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New omega vortex identification method

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A new vortex identification criterion called Ω-method is proposed based on the ideas that vorticity overtakes deformation in vortex. The comparison with other vortex identification methods like Q-criterion and λ_2-method is conducted and the advantages of the new method can be summarized as follows: (1) the method is able to capture vortex well and very easy to perform; (2) the physical meaning of Ω is clear while the interpretations of iso-surface values of Q and λ_2 chosen to visualize vortices are obscure; (3) being different from Q and λ_2 iso-surface visualization which requires wildly various thresholds to capture the vortex structure properly, Ω is pretty universal and does not need much adjustment in different cases and the iso-surfaces of Ω=0.52 can always capture the vortices properly in all the cases at different time steps, which we investigated; (4) both strong and weak vortices can be captured well simultaneously while improper Q and λ_2 threshold may lead to strong vortex capture while weak vortices are lost or weak vortices are captured but strong vortices are smeared; (5) Ω=0.52 is a quantity to approximately define the vortex boundary. Note that, to calculate Ω, the length and velocity must be used in the non-dimensional form. From our direct numerical simulation, it is found that the vorticity direction is very different from the vortex rotation direction in general 3-D vortical flow, the Helmholtz velocity decomposition is reviewed and vorticity is proposed to be further decomposed to vortical vorticity and non-vortical vorticity.

vorticity, vortex, vortex identification, turbulence

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1 Introduction

Vorticity is defined as the curl of velocity and interpreted as twice the local angular velocity of the fluid element. Because of the much simpler governing equations than velocity, vorticity received considerable attention from investigators. Classical fluid dynamics believes that vorticity cannot be generated nor destroyed within the interior of fluids, and it is transported inside the flow by advection and diffusion [1]. The physical meaning and properties of vorticity make itself of great value in investigating vortices dominant flows. Therefore, many researchers have tried to utilize vorticity magnitude to educe coherent structures and identify vortex cores in turbulent flows. As pointed out by Jeong and Hussain [2], however, this approach is not always successful, especially if the background shear is comparable to the vorticity magnitude within the vortex. It has been recognized that vorticity does not represent global rotation, i.e. vortices. For example, a laminar boundary layer possesses vorticity, but there is clearly no rotational motion in the laminar boundary layer.

In spite of the stated consensus about vorticity, several con-

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cepts are still rather confusing in fluid mechanics. A flow is called irrotational if $\nabla \times \boldsymbol{V} = 0$ in all space, while a rotational flow simply indicates the vorticity is not zero somewhere [3]. The previous example of the laminar boundary layer can serve as a counterexample in which vorticity exists without any rotational motion or vortices. Evidently, there is a difference between vorticity (local quantity) and vortices (group rotation). Thus, the effort to distinguish vorticity and vortices quantitatively is justified and necessary.

On the other hand, vortex definition and identification have been a longstanding issue. Robinson [4] proposed a rather accurate definition: a vortex exists when instantaneous streamlines mapped onto a plane normal to the vortex core exhibit a roughly circular or spiral pattern, when viewed from a reference frame moving with the center of the vortex core. The definition, however, suffers from a requirement to identify the vortex core as a priori. In the meantime, several vortex identification methods are introduced trying to fulfil the need to investigate the vortex structures in turbulent flows.

Perry and Chong [5] suggested vortices exist where eigenvalues of velocity gradient tensor $\nabla \boldsymbol{V}$ are complex, which implies the streamline pattern is spiral or closed viewed from a reference frame moving with the point. The method, named the $\widetilde{\Delta}$-method, was further developed by Zhou et al. [6]. They suggested employing iso-surfaces of imaginary part of the complex eigenvalue to capture vortices. Around the same time, the famous Q-criterion was introduced by Hunt et al. [7], in which an eddy is defined as the region with positive second invariant Q of the velocity gradient tensor. The idea behind this method is Q represents the balance between shear strain rate and vorticity magnitude since it can be derived that $Q = 1/2(\|\Omega\|^2 - \|S\|^2)$ where S and Ω are the symmetric and antisymmetric components of $\nabla \boldsymbol{V}$. Another well-known scheme is the λ_2 method, introduced by Jeong and Hussain [2]. They suggested the usage of second eigenvalue of the symmetric tensor $S^2 + \Omega^2$ trying to capture the pressure minimum in a plane normal to the vortex axis.

All these methods have achieved some success. As demonstrated by Pierce et al. [8], the $\widetilde{\Delta}$, Q, λ_2 criteria can produce the same images when applied to DNS data of a transitional boundary layer provided appropriately iso-surface thresholds are chosen respectively. These criteria, however, suffer from some common issues. First, a case related threshold is required; second, the physical meaning of $\widetilde{\Delta}$, Q and λ_2 is unclear; third, inappropriate thresholds may lead to strong vortices captured while weak ones are skipped. Note that if we use Q-criterion, different thresholds will lead to different vortex structures. It is really very hard, if not impossible, to judge which threshold is correct and which is incorrect. In this paper, a new vortex identification approach is proposed to overcome these weaknesses. The paper is organized as follows. In sect. 2, the concept of Helmholtz decomposition is revisited, and the necessity to decompose vorticity into a vortical part and a non-vortical part is addressed. A new vortex identification method is proposed in sect. 3. The new method is applied to the DNS results of late stages transitional boundary layer flow and LES results for micro vortex generator (MVG) case, and roughness induced hypersonic boundary layer transition flow in sect. 4. The paper ends with conclusions in sect. 5.

