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Recent advances in the extended finite element method (XFEM) and isogeometric analysis (IGA)

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The finite element method (FEM) is one of the most popular and efficient methods for computational modeling in scientific research and engineering [1-3]. To expand its application to more complex problems, lots of new FEM-based methods have been developed in recent decades. One major achievement is the significantly improvement on the flexibility of FEM in dealing with geometrically and/or physically complex problems by using new shape/interpolation functions. The extended finite element method (XFEM) and isogeometric analysis (IGA) are two examples of such successful developments, and both methods have their own advantages compared with the conventional FEM. In this paper, some recent developments in these two methods are presented.

The XFEM is an efficient numerical method for solving problems with discontinuities, such as strong discontinuity like crack and weak discontinuity like interface. In the XFEM, the element boundary does not need to coincide with crack surface or material interface. As a result, structured mesh can be used to model problems with complex crack and interface, and remeshing is not necessary when the discontinuity evolves.

The XFEM uses enriched shape functions with special characteristics to bring in the discontinuous information in the computational field. The map of XFEM and FEM is sketched in Figure 1(a), where N is the shape function of conventional FEM and Φ is the enriched function, $\boldsymbol{u}^e$ and $\boldsymbol{q}^e$ are the standard and enriched nodal displacement vector in the elements, respectively. Recently, the XFEM was enhanced to simulate dynamic crack branching [4], fracture in shell structures [5], and band structure of metamaterials with complex microstructures [6], which are all top challenges in computational fracture mechanics and composite modeling.

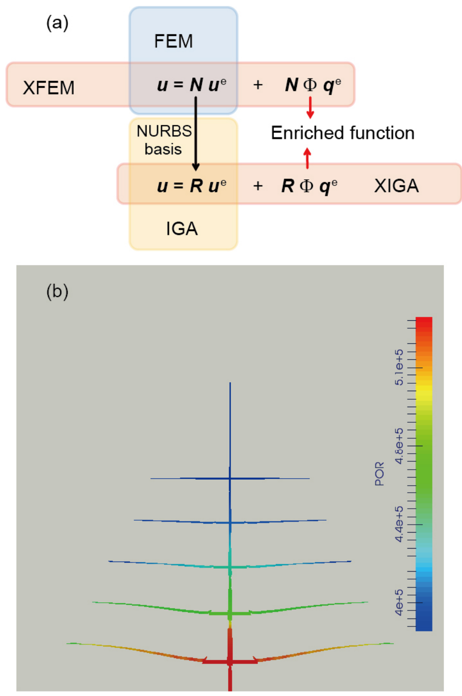

Figure 1 (Color online) (a) Sketch map of the methods; (b) hydraulic fracture network evolution morphology.

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One of the most successful applications of XFEM is the efficient modeling of the fracture network in hydraulic fracturing, which is a key stimulating technology for production improvement in the remarkable shale gas revolution. The formation and growth of the fracture network is a complicated process, involving rock deformation, crack propagation, and fluid diffusion and flow. The XFEM formulation is advanced to model this multi-physics coupling process. A deformation-flow-diffusion coupled XFEM computational model was developed [7], and Figure 1(b) shows a numerical result of the flow distribution and fracture network configuration under fluid pressure driven.

The IGA employs basis function generated from non-uniform rational B-splines (NURBS), which is a standard technology used in computer aided design (CAD). The IGA combines the geometric and analytical model, so that the complex data interaction is avoided, and the mesh refinement is simplified. With the NURBS interpolation, the IGA is very suitable to study high order problems of plates and shells. In addition, it is widely used in the optimization, for which the control points of NURBS are directly applied as the optimization objects. Recent advances in the IGA focus on the development of structural [8-11] and dynamic problems [12].

A curved beam element based on the Timoshenko model and NURBS interpolation was developed to analyze plane-curved beams and arches [8]. Thai et al. [9] used the IGA to model laminated composite and sandwich plates based on their new proposed inverse tangent shear deformation theory (ITSDT). An extended isogeometric element formulation based on the Kirchhoff-Love theory was developed to analyze the through-the-thickness cracks in thin shell structures [10]. In this approach, the XFEM was combined with the IGA, therefore the method was named as XIGA. Key concepts of the IGA and XIGA are illustrated in Figure 1(a) together with the FEM and XFEM, where R is the NURBS basis function.

Recently, Wang et al. [12] presented an ultra-accurate isogeometric dynamic analysis. The key ingredient of their methodology was the development of isogeometric higher order mass matrix. The dynamic isogeometric analysis demonstrated superiority systematically through frequency spectra, free vibration as well as transient analysis.

In general, the XFEM is suitable for solving discontinuous problems such as crack propagation; the IGA is good at modeling structures with complex geometries and/or high order deformation. Both methods are the successful expansions on the concept of FEM.

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