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Structural reliability analysis and reliability-based design optimization: Recent advances

QIU ZhiPing[1*], HUANG Ren[1], WANG XiaoJun[1] & QI WuChao[2]

[1] *Institute of Solid Mechanics, Beihang University, Beijing 100191, China;*
[2] *College of Aerospace Engineering, Shenyang Aerospace University, Shenyang 110136, China*

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We review recent research activities on structural reliability analysis, reliability-based design optimization (RBDO) and applications in complex engineering structural design. Several novel uncertainty propagation methods and reliability models, which are the basis of the reliability assessment, are given. In addition, recent developments on reliability evaluation and sensitivity analysis are highlighted as well as implementation strategies for RBDO.

uncertainty propagation, reliability assessment, reliability-based design optimization, hybrid reliability

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1 Introduction

In the process of traditional structural design, it is assumed that the parameters of structural models and operational environments involved in engineering problems are deterministic values. However, for most practical engineering cases, uncertainties in material properties, geometric dimensions, applied loads and other parameters are unavoidable and parameter perturbations may exhibit remarkable sensitivity to optimal structural designs [1]. The need to incorporate uncertainties in engineering design has long been realized. Despite traditional approaches being successfully implemented in many practical design situations, in which the uncertainties are included by introducing simplifying hypotheses such as the consideration of extreme values and/or the application of safety factors, the assumption of a deterministic model is certainly a simplification in that different types of uncertainties would cause various degrees changes in the performance and reliability of final designs.

In recent years, with the fast development of the computational technology, many researchers have focused on structural design methods with uncertainties which have been applied in various fields, such as aerospace structures [2–8], ship structures [9], civil structures [10–13], marine search and rescue [14] and acoustic analysis [15]. The reliability design is based on the concept of structural safety design, which considers uncertainties and ensures that the analyzed system will perform within prescribed margins with a quantitative measure of system safety. Despite the fact that an adequate level of reliability is a basic objective when design and construct structural systems or components, high levels of reliability are usually associated with large economical costs. An appropriate trade-off between an acceptable reliability level and economical design of the structure should be offered in engineering practice. In consequence, the concept of reliability-based design optimization (RBDO) has emerged.

Although recently introduced, the structural reliability

*Corresponding author (email: zpqiu@buaa.edu.cn)

design technologies have made strides in China during the past two decades. The "National Guideline on Medium- and Long-Term Program for Science and Technology Development (2006-2020)" demands advanced techniques, including the reliability technology, for both usage and lifespan prediction of major products and major facilities. Structural reliability technique is one of the most important aspects of reliability-based design in complex systems. Herein we do not intend to present an exhaustive review of the field of structural reliability design but to offer a brief survey on some of the most relevant contributions of Chinese researchers in the area, namely uncertainty propagation and reliability modeling, reliability analysis and RBDO. Thus, the following sections briefly summarize the novel uncertainty propagation approaches and reliability models, not limited in the stochastic methodology. We also focus on recent advances in reliability analysis methods, particularly on the reliability sensitivity analysis. Finally, we give an overview of recent developments in the context of reliability-based design optimization.

2 Recent advances in propagation of uncertainties and reliability modeling

The quantification and propagation of uncertainty in structural analysis of complex models demand large computational efforts compared with the corresponding deterministic analysis. The computational efficiency and accuracy are the basic contradiction in engineering computation. In this section, novel uncertainty propagation methods which can strongly reduce the cost of evaluation and improve the accuracy of calculation are introduced. These techniques are related not only to stochastic methods, but also in a non-probabilistic framework. Because the traditional reliability models based on probabilistic method are more developed, the recent progress of hybrid reliability models is only discussed herein.

2.1 Novel uncertainty propagation implementations

Parallel processing and model reduction are two of the most effective techniques to reduce the computational cost. In order to evaluate the response variability by parallel processing, one should choose a non-intrusive method for uncertainty propagation analysis, in which the deterministic analysis would be considered as a "black box" procedure, so that the model with uncertainties can be transformed into a new set of models with deterministic parameters. It would only then be convenient to implement the parallel computation.

In a stochastic context, Monte Carlo simulation is the most widely used non-intrusive method. The basic application of Monte Carlo procedures is the generation of independent samples for the structural properties and loads by making use of an appropriate random number generator. Parallel processing is particularly well suited to generate the response of independent realization since all tasks are completely independent and require little communication [16]. However, the computational efforts by using parallel Monte Carlo simulation would not be acceptable as the large amount of structural degrees of freedom and uncertain parameters of complex structures. The structural analysis for engineering practice is usually carried out almost exclusively by finite element method. Several stochastic finite element (SFE) procedures were developed to handle the expensive computational requirement of Monte Carlo simulation-based methods. In addition to the perturbation SFE and spectral SFE, the moment method-based SFE is another feasible approach. As the first two order moments of structural responses are the primary concerned quantities in the reliability analysis, Qiu et al. [17,18] developed a direct probabilistic approach which can avoid calculating the inverse of the mean matrix and reduce storage and computational complexities. It should be noted that no assumptions or approximations were made in deriving the moment equations, except the assumed existence of the various first and second moment values. Therefore, the obtained moment equations can be considered to be exact. The parallel arithmetic was used, which can decrease the runtime effectively and increase computational efficiency.