2 Helmholtz decomposition revisit

According to Helmholtz [9], a vector field in three dimensions can be resolved into the sum of an irrotational (curl-free) vector and a solenoidal (divergence-free) vector field; this is known as the Helmholtz velocity decomposition. Based on the theorem, fluid motion can be decomposed into a symmetric part and an antisymmetric part (Figure 1):

$$\boldsymbol{V}(\boldsymbol{X}+\mathrm{d}\boldsymbol{X}) = \boldsymbol{V}(\boldsymbol{X}) + \mathrm{d}\boldsymbol{V}, \tag{1}$$

$$\mathrm{d}\boldsymbol{V} = \mathrm{d}\boldsymbol{X}\cdot\nabla\boldsymbol{V}, \tag{2}$$

$$\begin{aligned}\nabla\boldsymbol{V} &= \frac{1}{2}(\nabla\boldsymbol{V}+\nabla\boldsymbol{V}^{\mathrm{T}}) + \frac{1}{2}(\nabla\boldsymbol{V}-\nabla\boldsymbol{V}^{\mathrm{T}}) \\ &= \boldsymbol{\xi} + \frac{1}{2}(\nabla\boldsymbol{V}-\nabla\boldsymbol{V}^{\mathrm{T}}),\end{aligned} \tag{3}$$

$$\mathrm{d}\boldsymbol{V} = \mathrm{d}\boldsymbol{X}\cdot\boldsymbol{\xi} - \mathrm{d}\boldsymbol{X}\times\boldsymbol{\omega}, \tag{4}$$

where $\boldsymbol{\omega} = \frac{1}{2}\nabla\times\boldsymbol{V}$.

In general, people believe the first symmetric part is deformation while the second antisymmetric part is rotation. However, as discussed in sect. 1, the antisymmetric part is actually related to vorticity rather than the global rotation. Vorticity could be considered as self-rotation for an infinitesimal particle, but not vortex. For example, both laminar and turbulent boundary layer flows possesses vorticity. Vorticity of laminar boundary layer flow is concentrated near the wall surface and should be viewed as irrotational since the streamlines and pathlines are all parallel and straight. Nevertheless, vorticity of turbulent flow is mainly rotational with different sizes of

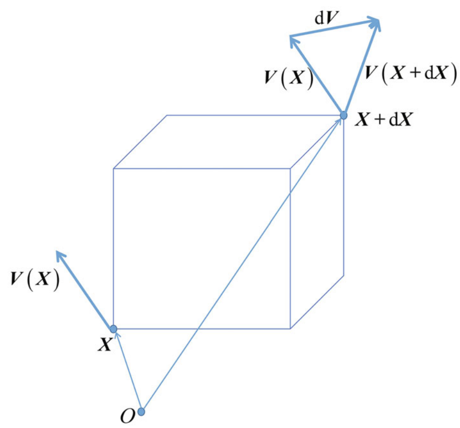

Figure 1 (Color online) Helmholtz velocity decomposition.

vortices. Thus, the definitions of irrotational flow as vorticity free field and rotational flow otherwise have long been a source of confusion and misunderstanding. On the other hand, vorticity should be further decomposed into two parts: one is vortical vorticity contributed to rotation and the other one is non-vortical vorticity like the vorticity without rotation in laminar boundary layer flows (Blasius solution).

$$\nabla\times\boldsymbol{V}=\boldsymbol{R}+(\nabla\times\boldsymbol{V}-\boldsymbol{R}). \tag{5}$$

$\boldsymbol{R}$ is the vortical vorticity as part of vortex, and therefore $\nabla\times\boldsymbol{V}-\boldsymbol{R}$ represents the vorticity not related to the global rotation, namely non-vortical vorticity. In general, the directions of $\boldsymbol{R}$ and $\nabla\times\boldsymbol{V}$ are different, sometimes very different based on our observation in turbulent boundary layer data from a high order DNS [10]. Therefore, a decomposition can be made inside a vortex as follows:

$$\begin{aligned}\nabla\times\boldsymbol{V}&=\left(\nabla\times\boldsymbol{V}\cdot\frac{\boldsymbol{R}}{\|\boldsymbol{R}\|_2}\right)\frac{\boldsymbol{R}}{\|\boldsymbol{R}\|_2}\\&\quad+\left(\nabla\times\boldsymbol{V}-\left(\nabla\times\boldsymbol{V}\cdot\frac{\boldsymbol{R}}{\|\boldsymbol{R}\|_2}\right)\frac{\boldsymbol{R}}{\|\boldsymbol{R}\|_2}\right)\\&=\sqrt{\Omega}\,\|\nabla\times V\|_2\frac{\boldsymbol{R}}{\|\boldsymbol{R}\|_2}\\&\quad+\left(\nabla\times\boldsymbol{V}-\sqrt{\Omega}\,\|\nabla\times V\|_2\frac{\boldsymbol{R}}{\|\boldsymbol{R}\|_2}\right),\end{aligned} \tag{6}$$

Ω is then defined as a ratio of the vortical vorticity over the total vorticity as $\Omega=\frac{(\nabla\times\boldsymbol{V}\cdot\boldsymbol{R})^2}{\|\nabla\times\boldsymbol{V}\|_2^2\cdot\|\boldsymbol{R}\|_2^2}$. The first term is vortical part and the second term is non-vortical part of the vorticity.

Then pure deformation can be more appropriately defined as flows with $\Omega=0$ while the rigidly rotational flow is defined as $\Omega=1$ since the vorticity direction is the rotational axial. Note that $\Omega\in[0,1]$. In particularly, we note that when the vorticity is aligned with vortex axial, the deformation will become very small. Like vorticity, the deformation is also an important scale to judge if flow is rotational or vortex is formed. In other words, both vorticity and deformation are important to define a "vortex" and it reasonably leads to consider the ratio of vorticity and deformation, which is called omega method described in next section.

