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Properties of quark matter in a new quasiparticle model with QCD running coupling

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The running of the QCD coupling in the effective mass causes thermodynamic inconsistency problem in the conventional quasiparticle model. We provide a novel treatment which removes the inconsistency by an effective bag constant. The chemical potential dependence of the renormalization subtraction point is constrained by the Cauchy condition in the chemical potential space. The stability and microscopic properties of strange quark matter are then studied within the completely self-consistent quasiparticle model, and the obtained equation of state of quark matter is applied to the investigation of strange stars. It is found that our improved model can describe well compact stars with mass about two times the solar mass, which indicates that such massive compact stars could be strange stars.

quark matter, quasiparticle model, thermodynamic inconsistency

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1 Introduction

Due to the non-perturbative nature of strong interactions at low energy scales [1], understanding the cold and strongly interacting quark matter from models inspired by quantum chromodynamics (QCD) is one of the most interesting subjects in modern nuclear physics [2]. It is generally believed that some neutron stars are in part or as a whole composed of deconfined matter, and thus provide a natural laboratory for studying the properties of quark matter [3, 4]. Recent observations [5] showed that the binary millisecond pulsar J1614-2230 has a mass of about 1.97 times the solar mass. The authors concluded that only strongly interacting quark matter can support such a large mass. Therefore, this precise mass measurement gives strong constraints on the equation of state (EOS) of quark matter.

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Strange quark matter (SQM), composed of deconfined up, down, and strange quarks, was conjectured by Witten [6] as the ground state of strong interactions. Soon Fahri and Jaffe [7] studied the properties of SQM in the framework of MIT bag model and found that SQM is absolutely stable for a wide range parameters. If this is really the case, SQM can be found inside compact objects [8-11], or in the form of strangelets [12].

Bombaci [13] compared theoretical results with the semiempirical mass-radius relation extracted for the X-ray burst source 4U 1820-30, and found that the source could be a kaon-condensed nucleon star or a strange star. Not long ago, based on a variety of scintillation phenomena observed from pulsars and quasars, the authors of ref. [14] proposed that the pulsar scintillations may be caused by an ionization agent constituted by positively charged lumps of SQM.

For strongly interacting quark matter, an important problem is how to describe the strong interactions between quarks appropriately. Because the fundamental theory of strong interactions is not exactly solvable at present, one has to turn to various phenomenological models characterizing the properties of QCD. Over the decades there have been many QCD inspired effective models [15-21]. The simplest as well as most popular used model is the MIT bag model [7]. This model treats quarks as a free Fermi gas plus an additional bag constant, sometimes with perturbative interactions included. The advantage of the bag model is obvious: it has a simple physical description of quark matter, and particularly useful when we face with a complex quantum many-body system, such as quark system.

However, quark masses vary with environment due to the strong coupling between quarks. Therefore, Fowler et al. [22] initially introduced a quark mass-density-dependent model (QMDD), which mimicked the strong interactions between quarks by the density dependence of quark masses. Afterwards, many further works have been done in this direction [23-28]. The thermodynamics treatment of this kind models has caused a lot of puzzles, and it is now clear that the quark chemical potential becomes effective if quark mass is density dependent while the quark number density and the system free energy density keep the same form of a free-gas system [29, 30].

By considering quarks as quasiparticles with an effective mass, the quasiparticle model has been widely used to study the properties of quark matter and quark-gluon plasma [31-33]. Unlike the QMDD model, the quark masses are not dependent on the density but on the chemical potential(s) and/or temperature [34-38]. The chemical-potential dependent quark masses are derived by an effective quark propagator with one-loop self-energy computed within the hard-dense-loop approximation [39, 40].

Because the quark masses are chemical potential and/or temperature dependent, thermodynamics formulas need special treatments. Originally quasiparticles were introduced as propagating in a refractive medium [41]: the pressure was written in an ideal gas form, while other quantities had extra terms in order to meet thermodynamic consistency [42]. In later calculations both the pressure and energy density have been taken in the form of an ideal gas [43, 44]. The, however, caused a serious thermodynamic inconsistency problem, as pointed out in ref. [45]. The inconsistency can be solved using different methods, among them the the Gorenstein and Steffens' approach [46] being the most popular. This approach was originally introduced for a gluon gas at finite temperature, and it has been extended to the case of quark matter with one chemical potential [47].

The effective mass in quasiparticle model depends not only explicitly on chemical potential but also on the QCD running coupling. The main purpose of the present paper is to include both the explicit dependence on chemical potentials and the running of the QCD coupling in a self-consistent way. It is found that the relation between the renormalization subtraction point and chemical potentials should satisfy a Cauchy-type equation. Accordingly, the SQM EOS becomes stiffer, which makes the maximum mass of strange stars as large as about two times the solar mass.

The organization of this paper is as follows. In sect. 2 , we study the thermodynamic self-consistency of the quasiparticle model with medium effects and the running coupling taken into account. Then, the improved model is used to study the stability and macroscopic properties of SQM in sect. 3. We apply the obtained new EOS to the investigation of strange stars in sect. 4. Finally, conclusions and discussions are given in the last sect. 5.