The K-L expansion and polynomial chaos expansion, which are the basis of the spectral SFE, are conventionally adopted for the model reduction in SFE analysis. The K-L expansion is a spectral representation method of a stochastic process which can only deal with the uncertainty by Gaussian random field. The response with non-Gaussian properties can be approximated by polynomial chaos expansion which is a Galerkin projection scheme based on the Wiener integral representation [19]. However, the polynomial chaos expansion suffers from the reputed excess of dimensionality, which indicates an exponential growth of computational efforts and the number of terms P as a function of the number m of independent random quantities. Zhang et al. [20] introduced a novel SFE method with dimensional reduction integration. The moment computation of random response was derived by numerical integration based on weighting function of orthogonal polynomial and probability density function. The C-type Gram-Charlier series were used to calculate moments of uncertain response approximately. As a result of the model reduction, it was not necessary need to solve the Hessian matrix with respect to random variables, and the computational efficiency was considerably improved.

The Taylor series-based interval method can be used to perform a non-intrusive interval finite element (IFE) procedure, which can only solve problems with small levels of uncertainty. Wu et al. [21] introduced Chebyshev series expansions into IFE for uncertain nonlinear dynamic systems analysis. The proposed non-intrusive method offered higher accuracy approximation solutions and the Chebyshev inclusion function was developed to effectively control the overestimation of interval computation. Qi et al. [22] proposed the collocation IFE based on the first class Chebyshev polynomial approximation for interval analysis. The application of collocation IFE was not limited to small levels of uncertainty, and the truncation order of polynomials only contributed to computational accuracy. A higher order truncation would effectively add no computational effort. In addition, the global optimization strategies are often considered in that it can give an exact result in interval analysis, but the numerical cost would be cumbersome. Wang et al. [23] introduced a smallest interval-set for uncertain parameters quantification, in which the global optimization algorithm was used for the best interval-set searching to determine the intervals of the uncertain parameters. By combined the interval analysis method the structural response would be improved, as the presented strategy obtained more accurate uncertain parameters estimations because of insufficient experimental data. By introducing a mathematical programme method, Qiu et al. [24] investigated an inequality model for solving interval dynamic response of structures with interval parameters. Thus, the optimization strategies can take advantage of precision amelioration in interval operation procedure more effectively.

2.2 Hybrid reliability models

After experiencing more than half a century of development, the classical probabilistic reliability theory is considered well formulated. The probabilistic reliability model, when the adequate information and high precision calculation model are available, is a good safety assessment model and has a significant function in engineering practices. Recently there has been some progress in the fields of the fuzzy reliability model and non-probabilistic reliability modelling. Along with new mathematical knowledge in engineering, studies of the effect of uncertainties on structural reliability are more comprehensive. In order to allow the uncertainty models to be pertinent to actual conditions, the hybrid uncertain models are dealt with in the framework of interval probabilities, fuzzy randomness and fuzzy interval. The hybrid reliability modeling is considered as an approach, which exhibits both the merits of two different uncertainity models.

The interval-probability hybrid reliability approach is a challenging topic in engineering in that the information available is frequently not sufficient to formulate clear probabilistic models with a substantial confidence level. Two differing types of methods for uncertainty quantification in the framework of interval probabilities are introduced in previous research. In the first-type hybrid model, Wang et al. [25,26] combined the probability model and convex model to describe the parameters with and without sufficient uncertainty information, respectively. The optimal criterion enumerating the main failure modes and the relationship of failure modes were presented in structural system with hybrid uncertain parameters. The hybrid reliability of structural system was obtained through analyzing the correlation of each failure mode of hybrid structural system. The advantage of this type model is that it can obtain the reliability interval of the hybrid structural system to replace a certain value or a simple reliability index, which is more credible and significant in the case when the statistic data of uncertain parameters are scant or uncertain information is unclear. In the second case, all the uncertain parameters are quantified through the probability model, while some of the distribution parameters can only be given variation intervals since lacking information. These intervals can be easily determined through the interval estimation technique in statistics theory [27–29]. The well-developed random methods, like reliability index approach and the performance measurement approach, can be used for reliability analysis. However the dependent of random variables and different failure modes still need to be considered.