After revisiting Helmholtz decomposition, we believe the fluid particle motion can be more appropriately decomposed into four parts: translation, deformation, vortical part of vorticity and non-vortical part of vorticity. Vorticity doesn't directly represent rotation even though rigid body rotation must possess vorticity. Therefore, vorticity could be small while rotation is strong and vorticity could be large while rotation is weak or none.

Boundary layer transition from laminar flow on a flat plate, in which Ω is small, implying all the vorticity is non-vortical, to turbulent flow in which Ω is large in many places inside of the boundary layer, representing part of the vorticity is vortical, is basically a process in which non-vortical vorticity transfers to vortical vorticity.

3 New vortex identification method

Based on the idea of splitting vorticity into a vortical part and non-vortical part, a parameter Ω is introduced to represent the ratio of vortical vorticity over the whole vorticity inside a vortex core. In order to find Ω, eq. (3) can be rewritten as following:

$$\begin{aligned}\nabla\boldsymbol{V}&=\frac{1}{2}(\nabla\boldsymbol{V}+\nabla\boldsymbol{V}^{\mathrm{T}})+\frac{1}{2}(\nabla\boldsymbol{V}-\nabla\boldsymbol{V}^{\mathrm{T}})\\&=\boldsymbol{A}+\boldsymbol{B},\end{aligned} \tag{7}$$

where $\boldsymbol{A}$ is symmetric and $\boldsymbol{B}$ is anti-symmetric. As stated above, $\boldsymbol{A}$ represents deformation while $\boldsymbol{B}$ is related to the whole vorticity. The square of Frobenius norm of $\boldsymbol{A}$ and $\boldsymbol{B}$ is then introduced as given below:

$$a=\mathrm{trace}(\boldsymbol{A}^{\mathrm{T}}\boldsymbol{A})=\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\boldsymbol{A}_{ij}\right)^2,$$

$$b=\mathrm{trace}(\boldsymbol{B}^{\mathrm{T}}\boldsymbol{B})=\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\boldsymbol{B}_{ij}\right)^2.$$

Now an estimate of the ratio Ω is introduced which shows vortex is formed when the vorticity is strong but deformation is weak:

$$\Omega=\frac{b}{a+b}.$$

We pick

$$\Omega=\frac{b}{a+b+\varepsilon}, \tag{8}$$

where ε is a small positive number used to avoid division by zero. Note that the length and velocity must be used in the nondimensional form to calculate Ω. Otherwise ε needs to be adjusted to a large number depending on the length and velocity dimension. First, $0\le\Omega\le1$ since both a and b are not negative. Second, in terms of a 2D rigid-body vortex with a uniform angular velocity ϕ, the velocity field is given as $\boldsymbol{V}=(-\phi y,\phi x)$. It is easy to verify $a=0$ and then $\Omega=1$ as expected. Apparently, we give an approximation that

$$\Omega=\frac{(\nabla\times\boldsymbol{V}\cdot\boldsymbol{R})^2}{\|\nabla\times\boldsymbol{V}\|_2^2\cdot\|\boldsymbol{R}\|_2^2}\approx\frac{b}{a+b}, \tag{9}$$

which shows when vorticity is aligned with rotation the deformation is small and vortex is really an area where projection of vorticity in the rotating axial is about the same.

Confusions may be raised about the numerator in eq. (8), that why the sum of deformation and vorticity becomes the denominator. The reason is $\boldsymbol{A}_{ij}$ (deformation) and $\boldsymbol{B}_{ij}$ (vorticity) are not independent. More specifically, the shear strain rate decreases as the vortical vorticity grows. Thus, the shear stresses will decrease accordingly for Newton fluids. Therefore, the dissipation of vortical flow is lower than the corresponding non-vortical flow like laminar boundary layer. The rotation state of fluid is more stable and transition from laminar flow to turbulent flow is a process moving toward to a more stable state. Unlike solid body which can have rotation without any deformation, vortex in fluid flow is always a mixture of vorticity and deformation. This is the source of difficulties to give a rigorous definition for "vortex". The omega criteria just shows the vorticity overtakes the deformation when the "vortex" is formed.

4 Application of the new vortex identification method

After obtaining a formula to estimate the ratio Ω, it is tempting to take advantage of it to visualize vortices in turbulent flows. The new method is first implemented on direct numerical simulation data of a transitional boundary layer [10, 11] at both an early and a late time steps. MVG (Micro Vortex Generator) and RIT (Roughness Induced Transition) cases are also used to evaluate the new method. The Q-criterion and λ_2 method are also implemented on the same data sets for comparison.

4.1 Boundary layer transition

Figure 2 shows the iso-surfaces of λ_2=−0.001, λ_2=−0.0001 and λ_2=−0.01, a method proposed by Jeong and Hussain [2]. Figure 3 gives the iso-surfaces of Q=0.001, Q=0.0001 and Q=0.01.

