SCIENCE CHINA
Physics, Mechanics & Astronomy

• Research Paper •

December 2011 Vol.54 No.12: 2284–2295
doi: 10.1007/s11433-011-4534-3

Relative position determination of a lunar rover using the biased differential phase delay of same-beam VLBI

CHEN Ming[1,2*], LIU QingHui[1*], WU YaJun[1], ZHAO RongBing[1] & DAI ZhiQiang[1]

[1] *Shanghai Astronomical Observatory, Chinese Academy of Sciences, Shanghai 200030, China;*
[2] *Beijing Aerospace Control Center, Beijing 100094, China*

Received January 24, 2011; accepted March 24, 2011; published online October 28, 2011

When only data transmission signals with a bandwidth of 1 MHz exist in the rover, the position can be obtained using the differential group delay data of the same-beam very long baseline interferometry (VLBI). The relative position between a lunar rover and a lander can be determined with an error of several hundreds of meters. When the guidance information of the rover is used to determine relative position, the rover's wheel skid behavior and integral movement may influence the accuracy of the determined position. This paper proposes a new method for accurately determining relative position. The differential group delay and biased differential phase delay are obtained from the same-beam VLBI observation, while the modified biased differential phase delay is obtained using the statistic mean value of the differential group delay and the biased phase delay as basis. The small bias in the modified biased phase delay is estimated together with other parameters when the relative position of the rover is calculated. The effectiveness of the proposed method is confirmed using the same-beam VLBI observation data of SELENE. The radio sources onboard the rover and the lander are designed for same-beam VLBI observations. The results of the simulations of the differential delay of the same-beam VLBI observation between the rover and the lander show that the differential delay is sensitive to relative position. An approach to solving the relative position and a strategy for tracking are also introduced. When the lunar topography data near the rover are used and the observations are scheduled properly, the determined relative position of the rover may be nearly as accurate as that solved using differential phase delay data.

same-beam VLBI, biased differential phase delay, relative position determination, lunar rover

PACS: 95.55.Pe, 95.55.Jz, 95.55.Br, 95.75.Kk

1 Introduction

The moon is, on average, 3.8×10^5 km away from the earth, and is an ideal springboard and transfer platform for deep space exploration and expansion in outer space. The long-term schedule of the Chinese unmanned lunar exploration plan is divided into three stages: orbiting, landing and returning [1]. The first Chinese lunar explorer, CE-1, was launched on 24 October, 2007, whereas the CE-2 satellite was launched on 1 October, 2010. The CE-2 satellite was the forerunner of the succeeding lunar explorations and is intended to take photos of the lunar surface, obtain topography data on the polar region, and prepare for soft landing on the lunar surface. During the orbiting stage, the explorers fly around the Moon and take photos of the landing area. In the landing stage, the main tasks are to soft land on the lunar surface and automatically explore the area. In the landing stage, a lunar rover and a lander are used; such equipment requires a tracking and controlling system that simultaneously manages multi-explorers. The position of the rover relative to the lander must be accurately determined. The automatic exploration and remote operation technologies of the rover are innovations that are keys in the landing stage [2]. By exploring the topography and construction of the local area, scientists can realize an important objec-

*Corresponding author (email: chenming@shao.ac.cn; liuqh@shao.ac.cn)

tive—establishing the relationship between the local topography and the evolution of the local construction [3]. Moreover, precise position determination of rover position can be used to estimate its movement, ascertain accurate position on the lunar surface, and set up a local topography model. Hence, the landing stage presents significant implications for engineering and science.

Conventional methods determining the position or the orbit of explorers include measuring range, Doppler, and angle. In deep space exploration, very long baseline interferometry (VLBI) is a precise approach to angle measurement. VLBI can provide plane-of-sky position information and can therefore be used to determine the relative position of a rover and a lander. The group delay with an error of several nanoseconds can typically be obtained using a phase/bandwidth formula because the bandwidth is usually less than 10 MHz for a spacecraft. To improve the accuracy of the group delay, multi-frequency signals can be adopted because the frequency range between the signals corresponds to the bandwidth. Hence, group delay with an error of 0.1 ns (30 mm) can be obtained using differential one-way range (DOR) signals, which have a bandwidth of nearly 40 MHz. In a previous study [4], we proposed a method for determining the relative position of a lunar rover using high-accuracy multi-frequency same-beam VLBI observation. We designed the radio sources on board the explorers that are used for multi-frequency same-beam VLBI observation. We also analyzed the error of the relative position determination of the lunar rover using same-beam VLBI observations. The analysis indicated that the differential phase delay between the lunar rover and the lander, with an error of 1 mm, can be obtained using multi-frequency same-beam VLBI when the separation angle of the two explorers is small. The two-dimensional relative position in the plane-of-sky between the lunar rover and the lander, with an error of 1 m in the distance of the Moon, can be determined using the differential phase delay data from four Chinese VLBI stations (Beijing, Shanghai, Kunming, and Urumqi stations). The relative position of the lunar rover, with an error of 10 m, can be determined using differential phase delay and the range data of the lander when the lunar topography near the rover with an error of 10 m can be obtained.

