SCIENCE CHINA
Physics, Mechanics & Astronomy

• Article •

July 2014 Vol.57 No.7: 1318–1330
doi: 10.1007/s11433-014-5446-9

A collocation reliability analysis method for probabilistic and fuzzy mixed variables

XU MengHui* & QIU ZhiPing

Institute of Solid Mechanics, Beihang University, Beijing 100191, China

Received June 23, 2013; accepted December 6, 2013; published online May 13, 2014

The main bottleneck of the reliability analysis of structures with aleatory and epistemic uncertainties is the contradiction between the accuracy requirement and computational efforts. Existing methods are either computationally unaffordable or with limited application scope. Accordingly, a new technique for capturing the minimal and maximal point vectors instead of the extremum of the function is developed and thus a novel reliability analysis method for probabilistic and fuzzy mixed variables is proposed. The fuzziness propagation in the random reliability, which is the focus of the present study, is performed by the proposed method, in which the minimal and maximal point vectors of the structural random reliability with respect to fuzzy variables are calculated dimension by dimension based on the Chebyshev orthogonal polynomial approximation. First-Order, Second-Moment (FOSM) method which can be replaced by its counterparts is utilized to calculate the structural random reliability. Both the accuracy and efficiency of the proposed method are validated by numerical examples and engineering applications introduced in the paper.

reliability, membership function, probabilistic variable, fuzzy variable, mixed uncertain variables, FOSM

PACS number(s): 46.70.-p, 02.60.Cb, 02.30.Mv, 02.50.Cw, 05.10.Ln

Citation: Xu M H, Qiu Z P. A collocation reliability analysis method for probabilistic and fuzzy mixed variables. Sci China-Phys Mech Astron, 2014, 57: 1318–1330, doi: 10.1007/s11433-014-5446-9

1 Introduction

Reliability is one of the major concerns in engineering practices since the occurrence of failures may lead to catastrophic consequences. A variety of reliability analysis methods have been proposed over the last three decades and have stimulated the interest in the probabilistic optimum design of structures based on the consideration that the structural reliability is solely determined by the parametric randomness. With the theoretical advancements of uncertainty analysis theory, the nature of uncertainty is gradually recognized, which subsequently promotes the development in the structural reliability analysis. Uncertainties present in realistic problems can be classified into two categories, namely, aleatory and epistemic uncertainties depending on the amount and type of information available. The aleatory uncertainty modeled as randomness using probabilistic theory is the natural variation of the structure while the epistemic uncertainty modeled as fuzziness using possibility theory is due to the lack of knowledge about the parameter. It is noted that both randomness and fuzziness always coexist in engineering practices, e.g. the preliminary design of an airfoil where sufficient information required to model random variation is typically unavailable. However, as the design progresses additional information can be used to modify some of these parametric variations initially modeled as fuzziness and then coexistence of randomness and fuzziness occurs. Accordingly, the reliability analysis for probabilistic and fuzzy mixed variables is urgent.

Reliability analysis for the probabilistic and fuzzy mixed

*Corresponding author (email: shandongxumenghui@163.com)

variables originates from the random reliability analysis combining possibility theory dealing with fuzziness in the structure. The classical random reliability has been studied deeply and then numerous methods have been proposed, e.g. sampling method [1–5], moment method [6–9] and MPP method [10–12]. Following the pioneering work of Braibant et al. [13] who used possibilistic approaches for the structural optimization and design, vertex method which is only well suited for monotonic problems was proposed [14] and optimization techniques with high accuracy as well as unaffordable computational efforts for implicit performance functions have been employed to accomplish the fuzziness propagation for fuzzy reliability analysis [15,16]. All methods aforementioned consider only either random or fuzzy variables but do not accommodate a combination of variables. Considering mixed uncertainties in the structure, a few typical methods have been proposed. Möller et al. [17] developed an effective method for estimating the membership function of the safety index from which the calculation of the structural failure probability is prone to errors. Adduri et al. [18,19] proposed a system reliability analysis method utilizing the transformation of membership functions for the random reliability analysis and a non-complete second-order response surface model with limited accuracy for the limit-state function, which cannot handle the cross-terms of random and fuzzy variables. Besides, Lu et al. [20–22] proposed a methodology combining the saddle-point approximation for linear performance function with the line or truncated importance sampling method to improve the computation efficiency in random reliability analysis but optimization techniques with much computational efforts are adopted to capture the minimum and maximum of the random reliability with respect to (w.r.t.) fuzzy variables at each alpha-level, in which local extremum may yet be obtained.

It is noted that the efficiency of the reliability analysis method for probabilistic and fuzzy mixed variables can be enhanced by improving the random reliability analysis method, which is the motivation of existing methods [18–22], and by accomplishing fuzziness propagation based on the random reliability analysis efficiently, which have not been reported yet but motivated the present study. In this paper, an efficient reliability analysis method with high accuracy for probabilistic and fuzzy mixed variables based on the First-Order Second-Moment (FOSM) method for the classical random reliability analysis is proposed to overcome the potential obstacles mentioned above. The minimum and maximum of the structural reliability at each alpha-level can be calculated dimension and dimension w.r.t. fuzzy variables based on the Chebyshev orthogonal polynomial approximation instead of optimization techniques.

2 Problem statement

For simplicity, the independent probabilistic variables, into which the dependent ones could be transformed, are merely considered. And it is noted that fuzzy variables are, in general, independent of each other in engineering practices. The vector consisting of probabilistic and fuzzy mixed variables for the reliability analysis can be denoted as $\boldsymbol{x}=(\boldsymbol{x}_S, \boldsymbol{x}_F)$ where $\boldsymbol{x}_S=(x_{s1},x_{s2},\dots,x_{sn})$ is a vector with probabilistic element whose probability density function is expressed as $f_{x_{si}}(x_{si})$ w.r.t. x_{si} (i=1,2,…,n) while $\boldsymbol{x}_F=(x_{f1},x_{f2},\dots,x_{fm})$ is a vector with fuzzy element whose membership function is denoted as $\mu_{x_{fj}}(x_{fj})$ w.r.t. x_{fj} (j=1,2,…,m). Thus, the structural performance function can be expressed as $z=g(\boldsymbol{x}_S,\boldsymbol{x}_F)$ and then the reliability is obtained as follows:

$$P_r = P\{z \geqslant 0\} = 1 - P\{z < 0\} = 1 - P_f, \quad (1)$$

where P_f is the failure probability of the structure. It is noted that the resulting reliability is fuzzy due to the fuzziness of the parameters $\boldsymbol{x}_F$ based on the random reliability theory. Theoretically there are two pivotal procedures for accurately estimating the membership function of the reliability, i.e. calculating the random reliability in the reduced space spanned by the probabilistic variables with the fuzzy variables being fixed at given values and capturing the minimum and maximum of the random reliability within the specified bounds of fuzzy variables at each alpha-level. It is the classical random reliability analysis essentially for the former that has been extensively researched with plentiful and substantial achievements and the fuzziness propagation analysis for the latter which is the focus of the present study.

Zadeh's extension principle extending a classical crisp function to a fuzzy mapping plays an important role in the fuzziness propagation analysis. It is supposed that V is a Cartesian product of universes $V = V_1 \times V_2 \times \cdots \times V_n$ and $\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n$ are n fuzzy sets in $V_1, V_2, \dots, V_n$, respectively. Let $f(x_1, x_2, \dots, x_n)$ be a crisp function mapping from V to a universe set W and then the extension principle induces a fuzzy set $\tilde{b}$ in W by

$$\tilde{b} = \left\{ \left(y, \mu_{\tilde{b}}(y)\right) \middle| y = f(x_1, x_2, \dots, x_n), (x_1, x_2, \dots, x_n) \in V \right\}, \quad (2)$$

where

$$\mu_{\tilde{b}}(y) = \max_{y=f(x_1,x_2,\dots,x_n)} \min\left\{ \mu_{\tilde{a}_1}(x_1), \mu_{\tilde{a}_2}(x_2), \dots, \mu_{\tilde{a}_n}(x_n) \right\}. \quad (3)$$

As the mathematical basis of the present method for the fuzziness propagation analysis, the following property of Zadeh's extension principle is firstly obtained.