The models with fuzzy randomness describe imprecise probabilities as randomness measures of fuzzy sets. The associated fuzzy probabilities represent weighted bounds of probability. With the interpretation of a fuzzy set as a set of α-level sets via α-discretization, the relationship to interval probabilities in the form of sets of probability measures becomes apparent [30]. The fuzzy random quantity $\tilde{X}$ is defined as the fuzzy result of the mapping from the space of the random elementary events Ω to the set of all fuzzy quantities, and each real-valued random quantity X on the fundamental set $\boldsymbol{X} = \mathbb{R}^n$ is referred to as an original of $\tilde{X}$ (see Figure 1). The reliability is fuzzy in the fuzzy randomness hybrid reliability model, which could be obtained by the integration on the fuzzy joint probability density function over the fuzzy safe region. As the strong nonlinear and multi-dimensional limit state functions, some approximation methods like λ level cut set method are used to degenerate fuzzy variables into interval variables [31]. After the boundaries of performance and cumulative distribution function are estimated, the reliability can be calculated by accounting for the relationship between the cumulative distribution and reliability. The fuzzy randomness hybrid reliability model has obvious advantages when model parameters may not be precisely represented because of factors in engineering practices, such as lack of sufficient data, data of fuzziness and unknown or non-constant reproduction conditions. However the computational efficient of exiting fuzzy randomness hybrid approaches is always a difficult problem because the fundamental mathematical theory of this type uncertain problems is not considered ideal.

Ni at al. [32] introduced a new hybrid reliability model which contains randomness, fuzziness, and non-probabilistic uncertainty. The presented hybrid model utilized available data and the advantages inherent in each individual

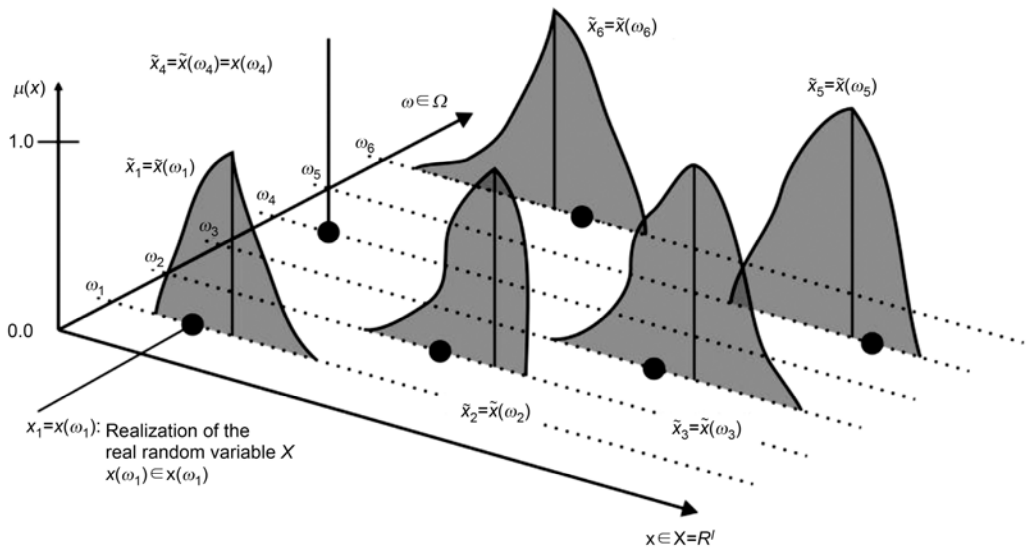

Figure 1 Fuzzy random variable [30].

model fully and effectively and may reflect, with validity, the real state of structural safety.

3 Recent advances in reliability analysis

The primary difficulties in structural reliability analysis are in the performance of three aspects: multidimensional uncertainties, nonlinear and implicit expression of limit state function. Both of reliability assessment and sensitivity analysis methods are introduced to solve these problems, which are primarily based on probabilistic reliability model.

3.1 Reliability assessment approaches

The reliability assessment approaches can be divided into three types [33]: the first case is approximate analytical method, such as the point estimation method, etc.; the second case is numerical simulation method including Monte Carlo simulation and some other sampling methods; the last case is the indirect method as the response surface method, which constructs an approximate model to reduce the computational cost when the limit state functions are implicit.

Gong et al. [34] introduced a robust iterative algorithm for structural reliability analysis named as finite-step-length iterative algorithm, which was built based on the modification of the HL-RF iterative algorithm in the traditional first-order reliability method and aimed to deal with the computational convergence for highly nonlinear limit state surfaces. When the step length tends to infinity, the finite-step-length iterative algorithm was converted into a particular case, that is, HL-RF method. As the process of line search for obtaining the step length was not needed, the proposed algorithm was considered to be easier than other optimization schemes.