To use the method described above, radio sources on board the lunar rover and the lander should be specially designed. The volume of the lander is usually larger because it can contain more payloads and supply stronger power. Hence, supporting the radio source on board is more easily achieved by the lander. On the contrary, the lunar rover is usually smaller and constrained by strict limitations in payload volume and weight. The power supplied to the payloads is also limited. This is especially true during the early landing exploration phase, when the volume of the lunar rover is so small that it cannot contain the radio source designed for multi-frequency same-beam VLBI observation. Only the data transmission signals can be received. The bandwidth of the data transmission signals is usually 1 MHz. Accurate determining the relative position of the lunar rover is difficult to accomplish merely through the traditional differential group delay of VLBI observation. The group delay data normally contain obvious system and random errors. The bandwidth of the signals of the lander is approximately 40 MHz; thus group delay with an error of less than 1 nanosecond can be obtained. However, the group delay of the lunar rover with an error of several nanoseconds can be obtained using data transmission signals with a bandwidth of approximately 1 MHz. The differential group delay between the lunar rover and the lander with an error of several nanoseconds can be obtained. The relative position of the lunar rover can be determined with an error of several hundreds of meters. The three-dimensional instantaneous position of the landing point of the CE-1 satellite can be determined, with an error of approximately 550 m [5]. The distance that the lunar rover can travel in one day is several tens of meters. If the relative position with an error higher than this value can be determined, evaluating whether the lunar rover moves within one day will be difficult.

In this article, a new method for accurately determining the relative position of the lunar rover is proposed. According to this method, the differential group delay and biased differential phase delay of the lunar rover and the lander can be obtained first using the same-beam VLBI observation data. The bias of the biased differential phase delay can be estimated using the statistic mean value of the differential group delay as basis. The residual bias is estimated together with the solved-for state variables when obtaining the relative position. In the same-beam VLBI observation, the residual correlation phase can be obtained by correlating the received signals. The differential residual correlation phase can be obtained by differencing the residual correlation phase of the two explorers. To acquire the differential phase delay from the differential residual correlation phase, cycle ambiguity should be addressed. Cycle ambiguity cannot be solved for a one-frequency signal, and remains at the same level for a continuous observation period. The differential delay obtained from the differential residual correlation phase includes the bias caused by the cycle ambiguity of the phase, which is called the biased differential phase delay. The random error of the biased differential phase delay is very small. For example, if the frequency of the signal in X-band is 8.4 GHz and the random error of the phase is 3.0 degrees, the random error of differential delay is approximately 1.0 picoseconds (1 ps = 0.001 ns). The movement of the lunar rover can be observed with a resolution of several meters through the biased differential phase delay. However, because of the uncertainty of the cycle ambiguity contained in the differential residual correlation phase, the bias of the biased differential phase delay is uncertain, and the scope of the bias cannot be determined as well. The accuracy of the differential group delay is known. The statistic mean value of the differential group delay reflects the system error between the differential group delay and the true value. The statistic mean value of the differential group delay can be used to modify the bias of the biased differential phase delay and obtain modified biased differential phase delay. The bias of the modified biased differential phase delay can be estimated during the calculation of the relative position of the lunar rover.

The effectiveness of the method is confirmed using the same-beam VLBI observation data of SELENE satellites. The examples of radio sources on the lunar rover and the lander are designed in accordance with the fact that radio sources can influence the accuracy of the observables. To show the sensitivity of the differential delay to the relative position, we simulate the differential delay between the lunar rover and the lander. We also present a method for solving the relative position of the lunar rover and the bias of the modified biased differential phase delay.