2 Thermodynamic self-consistency of the quasiparticle model with medium effects

Medium effects play an important role in describing the properties of quark matter via the concept of effective masses. The effective quark masses in the quasiparticle model are derived in the zero momentum limit of the dispersion relation from an effective quark propagator by resuming one-loop self-energy diagrams in the hard-dense-loop approximation [39, 48]

$$m_i = \frac{m_{i0}}{2} + \sqrt{\frac{m_{i0}^2}{4} + \frac{2\alpha_s}{3\pi}\mu_i^2}, \tag{1}$$

where m_i, m_{i0} and μ_i are, respectively, the effective mass, current mass and chemical potential of the corresponding quark flavor with i going over the up (u), down (d), and strange (s) quarks, while α_s is the QCD running coupling [49].

In the present paper the current quark masses of light quarks are assumed to be zero due to their small value compared to that of strange quarks. Consequently, eq. (1) is reduced to the following simple form:

$$m_{u/d} = \sqrt{\frac{2\alpha_s}{3\pi}}\mu_{u/d}. \tag{2}$$

We would like to comment that the expressions for effective quark masses we use in the present paper should not be taken in the limit of vanishing chemical potentials. There are several reasons for this: first of all, if one takes formally the zero μ limit then the QCD running coupling would diverge, which clearly shows that the chemical potentials should be no less than the QCD scale. Moreover, the effective quark masses have been computed by using a hard-dense-loop ansatz to take into account the in-medium effects on the self-energy. Therefore, there is no contradiction with effective chiral perturbation theories where one takes the contribution of the chiral condensate to the in-medium mass which vanishes at the chiral phase transition.

For a system consisting of up, down, strange quarks and electrons, the contribution from the quasiparticle type i to the thermodynamic potential at zero temperature is

$$\Omega_i = \frac{d_i}{2\pi^2}\int_0^{\nu_i}\left(\sqrt{k^2 + m_i^2} - \mu_i\right)k^2\mathrm{d}k$$

$$= \frac{d_i}{24\pi^2}\Big[\mu_i\nu_i\Big(\mu_i^2 - \frac{5}{2}m_i^2\Big) + \frac{3}{2}m_i^4\mathrm{arcsh}\Big(\frac{\nu_i}{m_i}\Big)\Big], \tag{3}$$

where $\nu_i = \sqrt{\mu_i^2 - m_i^2}$ is the Fermi momentum of particle type i, and d_i is the degeneracy factor with 2 for electrons and 6 for quarks.

In the framework of the quasiparticle model, the total thermodynamic potential density of SQM can then be written as:

$$\Omega = \sum_i \Omega_i(\mu_i, m_i) + B + B_0, \tag{4}$$

where $B \equiv B(\mu_u, \mu_d, \mu_s)$ is necessary to guarantee the thermodynamic consistency, and B_0 refers to the bag constant accounting for the vacuum energy.

For a quasiparticle Fermi system, the particle density of the particle type i should have the same form of a free-particle system with the current quark mass replaced by the effective quasiparticle mass, namely

$$n_i = -\frac{\partial\Omega_i}{\partial\mu_i} = \frac{d_i}{6\pi^2}(\mu_i^2 - m_i^2)^{3/2}. \tag{5}$$

If the QCD coupling α_s is treated as a free parameter, the effective mass of the quark flavor i depends merely on the corresponding flavor quark's chemical potential, i.e., $m_i = m_i(\mu_i)$, then we obviously have

$$n_i = -\frac{\partial\Omega_i}{\partial\mu_i} - \frac{\partial\Omega_i}{\partial m_i}\frac{\partial m_i}{\partial\mu_i} - \frac{\partial B}{\partial\mu_i}. \tag{6}$$

Since both eqs. (5) and (6) correspond to the number density of quasiparticle type i, comparison of these two equations naturally gives

$$\frac{\partial B}{\partial\mu_i} = -\frac{\partial\Omega_i}{\partial m_i}\frac{\partial m_i}{\partial\mu_i} \tag{7}$$

or

$$B = -\sum_i \int \frac{\partial\Omega_i}{\partial m_i}\frac{\partial m_i}{\partial\mu_i}\mathrm{d}\mu_i. \tag{8}$$

This treatment has been adopted by Wen et al. in ref. [50], and was previously done by Schertler et al. [48]. It can be easily proved that this approach is thermodynamically self-consistent when the QCD coupling is assumed to be a real constant.

However the QCD coupling runs with the energy scale, the evolution being given by the renormalization group equation

$$u\frac{\mathrm{d}\alpha_s(u)}{\mathrm{d}u} = \beta(\alpha_s(u)) = -\alpha_s^2(u)\sum_{j=0}^{N-1}\beta_j\alpha_s^j(u) \tag{9}$$

with N corresponding to the number of loops in the beta function. The renormalization subtraction point u is chosen according to the physical problem. In particle physics, it is taken to be the momentum transfer. In the present case, it should be a function of the chemical potentials μ_u, μ_d and μ_s. We therefore have

$$\alpha_s = \alpha_s(u(\mu_u, \mu_d, \mu_s)). \tag{10}$$

In the following, we show that the function $u(\mu_u, \mu_d, \mu_s)$ is constrained by a consistency condition.