Proposition Let $f(x_1, x_2, \dots, x_n)$ be a crisp function and $\tilde{a}_1, \tilde{a}_2, \dots, \tilde{a}_n$ be n fuzzy numbers. $\tilde{f}: \mathscr{F}^n \to \mathscr{F}$ is supposed to be a fuzzy-valued function induced by f through Zadeh's extension principle. If all membership functions $\mu_{\tilde{a}_i}(x_i)$ are continuous and the range of f is compact for all y, the alpha-cut of the resulting fuzzy number $\tilde{b}=\tilde{f}(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n)$ can be expressed as follows:

$$\tilde{b}_\alpha=\left\{f(x_1,x_2,\ldots,x_n): x_i\in\tilde{a}_{i,\alpha}, i=1,2,\ldots,n\right\}. \tag{4}$$

That is

$$\begin{aligned}b_\alpha^{\mathrm{l}}&=\min\left\{f(x_1,x_2,\ldots,x_n)\middle|x_i\in\left[\tilde{a}_{i,\alpha}^{\mathrm{l}},\tilde{a}_{i,\alpha}^{\mathrm{u}}\right], i=1,2,\ldots,n\right\},\\ b_\alpha^{\mathrm{u}}&=\max\left\{f(x_1,x_2,\ldots,x_n)\middle|x_i\in\left[\tilde{a}_{i,\alpha}^{\mathrm{l}},\tilde{a}_{i,\alpha}^{\mathrm{u}}\right], i=1,2,\ldots,n\right\},\end{aligned} \tag{5}$$

where b_α^{l} and b_α^{u} are the lower and upper bounds of the fuzzy number $\tilde{b}$ at the alpha-level, respectively.

Proof For an arbitrary real number y in the range of the crisp function f, if

$$y_0\in\left\{f(x_1,x_2,\ldots,x_n): x_i\in\tilde{a}_{i,\alpha}, i=1,2,\ldots,n\right\},$$

then a vector $\boldsymbol{x}=(x_1,x_2,\ldots,x_n)$ satisfying the equation $y_0=f(x_1,x_2,\ldots,x_n)$ can be obtained and $x_i\in\tilde{a}_{i,\alpha}$ ($i=1,2,\ldots,n$). Thus $\mu_{\tilde{a}_i}(x_i)\geqslant\alpha$ holds and the following equation could be obtained

$$\mu_{\tilde{b}}(y_0)=\max_{y_0=f(x_1,x_2,\ldots,x_n)}\min_{1\leqslant i\leqslant n}\mu_{\tilde{a}_i}(x_i)\geqslant\alpha,$$

i.e. $y_0\in\tilde{b}_\alpha$. Therefore, we can obtain that

$$\left\{f(x_1,x_2,\ldots,x_n): x_i\in\tilde{a}_{i,\alpha}, i=1,2,\ldots,n\right\}\subseteq\tilde{b}_\alpha.$$

On the other hand, if $y_0\in\tilde{b}_\alpha$, then

$$\max_{y_0=f(x_1,x_2,\ldots,x_n)}\min_{1\leqslant i\leqslant n}\mu_{\tilde{a}_i}(x_i)\geqslant\alpha.$$

Because the range of f is a compact set and $\min\limits_{1\leqslant i\leqslant n}\mu_{\tilde{a}_i}(x_i)$ is continuous on R^n, there exists a vector $\boldsymbol{x}=(x_1,x_2,\ldots,x_n)$ satisfying that $\min\limits_{1\leqslant i\leqslant n}\mu_{\tilde{a}_i}(x_i)\geqslant\alpha$ and $f(x_1,x_2,\ldots,x_n)=y_0$. Therefore, $\mu_{\tilde{a}_i}(x_i)\geqslant\alpha$ i.e. $x_i\in\tilde{a}_{i,\alpha}$ ($i=1,2,\ldots,n$) and

$$y_0\in\left\{f(x_1,x_2,\ldots,x_n): x_i\in\tilde{a}_{i,\alpha}, i=1,2,\ldots,n\right\}.$$

It says that

$$\tilde{b}_\alpha\subseteq\left\{f(x_1,x_2,\ldots,x_n): x_i\in\tilde{a}_{i,\alpha}, i=1,2,\ldots,n\right\}.$$

Therefore, we can obtain that

$$\tilde{b}_\alpha=\left\{f(x_1,x_2,\ldots,x_n): x_i\in\tilde{a}_{i,\alpha}, i=1,2,\ldots,n\right\}.$$

The proof is completed.

Remark 1 There are two key requirements for the above proposition, i.e. the continuous membership function for each fuzzy argument and the compact set for the mapping function to guarantee the existence of its maximum and minimum. Thus, the proposed method in this study is suited for the cases where the range of the structural random reliability is compact w.r.t. the fuzzy variables and the membership function for each fuzzy variable is continuous. And fortunately both of these requirements hold for realistic problems, which is implied in refs. [20,22] where the maximum and minimum of the random reliability at each alpha-level of the fuzzy variables have been calculated.

Accordingly, the bounds of the structural reliability at a specified alpha-level can be obtained within the space spanned by the basic fuzzy variables at the same alpha-level and subsequently the membership function of the structural reliability could be estimated. As stated in section 1, both vertex method and optimization techniques are not well suited for determining the bounds of the random reliability which is always an implicit nonlinear non-monotonic function at each alpha-level. An efficient tool to achieve the trade-off between the accuracy requirement and the computational efforts especially for the implicit nonlinear multivariate function is the surrogate model which has been used in existing methods for the random reliability analysis, e.g. the multi-point approximation (MPA) based on the two-point adaptive nonlinear approximation (TANA2) [18,19] and the saddle-point approximation based on the line sampling method [20–22]. As stated in sect. 1, potential obstacles in existing methods are summarized as follows:

(i) The aim of these approximations is to improve the efficiency of the random reliability analysis, e.g. capturing the most probable failure point (MPP) for MPA based on TANA2 and calculating the structural failure probability for linear performance functions obtained by the line sampling method for the nonlinear ones, which is not the research focus of the present study.

(ii) In refs. [18,19], the fuzziness propagation is performed based on the transformation of membership functions using intervening variables and a non-complete second-order polynomial omitting the cross-terms of probabilistic and fuzzy variables, which limits its accuracy and application scope.

(iii) In refs. [20–22], the fuzziness propagation is accomplished using the optimization techniques, which results in two potential issues, i.e. the unaffordable computational efforts and the resulting local optimum w.r.t. fuzzy variables.

Accordingly, a method with high accuracy, efficiency and universal applicability is urgently demanded for the fuzziness propagation in the reliability analysis for structures with probabilistic and fuzzy variables. However, the application scope of the methods in refs. [18,19] is limited due to the adoption of the non-complete polynomial approximation for the performance function. Moreover, the computational efforts of the optimization techniques for the fuzziness propagation adopted in refs. [20–22] are, in general, expensive.