2 Biased differential phase delay and differential group delay

2.1 Calculating the biased differential phase delay

In multi-frequency same-beam VLBI observations, two explorers with small separation angles are simultaneously observed within the main beam of the receiving telescopes. The residual correlation phase of each explorer between the two telescopes is first obtained, and then the differential residual correlation phase is determined from the residual correlation phases of the two explorers. The cycle ambiguity in the differential residual correlation phase is unknown. If the cycle ambiguity can be addressed, the differential correlation phase can be obtained. The differential phase delay can be acquired by dividing the differential correlation phase by the frequency of the signal. The differential group delay of the two frequencies can be determined by dividing the difference of the correlation phase of the two signals by the difference of the two frequencies. If the signal is of wideband type, the derivative correlation phase via the frequency is group delay. If the cycle ambiguity cannot be resolved, the differential phase delay from the differential correlation phase is biased. In a continuous observation period, the correlation phase is also continuous and the bias is a constant. The bandwidth of the signals is far lower than their frequencies; thus, the accuracy of the differential phase delay is much higher than that of the differential group delay. For the differential phase delay, the separation angle of the two explorers is small, and the influences of the atmosphere and the ionosphere are almost canceled in the differential correlation phase. The paths of the signals from the two explorers are the same in the receiving instruments, and each pair of frequency signals is recorded in the same channel. The delay in the instruments is almost canceled. The differential phase delay, which is the basic observable in multi-frequency same-beam VLBI observation, can provide information on the relative position of the plane-of-sky. The error of the differential phase delay is very small, which results in very high resolution of the relative position of the lunar rover.

The differential geometry delay between the lunar rover and the lander, $\Delta\tau_{\mathrm{g}}$, can be expressed as $\Delta\tau_{\mathrm{g}} = \tau_{\mathrm{g}}^{\mathrm{R}} - \tau_{\mathrm{g}}^{\mathrm{L}}$, where $\tau_{\mathrm{g}}^{\mathrm{R}}$ and $\tau_{\mathrm{g}}^{\mathrm{L}}$ are the geometry delays from the two explorers to the two receiving stations on Earth.

The residual correlation phases ϕ_i^{R} and ϕ_i^{L} (i: i^{th} frequency) of the rover and the lander can be obtained after eliminating the model value of the geometry delay. The relationship among the residual correlation phase, differential residual correlation phase, differential phase delay, and differential group delay can be expressed as follows.

The residual correlation phases of the rover and the lander can be expressed as:

$$\begin{cases} \phi_i^R = 2\pi f_i^R \tau^R - 2\pi N_i^R + \sigma_\phi, \\ \phi_i^L = 2\pi f_i^L \tau^L - 2\pi N_i^L + \sigma_\phi, \end{cases} \tag{1}$$

where f_i^R, f_i^L represent the frequency of the rover and the lander, respectively, τ^R, τ^L represent the residual delay of the rover and the lander, respectively, and σ_ϕ represents the phase noise.

From eq. (1), differential residual phase delay $\Delta\tau_P$ can be expressed as:

$$\Delta\tau_P = \frac{1}{2\pi f_i^R}\left(\Delta\phi_i + 2\pi\Delta N_i + \sigma'_\phi\right), \tag{2}$$

where $\Delta\phi_i = \phi_i^R - \phi_i^L \cdot f_i^R / f_i^L$, $\Delta N_i = N_i^R - N_i^L \cdot f_i^R / f_i^L$, and $\sigma'_\phi = \sqrt{1 + (f_i^R / f_i^L)^2} \cdot \sigma_\phi$.

From eq. (1), differential residual group delay $\Delta\tau_G$ can be expressed as:

$$\Delta\tau_G = \frac{1}{2\pi\left(f_i^R - f_j^R\right)}\left[\phi_i^R - \phi_j^R + 2\pi\left(N_i^R - N_j^R\right) + \sqrt{2}\sigma_\phi\right] - \frac{1}{2\pi\left(f_i^L - f_j^L\right)}\left[\phi_i^L - \phi_j^L + 2\pi\left(N_i^L - N_j^L\right) + \sqrt{2}\sigma_\phi\right]. \tag{3}$$

Disregarding the influence of the phase noise and the ionosphere, eq. (2) equals eq. (3). That is, $\Delta\tau_P = \Delta\tau_G = \Delta\tau$, where $\Delta\tau$ represents the differential residual delay.