Taking into account eq. (4) and the chain rule for derivative, we have

$$\begin{aligned} n_i &= -\frac{\mathrm{d}\Omega}{\mathrm{d}\mu_i}\Big|_{\mu_{j\neq i}} \\ &= -\frac{\partial\Omega_i}{\partial\mu_i} - \frac{\partial\Omega_i}{\partial m_i}\frac{\partial m_i}{\partial\mu_i} \\ &\quad -\sum_q \frac{\partial\Omega_q}{\partial m_q}\frac{\partial m_q}{\partial\alpha_s}\frac{\mathrm{d}\alpha_s}{\mathrm{d}u}\frac{\partial u}{\partial\mu_i} - \frac{\partial B}{\partial\mu_i} \end{aligned} \tag{11}$$

with $q = u, d, s$. Comparing the above equation with eq. (6) we notice additional terms in the former, namely terms involving derivative of the running coupling arising from chemical potential dependence of the coupling via the renormalization substraction.

Subtracting eq. (5) from eq. (11), we immediately have

$$\frac{\partial B}{\partial\mu_i} = -\frac{\partial\Omega_i}{\partial m_i}\frac{\partial m_i}{\partial\mu_i} - \sum_q \frac{\partial\Omega_q}{\partial m_q}\frac{\partial m_q}{\partial\alpha_s}\frac{\mathrm{d}\alpha_s}{\mathrm{d}u}\frac{\partial u}{\partial\mu_i}, \tag{12}$$

which means

$$\mathrm{d}B = F\mathrm{d}\mu_u + M\mathrm{d}\mu_d + J\mathrm{d}\mu_s, \tag{13}$$

where $F \equiv \partial B/\partial\mu_u$, $M \equiv \partial B/\partial\mu_d$, $J \equiv \partial B/\partial\mu_s$, or, explicitly

$$F = -\frac{\partial\Omega_u}{\partial m_u}\frac{\partial m_u}{\partial\mu_u} - \sum_q \frac{\partial\Omega_q}{\partial m_q}\frac{\partial m_q}{\partial\alpha_s}\frac{\mathrm{d}\alpha_s}{\mathrm{d}u}\frac{\partial u}{\partial\mu_u}, \tag{14}$$

$$M = -\frac{\partial\Omega_d}{\partial m_d}\frac{\partial m_d}{\partial\mu_d} - \sum_q \frac{\partial\Omega_q}{\partial m_q}\frac{\partial m_q}{\partial\alpha_s}\frac{\mathrm{d}\alpha_s}{\mathrm{d}u}\frac{\partial u}{\partial\mu_d}, \tag{15}$$

and

$$J = -\frac{\partial\Omega_s}{\partial m_s}\frac{\partial m_s}{\partial\mu_s} - \sum_q \frac{\partial\Omega_q}{\partial m_q}\frac{\partial m_q}{\partial\alpha_s}\frac{\mathrm{d}\alpha_s}{\mathrm{d}u}\frac{\partial u}{\partial\mu_s}. \tag{16}$$

In this case, therefore, the additional term should be given by a path integral as:

$$B = \int_{\mu_0}^{\mu}\mathrm{d}B = \int_{\mu_0}^{\mu}(F\mathrm{d}\mu_u + M\mathrm{d}\mu_d + J\mathrm{d}\mu_s), \tag{17}$$

where $\mu_0 = (0, 0, \mu_{s0})$ and $\mu = (\mu_u, \mu_d, \mu_s)$ are the starting and ending points for the integral respectively. μ_{s0} is determined by requiring the non-negative value of the Fermi momentum of strange quarks, i.e., $\nu_s = \sqrt{\mu_s^2 - m_s^2} \geqslant 0$, which leads to $\mu_{s0} = m_{s0}/(1 - 2\alpha_0/(3\pi))$ with $\alpha_0 = \alpha_0(0, 0, \mu_{s0})$ being the QCD running coupling at $\mu_u = \mu_d = 0$ and $\mu_s = \mu_{s0}$.

Because the thermodynamic potential is a state function, the integral in eq. (17) should be path independent. With this aim, the following Cauchy conditions should be imposed:

$$\frac{\partial F}{\partial \mu_d} = \frac{\partial M}{\partial \mu_u}, \quad \frac{\partial J}{\partial \mu_u} = \frac{\partial F}{\partial \mu_s}, \quad \frac{\partial J}{\partial \mu_d} = \frac{\partial M}{\partial \mu_s}. \tag{18}$$

Obliviously, if the QCD coupling constant was assumed to be a pure constant as in refs. [48, 50], the Cauchy conditions in eq. (18) would be always satisfied. However, the QCD coupling is running with the quark chemical potentials as shown in eq. (10). We thus need to find the conditions which ensure all the equalities in eq. (18).