It is quite tricky to choose the threshold for iso-surface visualization in both λ_2 and Q methods. The criteria for choosing appropriate thresholds in this paper are as follows: (1) the main structures should be clear; (2) do not lose too much small vortices. Based on these requirements, the vortical structures are relatively appropriately represented in Figures 2(a) and 3(a). It is argued Q is the balance between shear strain rate and vorticity magnitude while negative λ_2 denotes the pressure minimum in a plane perpendicular to the vortex axis. However, the physical meaning of these values is unclear. On the other hand, a conclusion can be drawn that larger magnitude Q (positive) or λ_2 (negative) capture stronger vortices. See Figures 2(b) and (c) and Figures 3(b) and (c). Comparing Figures 2(a) and (b), we can see that a smaller magnitude λ_2 can capture weaker vortices including small ring-like vortices downstream the first vortex ring and "clouds" above Λ-vortex which are denoted in Figure 2(b). However, this smaller magnitude λ_2 iso-surface smears many structures, makes them undistinguishable from each other. A similar situation occurs when Q-method is used as shown in Figure 3(b). Correspondingly, a larger magnitude threshold can lead to clear representation of the strong vortices while the weak vortices are all skipped, see Figures 2(c) and 3(c). Note that, in Figure 2(a), there are some "bulges" above the Λ-vortex which might be thought as part of the Λ-vortex while a vortex layer seems to lay upon the Λ-vortex in Figure 3(a), but these structures disappear in Figures 2(c) and 3(c), which seems clearer. However, the weak vortices beside the strong vortices also disappear.

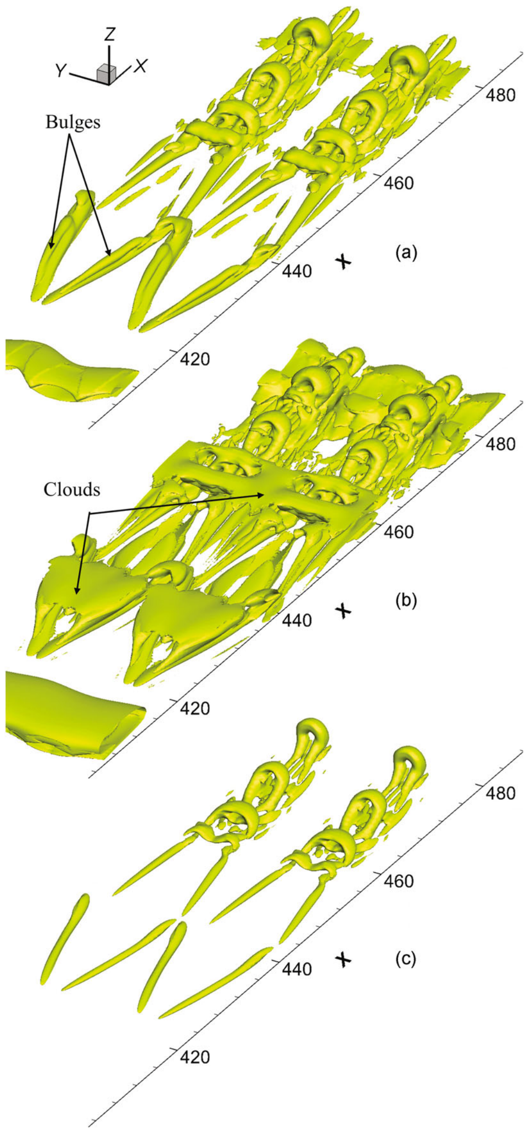

Figure 2 (Color online) Iso-surfaces of (a) λ_2 = −0.001, (b) λ_2= −0.0001, (c) λ_2=−0.01 at t = 6.8T , where T is the period of T-S wave.

Iso-surface of the new method Ω=0.52 is also shown in Figure 4. The threshold value is kind of fixed since the physical meaning, that the vorticity overtakes the deformation, is clear, although a light change of the value like 0.51 or 0.53 might also be appropriate value to represent the vortical structures. The main vortical structures captured by these three methods are basically the same. However, advantages of the Ω-method can be seen from Figure 4 that strong vortices are

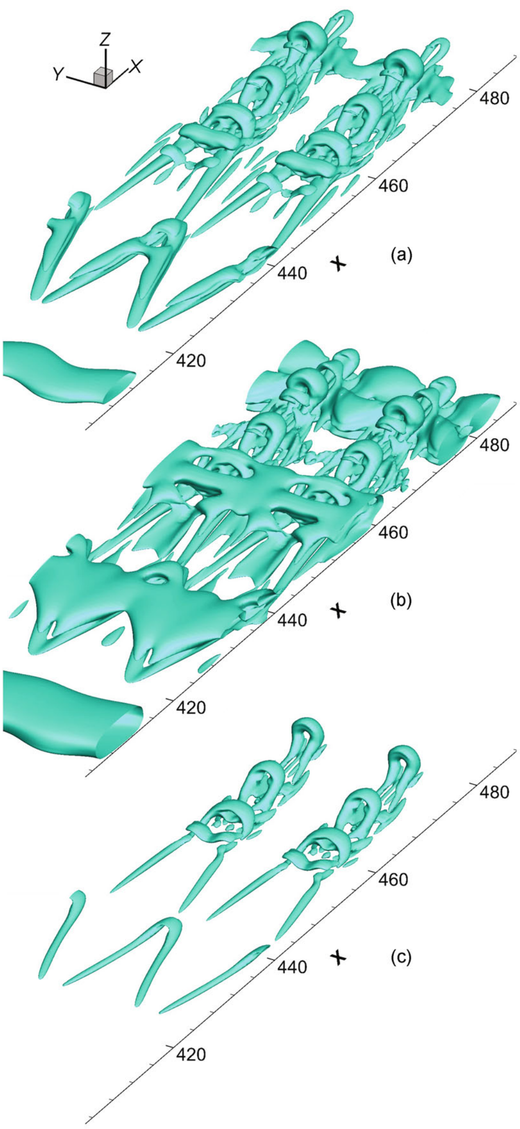

Figure 3 (Color online) Iso-surfaces of (a) $Q = 0.001$, (b) $Q = 0.0001$, (c) $Q = 0.01$ at $t = 6.8T$, where T is the period of T-S wave.