The signals from the rover are the data transmission signals with a bandwidth of 1 MHz. The phase at the midpoint of the bandwidth represents the phase of the signals. For the group delay of the signals, the signal phase is continuous in the frequency domain, and the difference of cycle ambiguity is zero in eq. (3). The group delay is the derivative of the phases from the frequencies, and is always obtained by weighted least square fitting. The signals of the lander include both the DOR frequencies and the data transmission signals. The group delay of the DOR frequencies can be obtained by bandwidth synthesis. Normally, the phases of the most proximate frequencies are selected to obtain the differential phases of the rover and the lander.

The differential residual delay can be expressed as, $\Delta\tau = \Delta\tau_{\text{gm}} + \Delta\tau_{\text{atm}} - \Delta\tau_{\text{ion}-i} + \Delta\tau_{\text{inst}}$, where $\Delta\tau_{\text{gm}}$ is the differential residual geometry delay. The differential residual geometry delay is the difference between real differential geometry delay $\Delta\tau_{\text{g}}$ and model delay, $\Delta\tau_{\text{m}}$, and can be expressed as $\Delta\tau_{\text{gm}} = \Delta\tau_{\text{g}} - \Delta\tau_{\text{m}}$, where $\Delta\tau_{\text{atm}}$, $\Delta\tau_{\text{ion}-i}$, and $\Delta\tau_{\text{inst}}$ represent the differential delay caused by the atmosphere, ionosphere, and instrument, respectively. However, because the separation distance between the rover and the lander is extremely short, the paths of the signals from the two explorers to the stations are near each other when they pass through the atmosphere. The delays caused by the atmosphere and the ionosphere are almost cancelled. The signals are received by the antenna from almost the same direction. The paths of the signals are the same in the instruments, and each pair of frequencies is recorded in the same channel. Hence, the delay from the instruments is almost canceled by the difference. The same-beam VLBI observation is clearly different from the conventional VLBI observation. Reference radio sources should be used in the latter. The differential delays caused by the instruments and the clock can be calibrated using the reference radio sources, whose positions are already well known. The calibrated delay is used to correct the delay of the observation object. In the same-beam VLBI observation, however, the influences of the transmission media and the receiving instruments are almost identical. Most of the error is canceled after the difference. This is the reason why the differential delay can achieve very high accuracy in same-beam VLBI. The differential residual delay in the same-beam VLBI observation is composed mainly of the differential geometry delay. The influences of the media and the instruments after difference primarily serve as system errors. If the separation angle is less than 0.1 degrees, the system error is approximately 0.5 mm [4].

If the cycle ambiguity of the differential residual phase delay, ΔN_i, is unknown, the biased differential residual phase delay can be calculated as follows:

$$\Delta\tau_P^{\text{bias}} = \frac{\Delta\phi_i}{2\pi f_i^R}. \tag{4}$$

The bias is expressed as, $\Delta\tau_P^{\text{bias}} - \Delta\tau_P = -\Delta N_i / f_i^{\text{R}} = -\left(N_i^{\text{R}} / f_i^{\text{R}} - N_i^{\text{L}} / f_i^{\text{L}}\right)$. If the cycle ambiguity of the phase is altered, the bias changes as well. The bias should be solved in the process of determining the relative position. The uncertainty of the cycle ambiguity makes the solving process more difficult. An excessively large bias influences the accuracy and convergence of the solved-for parameters. Obtaining the differential residual group delay is usually easily accomplished. A lander residual group delay with an error of 1 nanosecond or less can be obtained using multi-frequency signals. A rover residual group delay with an error of several nanoseconds can be determined using data transmission signals with a bandwidth of 1 MHz. The differential residual group delay with an error of several nanoseconds can be ascertained by differencing the two residual group delays. The bias of the biased differential phase delay can be estimated through the differential residual group delay, which is already known. The residual bias is resolved in the relative position determination process. The basic process can be described as follows. The mean value of the differential residual group delay, $\Delta\tau_G$, can be obtained by $\Delta\tau_{G-m} = E(\Delta\tau_G)$, which is used as the estimated value of the differential residual delay. The difference between the biased differential residual phase delay and the mean value can be obtained by the formula, $\Delta\tau_{P-G}^{\text{bias}} = \Delta\tau_P^{\text{bias}} - \Delta\tau_{G-m}$, which is the estimated value of the bias. The modified biased differential residual phase delay can be determined by subtracting the estimated value from the biased differential residual phase delay. The modified bias is then solved in the relative position determination process. The value of the modified bias depends on the system error of the differential residual group delay. If the system error is sufficiently small, the modified bias is close to zero. This influence should be considered when the radio sources are designed.