The first order partial derivative of the quasiparticle contribution to Ω_i reads

$$\frac{\partial \Omega_i}{\partial m_i} = \frac{d_i m_i}{4\pi^2}\left[\mu_i \nu_i - m_i^2 \mathrm{arcsh}\left(\frac{\nu_i}{m_i}\right)\right]. \tag{19}$$

After carrying out the calculation in eq. (18), the three equalities can be simplified as follows:

$$\frac{\partial n_d}{\partial \mu_u} = \frac{\partial n_u}{\partial \mu_d}, \quad \frac{\partial n_s}{\partial \mu_u} = \frac{\partial n_u}{\partial \mu_s}, \quad \frac{\partial n_s}{\partial \mu_d} = \frac{\partial n_d}{\partial \mu_s}. \tag{20}$$

In fact, eq. (20) can also be derived from the fundamental differential equation of thermodynamics, i.e., $\mathrm{d}\Omega = \sum_i n_i \mathrm{d}\mu_i$.

Finally the set of eq. (20) reduces to the following quasi-linear differential equations:

$$\mu_u^3 \frac{\partial u}{\partial \mu_d} = \mu_d^3 \frac{\partial u}{\partial \mu_u}, \tag{21a}$$

$$\mu_u^3 \frac{\partial u}{\partial \mu_s} = \frac{m_s \mu_s^2 \sqrt{\mu_s^2 - m_s^2}}{\sqrt{1 - \frac{2\alpha_s}{3\pi}(m_s - \frac{m_{s0}}{2})}} \frac{\partial u}{\partial \mu_u}, \tag{21b}$$

$$\mu_d^3 \frac{\partial u}{\partial \mu_s} = \frac{m_s \mu_s^2 \sqrt{\mu_s^2 - m_s^2}}{\sqrt{1 - \frac{2\alpha_s}{3\pi}(m_s - \frac{m_{s0}}{2})}} \frac{\partial u}{\partial \mu_d}. \tag{21c}$$

Now, let us focus on eq. (21a) first. Dimensional analysis suggests the general solution of this equation to be $f(\rho)$ with $\rho = \sqrt[4]{\mu_u^4 + \mu_d^4}$. This means that the solution of u is a composite function of μ_s and ρ. However, it is generally not possible to write the solution explicitly. In this case, we assume the solution of eqs. (21a)-(21c) is determined by the following implicit function:

$$\phi(u, \rho, \mu_s) = 0. \tag{22}$$

Taking the derivatives respectively with respect to μ_u and μ_s on both sides of eq. (22), we get

$$\frac{\partial u}{\partial \mu_u} = -\frac{\mu_u^3}{\rho^3}\frac{\phi_\rho}{\phi_u}, \quad \frac{\partial u}{\partial \mu_s} = -\frac{\phi_{\mu_s}}{\phi_u}, \tag{23}$$

where ϕ_ρ, ϕ_u and ϕ_{μ_s} indicate the derivatives of ϕ with respect to ρ, u and μ_s respectively. Substituting eq. (23) into eq. (21b), then we get

$$\rho^3 \phi_{\mu_s} = \frac{m_s \mu_s^2 \sqrt{\mu_s^2 - m_s^2}}{\sqrt{1 - \frac{2\alpha_s}{3\pi}(m_s - \frac{m_{s0}}{2})}} \phi_\rho. \tag{24}$$

To find a practically usable solution of eq. (24), the simplest choice is to take $\partial\phi/\partial\rho = 4\rho^3$. In this case, we have

$$\frac{\partial \phi}{\partial \mu_s} = \frac{4 m_s \mu_s^2 \sqrt{\mu_s^2 - m_s^2}}{\sqrt{1 - \frac{2\alpha_s}{3\pi}(m_s - \frac{m_{s0}}{2})}}. \tag{25}$$

Consequently, we can write ϕ in the following form:

$$\phi = \rho^4 + \int \phi_{\mu_s} \mathrm{d}\mu_s - \theta(u) = 0 \tag{26}$$

with $\theta(u)$ being an arbitrary function of u. Here, in order to satisfy the dimensional requirement and for simplicity, we choose $\theta(u) = N_\mathrm{f} u^4/C^4$ with C being a constant model parameter which might be determined by the stability of SQM.

In conclusion, the solution of the Cauchy conditions in eq. (18) is determined by the following equation:

$$\phi = \rho^4 + \lambda - \frac{N_\mathrm{f}}{C^4} u^4 = 0. \tag{27}$$

In the following $\alpha = \alpha_s/\pi$. The integral in eq. (26) is given by

$$\lambda = \int \frac{4 m_s \mu_s^2 \sqrt{\mu_s^2 - m_s^2}}{\sqrt{1 - \frac{2}{3}\alpha\left(m_s - \frac{m_{s0}}{2}\right)}} \mathrm{d}\mu_s = \frac{4}{b^2} \int (x + c_1)^2 \sqrt{x^2 - c_0^2} \mathrm{d}x. \tag{28}$$

In the second equality, we have used the notations

$$b \equiv \frac{2}{3}\alpha, c_0 \equiv \frac{b m_{s0}}{2(1-b)}, c_1 \equiv \frac{m_{s0}}{2}\frac{2-b}{1-b}, \tag{29}$$

and changed integration variable substitution from μ_s to x by

$$x = m_s - c_1 = \sqrt{\frac{m_{s0}^2}{4} + b\mu_s^2} + \frac{m_{s0}}{2(b-1)}, \tag{30}$$

which makes the integration easier; we get

$$\lambda = \frac{3x + 8c_1}{3b^2}(x^2 - c_0^2)^{3/2} + \frac{x}{2b^2}\left(4c_1^2 + c_0^2\right)\sqrt{x^2 - c_0^2} - \frac{c_0^2}{2b^2}(4c_1^2 + c_0^2)\ln\left(x + \sqrt{x^2 - c_0^2}\right). \tag{31}$$