3 Collocation reliability analysis method (CRAM)

The contradiction between the accuracy requirement and computational efforts is the main obstacle in existing reliability analysis methods for probabilistic and fuzzy mixed variables. By referring to the idea of surrogate model, a univariate function is adopted here to approximate the reduced reliability function $P_r^j = P_r\left(\boldsymbol{x}_S, x_{f1}^c, \ldots, x_{fj}, \ldots, x_{fm}^c\right)$ induced by the original reliability function $P_r = P_r\left(\boldsymbol{x}_S, \boldsymbol{x}_F\right)$ with one of the basic fuzzy variables varying and the others being set to their values at the maximum possibility with the denotation of the superscript 'c', and subsequently the minimal and maximal points w.r.t. x_{fj} at a specific alpha-level are calculated. The minimal and maximal point vectors of the original reliability function can be obtained dimension by dimension and then its bounds can be estimated at the corresponding alpha-level. It is noted that the aim of the approximate function is to capture the minimal and maximal point vectors rather than the extremum of the function of interest.

Remark 2 The idea of approximation adopted here is inspired by the manner for drawing a space curved surface. For example, a 2-dimensional space curved surface can be generated by either its four sidelines or two different curves with one being guiding curve defining the manner of the other's sliding. Accordingly, the minimal and maximal point vectors of the function defined by the space curved surface can be obtained dimension by dimension if the surface is smooth, i.e. the corresponding function is the first order continuous. And, the random reliability function of the fuzzy variables satisfies this requirement in the structural analysis for engineering problems, which is implied in ref. [20,22].

Chebyshev orthogonal polynomial expansion called 'the most economic expansion' is frequently used to approximate a univariate function. Therefore, the first class Chebyshev orthogonal polynomial is used in the present study to approximate the reduced reliability function w.r.t. the selected fuzzy variable at each alpha-level. The coefficients of expansion equation can be obtained according to the Gauss-Chebyshev quadrature formula through collocating Gauss quadrature points within the bound of the selected fuzzy variable at the corresponding alpha-level. The extreme points' distribution of the approximate function w.r.t. the selected fuzzy argument can be determined at the alpha-level and thus the minimal and maximal point vectors of the reliability function are subsequently obtained by traversing for each fuzzy variable, at which the interval of the structural reliability are calculated at the same alpha-level. The membership function of the structural reliability can be estimated efficiently by traversing alpha through specified discrete levels distributed within the interval [0,1].

Let $\boldsymbol{x}_F=(x_{f1},x_{f2},\ldots,x_{fm})$ be a fuzzy vector, where the component element x_{fj} (j=1,2,…,m) is a fuzzy number with membership function $\mu_j\left(x_{fj}\right)$. For a given alpha-level, we can obtain the alpha-cut of $\boldsymbol{x}_F$ as follows:

$$\begin{aligned}\boldsymbol{x}_{F,\alpha} &= \left[\underline{\boldsymbol{x}}_{F,\alpha}, \overline{\boldsymbol{x}}_{F,\alpha}\right] \\ &= \left(x_{f1}, x_{f2}, \ldots, x_{fm}\right)_\alpha \\ &= \left(x_{f1,\alpha}^I, x_{f2,\alpha}^I, \ldots, x_{fm,\alpha}^I\right),\end{aligned} \tag{6}$$

where

$$\begin{aligned}x_{fj,\alpha}^I &= \left\{x_{fj} \in \mathrm{R} \middle| \mu_j\left(x_{fj}\right) \geqslant \alpha\right\} = \left[\underline{x}_{fj,\alpha}, \overline{x}_{fj,\alpha}\right], \\ j &= 1,2,\ldots,m.\end{aligned} \tag{7}$$

The mid-point vector $\boldsymbol{x}_{F,\alpha}^c$ for the alpha-cut of fuzzy vector $\boldsymbol{x}_F$ is defined as:

$$\boldsymbol{x}_{F,\alpha}^c = \left(x_{fj,\alpha}^I\right)^c = \frac{\left(\underline{\boldsymbol{x}}_{F,\alpha} + \overline{\boldsymbol{x}}_{F,\alpha}\right)}{2}, \quad j = 1,2,\ldots,m, \tag{8}$$

and the deviation amplitude vector $\Delta\boldsymbol{x}_{F,\alpha}$ is defined as:

$$\Delta\boldsymbol{x}_{F,\alpha} = \left(\Delta x_{fj,\alpha}^I\right) = \frac{\left(\underline{\boldsymbol{x}}_{F,\alpha} - \overline{\boldsymbol{x}}_{F,\alpha}\right)}{2}, \quad j = 1,2,\ldots,m. \tag{9}$$

Therefore, the alpha-cut of the fuzzy vector denoted as $\boldsymbol{x}_{F,\alpha}$ could be decomposed into the combination of the mid-point vector $\boldsymbol{x}_{F,\alpha}^c$ and the deviation vector $\Delta\boldsymbol{x}_{F,\alpha}$ expressed as follows:

$$\begin{aligned}\boldsymbol{x}_{F,\alpha} &= \left[\underline{\boldsymbol{x}}_{F,\alpha}, \overline{\boldsymbol{x}}_{F,\alpha}\right] = \left[\boldsymbol{x}_{F,\alpha}^c - \Delta\boldsymbol{x}_{F,\alpha}, \boldsymbol{x}_{F,\alpha}^c + \Delta\boldsymbol{x}_{F,\alpha}\right] \\ &= \boldsymbol{x}_{F,\alpha}^c + \Delta\boldsymbol{x}_{F,\alpha}^I = \boldsymbol{x}_{F,\alpha}^c + \Delta\boldsymbol{x}_{F,\alpha} \cdot [-1,1] \\ &= \boldsymbol{x}_{F,\alpha}^c + \Delta\boldsymbol{x}_{F,\alpha} \circ \boldsymbol{e},\end{aligned} \tag{10}$$

where $\boldsymbol{e}$ is a m-dimensional vector with the absolute value of each element not being greater than 1. The Hadamard operator '$\circ$' represents the corresponding components in two vectors multiplied. We assume that the k^{th} element in vector $\boldsymbol{e}$ is a variable denoted as u ($u \in [-1,1]$), and the other $m-1$ elements are all fixed at 0. Accordingly, we could define that

$$\boldsymbol{U}^{(k)} = \underset{1,\ldots,k,\ldots,n}{\left(0,\ldots,u,\ldots,0\right)^{\mathrm{T}}}, \quad k = 1,2,\ldots,m, \tag{11}$$

and

$$\boldsymbol{x}_{\mathrm{F},\alpha}^{(k)} = \boldsymbol{x}_{\mathrm{F},\alpha}^{\mathrm{c}} + \Delta\boldsymbol{x}_{\mathrm{F},\alpha} \circ \boldsymbol{U}^{(k)}. \tag{12}$$

The first class Chebyshev polynomials can be expressed as:

$$T_p(u) = \cos(p \cdot \arccos(u)), \ -1 \leqslant u \leqslant 1, \tag{13}$$

where p is a nonnegative integer. The set $\{T_p(u)\}$ is a series of orthogonal polynomials with a weight function $1/\sqrt{1-u^2}$ within the interval $[-1,1]$, and satisfies recurrence relation as follows:

$$\begin{aligned} &T_0(u) = 1, \\ &T_1(u) = u, \qquad\qquad p = 1,2,\ldots. \\ &T_{p+1}(u) = 2uT_p(u) - T_{p-1}(u). \end{aligned} \tag{14}$$