2.2 Examples of calculating the biased differential phase delay and differential group delay

SELENE satellites were launched in October 2007, and they consist of a three-axis stabilized main satellite and two small spin satellites, called Rstar and Vstar [6]. The main satellite is controlled to maintain a nearly circular polar orbit of approximately 100 km in altitude. The orbits of the Rstar and Vstar are ellipses. Rstar serves as the relay satellite between the main satellite and the ground station. Rstar and Vstar transmit three carriers at 2212, 2218 and 2287 MHz for same-beam VLBI observations [7]. The differential phase delay between Rstar and Vstar is obtained from the differential correlation phases of the three carriers. The VLBI observations of SELENE were mainly executed by four Japanese VERA stations, ISGK, MZSW, IRIK, and OGSW. Four international stations (25 m, Shanghai, China; 25 m, Urumqi, China; 26 m, Hobart, Australia; 20 m, Wettzell, Germany) were involved in parts of the observations. The same-beam VLBI observations were executed from November 2007 to February 2009. The differential phase delays were obtained from the VLBI observation data, and were used to solve lunar gravity [8]. The three frequencies of Rstar and Vstar are proximate to one another. The frequency differences of each pair of frequencies were almost the same and less than 50 kHz. Hence, $f_i^R / f_i^V \approx 1$. To obtain the differential phase delay, the cycle ambiguity must be solved, and the necessary constraints for solving the cycle ambiguity must be satisfied [9]. The simple procedure of solving cycle ambiguity is described as follows. The cycle ambiguity $\Delta N_{21}=\Delta N_2-\Delta N_1$ is first resolved in the frequency range of 6 MHz. When the error of the predicted geometric delay is less than 83 ns, ΔN_{21} is 0 or 1. The value of 83 ns corresponds to an error of 4000 m in the position near the Moon for a baseline of 2000 km, which can be easily satisfied. Then, the cycle ambiguity of ΔN_{31} is resolved in the frequency range of 75 MHz based on the value of ΔN_{21}. Finally, the cycle ambiguity of ΔN_1, ΔN_2, and ΔN_3 are resolved based on the value of ΔN_{31}, with a requisite error of the differential correlation phase of less than 4.3 degrees. Although 4.3 degrees is strict, it is easily satisfied in the same-beam differential VLBI observations because the maximum separation distance between the rover and the lander is only several kilometers. In SELENE same-beam VLBI observations, more than 85% of the cycle ambiguities can be successfully resolved. A differential phase delay with an error of 1 mm can be obtained when the separation angle is small [10]. The relationships among the differential group delay, differential phase delay, and biased differential phase delay are analyzed using the accurate differential phase delay observation data of Rstar and Vstar.

Figure 1(a) shows that the differential residual phase delay can reflect the change in the delay from the two explorers to the stations with very high resolution. Figure 1(b) illustrates the entire history of the differential delay in the observation period. The varying scope may reach several kilometers. The error of the differential phase delay is 1 mm for a small separation angle and several millimeters for a larger one; this error is far smaller than that of the differential group delay. Figures 1(c) and 1(d) show the accuracy of the group delay in different bandwidths. As shown in these figures, the system and random errors are clearly different for varied bandwidths. The differential group delay is more accurate for a wider bandwidth.

Figure 2 shows that the random error of the delay clearly decreases with the increase in the integral period. However, the system error is nearly unchanged. The difference between the biased differential phase delay and the differential phase delay of f1 is a constant bias.

Figure 3(a) shows that the system error of the differential group delay of f1–f2 at baseline VERAISGK-VERAMZSW is −0.582 m. The bias of the biased differential phase delay