The saddlepoint approximation (SA) based method can be used to deal with the reliability problems with non-Gaussian random variables [35]. Song et al. [36] compared three SA based structural reliability analysis methods and chose the most ideal case for complex reliability problems. The first case was SA based reliability bounds theory, which can only obtain the bounds of system failure probability and be only acceptable for the linear limit state function. The second case was SA based Nataf approximation, which can give the estimation of system failure probability, and the error mostly resulted from the approximation of Nataf distribution for the joint PDF of the structural system performance functions and the linearization of the performance functions. The last case was SA based line sampling as a numerical simulation method, which could obtain the estimate of system failure probability considering the influence of nonlinearity of limit state function. This could be acceptable for the structural system with multiple failure modes. These potential advantages can ensure that the SA based line sampling method may have wide application, not only in the stochastic context, but also in the fuzzy reliability analysis to replace the direct Monte Carlo method [37].

The importance sampling method is widely used for failure probability computation, which can improve the sampling efficiency and reduce the estimate variance of failure probability by using an importance sampling density function. As the selection of importance sampling density function would greatly affect the sampling efficiency, one of the methods for constructing sampling function is using the kernel density estimator method. By initial sampling for the failure region, the optimal importance sampling density can be approximated by fitting the initial sample data. Dai et al. [38] introduced an adaptive Markov chain simulation method for initial sampling, which was based on adaptive Metropolis algorithm as a replacement of classic Metropolis algorithm for sampling regions with high likelihood of failure. The importance sampling density function was constructed by support vector density, which can approximate the sampling density with fewer samples in comparison using the conventional kernel density estimation.

To deal with the problems with limited information on uncertainty, numerous studies based on non-probabilistic reliability models have been developed. In recent years, researchers have focussed on introducing some well-established techniques from traditional probability-based reliability analysis into the non- probabilistic reliability analysis. Some effective methods with better practicability have been developed and successfully applied to some industrial problems. The classical methods such as the Monte Carlo simulation, the first order and second order approximation are the most commonly used [39,40]. Nevertheless, it must be pointed out that the probability model and non-probabilistic model are effectively two types of completely different means of dealing with the uncertainty, as the former is based on the detailed distributions of the parameters while the latter does not need the precise probability distributions information of the uncertain domain. Therefore, some severe problems will be inevitably caused when the probabilistic reliability-based analysis techniques are introduced into solving the non-probabilistic model problems, such as the application field of the existing methods, computational efforts and accuracy.

3.2 Reliability sensitivity analysis approaches

In the traditional structural reliability analysis, the reliability is indicated by a reliability index, which is defined as obtaining a minimum distance from the failure surface to the origin in the standard parameter space. Therefore, reliability analysis can be formulated as an optimization problem, and the reliability sensitivity refers to the partial derivative of the failure probability P_f with respect to the distribution parameter of the basic random variables $\boldsymbol{x}$, that is, $\partial P_f / \partial \boldsymbol{x}$, which measures the effect of basic variable distribution parameters on the failure probability. In general, the reliability sensitivity analysis can be divided into local sensitivity analysis and global sensitivity analysis, the latter also being termed the important measure analysis. Similar to the reliability assessment, reliability sensitivity analysis approaches are based on probabilistic model which may include three aspects as approximate analytical method, numerical simulation and equivalent function method such as the response surface method [33].

Researchers have generally focussed on the reliability sensitivity analysis problems with nonlinear and implicit limit state functions, and improvements on the traditional methods are used to solve problems. Zhang et al. [41] introduced a modified perturbation method to improve the accuracy of reliability sensitivity analysis for systems with strong nonlinear performance functions. By combining the matrix differential method and the Kronecker algebra theory, the mathematical expression of reliability sensitivity based on the perturbation method can be modified to be suitable for the nonlinear problems.

The traditional response surface method based on polynomial approximation usually cannot obtain a high-quality equivalent model for the highly nonlinear limit state function. Zhao et al. [42] employed the support vector regression algorithm (SVR) to construct the equivalent limit state function. The SVR is an application of support vector machine (SVM) [43] in regression estimation, which is grounded in the framework of statistical learning theory and can overcome for the lack of the problems because of local minima, overfitting, and an inconveniently large number of tunable parameters in other learning algorithms. After scaling the training set using neural networks, the support vector regression based response surface method has been developed to approximate the actual limit state function as:

$$\bar{g}(\boldsymbol{y}) = \sum_{i=1}^{l}\left(\alpha_i - \alpha_i^*\right) K\left(\boldsymbol{y}^{(i)}, \boldsymbol{y}\right) + b, \tag{1}$$

where α_i and α_i^* are Lagrange multipliers, K is a kernel function, and $\boldsymbol{y}^{(i)}$ is the ith support vector obtained from the SVR training. As the SVM method is dependent on the optimization technology, the application to the reliability analysis would be converted into a nested optimization problem, and the computational cost would rise remarkably for a large scale structural reliability analysis.