Let $C[-1,1]$ be the family of all continuous functions within the interval $[-1,1]$ and $\boldsymbol{T}_p = \mathrm{Span}\{T_0, T_1, \ldots, T_p\}$ be a subspace included in it. The reduced reliability function induced by the original reliability function $P_r(\boldsymbol{x}_S, \boldsymbol{x}_F)$ can be denoted as:

$$\begin{aligned} &P_{\mathrm{r}}\left(\boldsymbol{x}_{\mathrm{S}}, x_{\mathrm{f}1}^0, \ldots, x_{\mathrm{f}(j-1)}^0, x_{\mathrm{f}j}, x_{\mathrm{f}(j+1)}^0, \ldots, x_{\mathrm{f}m}^0\right) \\ &= P_{\mathrm{r}}\left(\boldsymbol{x}_{\mathrm{S}}, x_{\mathrm{f}1}^0, \ldots, x_{\mathrm{f}(j-1)}^0, u^{(j)}, x_{\mathrm{f}(j+1)}^0, \ldots, x_{\mathrm{f}m}^0\right) \\ &= P_{\mathrm{r}}\left(u^{(j)}\right). \end{aligned} \tag{15}$$

For simplicity, $P_{\mathrm{r}}\left(u^{(j)}\right)$ in eq. (15) is denoted as $P_{\mathrm{r}}(u)$ in the sequel, the approximation of which is denoted as $S_p(u) \in \boldsymbol{T}_p$ expressed in the following:

$$S_p(u) = \frac{a_0}{2} + \sum_{j=1}^{p} a_j T_j(u), \tag{16}$$

where the coefficients are obtained according to the Gauss-Chebyshev quadrature formula as:

$$a_0 = \frac{2}{\pi}\int_{-1}^{1} \frac{P_{\mathrm{r}}(u)}{\sqrt{1-u^2}}\mathrm{d}u \approx \sum_{l=1}^{L} A_l P_{\mathrm{r}}(u_l), \tag{17}$$

$$a_j = \frac{2}{\pi}\int_{-1}^{1} \frac{P_{\mathrm{r}}(u)T_j(u)}{\sqrt{1-u^2}}\mathrm{d}u \approx \sum_{l=1}^{L} A_l P_{\mathrm{r}}(u_l)T_j(u_l), \quad j = 1,2,\ldots,p, \tag{18}$$

where quadrature nodes u_l, $l = 1,2,\ldots,L$ are zero roots of $T_L(u)$ and A_l are quadrature coefficients. The expressions of u_l and A_l are listed, respectively, as:

$$u_l = \cos\left(\frac{2(L-l)+1}{2L}\pi\right), \quad l = 1,2,\ldots,L, \tag{19}$$

$$A_l = \int_{-1}^{1} \frac{T_L(u)}{\sqrt{1-u^2}\,(u-u_l)T_L'(u_l)}\mathrm{d}u = \frac{\pi}{L}, \quad l = 1,2,\ldots,L. \tag{20}$$

Substituting eqs. (20) into eqs. (17) and (18), the coefficients of the expansion equation is calculated as:

$$a_0 = \frac{2}{L}\sum_{l=1}^{L} P_{\mathrm{r}}(u_l), \tag{21}$$

$$a_j = \frac{2}{L}\sum_{l=1}^{L} P_{\mathrm{r}}(u_l)T_j(u_l), \quad j = 1,2,\ldots,p. \tag{22}$$

Thus the following formula is obtained by substituting eqs. (21) and (22) into eq. (16) as:

$$S_p(u) = \frac{1}{L}\sum_{l=1}^{L} P_{\mathrm{r}}(u_l) + \frac{2}{L}\sum_{j=1}^{p}\sum_{l=1}^{L} P_{\mathrm{r}}(u_l)T_j(u_l)T_j(u), \tag{23}$$

which can be rewritten in the matrix form as:

$$S_p(u) = \frac{2}{L}\left(P_{\mathrm{r}}(u_1), P_{\mathrm{r}}(u_2), \ldots, P_{\mathrm{r}}(u_L)\right) \times \begin{pmatrix} 1/2 & T_1(u_1) & T_2(u_1) & T_p(u_1) \\ 1/2 & T_1(u_2) & T_2(u_2) & T_p(u_2) \\ 1/2 & & & \\ 1/2 & T_1(u_L) & T_2(u_L) & T_p(u_L) \end{pmatrix} \begin{pmatrix} T_0(u) \\ T_1(u) \\ \vdots \\ T_p(u) \end{pmatrix}. \tag{24}$$

The following denotations will be used in the sequel.

$$\boldsymbol{P} = \left(P_{\mathrm{r}}(u_1), P_{\mathrm{r}}(u_2), \ldots, P_{\mathrm{r}}(u_L)\right), \tag{25}$$

$$\boldsymbol{T} = \begin{pmatrix} 1/2 & T_1(u_1) & T_2(u_1) & \cdots & T_p(u_1) \\ 1/2 & T_1(u_2) & T_2(u_2) & \cdots & T_p(u_2) \\ 1/2 & \vdots & \vdots & \ddots & \vdots \\ 1/2 & T_1(u_L) & T_2(u_L) & \cdots & T_p(u_L) \end{pmatrix}, \tag{26}$$

$$\boldsymbol{T}(u) = \left(T_0(u) \quad T_1(u) \quad \cdots \quad T_p(u)\right)^{\mathrm{T}}, \tag{27}$$

$$\boldsymbol{C} = \boldsymbol{PT}. \tag{28}$$

Finally, the approximate polynomial $S_p(u)$ can be rewritten as:

$$S_p(u) = \boldsymbol{CT}(u), \tag{29}$$

where $\boldsymbol{C}$ is a row vector and $\boldsymbol{T}(u)$ is a column vector.

It is necessary to obtain the extreme points of the resulted approximate function $S_p(u)$ in order to calculate the minimum and maximum of the reliability function $P_{\mathrm{r}}(\boldsymbol{x}_{\mathrm{S}}, \boldsymbol{x}_{\mathrm{F}})$ when the k^{th} element of $\boldsymbol{x}_{\mathrm{F},\alpha}$ varies within the bounds determined by the given alpha-level, which can be accomplished through seeking the zero roots of derived function $S_p'(u)$ and combining the bounds of $\boldsymbol{x}_{\mathrm{F},\alpha}$. The minimum and maximum point vectors for the k^{th} fuzzy variable are regarded as $x_{\text{f}k,\alpha}^{\max}$ and $x_{\text{f}k,\alpha}^{\min}$, respectively. Then traversing k from 1 to m, we can obtain the maximum and minimum point vectors of the structural reliability function w.r.t. fuzzy variables $\boldsymbol{x}_{\text{F}}$ at the alpha-level.

The maximum and minimum point vectors can be captured as follows:

$$\boldsymbol{x}_{\text{F},\alpha}^{\max}=\left(x_{\text{f}1,\alpha}^{\max},x_{\text{f}2,\alpha}^{\max},\ldots,x_{\text{f}m,\alpha}^{\max}\right)^{\text{T}}, \quad (30)$$

$$\boldsymbol{x}_{\text{F},\alpha}^{\min}=\left(x_{\text{f}1,\alpha}^{\min},x_{\text{f}2,\alpha}^{\min},\ldots,x_{\text{f}m,\alpha}^{\min}\right)^{\text{T}}. \quad (31)$$

Then the minimum and maximum of the structural reliability function $P_{\text{r}}\left(\boldsymbol{x}_{\text{S}},\boldsymbol{x}_{\text{F}}\right)$ w.r.t. fuzzy variables $\boldsymbol{x}_{\text{F}}$ at the alpha-level can be derived from the point vectors in eqs. (30) and (31), respectively, i.e.

$$P_{\text{r},\alpha}^{\max}=P_{\text{r}}\left(\boldsymbol{x}_{\text{S}},\boldsymbol{x}_{\text{F},\alpha}^{\max}\right), \quad (32)$$

$$P_{\text{r},\alpha}^{\min}=P_{\text{r}}\left(\boldsymbol{x}_{\text{S}},\boldsymbol{x}_{\text{F},\alpha}^{\min}\right). \quad (33)$$

The maximum and minimum of the structural reliability at the alpha-level have been calculated according to eqs. (32) and (33), respectively, and then the membership function of the structural reliability can be determined by traversing alpha in a specified set consisting of discrete values distributed within the interval [–1,1].