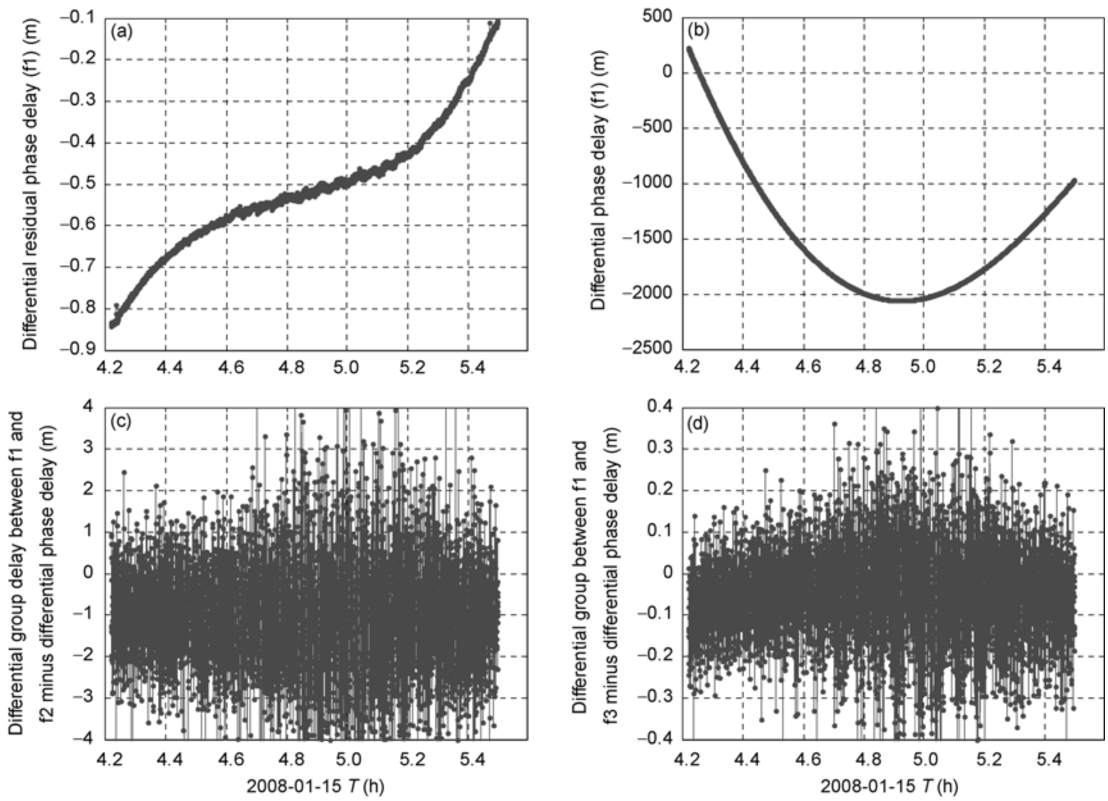

Figure 1 Differential delay and comparison of the same-beam VLBI observation from the baseline VERAISGK-VERAIRIK (converted to distance unit); (a) differential residual phase delay of f1 obtained from the differential correlation phase; (b) differential phase delay of f1 obtained from the differential residual phase delay plus the model value of the differential delay; (c) differential group delay between f1 and f2 minus the differential phase delay of f1; mean value is −0.973 m; standard deviation is 1.502 m; (d) differential group delay between f1 and f3 minus the differential phase delay of f1; mean value is −0.050 m; standard deviation is 0.123 m; integral time is 1 s, and data rate is 1 point per second.