Considering the symmetric case, i.e., $m_s \to 0$ in eq. (27), we obtain a simple form for the renormalization subtraction as:

$$u = C\sqrt[4]{\frac{\mu_u^4 + \mu_d^4 + \mu_s^4}{N_\mathrm{f}}}, \tag{32}$$

which is a good approximation for SQM at high density since the current quark mass of a strange quark becomes negligible in that case. When considering merely two-flavor massless quarks, eq. (32) becomes

$$u = C\sqrt[4]{\frac{\mu_u^4+\mu_d^4}{N_{\rm f}}}, \tag{33}$$

which is valid for non-strange quark matter at zero temperature [51].

The partial derivative of λ with respect to the renormalization substraction u is

$$\begin{aligned}\frac{\partial\lambda}{\partial u} = -\Bigg[&\frac{m_{s0}}{2b^2}\frac{\sqrt{x^2-c_0^2}}{(1-b)^2}\Big(6c_0c_1+2c_0m_s+4c_1m_s\\ &+\frac{4}{3}(x^2+2c_0^2)+\frac{c_0x(4c_1^2+c_0^2)}{2(x^2-c_0^2)}\\ &-\frac{c_0^3}{2}\frac{4c_1^2+c_0^2}{x^2-c_0^2}\frac{1}{x+\sqrt{x^2-c_0^2}}\\ &+\frac{4c_0c_1(c_0+c_1)+2c_0^3}{\sqrt{x^2-c_0^2}}\ln\left(x+\sqrt{x^2-c_0^2}\right)\Big)\\ &-\frac{4m_s^2\mu_s^2\sqrt{x^2-c_0^2}}{b^2(2m_s-m_{s0})}+\frac{2}{b}\lambda\Bigg]\frac{2}{3\pi}\frac{{\rm d}\alpha_s}{{\rm d}u}.\end{aligned} \tag{34}$$

Also, one can easily get the partial derivatives of ϕ with respect to μ_u and μ_d respectively, i.e.,

$$\frac{\partial\phi}{\partial\mu_u} = 4\mu_u^3 \quad\text{and}\quad \frac{\partial\phi}{\partial\mu_d} = 4\mu_d^3. \tag{35}$$

As a result, the partial derivatives of the renormalization substraction u with respect to the respective chemical potential μ_q ($q = u, d, s$) in eqs. (14)-(16) are

$$\frac{\partial u}{\partial\mu_q} = \frac{\partial\phi/\partial\mu_q}{4N_f u^3/C^4 - \partial\lambda/\partial u}. \tag{36}$$

For the u dependence of the coupling, we adopt the following analytic form [52]:

$$\alpha_s = \frac{1}{\beta_0}\left[\frac{1}{\ln(u^2/\Lambda^2)}+\frac{\Lambda^2}{\Lambda^2-u^2}\right] \tag{37}$$

with $\beta_0 = (33-2N_f)/12\pi$ and Λ the QCD energy scale. In the present paper, we take $\Lambda = 147$ MeV [49].

The derivative of the QCD coupling with respect to the renomination substraction u on the right-hand side of eq. (34) is

$$\frac{{\rm d}\alpha_s}{{\rm d}u} = \frac{2}{\beta_0 u}\left[\frac{u^2\Lambda^2}{(\Lambda^2-u^2)^2}-\frac{1}{\ln^2(u^2/\Lambda^2)}\right]. \tag{38}$$

Substituting eq. (34) into eq. (17), then the additional term B can be derived by numerical integration. Furthermore, the energy density and pressure of SQM can be obtained by

$$E = \Omega + \sum_i \mu_i n_i, \quad P = -\Omega. \tag{39}$$

Now, a new treatment of the quasiparticle model has been proposed. In this treatment, the thermodynamic inconsistency problem is removed by an additional effective bag constant.

3 Properties of strange quark matter

In the previous section, we have presented a completely self-consistent quasiparticle model. In this section, we will use it to study the stability window and bulk properties of SQM.

For SQM in beta equilibrium maintained by weak reactions such as $s, d \leftrightarrow u + e^- + \overline{\upsilon}_e$ and $s + u \leftrightarrow u + d$, the chemical potentials satisfy the conditions

$$\mu_s = \mu_d = \mu_u + \mu_e. \tag{40}$$

In addition, the charge neutrality condition should be imposed to the bulk SQM, i.e.,

$$\frac{2}{3}n_u - \frac{1}{3}n_d - \frac{1}{3}n_s - n_e = 0. \tag{41}$$

At the same time, the baryon number density is given by

$$n_b = \frac{1}{3}(n_u + n_d + n_s). \tag{42}$$

For a given n_b, the chemical potentials μ_u, μ_d, μ_s and μ_e can be obtained by numerically solving the coupled eqs. (27) and (40)-(42). Accordingly, the chemical potential dependent bag constant B can be derived by performing the integration in eq. (17). Also, with the given expression of thermodynamic potential density in eq. (4), the EOS of strange quark matter can be calculated from eq. (39).