Lu et al. [44–46] summarized the existing importance measures which can be classified as three categories, that is, non-parameter techniques (correlation coefficient model), variance based methods, and moment independent model. The importance measure methods have been extended to the study of importance analysis in the presence of epistemic and aleatory uncertainties [47–49], that is, hybrid uncertainties. The common importance measures are defined based on the expectation, variance and moment-independent of uncertain response $\boldsymbol{Y}$, respectively. The conditional expectation, conditional variance and conditional probability density functions of the uncertain response $\boldsymbol{Y}$can be defined as $E(\boldsymbol{Y}|\boldsymbol{x})$, $Var(\boldsymbol{Y}|\boldsymbol{x})$ and $f_{Y|x}(y)$, respectively. Thus the importance measures of the random input variables can be expressed as:

$$\mathrm{IM}^e = \sqrt{\int_{-\infty}^{+\infty}\left(E\left(\boldsymbol{Y}\right) - E\left(\boldsymbol{Y}\middle|\boldsymbol{x}\right)\right)^2 f_{\boldsymbol{x}}(x)\mathrm{d}x}, \tag{2}$$

$$\mathrm{IM}^v = \sqrt{\int_{-\infty}^{+\infty}\left(Var\left(\boldsymbol{Y}\right) - Var\left(\boldsymbol{Y}\middle|\boldsymbol{x}\right)\right)^2 f_{\boldsymbol{x}}(x)\mathrm{d}x}, \tag{3}$$

$$\mathrm{IM}^{M-I} = \frac{1}{2}\int_{-\infty}^{+\infty}\left[\int_{-\infty}^{+\infty}\left|f_{\boldsymbol{Y}}\left(y\right) - f_{\boldsymbol{Y}|\boldsymbol{x}}\left(y\right)\right|\mathrm{d}y\right] f_{\boldsymbol{x}}(x_i)\mathrm{d}x_i, \tag{4}$$

where IM^e, IM^v and IM^{M-I} are the critical measures on expectation, variance and moment-independent of uncertain response $\boldsymbol{Y}$, respectively. For the fuzzy variables $\boldsymbol{x}$, we know that both $E(\boldsymbol{Y})$ and $Var(\boldsymbol{Y})$ are fuzzy variables, which can be characterized by the membership functions. Then the unconditional membership function and the conditional membership function can be used to replace the first two moment functions in eqs. (2) and (3) for critical measures of fuzzy variables.

4 Recent advances in reliability-based optimization

RBDO is a methodology that allows solving optimization problems and explicitly modeling the effects of uncertainty. A RBDO problem can be defined by the following mathematical formulation:

$$\begin{cases} \min\limits_{\substack{\boldsymbol{x}\in\Omega\\ \boldsymbol{d}\in\boldsymbol{D}}} & E\left[f(\boldsymbol{d},\boldsymbol{x})\right], \\ \text{subject to} & P(r_j(\boldsymbol{d},\boldsymbol{x}) \leqslant b_j) \geqslant p_j; \quad j=1,\cdots,J, \\ & s_m(\boldsymbol{d}) \leqslant c_m; \qquad m=1,\cdots,M, \\ & d_l^- \leqslant d_l \leqslant d_l^+; \quad l=1,\cdots,L. \end{cases} \tag{5}$$

where $E\{f(\boldsymbol{d},\boldsymbol{x})\}$ is the expectation of the cost function as an objective function, $\boldsymbol{d}\in\boldsymbol{D}$ denotes the vector of design variables and $\boldsymbol{x}\in\Omega$ denotes the vector of uncertain parameters, $P(\cdot)$ is an operator to denote the probability of occurrence, p_j denotes the tolerable threshold that the j-th uncertain constraint should be satisfied. s_m is the m-th determined constraint and $\left[d_l^-, d_l^+\right]$ denotes the design domain boundary interval of the l-th design variable.

As discussed above, the reliability analysis is an optimization problem in itself, but sometimes is found as a nested optimization problem when SVM-based like approaches are used. Thus a RBDO problem is a double-loop problem, as shown in Figure 2. The reliability evaluation algorithm is nested within the optimization loop. The numerical costs are usually unaffordable for this type of problem. Therefore, how to simplify the double-loop optimization framework for numerical efforts reduction is a major difficulty for solving RBDO problems. Two main aspects of this type of problem will be discussed below. The first one is the reliability constraints approximate method, and the second one is double-loop decoupling method.