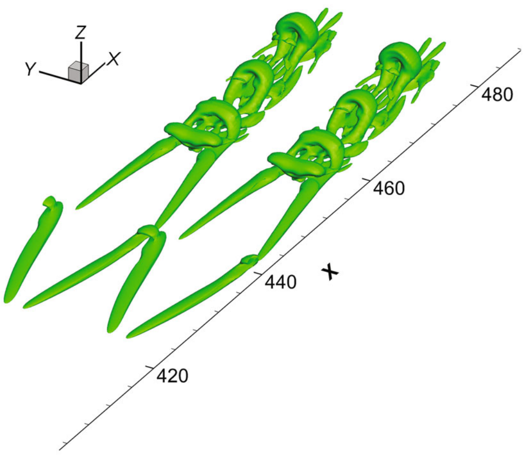

Figure 4 (Color online) Iso-surfaces of $\Omega = 0.52$ at $t = 6.8T$, where T is the period of T-S wave.

captured as clear as Figures 2(c) and 3(c) while the weak vortices are also well represented. And there are not any "bulge" structures and extra vortex layers above the Λ-vortex in Figure 4. The reason is that Ω is a ratio while both λ_2 and Q are related to so-called vortex strength. Therefore, the Ω can represent vortices with much larger vortex strength latitude.

This superiority of the new Ω-method might be very helpful when analyzing the transition mechanism as large magnitude of λ_2 and Q could cause faked "vortex breakdown" since weak vortices may be missed [10].

At a much later time step, iso-surfaces of $\lambda_2 = -0.001$, $Q = 0.001$ and $\Omega = 0.52$ is shown in Figure 5. The choosing of thresholds for λ_2 and Q-methods is based on the criteria stated above. Again, the vortical structures captured by all the three methods are basically the same. Ring-like vortices,

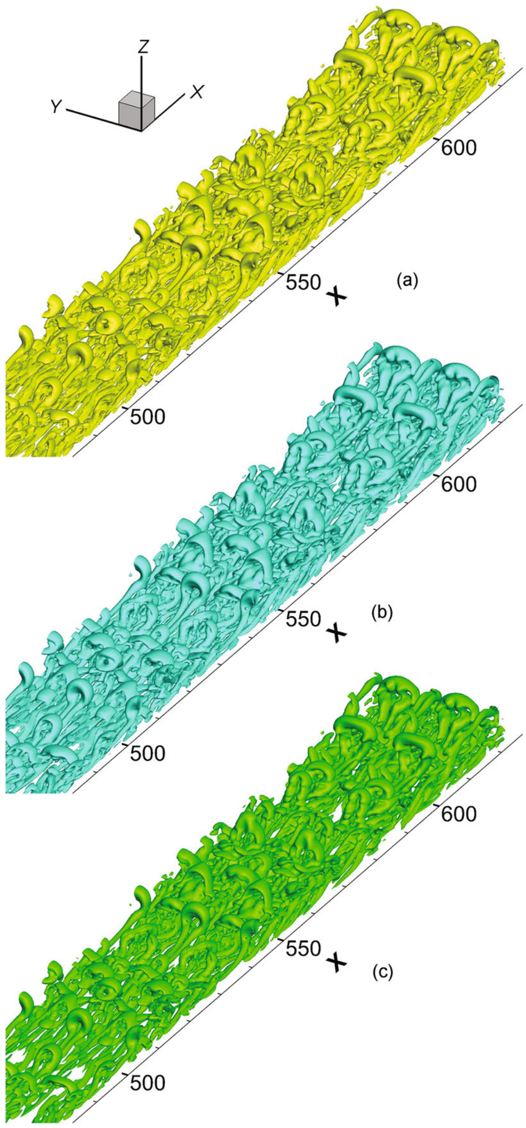

Figure 5 (Color online) Iso-surfaces of (a) $\lambda_2 = -0.001$, (b) $Q = 0.001$, (c) $\Omega = 0.52$ at $t = 12.95T$, where T is the period of T-S wave.