For any model-given energy density E and pressure P in perfect chemical equilibrium, the following equality

$$\triangle = P - n_b^2\frac{\rm d}{{\rm d}n_b}\left(\frac{E}{n_b}\right) = 0 \tag{43}$$

has to be fulfilled [30]. This means that the pressure must be exactly zero at the extreme point of the energy per baryon for any consistent model. In Figure 1, we show energy per baryon as well as pressure versus baryon density for the model with thermodynamic inconsistency and for our improved model. The difference between these two models is that the former one does not include the contribution from eq. (17). In fact, the contribution term in eq. (17) is necessary for maintaining thermodynamic consistency. It is obvious in our treatment that the energy per baryon and pressure are located at the same baryon density.

The important feature known as asymptotic freedom indicates that the interactions between quarks become weaker at shorter distance. Because of asymptotic freedom it is supposed that quarks behave as almost free particles at high energy scales. The normalized pressure $P/P_{\rm free}$ with different parameter sets is shown in Figure 2, where the free quark gas

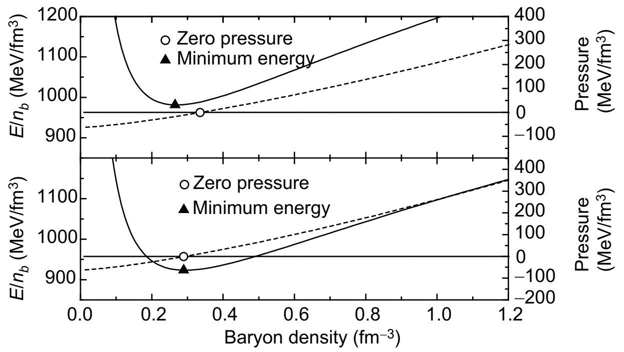

Figure 1 Density behavior of the energy per baryon and pressure in the models without (upper panel) and with (lower panel) the additional term.

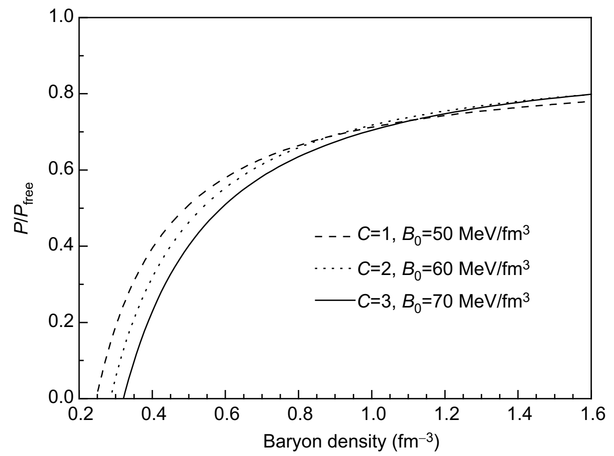

Figure 2 The pressure relative to the free strange quark gas pressure.

pressure is $P_{\rm free} = -\sum_i \Omega_i(\mu_i, m_{i0})$. The curves show qualitatively similar behaviors that they all trend to the free strange quark gas result at high baryon density.

We have to pay special attention to the stability of SQM since it may be the true ground state of strong interactions. It is a well-established fact that two-flavor quark matter is less stable compared to the most stable ^{56}F$_e$ nucleus at zero temperature [7]. For the ^{56}F$_e$ nucleus, its energy per nucleon $\epsilon_0(^{56}{\rm F_e})\approx$ 930 MeV. Therefore, energy per baryon of two-flavor quark matter should be greater than 930 MeV at zero temperature so as not to contradict the standard nuclear physics. Meanwhile, the energy per baryon of three-flavor quark matter should be lower than 930 MeV so that SQM can have a change to be absolutely stable:

$$\left.\frac{E}{n_b}\right|_{\rm ud} \geqslant \epsilon_0(^{56}{\rm F_e}) \geqslant \left.\frac{E}{n_b}\right|_{\rm SQM}. \tag{44}$$

If the energy per baryon of SQM satisfy

$$\epsilon(n) \geqslant \left.\frac{E}{n_b}\right|_{\rm SQM} \geqslant \epsilon_0(^{56}{\rm F_e}), \tag{45}$$

SQM is metastable, where $\epsilon(n) \approx 939$ MeV is the rest mass of a single nucleon. Otherwise, SQM is unstable. Only if SQM is absolutely stable or metastable can it be the true ground state of strong interactions as well as likely to exist in the interior of compact stars.

From eqs. (44) and (45) we can numerically derive the parameter range for SQM. In Figure 3, the stability window of SQM is shown in the C-B_0 plane. SQM is unstable in the uppermost region since its energy per baryon is larger than the rest mass of a single nucleon, whereas the lowest region is forbidden. In the region bounded by dotted and solid lines, SQM is absolutely stable.

To study the microscopic properties of SQM and structure of compact stars, we have selected several typical values of the parameters in the absolutely stable parameter region, i.e., (C, B_0) =(1,50), (2,60), (3,70), where C is dimensionless while B_0 is in unit of MeV $\cdot$ fm^{-3}. The selected parameter pairs are labeled in Figure 3 with solid dots.