The performance measure approach is the most used approach to formulate reliability constraints in the RBDO problems. The reliability constraints in eq. (5) can be transformed into equivalent constraints such that the minimum target performance value should be non-negative, for example,

$$\alpha_j\left(\boldsymbol{d}\right) > 0, \tag{6}$$

where the minimum target performance value $\alpha_j(\boldsymbol{d})$ is obtained by the following optimal problem:

$$\begin{cases} \alpha_j\left(\boldsymbol{d}\right) = \min \quad G_j(\boldsymbol{d},\boldsymbol{x}) = b_j - r_j(\boldsymbol{d},\boldsymbol{x}), \\ \text{subject to} \quad \|\boldsymbol{x}\| = p_j. \end{cases} \tag{7}$$

The equality constraint of the optimization problem in eq. (7) imposes the prescribed tolerable threshold by setting the norm of $\boldsymbol{x}$ equal to p_j. Because it is much simpler to solve an optimization problem with an equality constraint, the performance measure approach is numerically stable. Kang et al. [51–53] extended the performance measure approach to RBO problems with interval and probability-interval hybrid uncertainties.

The decoupling methods are used to decompose the intrinsic relations of the two loops in RBDO problems, by which the reliability analysis is not necessary to be performed each time for an optimization loop. The sequential approximate programming approach is the most commonly used decoupling method, which converts the nested RBDO problem to serial deterministic optimization problems. The deterministic optimization is performed using original ob-

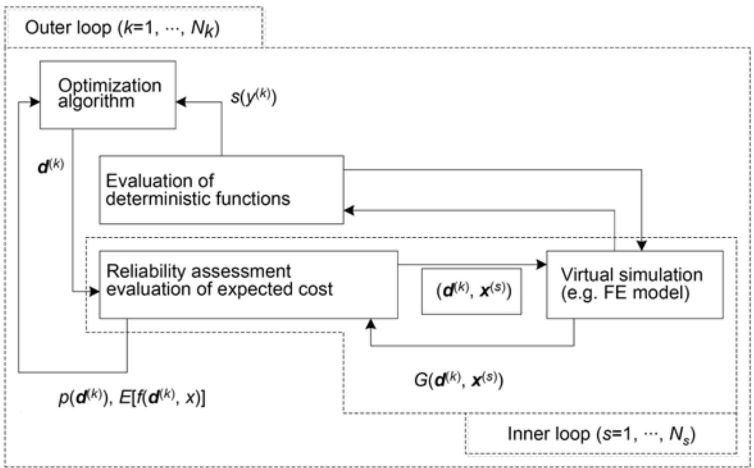

Figure 2 Schematic representation of a RBO problem [50].

jective function and convented constraints to find new design variables. An iteration scheme is used to update the parameters $\boldsymbol{x}_{,j}$ in constraints consistently with the values of the current design variables. These procedures will continue until the convergence criteria are satisfied [53]. Sometimes the linearization-based approach is used to approximate the minimum target performance value [52]. By approximating the limit-state function G_j with the first-order Taylor expansion about $\boldsymbol{x}_{,j}^{(k)}$, which is the approximate solution of eq. (7) in the k-th iteration, we can rewrite eq. (7) such that

$$\begin{cases} \alpha_j\left(\boldsymbol{d}^{(k)}\right) = \min\left\{ G_j(\boldsymbol{d}^{(k)}, \boldsymbol{x}_{,j}^{(k)}) + \left.\dfrac{\partial G_j}{\partial \boldsymbol{x}}\right|_{\boldsymbol{d}^{(k)}, \boldsymbol{x}_{,j}^{(k)}} \right\}, \\ \text{subject to} \quad \|\boldsymbol{x}\| = p_j. \end{cases} \tag{8}$$

5 Conclusion

Herein we have presented a general overview of the recent advances in structural reliability analysis and reliability-based design optimization. The methods for propagation of uncertainties, reliability modeling, reliability assessment, reliability sensitivity analysis and reliability-based optimization are discussed, which are not limited to problems with random uncertainties.

The uncertainty propagation is the basis for many types of uncertain analysis. To deal with the problems of computational efforts, the parallel processing and model reduction are two of most effective techniques. The non-intrusive methods should be developed for parallel processing, by which uncertainity modeling can be transformed into a new set of models with deterministic parameters. The model reduction techniques are effective for problems with a large number of uncertain parameters. However the retained terms need be validated to be considered the most influencing factors.

The hybrid reliability modeling is considered as an approach possessing both the merits of different two uncertain models as interval probabilities, fuzzy randomness and fuzzy interval. The hybrid reliability models are more consistent to actual conditions, which has an important supplementary role in structural reliability design.

The approaches for reliability assessment and reliability sensitivity analysis are similar, which can be divided into three parts as approximate analytical method, numerical simulation method and equivalent function method. The last type of methods like SVM may obtain the most accurate results, particularly for problems with non-linear limit and implicit state function. The reliability-based optimization is a nested optimization problem, and researchers have focussed on how to simplify the double-loop optimization framework for the reduction of numerical efforts.

For the strong demand for structural reliability design technology, this review paper, together with the collection of approaches in this special issue, may help to highlight the benefits of structural reliability design methods not only in academia, but also in practical engineering conditions.