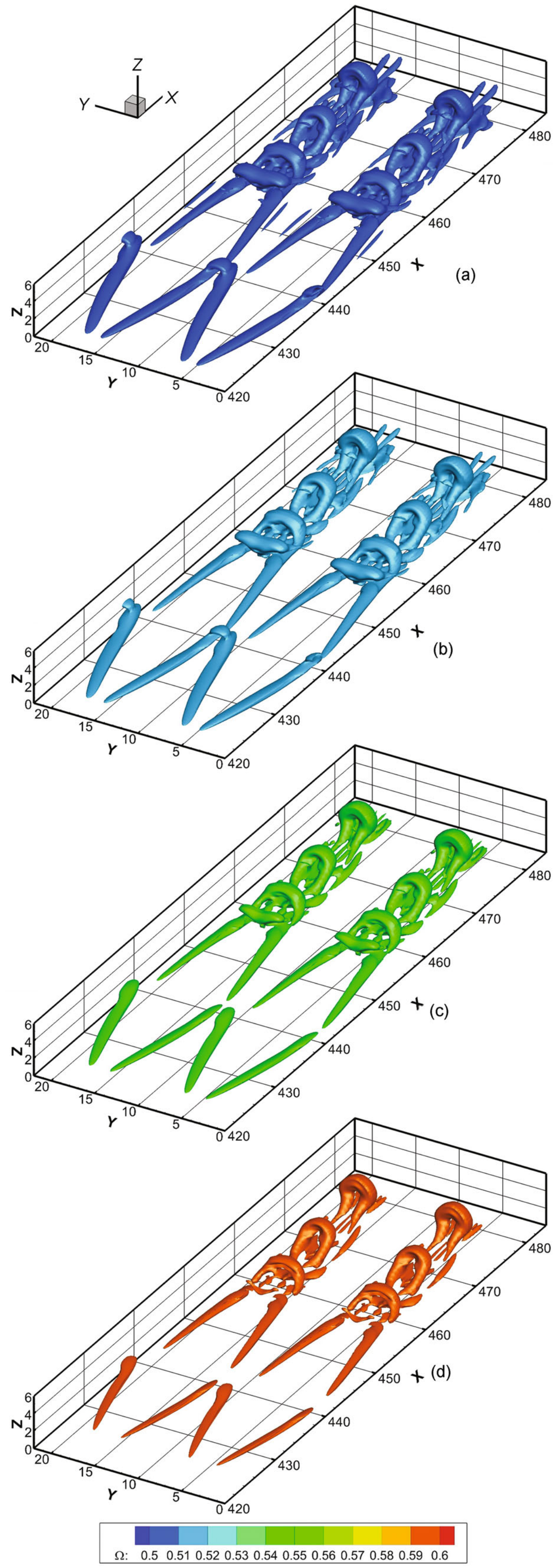

Figure 6 (Color online) Iso-surfaces of (a) Ω= 0.51, (b) Ω= 0.52, (c) Ω= 0.55, (d) Ω= 0.6 at $t = 6.8T$, where T is the period of *T-S* wave.

Λ-vortex and stream-wise vortices are all clearly represented.

4.2 Vortex visualization by different Ω

Figure 6 shows the vortical structures in early Λ-vortex development at $t = 6.8T$ and Figure 7 visualizes the vortex structure in late boundary transition at $t = 12.9\ T$, where T is the *T-S* wave period. In both cases, early and late, four different Ω values are selected to do the visualization, which vary from 0.51 to 0.6. This shows the vortical structures identified by Ω method are not very sensitive with the change of Ω value, although some minor structures will vanish as Ω increases.

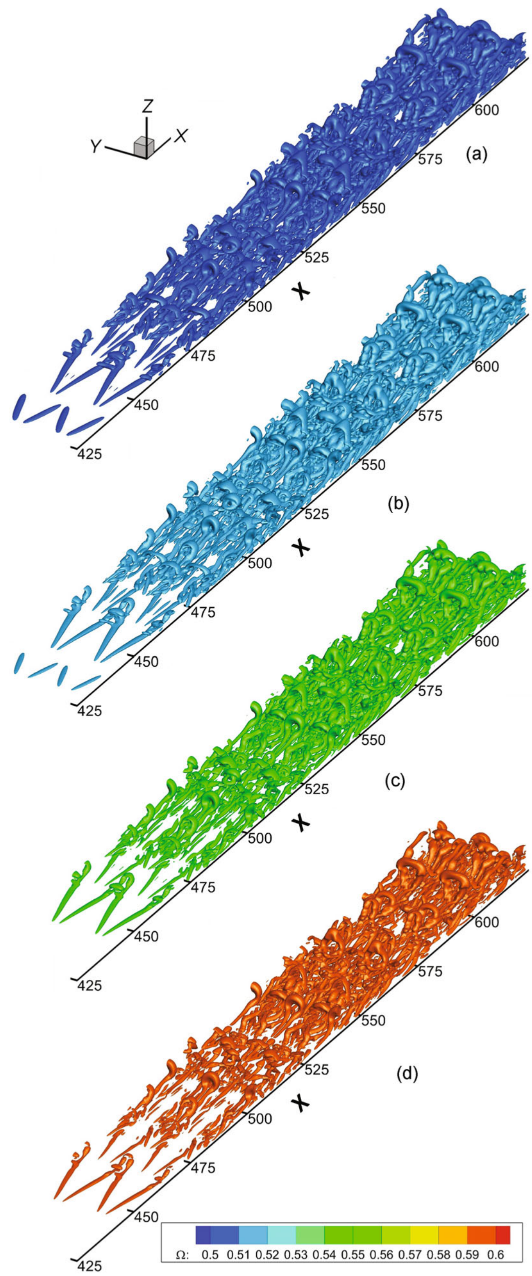

Figure 7 (Color online) Iso-surfaces of (a) Ω= 0.51, (b) Ω= 0.52, (c) Ω= 0.55, (d) Ω= 0.6 at $t = 12.9T$, where T is the period of *T-S* wave.

4.3 MVG case

MVG (Micro vortex generator) is a passive control device aiming to alleviate flow separation. Our previous study [12] shows that it is the ring-like vortical structures that functions to ease the separation. The computational domain is shown in Figure 8. The visualization of these three methods with same thresholds as the DNS case at a certain time step are illustrated in Figure 9. As shown by Figure 9(a), the λ_2-method fails to capture the vortical structures in the downstream and produce some noise in the space. Although the downstream vortical structures are relatively well captured by Q-method with a same threshold, there are too much noise in the space (Figure 9(b)). However, with the same Ω=0.52, the Ω-method is able to well capture all the vortices without any noise. It should be noticed that the MVG case and DNS case are quite different in many aspects like speed, compressibility and other condition. Despite the differences, the Ω-method well captured the vortical structures with the same value of Ω=0.52, while the other two methods fail to accomplish.