In Figure 4 we plot the energy per baryon versus baryon

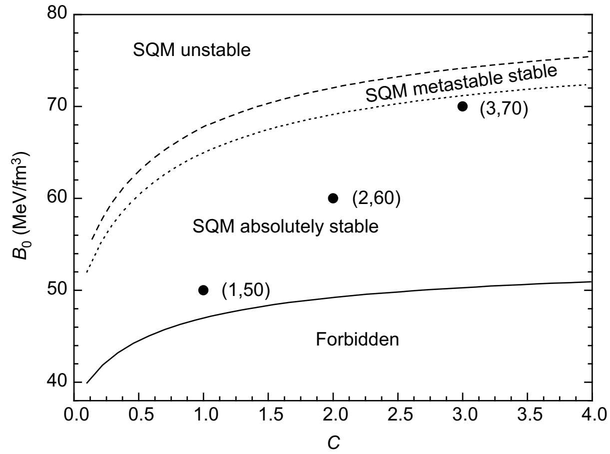

Figure 3 The stability window in the C-B_0 plane for SQM at zero temperature. SQM is absolutely stable in the region bounded by the solid and dotted lines.

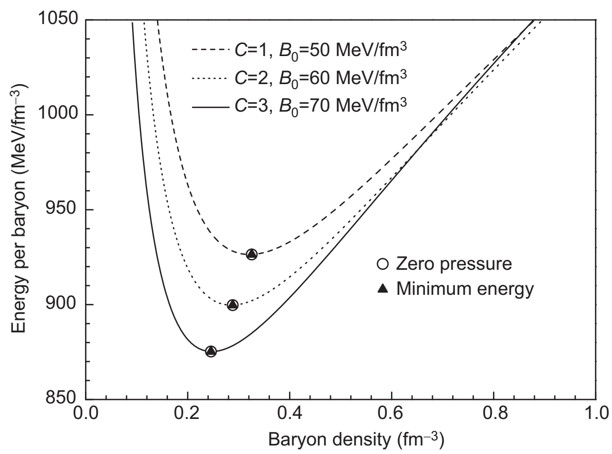

Figure 4 The energy per baryon as a function of the baryon density with different parameter sets. It is clear that the zero pressure (the circle) are exactly located at the minimum energy per baryon (the triangle).

number density for several parameters sets. It is evident from the figure that the minimum energy per baryon corresponds to a vanishing pressure. Moreover, a further study shows that larger value of C trends to have a lower value of the minimum energy per baryon and the corresponding baryon number density becomes larger for the same value of the bag constant.

We define $B^* = B + B_0$ as the effective bag constant, which is the sum of eq. (17) and the vacuum energy B_0. In Figure 5, we plot the effective bag constant B^* as a function of the baryon number density with the parameter pairs selected in Figure 3. It is obvious the effective bag constant decrease monotonically with increasing density.

This is understandable. The additional function B, with its expression derived by the requirement of thermodynamic consistency, arises because quarks acquire effective masses in a medium. The effective masses depend on the QCD running coupling that reflects interactions. The running bag constant, therefore, reflect the interaction between the vacuum and matter. At lower densities, the interaction strength is larger due to confinement. With increasing densities, however, the interactions becomes weaker due to the asymptotic freedom of the strong interaction, and the B value is according smaller.

4 Mass-radius relation of strange stars

Compact stars provide natural laboratories to explore the properties of quark matter and deconfinement phase transition at high baryon number density. The study of deconfinement transition in compact stars is a hot topic in modern physics [53]. Because strange quark matter can be self-bounded, some neutron stars might be composed entirely of SQM, which would be named as strange stars.

The structure of strange stars depends crucially on the EOS of quark matter. Recently, the mass measurement of compact stars PSR J1614-2230 and PSR J0348-0432 gave the gravitational mass $1.97 \pm 0.04 M_\odot$ [5] and $2.01 \pm 0.04 M_\odot$ [54] respectively, where $M_\odot$ is the solar mass. The observation data offer tight constraints on the EOS of cold quark matter as well as on the model parameters.

The equilibrium structure of a static spherically symmetric strange star is determined by the Tolman-Oppenheimer-Volkoff (TOV) equation

$$\frac{\mathrm{d}P(r)}{\mathrm{d}r} = -\frac{GmE}{r^2}\frac{(1+P/E)(1+4\pi r^3 P/m)}{1-2Gm/r}, \tag{46}$$

and the subsidiary condition

$$\frac{\mathrm{d}m(r)}{\mathrm{d}r} = 4\pi r^2 E, \tag{47}$$

where $G = 6.7\times 10^{-45}$ MeV^{-2} is the gravitational constant, r is the radial coordinate of the strange star and m is the gravitational mass contained within the radius r.

The TOV equation can be solved numerically by integration from the center of the star up to the radius, the latter being defined as the coordinate $r = R$ at which the pressure vanishes; the boundary conditions for the TOV equation are thus

$$n_b(r=0) = n_c, \quad m(r=0) = 0, \quad P(r=R) = 0, \tag{48}$$

where n_c is the central baryon number density. The EOS of quark matter in the preceding section is necessary for the calculation. Then the radius R of the strange star is determined by the stability condition, i.e., the third equality in eq. (48). One can also derive the gravitational mass of the star by $M = m(r = R)$. The detailed solving process can be found in ref. [55].