The error from the calculation for coefficients of the expansion equation can be estimated as:

$$R(L)=\frac{S_p^{(2L)}(\eta)}{a_L^2(2L)!}\int_{-1}^{1}\frac{T_L^2(u)}{\sqrt{1-u^2}}\mathrm{d}u =\frac{2\pi}{2^{2L}(2L)!}S_p^{(2L)}(\eta),\quad \eta\in(-1,1). \quad (34)$$

Thus, the value $R(L)$ can be reduced to 0 if L is not less than $(p+1)/2$, where L is the number of collocation points. It is suggested that 3–11 are chosen for p in engineering application and thus the value of L can be determined to reduce the error $R(L)$ to 0 subsequently. Accordingly, the total error of the present method can be reduced to the truncation error when Chebyshev orthogonal polynomials are used to approximate the reduced reliability function. The aim of polynomial approximation in this study is different from that of the traditional one within the space spanned by the interval variables obtained at the alpha-level of fuzzy variables: the latter where non-complete polynomials are always used to approximate the accurate function of interest for capturing its extremum, limits the accuracy, while the former instead is to calculate the minimal and maximal point vectors of the reliability function dimension by dimension with better precision due to higher order polynomial's adoption. Figure 1 shows the flowchart of CRAM for a probabilistic and fuzzy mixed structure.

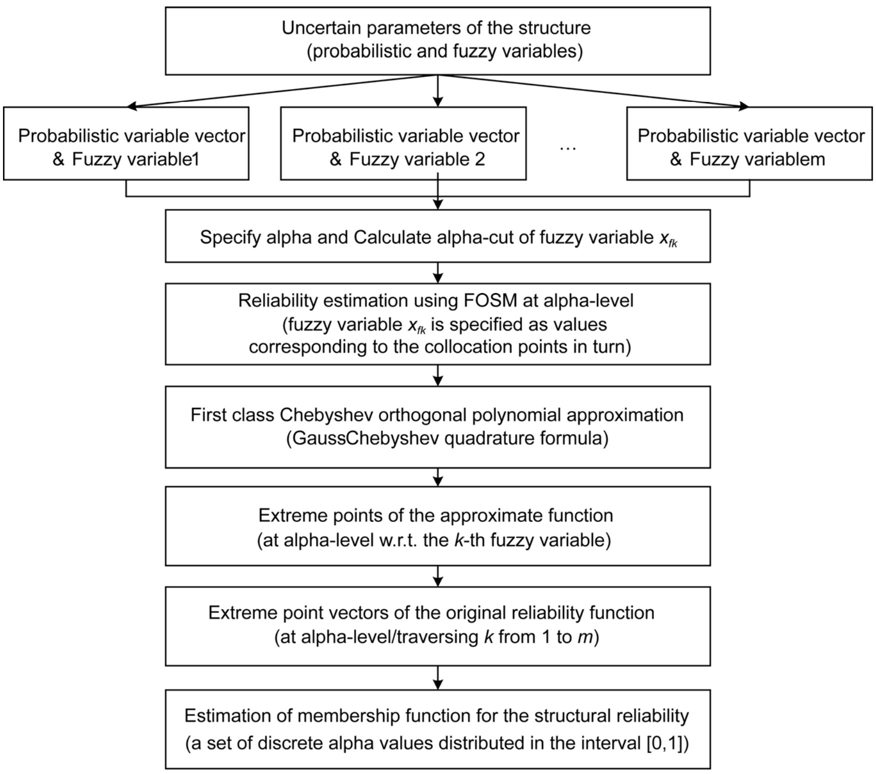

Figure 1 Flowchart of CRAM.

Remark 3 The focus of the present study is the fuzziness propagation rather than the calculation of the random reliability and thus FOSM is employed for simplicity. It is noted that more efficient methods for the random reliability analysis, e.g. the method based on Markov chain and the saddle-point approximation, can also be adopted to replace FOSM and then the reliability analysis methods for mixed uncertainties can be established subsequently.

4 Examples

As emphasized in the preceding text, the aim of the present study is to accomplish the fuzziness propagation in the structural reliability analysis and thus the validation of the accuracy of CRAM can be performed based on the same procedure for the calculation of the random reliability, e.g. FOSM adopted in the paper with considerations of both highlighting the idea of CRAM and simplifying the complexity in the random reliability calculation. The improvement in methods for the random reliability analysis and combining CRAM suggests a novel category methods for the reliability analysis of structures with probabilistic and fuzzy mixed variables.

Following FOSM for the classical random reliability, the fuzziness is propagated by CRAM and Monte Carlo (MC) simulation at each alpha-level in four numerical examples, where the validation of the accuracy of CRAM is performed by the comparison between results obtained by MC and CRAM. And an application of CARM is also introduced. The implications of alphabetical combinations in legends in the sequel are explained simultaneously as follows.

LMF-CRAM: the estimated left membership function by CRAM,

RMF-CRAM: the estimated right membership function by CRAM,

LMF-MC: the left membership function obtained by MC,

RMF-MC: the right membership function obtained by MC.

Example 1 Consider a linear performance function w.r.t. probabilistic and fuzzy mixed variables expressed as:

$$g\left(\boldsymbol{x}_{\mathrm{S}},\boldsymbol{x}_{\mathrm{F}}\right)=5x_{\mathrm{s}1}-2x_{\mathrm{s}2}+4x_{\mathrm{s}3}-8x_{\mathrm{f}},$$

where x_{s1}, x_{s2} and x_{s3} are independent normal random variables satisfying x_{s1}~$N(80,10)$, x_{s2}~$N(170,15)$ and x_{s3}~$N(80,5)$, respectively. The variable x_f is fuzzy with its membership function $\mu(x_f)$ expressed as:

$$\mu(x_{\mathrm{f}})=\begin{cases}(x_{\mathrm{f}}-21.15)/8.85, & 21.15\leqslant x_{\mathrm{f}}\leqslant 30,\\ (x_{\mathrm{f}}-38.85)/(-8.85), & 30\leqslant x_{\mathrm{f}}\leqslant 38.85,\\ 0, & \text{otherwise.}\end{cases}$$

It is noted that the result obtained by MC with the probability convergence feature is, in general, a subset of the accurate one and the former gradually converges to the latter with the increase in the number of samples which should thus be determined prior to utilizing MC as the reference method for the validation of CRAM. The lower and upper bounds of the random reliability w.r.t. the fuzzy variable at discrete alpha-levels including 0, 0.2, 0.4, 0.6, 0.8, and 1.0 simulated by MC with different numbers of samples are shown in Table 1.

It can be seen that the ranges are almost unchanged if the number of samples is up to 10^5 at each alpha-level. Accordingly, the bounds calculated by MC with 10^5 samples are chosen as the reference bounds in this example. The membership function of the reliability obtained by CRAM as well as MC is shown in Figure 2. One hundred thousand points were sampled for the fuzzy variable x_f at each alpha-level in MC. And conclusions as follows can be obtained.

(i) The membership function obtained by CRAM is exactly identical with that by MC, which suggests that CRAM can capture the minimum and maximum of the random reliability w.r.t. fuzzy variables at each alpha-level accurately.

(ii) The computational time of the CRAM is 0.0640 seconds while MC 128.3410 seconds on the same computing platform, which demonstrates the efficiency of CRAM by the significant efficiency gap (with the order of 10^3) between the two methods. The trade-off between the accuracy requirement and computational efforts is commendably achieved by CRAM.