By adjusting the thresholds of λ_2 and Q based on the criteria described above, the vortical structures in this case can also be properly represented as in Figure 10. However, the new threshold of λ_2 is 100 times larger than the appropriate λ_2 in the DNS case and the new threshold of Q is 500 times larger than the appropriate Q in the DNS case. In addition, some literature shows vortices can be properly visualized by iso-surfaces of Q=50000 (Duan et al. [13]). A rather fixed threshold in Ω-method is one of the major advantages over other vortex identification methods.

4.4 RIT case

In recent years, RIT (Roughness Induced Transition) in hypersonic boundary layer has becomes a research hotspot. The DNS results in our previous study [13] show that the horseshoe vortex and the shear layer instabilities which come from the "jet" upstream of the roughness, dominate the flow transition procedure downstream.

Figure 11 depicts the vortex structures around the cylindrical roughness element, through the Q-criterion with different thresholds and the Ω-criterion. It should be note that the reference length in this study is 1 m, so the iso-surfaces of Q= 1000 and 10000 are used to visualized the vortices. But the Ω=0.52 iso-surface, which is the same with boundary layer flow and MVG cases, is capable to well illustrate the vortices. The Ω-method clearly shows the vortices around cylindrical roughness and the ring-like vortices downstream simultaneously, while the Q=10000 fails to illustrate the streamwise vortex structure in the side region.

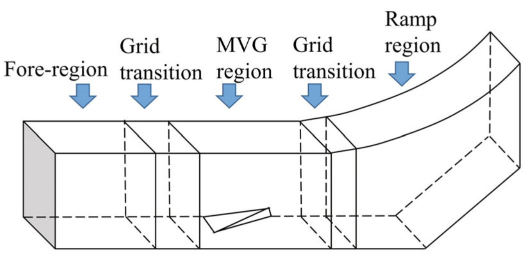

Figure 8 (Color online) The computational domain of MVG case.

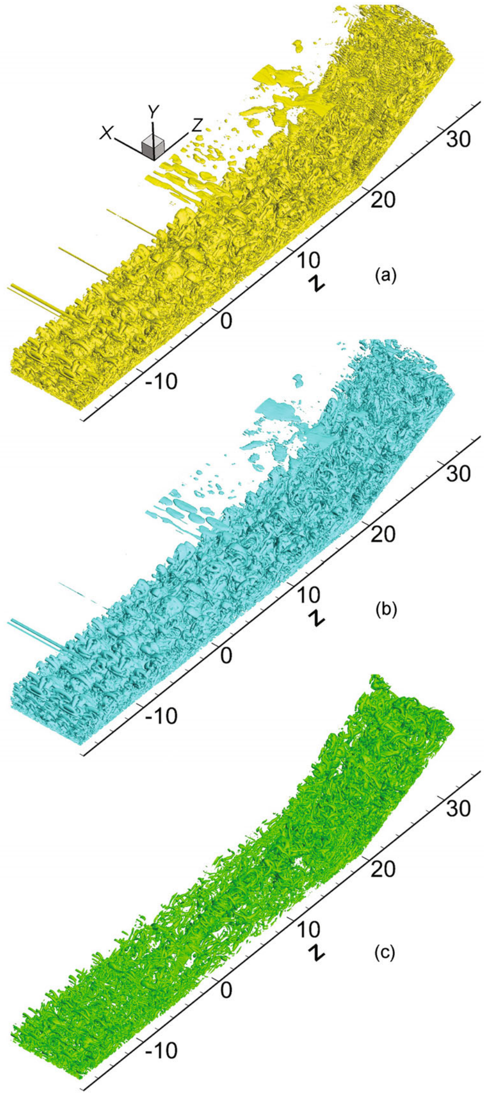

Figure 9 (Color online) Iso-surfaces of (a) λ_2=–0.001, (b) Q=0.001, (c) Ω=0.52.

5 Concluding remarks

Helmholtz fluid velocity decomposition of velocity field is revisited in this paper. It is demonstrated that the antisymmetric part of velocity gradient is not directly related to rotational motions in general. Based on the above analysis, a further decomposition of vorticity into a vortical part and non-vortical part is proposed. A new parameter Ω is introduced to repre-

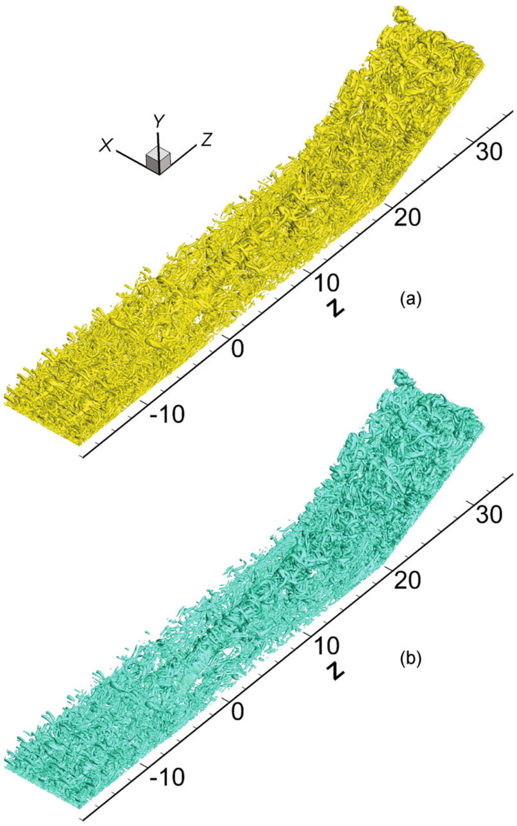

Figure 10 (Color online) Iso-surfaces of (a) $\lambda_2 = -0.1$, (b) $Q = 0.5$.