In Figure 6, the mass-radius relation of strange stars is shown for the selected parameters indicated in the legend. We notice that all the curves show the same qualitative behavior: firstly the mass increases with the radius up to the maximum mass $M_{\rm max}$, then it decreases and the star becomes mechanically unstable [55]. In addition, it is found that the maximum mass of strange stars decreases with the increasing bag constant and the parameter C, in agreement with the idea that quark matter with a stronger interaction can support a

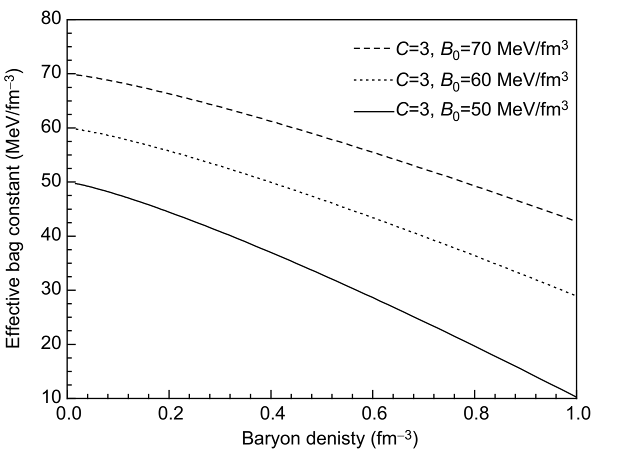

Figure 5 The density behavior of the effective bag constant B^* in our model with different values of C and B_0.

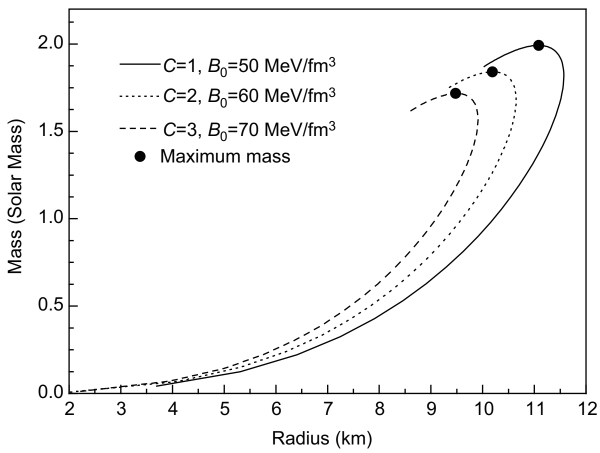

Figure 6 The mass-radius relation of strange stars for different parameter sets of C and B_0.

heavier star compared with the relatively free quarks. For the typical parameter pair (C, B_0)= $(1, 50\ \mathrm{MeV/fm^3})$, the maximum gravitational mass is very close to two times the solar mass. This means that such massive stars could be strange stars.

To study the density distribution within a strange star, we display the density profiles in Figure 7 for several typical parameter sets. The solid line in each panel represents the highest acceptable central baryon number density that corresponds to the maximum mass for a certain set of parameters. We can see that all the curves decrease smoothly and monotonically with increasing radius from the central outward until the star surface.

5 Conclusions

The thermodynamic self-consistency plays a curial role in the phenomenological models. In the present work, we particularly focused on the thermodynamic consistency in the quasiparticle model when the effective mass depends on both the chemical potentials and the running QCD coupling.

Because the QCD coupling α_s is a running coupling, in order to take into account this QCD feature we assume a quark chemical potential dependence of α_s, and we fix this dependence by a choice of a subtraction at the renormalization point depending on the chemical potential. Then the relevant functional form is constrained by the thermodynamic self-consistency requirement. We found that the effective bag constant decreases monotonously with increasing density.

Within the improved quasiparticle model, we have explored the stability and EOS of SQM in beta equilibrium at zero temperature. We have found that SQM could be absolutely stable in a reasonable parameter region. In addition, the derived EOS of quark matter is applied to study the structure of strange stars. We have shown that the recently discovered massive compact stars [5,54] with mass around two times the solar mass can be well described within our model if reasonable parameter sets are adopted. In other words, our calculations implies that such massive stars could be strange stars.

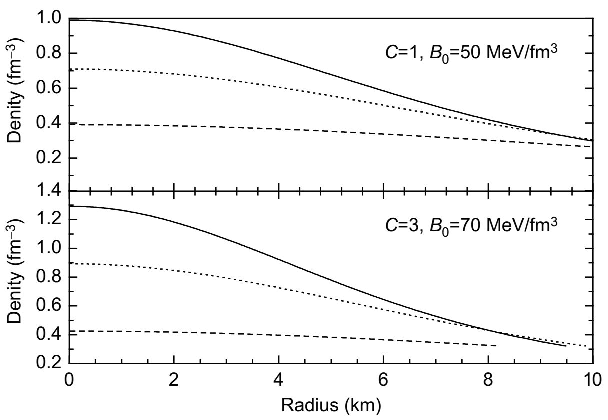

Figure 7 Density profiles of strange stars for different sets of parameters. The solid curve in each panel corresponds to the central density for the strange star with maximum mass.

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