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1 Guo X, Cheng G D. Recent development in structural design and optimization. Acta Mech Sin, 2010, 26: 807–823
2 Yang C, Xiao Z P, Wan Z Q, et al. Aeroelastic optimization design for wing with maneuver load uncertainties. Sci China-Tech Sci, 2010, 53: 3102–3109
3 Dai Y T, Wu Z G, Yang C. Identification and robust limit-cycle-oscillation analysis of uncertain aeroelastic system. Sci China-Tech Sci, 2011, 54: 1841–1848
4 Liang Y, Cheng X Q, Li Z N, et al. Multi-objective robust airfoil optimization based on non-uniform rational B-spline (NURBS) representation. Sci China-Tech Sci, 2010, 52: 2708–2717
5 Tang Z C, Lu Z Z, Li D W, et al. Nonprobabilistic reliability analysis for an inside flap of an aircraft. J Aircraft, 2012, 49: 250–255
6 Gu Y S, Zhang X P, Yang Z C. Robust flutter analysis based on genetic algorithm. Sci China-Tech Sci, 2012, 55: 2474–2481
7 Guan X R, Xu C, Wang Y J, et al. Influence of nose-perturbation location on behavior of vortical flow around slender body at high incidence. Sci China-Tech Sci, 2009, 52: 1933–1946
8 Yang C, Xiao Z P, Wan Z Q, et al. Aeroelastic optimization design for wing with maneuver load uncertainties. Sci China-Tech Sci, 2010, 53: 3102–3109
9 An H, An W G, Zhao W T. Reliability analysis of ship structure system with multi-defects. Sci China-Phys Mech Astron, 2010, 53: 2113–2124
10 Liu Y, Duan Z D. Fuzzy finite element model updating of bridges by considering he uncertainty of the measured model parameters. Sci China-Tech Sci, 2012, 55: 3109–3117
11 Guo T, Frangopol D M, Chen Y W, et al. Fatigue reliability assessment of steel bridge detail integrating weigh-in-motion data and probabilistic finite element analysis. Comput Struct, 2012, 112-113: 245–257
12 Deng Y, Ding Y L, Li A Q, et al. Fatigue reliability assessment for bridge welded details using long-term monitoring data. Sci China-Tech Sci, 2011, 54: 3371–3381
13 Bai J L, Ou J P. Seismic failure mode improvement of RC frame structure based on multiple lateral load patterns of pushover analyses. Sci China-Tech Sci, 2011, 54: 2825–2833
14 Ni Z, Qiu Z P, Su T C. On predicting boat drift for search and rescue. Ocean Eng, 2010, 37: 1169–1179
15 Xia B Z, Yu D J. Modified sub-interval perturbation finite element method for 2D acoustic field prediction with large uncertain-but-bounded parameters. J Sound Vib, 2012, 331: 3774–3790
16 Schueller G I, Pradlwater H J. Uncertain linear systems in dynamics: Retrospective and recent developments by stochastic approaches. Eng Struct, 2009, 31: 2507–2517
17 Qiu Z P, Wang J. New direct probabilistic approach for static responses of structures with uncertainties. Int J Comput Math, 2011, 88: 714–730
18 Qiu Z P, Wu D. A direct probabilistic method to solve state equations under random excitation. Probabilistic Eng Mech, 2010, 25: 1–8
19 Wang X D, Kang S. Application of polynomial chaos on numerical simulation of stochastic cavity flow. Sci China-Tech Sci, 2010, 53: 2853–2861
20 Zhang X F, Pandey M D, Zhang Y M. A numerical method for structural uncertainty response computation. Sci China-Tech Sci, 2011, 54: 3347–3357

21 Wu J L, Zhang Y Q, Chen L P, et al. A Chebyshev interval method for nonlinear dynamic systems under uncertainty. Appl Math Modelling, in press

22 Qi W C, Qiu Z P. A collocation interval analysis method for interval structural parameters and stochastic excitation. Sci China-Phys Mech Astron, 2012, 55: 66–77

23 Wang X J, Wang L, Qiu Z P. Response analysis based on smallest interval-set of parameters for structures with uncertainty. Appl Math Mech, 2012, 33: 1153–1166

24 Qiu Z P, Ni Z. An inequality model for solving interval dynamic response of structures with uncertain-but-bounded parameters. Appl Math Modelling, 2010, 34: 2166–2177

25 Wang J, Qiu Z P. The reliability analysis of probabilistic and interval hybrid structural system. Appl Math Modelling, 2010, 34: 3648–3658

26 Qiu Z P, Wang J. The interval estimation of reliability for probabilistic and non-probabilistic hybrid structural system. Eng Failure Anal, 2010, 17: 1142–1154