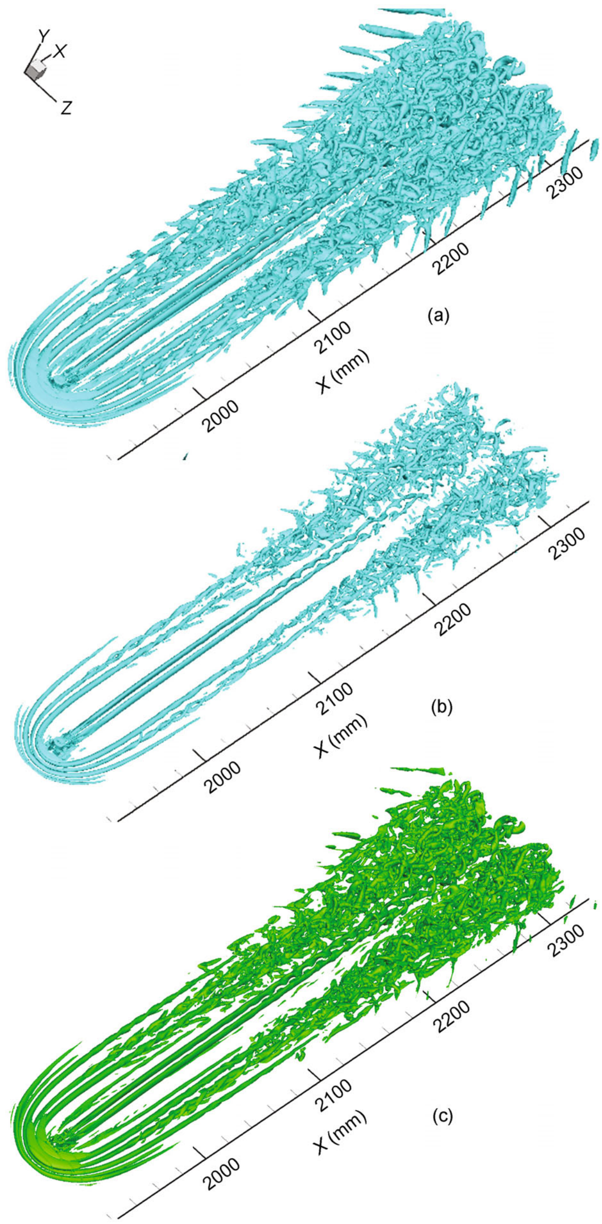

Figure 11 (Color online) Iso-surfaces of (a) Q=1000, (b) Q=10000, (c) Ω=0.52.

sent the ratio of vorticity square over the sum of vorticity square and deformation square. As the vortical vorticity is closely related to vortex motions, it is natural to visualize vortices using the idea described above. An estimation for the ratio Ω is proposed then. It is found the iso-surfaces of Ω=0.52 can well capture the vortices for all cases we studied and there is no need to empirically choose a threshold like λ_2 and Q-criterion.

Previous vortex identification methods including $\widetilde{\Delta}$, Q, λ_2 criteria are aiming to identify swirling strength. However, physical meanings of the values of $\widetilde{\Delta}$, Q, λ_2 are unclear. What's more, an arbitrary threshold introduces a degree of ambiguity. For example, some literature shows vortices visualized by iso-surfaces of Q=50000 (Duan et al. [13]) while the vortices is well visualized in Figure 3 by iso-surfaces of Q=0.001. The choosing of threshold seems arbitrary without a framework. Inappropriate thresholds may lead to false interpretation of turbulence physics. On the other hand, the threshold of this new method is always a little over 0.5 and has a clear physical meaning that is the vorticity overtakes the deformation. The reason why Ω=0.52 is selected is both theoretical and empirical selections. Theoretically Ω=0.52 means vorticity overtakes deformation. Empirically Ω=0.52 works for all cases we studied. These features of the new vortex identification method could be valuable and helpful for study of turbulence physics and many other vortex dominant flows in science and engineering.

The new method has the following features that previous methods like Q-criterion, λ_2-method do not possess.

(1) The Ω method is very easy to perform;

(2) The physical meaning of Ω, as a ratio of vorticity square over the sum of vorticity square and deformation square, is clear that the dissipation is low and the vorticity is locally dominant, while the values of Q and λ_2 are quite unclear;

(3) An arbitrary threshold is avoided by choosing a value Ω=0.52 while both Q and λ_2 methods require proper selection of a wildly changed threshold, case by case;

(4) Both strong and weak vortices can be well captured by the new method simultaneously due to the physical meaning

of Ω which is a ratio, while improper Q and λ_2 threshold may lead to strong vortex capture while weak vortices are lost or weak vortices are captured but strong vortices are smeared;

(5) Ω=0.52 is a pretty good quantity to define the vortex boundary.

A short Fortran subroutine is attached as appendix to calculate Ω from velocity field for uniform and non-uniform grids. A micro file is also attached for Techplot users to calculate Ω. Note that the length and velocity must be used in the nondimensional form to calculate Ω. Otherwise ε in the denominator of Ω needs to be adjusted to a large number depending on the length and velocity dimension.

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Supporting Information

The supporting information is available online at phys.scichina.com and http://link.springer.com/journal/11433. The supporting materials are published as submitted, without typesetting or editing. The responsibility for scientific accuracy and content remains entirely with the authors.

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