Example 2 The performance function considered for an engineering structure is given as:

Table 1 The lower and upper bounds calculated by MC

Alpha-level	10^4 samples		10^5 samples		10^6 samples	
	LB	UB	LB	UB	LB	UB
0.0	0.880259	0.999829	0.880155	0.999829	0.880154	0.999829
0.2	0.921666	0.999581	0.921664	0.999583	0.921663	0.999583
0.4	0.951235	0.999033	0.951234	0.999034	0.951233	0.999034
0.6	0.971122	0.997880	0.971121	0.997881	0.971120	0.997881
0.8	0.983748	0.995594	0.983746	0.995594	0.983746	0.995594
1.0	0.991313	0.991313	0.991313	0.991313	0.991313	0.991313

a) The abbreviations 'LB' and 'UB' represent the lower and the upper bounds, respectively, which will be used in the sequel.

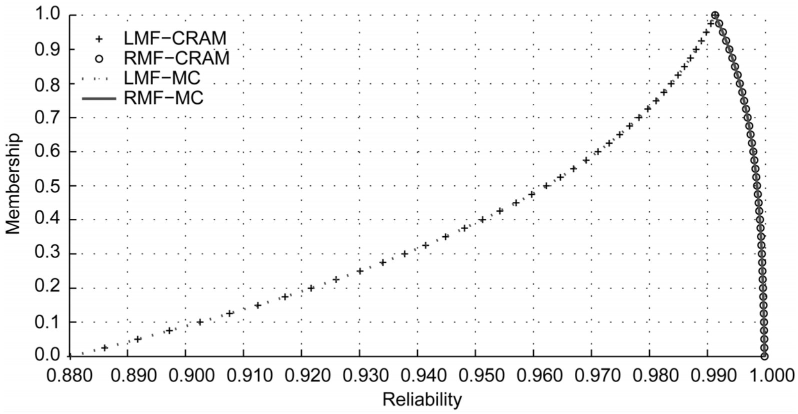

Figure 2 Membership function of the reliability.

$$g(\boldsymbol{x}) = x_1^2 + 5x_1 + 2x_2^2 + 8x_2 + x_3^2 - 8x_3 + x_4^2 - 10x_4 - 200,$$

where x_1 and x_2 are assumed to be normally distributed with a mean of 10.0 and a standard deviation of 2.0. The variables x_3 and x_4 are assumed to be fuzzy with their membership functions given as:

$$\mu_{x_3}(x_3) = \begin{cases} (x_3 - 5)/5, & 5 \leqslant x_3 \leqslant 10, \\ (x_3 - 15)/(-5), & 10 \leqslant x_3 \leqslant 15, \\ 0, & \text{otherwise}, \end{cases}$$

$$\mu_{x_4}(x_4) = \begin{cases} (x_4 - 5)/5, & 5 \leqslant x_4 \leqslant 10, \\ (x_4 - 15)/(-5), & 10 \leqslant x_4 \leqslant 15, \\ 0, & \text{otherwise}. \end{cases}$$

The lower and upper bounds of the random reliability w.r.t. fuzzy variables simulated by MC with different numbers of samples at discrete alpha-levels including 0, 0.2, 0.4, 0.6, 0.8, and 1.0 are listed in Table 2, from which the number of samples can be determined as 10^5 to guarantee the accuracy of the reference membership function.

Figure 3 shows the membership functions of the reliability estimated using CRAM as well as that obtained using MC. It is shown that CRAM could accurately capture the minimum and maximum of the random reliability w.r.t. fuzzy variables at each alpha-level and thus estimates the membership function of the reliability accurately. However, the consuming time of CRAM and MC for this problem is widely different, i.e. 0.0780 s for CRAM and 114.9250 s for MC, which demonstrates the efficiency of CRAM to some extent by the significant difference (with the order of 10^3) in computational efforts. Accordingly, the accuracy requirement could be satisfied by CRAM with limited computational efforts, which suggests a potential way for the structural reanalysis.

Example 3 A three-span beam is shown in Figure 4. The performance function is established under the constraint of the critical deflection of the beam not being greater than $L/360$, i.e.

$$g(\boldsymbol{x}_S, \boldsymbol{x}_F) = L/360 - 0.0069wL^4/EI,$$

where w and L are independent normal random variables satisfying $w \sim N(5,0.001)$ and $L \sim N(38,4)$, respectively, while E and I are two fuzzy variables with membership functions $\mu_1(E)$ and $\mu_2(I)$ as follows.

$$\mu_1(E) = \begin{cases} (E - 1.95\times10^7)/0.05\times10^7, & 1.95\times10^7 \leqslant E \leqslant 2\times10^7, \\ (E - 2.05\times10^7)/(-0.05\times10^7), & 2\times10^7 \leqslant E \leqslant 2.05\times10^7, \\ 0, & \text{otherwise}, \end{cases}$$

Table 2 The lower and upper bounds calculated by MC

Alpha-level	10^4 samples		10^5 samples		10^6 samples	
	LB	UB	LB	UB	LB	UB
0.0	0.995211	1.000000	0.995203	1.000000	0.995200	1.000000
0.2	0.995927	1.000000	0.995927	1.000000	0.995926	1.000000
0.4	0.997045	1.000000	0.997040	1.000000	0.997039	1.000000
0.6	0.998223	0.999975	0.998221	0.999978	0.998221	0.999979
0.8	0.999130	0.999900	0.999130	0.999901	0.999129	0.999901
1.0	0.999662	0.999662	0.999662	0.999662	0.999662	0.999662

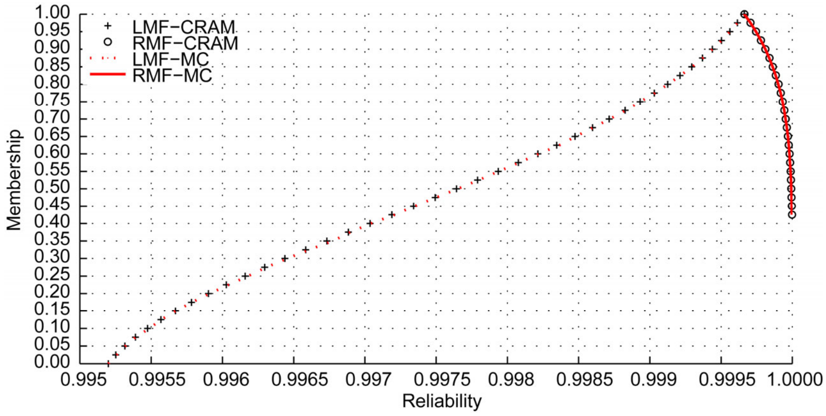

Figure 3 Membership function of the reliability.

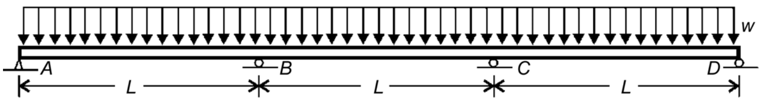

Figure 4 Sketch of a three-span beam.

$$\mu_2(I)=\begin{cases}\left(I-7.8\times10^{-4}\right)/0.2\times10^{-4}, & 7.8\times10^{-4}\leqslant I\leqslant 8.0\times10^{-4},\\ \left(I-8.2\times10^{-4}\right)/\left(-0.2\times10^{-4}\right), & 8.0\times10^{-4}\leqslant I\leqslant 8.2\times10^{-4},\\ 0, & \text{otherwise.}\end{cases}$$

The lower and upper bounds of the random reliability of the three-span beam w.r.t. fuzzy variables E and I simulated by MC with different numbers of samples at discrete alpha-levels including 0, 0.2, 0.4, 0.6, 0.8, and 1.0 are recorded in Table 3 and thus the number of samples for MC can be determined as 10^5 for the accuracy validation of CRAM.

