horizon radius r_h can be extremely small for a near-extremal black hole, we can set $r_h = r_0 + \epsilon$, where $0 < \epsilon \ll 1$. We also set $dt \sim \epsilon$ because the process considered here occurs in an infinitesimal time interval. Then eq. (40) can be rewritten as:

$$f_{\min}^{\text{final}} = -\epsilon^2 + \Gamma q^2 \left(\frac{\omega}{q} - \tilde{\Phi} \right) \left(\frac{\omega}{q} - \Phi_h \right) (n^2 + r_h^2)\epsilon. \quad (43)$$

Eq. (43) can be regarded as a quadratic function in term of ω/q . Due to

$$\begin{aligned} \Gamma &= -\frac{2[n^4 + n^2(4r_h^2 - 4r_h\epsilon - \epsilon^2) + 3r_h^2(r_h^2 - 4r_h\epsilon + 3\epsilon^2)]}{3r_h^2(r_h - 3\epsilon) + n^2(5r_h + \epsilon)} \\ &= -\frac{2(n^4 + 4n^2r_h^2 + 3r_h^4)}{3r_h^3 + 5n^2r_h} + O(\epsilon) < 0, \end{aligned} \quad (44)$$

if the maximum value of the final state f_{\min}^{final} is nonpositive, the other values of the final state are always negative, which

means that the event horizon of the black hole still exists. When the wave mode with ω/q satisfies

$$\frac{\omega}{q} = \frac{\tilde{\Phi} + \Phi_h}{2}, \quad (45)$$

f_{\min}^{final} is a maximum and can be expressed as:

$$f_{\min}^{\text{final}} = -\epsilon^2 + \frac{q^2(n^2 - 3r_h^2)^2(n^2 + r_h^2)^2\Phi_h^2}{2r_h^3(5n^4 + 18n^2r_h^2 + 9r_h^4)}\epsilon^3 < 0, \quad (46)$$

where the higher-order terms of ϵ have been omitted. Therefore, it is clear that the minimum values of the metric function f_{\min}^{final} (43) are always negative. Furthermore, we can find that $0 > f_{\min}^{\text{final}} > f_{\min}$, which means that the final state is still a near-extremal black hole and it is closer to the extremal state than the initial state. The result shows that the event horizon still exists and the black hole cannot be overcharged. So the event horizon of a near-extremal NUT black hole cannot be destroyed after the scalar field scattering and its final state is still a black hole.

Thus, both near-extremal and extremal Reissner-Nordström Taub-NUT black holes cannot be overcharged and their horizons will not disappear after the scattering of the test scalar field.

6 Destroy the event horizon with test particle

Another method to destroy the black hole event horizon is by throwing a test charged particle into a near-extremal or extremal charged Taub-NUT black hole. This gedanken experiment was proposed by Wald [30] for the first time. By ignoring back reaction, Hubeny [34] found that the near-extremal Reissner-Nordström black hole can be overcharged after capturing the test particle. Further, Jacobson and Sotiriou [35] found that the event horizon of the near-extremal Kerr black hole can be destroyed by the test particle without considering radiative and self-force effects. In this section, we will examine whether the event horizon of the charged Taub-NUT black hole still exists after absorbing a test charged particle.

The Lagrangian of a test particle with rest mass μ_m and charge δQ is

$$L = \frac{1}{2}\mu_m g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} + \delta Q A_\mu \frac{dx^\mu}{d\tau}, \quad (47)$$

where τ is the proper time of the test particle. From the Lagrangian (47), the equation of motion of the test particle can be derived as:

$$\frac{d^2 x^\mu}{d\tau^2} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = \frac{\delta Q}{\mu_m} F_{\nu}^\mu \frac{dx^\nu}{d\tau}. \quad (48)$$

In this paper, we consider that the particle is dropped on the equatorial plane ($\theta = \pi/2$) and along the radial direction, so the components P_ϕ and P_θ of the angular momentum of

the particle vanish. The energy and the angular momentum are

$$\delta E = -\frac{\partial L}{\partial \dot{t}} = -\mu_m g_{t\nu} \frac{dx^\nu}{d\tau} - \delta Q A_t, \quad (49)$$

$$P_\phi = \frac{\partial L}{\partial \dot{\phi}} = \mu_m g_{\phi\nu} \frac{dx^\nu}{d\tau} + \delta Q A_\phi = 0, \quad (50)$$

$$P_\theta = \frac{\partial L}{\partial \dot{\theta}} = \mu_m g_{\theta\theta} \frac{d\theta}{d\tau} = 0. \quad (51)$$

Then we investigate the conditions to destroy the event horizon of the black hole. Obviously, the test particle should be able to enter into the event horizon and the black hole will be overcharged after absorbing the test particle, which can give the relations between the energy δE and the charge δQ of the test particle.

The condition for the test particle entering the event horizon requires that its motion outside the event horizon is time-like and future directed, i.e.,

$$g_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1, \quad (52)$$

$$\frac{dt}{d\tau} > 0. \quad (53)$$

Substituting eqs. (49), (50), and (51) into eq. (52), we get

$$\begin{aligned} & g_{\phi\phi} \delta E^2 + 2(A_t g_{\phi\phi} - A_\phi g_{t\phi}) \delta Q \delta E \\ & + (A_\phi^2 g_{tt} - 2A_t A_\phi g_{t\phi} + A_t^2 g_{\phi\phi}) \delta Q^2 \\ & = (\mu_m^2 + g^{rr} P_r^2) (g_{t\phi}^2 - g_{tt} g_{\phi\phi}). \end{aligned} \quad (54)$$