27 Jiang C, Li W X, Han X, et al. Structural reliability analysis based on random distributions with interval parameters. Comput Struct, 2011, 89: 2292–2302

28 Jiang C, Han X, Li W X, et al. A hybrid reliability approach based on probability and interval for uncertain structures. J Mech Design, 2012, 134: 1–11

29 Jiang C, Lu G Y, Han X, et al. A new reliability analysis method for uncertain structures with random and interval variables. Int J Mech Mater Des, 2012, 8: 169–182

30 Moller B, Beer M. Engineering computation under uncertainty–Capabilities of non-traditional models. Comput Struct, 2008, 86: 1024–1041

31 Wang Z L, Huang H Z, Li Y, et al. An approach to system reliability analysis with fuzzy random variables. Mech Machine Theor, 2012, 52: 35–46

32 Ni Z, Qiu Z P. Hybrid probabilistic fuzzy and non-probabilistic model of structural reliability. Comput Ind Eng, 2010, 58: 463–467

33 Lu Z Z, Song S F. Reliability and Reliability Sensitivity Analysis of Structures and Mechanisms (in Chinese). Beijing: Science Press, 2009

34 Gong J X, Yi P. A robust iterative algorithm for structural reliability analysis. Struct Multidisc Optim, 2011, 43: 519–527

35 Yuan X K, Lu Z Z, Qiao H W. Conditional probability Markov chain simulation based reliability analysis method for nonnormal variables. Sci China-Tech Sci, 2010, 53: 1434–1441

36 Song S F, Lu Z Z. Saddlepoint approximation based structural reliability analysis with non-normal random variables. Sci China-Tech Sci, 2010, 53: 566–576

37 Yi L L, Lu Z Z, Song S F. Saddlepoint approximation based line sampling method for uncertainty propagation in fuzzy and random reliability analysis. Sci China-Tech Sci, 2010, 53: 2252–2260

38 Dai H Z, Zhang H, Wang W. A support vector density-based importance sampling for reliability assessment. Reliability Eng Syst Safety, 2012, 106: 86–93

39 Jiang C, Bi R G, Lu G Y, et al. Structural reliability analysis using non-probabilistic convex model. Comput Methods Appl Mech Eng, 2012, 254: 83–98

40 Zhang H, Dai H Z, et al. Structural reliability analysis on the basis of small samples: An interval quasi-Monte Carlo method. Mech Syst Signal Processing, 2012, 37: 137–151

41 Zhang Y M, Zhu L S, Wang X G. Advanced method to estimate reliability-based sensitivity of mechanical components with strongly nonlinear performance function. Appl Math Mech-Engl Ed, 2010, 31: 1325–1336

42 Zhao W, Liu J K, Ye J J. A new method for parameter sensitivity estimation in structural reliability analysis. Appl Math Comput, 2011, 217: 5298–5306

43 Wu J, Deng C, Shao X Y, et al. A reliability assessment method based on support vector machines for CNC equipment. Sci China-Tech Sci, 2009, 52: 1849–1857

44 Hao W R, Lu Z Z, Tian L F. Importance measure of correlated normal variables and its sensitivity analysis. Reliability Eng Syst Safety, 2012, 99: 151–160

45 Song S F, Lu Z Z, Zhang W, et al. Uncertainty importance measure by fast Fourier transform for wing transonic flutter. J Aircraft, 2011, 48: 449–455

46 Cui L J, Lu Z Z, Zhao X P. Moment-independent importance measure of basic random variable and its probability density evolution solution. Sci China-Tech Sci, 2010, 53: 1138–1145

47 Wei P F, Lu Z Z, Hao W R. Importance measure analysis in the presence of epistemic and aleatory uncertainties. J Aircraft, 2011, 48: 1733–1740

48 Li L Y, Lu Z Z, Li W. Importance measure system of fuzzy and random input variables and its solution by point estimates. Sci China-Tech Sci, 2011, 54: 2167–2179

49 Li L Y, Lu Z Z, Li W. State dependent parameter method for importance analysis in the presence of epistemic and aleatory uncertainties. Sci China-Tech Sci, 2012, 55: 1608–1617

50 Valdebenito M A, Schueller G I. A survey on approaches for reliability-based optimization. Struct Multidisc Optim, 2010, 42: 645–663

51 Kang Z, Luo Y J, Li A. On non-probabilistic reliability-based design optimization of structures with uncertain-but-bounded parameters. Struct Safety, 2011, 33: 196–205

52 Kang Z, Luo Y J. Reliability-based structural optimization with probability and convex set hybrid models. Struct Multidisc Optim, 2010, 42: 89–102

53 Luo Y J, Li A, Kang Z. Reliability-based design optimization of adhesive bonded steel–concrete composite beams with probabilistic and non-probabilistic uncertainties. Eng Struct, 2011, 33: 2110–2119