The resulting membership function of the reliability of the three-span beam by CRAM as well as that by MC is shown in Figure 5. As emphasized at the beginning of this section, FOSM is adopted for the estimation of the random reliability to highlight the procedure of CRAM. However, it is noted that the structural reliability for this problem cannot be calculated by the method in refs. [18,19] due to the obstacle in the cross-term of the probabilistic and fuzzy variables in the performance function, which is common in engineering practices. The good agreement between the membership functions estimated by CRAM and MC in Figure 5 demonstrates the accuracy of CRAM. It can be recognized that the application scope, which is limited by the existence of cross-terms of probabilistic and fuzzy variables or the adoption of optimization techniques for capturing the minimum and maximum of the random reliability at each alpha-level of fuzzy variables, is enlarged by CRAM. Besides, the wide difference (with the order of 10^4) in the computation time for the two methods, i.e. 0.0160 s for CRAM and 152.7710 s for MC, demonstrates the efficiency of CRAM to some extent.

Example 4 The top chord and compression bars of the roof truss shown in Figure 6 are made of steel reinforced concrete while bottom chord and pull bars are made of steel. A uniform load q acts on the roof truss, which can be transformed as the node load $P=ql/4$. The performance function under the constraint that the critical deflection of the roof truss at the node C in Fig.6 cannot be greater than 3 centimeters (cm) is established as:

$$g\left(\boldsymbol{x}_S,\boldsymbol{x}_F\right)=0.03-\frac{ql^2}{2}\left(\frac{3.81}{A_cE_c}+\frac{1.13}{A_sE_s}\right),$$

where

A_c cross section area of the steel reinforced concrete bar,

A_s cross section area of the steel bar,

E_c elastic modulus of the steel reinforced concrete,

E_s elastic modulus of the steel,

and A_c, A_s, q and l are independent random variables listed in Table 4 while E_c and E_s are fuzzy variables with membership functions $\mu_1(E_c)$ and $\mu_2(E_s)$ defined as follows:

$$\mu_1(E_c)=\begin{cases}\left(E_c-1.88\times10^{10}\right)/1.2\times10^{9}, & 1.88\times10^{10}\leqslant E_c\leqslant 2\times10^{10},\\ \left(E_c-2.12\times10^{10}\right)/\left(-1.2\times10^{9}\right), & 2\times10^{10}\leqslant E_c\leqslant 2.12\times10^{10},\\ 0, & \text{otherwise,}\end{cases}$$

Table 3 The lower and upper bounds calculated by MC

Alpha-level	10^4 samples		10^5 samples		10^6 samples	
	LB	UB	LB	UB	LB	UB
0.0	0.997000	0.999971	0.996991	0.999972	0.996991	0.999972
0.2	0.997964	0.999951	0.997964	0.999952	0.997964	0.999952
0.4	0.998652	0.999918	0.998652	0.999918	0.998651	0.999919
0.6	0.999119	0.999862	0.999117	0.999864	0.999116	0.999864
0.8	0.999433	0.999777	0.999432	0.999777	0.999432	0.999777
1.0	0.999605	0.999605	0.999605	0.999605	0.999605	0.999605

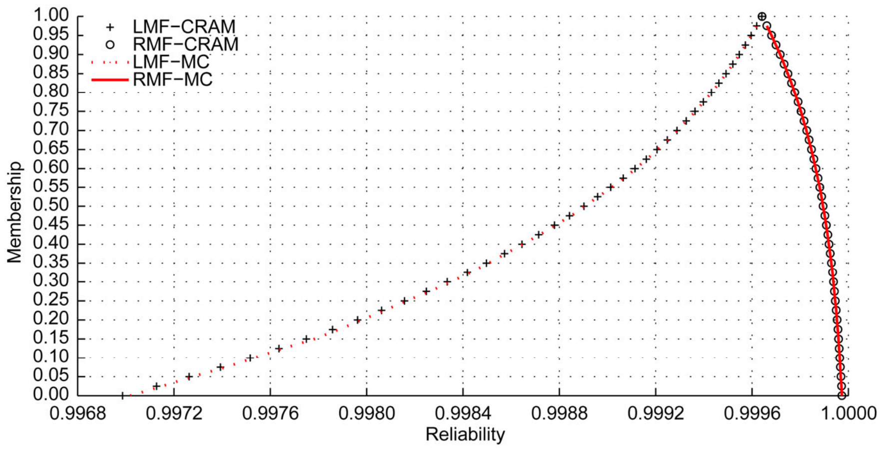

Figure 5 Membership function of the reliability of the three-span beam.

$$\mu_2(E_s) = \begin{cases} (E_s - 0.94\times10^{11})/(6\times10^9), & 0.94\times10^{11} \leqslant E_s \leqslant 1\times10^{11}, \\ (E_s - 1.06\times10^{11})/(-6\times10^9), & 1\times10^{11} \leqslant E_s \leqslant 1.06\times10^{11}, \\ 0, & \text{otherwise}. \end{cases}$$

The lower and upper bounds of the random reliability of the roof truss w.r.t. fuzzy variables calculated by MC at discrete alpha-levels including 0, 0.2, 0.4, 0.6, 0.8, and 1.0 are listed in Table 5 and thus the number of samples for MC can be chosen as 10^5 to validate the accuracy of CRAM.

The resulting membership function of the reliability for the roof truss using CRAM as well as that by MC is shown in Figure 7, where the accuracy of CRAM is demonstrated by the good agreement between membership functions obtained by the two methods. It is suggested that CRAM is well suited for the reliability analysis of structures with the performance function containing cross-terms of probabilistic and fuzzy variables. Moreover, the consuming time for the two methods is significantly different, i.e. 0.1090 s for CRAM and 160.8360 s for MC on the same computing platform, and then the efficiency of CRAM can be shown by the great gap (with the order of 10^3).

Example 5 Figure 8 shows a finite element model for the wing box of an airfoil of a regional aircraft. The membership functions of the reliability of the skin and web are discussed. According to the procedure of CRAM, the minimum and maximum of the structural random reliability can be calculated in parallel for both the fuzzy variables and discrete alpha-levels and thus the total computational efforts of CRAM is not proportional to the dimension of the fuzzy variable vector. Therefore, the fuzziness in elastic modulus of the constituent materials is considered for the validation for the applicability of CRAM. Membership functions of the elastic modulus are expressed as follows:

$$\mu_1(E_1) = \begin{cases} E_1 - 70, & 70 \leqslant E_1 \leqslant 71, \\ 72 - E_1, & 71 \leqslant E_1 \leqslant 72, \\ 0, & \text{otherwise}, \end{cases} \quad \text{(GPa)}$$

$$\mu_2(E_2) = \begin{cases} (E_2 - 71)^2, & 71 \leqslant E_2 \leqslant 72, \\ ((74 - E_2)/2)^2, & 72 \leqslant E_2 \leqslant 74, \\ 0, & \text{otherwise}. \end{cases} \quad \text{(GPa)}$$

Theoretically speaking, the reliability can be recognized as the result of the mathematical operations between the structural responses and the corresponding limit states and thus the reliability analysis of structures in engineering practices is always based on the finite element analysis (FEA), which usually involves the cross-terms of the structural and external parameters. Furthermore, optimization techniques for the minimum and maximum of the structural reliability w.r.t. fuzzy variables at each alpha-level are

Table 4 Probabilistic parameters of the roof truss

Variable (unit)	Type	Mean	Standard deviation
uniform load q (N/m)	normal	2×10^{4}	1.4×10^{3}
length of bar l (m)	normal	12	0.12
cross section area A_s (m^2)	normal	9.82×10^{-4}	5.892×10^{-5}
cross section area A_c (m^2)	normal	4×10^{-2}	4.8×10^{-3}