The energy of the particle can be obtained as:

$$\begin{aligned} \delta E = & \frac{1}{g_{\phi\phi}} \delta Q A_\phi g_{t\phi} - \delta Q A_t \\ & + \frac{1}{g_{\phi\phi}} \sqrt{(g_{t\phi}^2 - g_{tt} g_{\phi\phi}) (\delta Q^2 A_\phi^2 + \mu_m^2 g_{\phi\phi} + g_{\phi\phi} g^{rr} P_r^2)}. \end{aligned} \quad (55)$$

Since the trajectory of the charged particle outside the event horizon should be future directed (53), the condition for the test particle entering into the event horizon can be obtained as:

$$\delta E > -A_t \delta Q = \Phi_h \delta Q \equiv \delta E_{\min}. \quad (56)$$

Furthermore, the condition for the black hole being overcharged requires that the minimum value of the metric function f_{\min}^{final} is positive. From eq. (37), this condition to first order is

$$f_{\min}^{\text{final}} = f_{\min}(M, Q, N) + \Gamma \delta M - \Gamma \tilde{\Phi} \delta Q > 0. \quad (57)$$

Noting that $\Gamma < 0$ and $f_{\min} \leq 0$, the condition (57) can be rewritten as:

$$\delta E < \tilde{\Phi} \delta Q - \frac{f_{\min}}{\Gamma} \equiv \delta E_{\max}. \quad (58)$$

When both conditions (56) and (58) are satisfied simultaneously, the event horizon of the black hole will be destroyed.

For an extremal charged Taub-NUT black hole, we have $f_{\min} = 0$ and $r_h = r_0 = M$. With the expression of $\tilde{\Phi}$ in eq. (42), we can get

$$\tilde{\Phi} = \Phi_h. \quad (59)$$

Therefore, we have

$$\delta E_{\min} = \Phi_h \delta Q = \delta E_{\max}. \quad (60)$$

It is clear that both conditions (56) and (58) cannot be satisfied simultaneously. This means that the test charged particle which is able to destroy the event horizon will feel repulsive force from the black hole and cannot enter into the event horizon. Thus, the event horizon of the extremal charged Taub-NUT black hole cannot be destroyed by the test charged particle.

For a near-extremal charged Taub-NUT black hole, the metric function $f_{\min} < 0$. The conditions (56) and (58) become

$$\delta E > \Phi_h \delta Q = \delta E_{\min}, \quad (61)$$

$$\delta E < \tilde{\Phi} \delta Q - \frac{f_{\min}}{\Gamma} = \delta E_{\max}, \quad (62)$$

where $\tilde{\Phi}$ can be rewritten as:

$$\tilde{\Phi} = \Phi_h + \frac{2r_h \Phi_h + Q}{n^2 + 3r_h^2} \epsilon + O(\epsilon^2) > \Phi_h. \quad (63)$$

There exists an available range of the energy δE and the charge δQ for the test particle satisfying the two inequations (61) and (62) to destroy the event horizon of the black hole. Plot of this range is shown in Figure 3. As a result, the event horizon of the near-extremal charged Taub-NUT black hole can be destroyed for some charged particles.

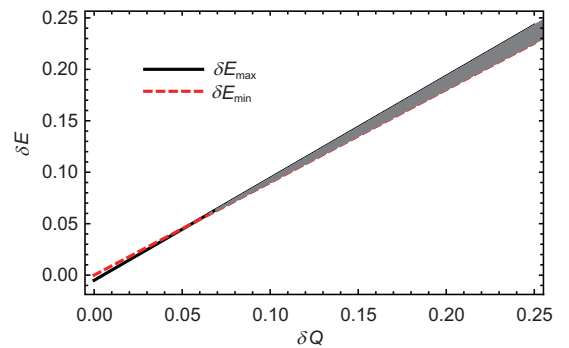


Figure 3 (Color online) Energy bounds δE_{\max} (black solid line) and δE_{\min} (red dashed line) to destroy the event horizon of a near-extremal charged Taub-NUT black hole by dropping a particle with charge δQ . Here we have set the mass $M = 1$, the charge parameter $e = 1$, and the NUT parameter $n = 0.1$. The grey region stands for $\delta E_{\max} > \delta E_{\min}$.

The results show that the event horizon cannot be destroyed for an extremal charged Taub-NUT black hole, while can be destroyed for a near-extremal one. We emphasize here that the weak cosmic censorship conjecture is not violated although the event horizon is destroyed. The reason is that the charged Taub-NUT black hole is regular and there is no spacetime singularity in the whole spacetime, although the existence of the string singularities which are coordinate singularities rather than spacetime singularities. Therefore, no naked singularity will appear after destroying the event horizon and the interior of the black hole might be accessible to distant observers.

7 Conclusion

The weak cosmic censorship conjecture has become one of the foundations of black hole physics. It might ultimately turn out to be true. However, the destruction of the event horizon of a regular black hole does not lead to the appearance of naked singularity and this does not cause the loss of predictability. The disappearance of the event horizon of a regular black hole is not forbidden by the weak cosmic censorship conjecture, and this might provide us the possibility to access regions behind the event horizon and might provide us observable information to build a consistent theory of quantum gravity.

In this paper, we investigated the possibility of destroying the event horizon of the charged Taub-NUT black hole by a test charged scalar field and a test charged particle, respectively. For the test scalar field scattering, both the near-extremal and extremal charged Taub-NUT black holes cannot be overcharged. For the test particle absorption, the result suggests that the event horizon of the extremal charged Taub-NUT black hole cannot be destroyed while the near-extremal charged Taub-NUT black hole can be overcharged. However, if we assume that the test particle crosses the event horizon of the black hole in a continuous path, i.e. the charge and energy are transferred to the black hole gradually, the horizon may be still stable as indicated by Gwak [54].

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