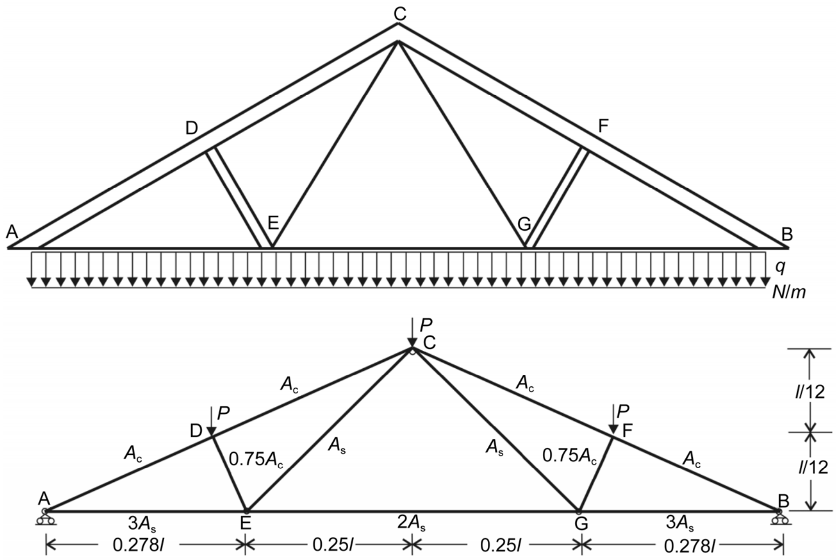

Figure 6 Sketch of a roof truss.

Table 5 The lower and upper bounds calculated by MC

Alpha-level	10^4 samples		10^5 samples		10^6 samples	
	LB	UB	LB	UB	LB	UB
0.0	0.988184	0.999963	0.988182	0.999962	0.988182	0.999963
0.2	0.992592	0.999925	0.992591	0.999926	0.992591	0.999926
0.4	0.995409	0.999855	0.995409	0.999855	0.995408	0.999856
0.6	0.997236	0.999725	0.997236	0.999726	0.997236	0.999726
0.8	0.998391	0.999488	0.998390	0.999490	0.998389	0.999490
1.0	0.999082	0.999082	0.999082	0.999082	0.999082	0.999082

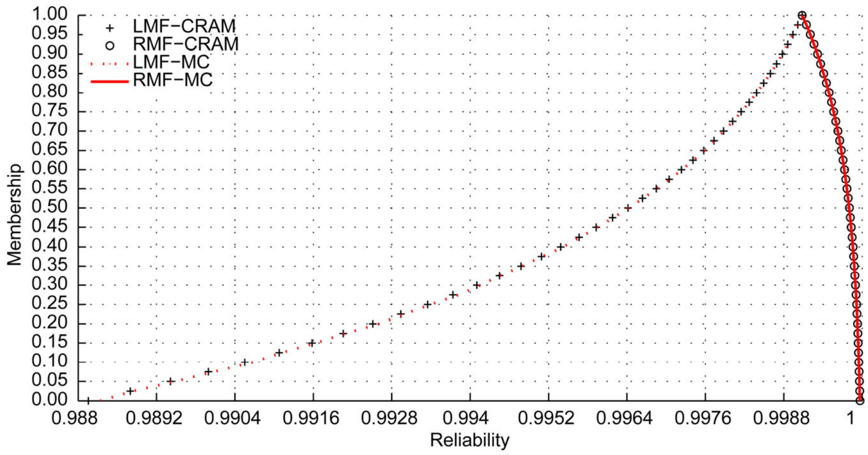

Figure 7 Membership function of the reliability of the roof truss.

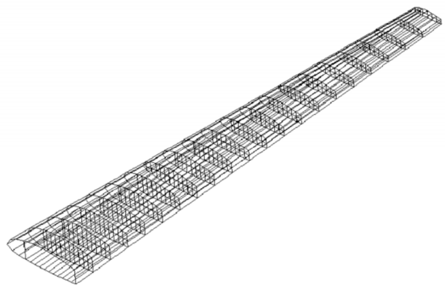

Figure 8 Finite element model of the wing box of an airfoil.

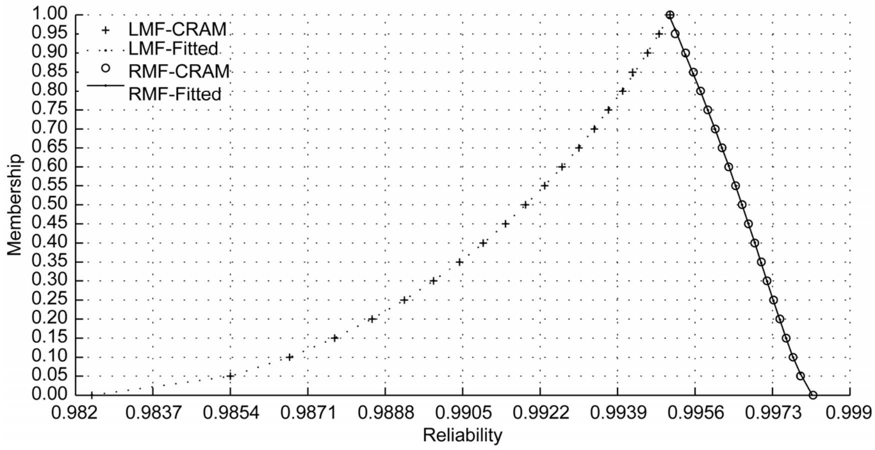

Figure 9 Membership function of the reliability of the wing box.

always haunted considering the expensive computational cost of FEA for complex structures. Following the validation of the accuracy and efficiency of CRAM, the membership function of the reliability of the wing box is shown in Figure 9, where 'LMF-Fitted' and 'RMF-Fitted' represent 'the fitted left membership function' and 'the fitted right membership function'. It is noted that the finite element analysis for structures involved in CRAM is in the form of a black box and thus the conventional FE softwares, e.g. MSC.Patran/Nastran and Ansys, could be utilized to perform the reliability analysis for complex structures with probabilistic and fuzzy variables by CRAM.

5 Conclusions

A novel reliability analysis method (CRAM) for structures with probabilistic and fuzzy mixed variables has been proposed based on FOSM for the classical random reliability and Chebyshev orthogonal polynomial approximation for the fuzziness propagation. The focus of CRAM is not the calculation of the structural random reliability but the fuzziness propagation in random reliability which is performed by capturing the minimal and maximal point vectors instead of its extremum w.r.t. fuzzy variables at each alpha-level. After the validation of CRAM, the conclusions can be obtained.

(i) Compared with results obtained by MC, the accuracy and efficiency requirements in engineering practices can be satisfied by CRAM.

(ii) CRAM can effectively perform the reliability analysis of structures with cross-terms of probabilistic and fuzzy variables in their performance function. Besides, optimization techniques with unaffordable computational efforts for the analysis of complex structures are avoided. Accordingly, CRAM is well suited for engineering applications.

(iii) The efficiency of CRAM can be significantly improved by the parallel computing of the random reliability for both the fuzzy variable vector and the discrete alpha-levels in need.

(iv) FOSM for the random reliability analysis adopted in the present study can be replaced by its counterparts in the field of the random reliability analysis and thus a category method for the reliability analysis of structures with probabilistic and fuzzy variables is suggested by CRAM.

(v) A potential structural reanalysis method is suggested by CRAM considering variations in the structural controllable parameters.

This work was supported by the Defense Industrial Technology Development Program (Grant Nos. A2120110001 and B2120110011), '111' Project (Grant No. B07009), the National Natural Science Foundation of

China (Grant No. 90816024). Besides, the authors wish to express their many thanks to the reviewers for their useful and constructive comments.

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