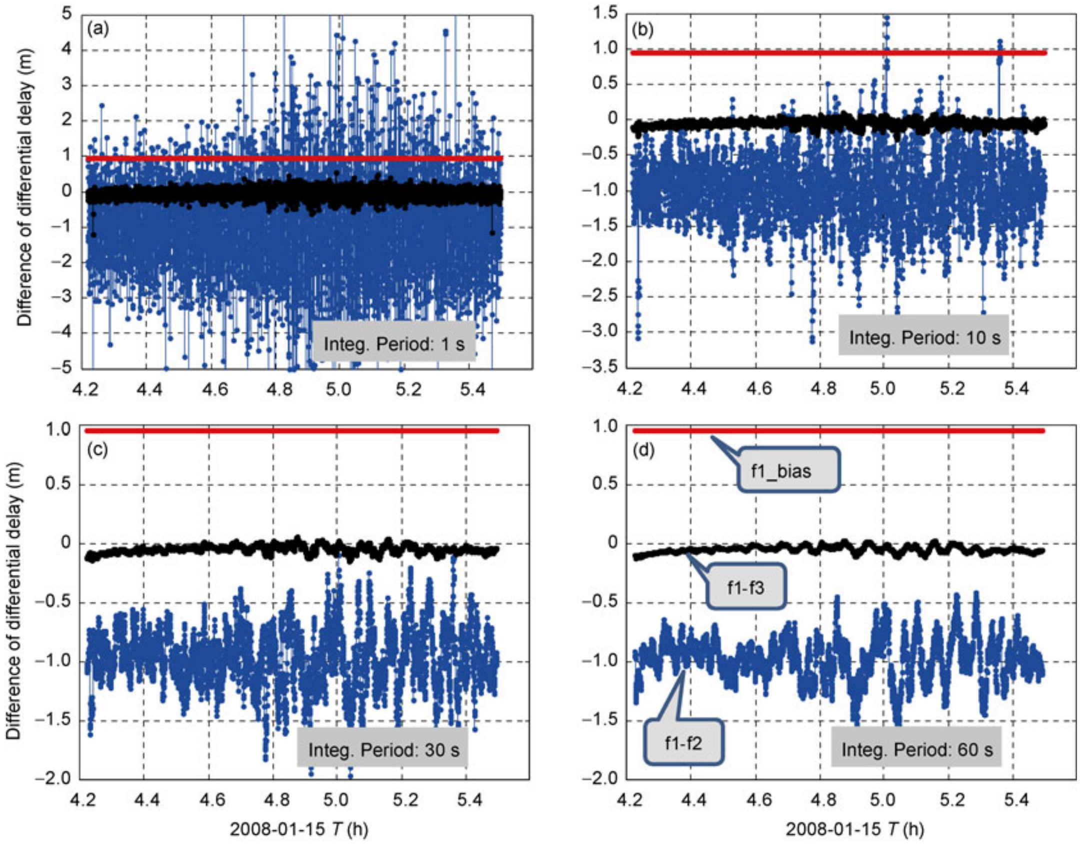

Figure 2 (Color online) Differential group delay and biased differential phase delay were compared with the differential phase delay for baseline VERAISGK-VERAIRIK with different integral periods. (a) Difference between the differential group delay of f1–f2 and the differential phase delay of f1, difference between the differential group delay of f1–f3 and the differential phase delay of f1, and difference between the biased differential phase delay of f1 and the differential phase delay of f1. The bias of f1 is 0.949 m and the integral period is 1 s. (b) The integral period is 10 s. The mean value of the differential group delay of f1–f2 minus the differential phase delay is –0.973 m and the standard deviation is 0.470 m. The mean value of the differential group delay of f1–f3 minus the differential phase delay is –0.050 m and the standard deviation is 0.045 m. (c) The integral period is 30 s. The mean value of the differential group delay of f1–f2 minus the differential phase delay is –0.974 m and the standard deviation is 0.258 m. The mean value of the differential group delay of f1–f3 minus the differential phase delay is –0.050 m and the standard deviation is 0.032 m. (d) The integral period is 60 seconds. The mean value of the differential group delay of f1–f2 minus the differential phase delay is –0.973 m and the standard deviation is 0.180 m. The mean value of the differential group delay of f1–f3 minus the differential phase delay is –0.050 m and the standard deviation is 0.027 m. The moving smooth method is used when integrating the data. The data rate is 1 point per second.

is 1.491 m. Figure 3(b) indicates that the system error of the differential group delay of f1–f2 at baseline VERAMZSW-VERAIRIK is –0.408 m. The bias of the biased differential phase delay is –0.542 m. Meanwhile, Figures 2(d), 3(a), and 3(b) show that the bias of the biased differential phase delay varies in a large scope. This variance is caused by the uncertainty of the cycle ambiguity of the phase. The three biases of the biased differential phase delay are closed for the three stations. It means that the sum of the biases equals zero. Given that the differential phase delay is closed [11] and the model value of the delay for each station is unique, the differential residual phase delay is also closed. The phase-closure characteristic of the differential phase delay of the multi-baseline results in closed biases. This constraint can be used to verify the solved biases from each baseline.

The differential group delay of f1–f2 includes a system error at each baseline. The system error varies from 0.4 to 1.0 m. The system error is caused by the system error of the differential phase. For the same-beam VLBI observations in SELENE, the system error of the differential phase arises from the separation angle between Rstar and Vstar, the elevation of the antenna, and the frequency-phase characteristics of the receiver. If the separation angle between Rstar and Vstar is large, the influence of the ionosphere, atmosphere, and antenna phase is remarkable. If the elevation of the station is low, the influences from the ionosphere and the atmosphere increase. The influence of the instruments is related to the difference of the frequencies from the two explorers. If the difference of the frequencies is large, the influence is also large. The phase variation relative to the direction in the main beam of the receiving antenna is not clearly observable. Such influence is cancelled by differencing the phases of the two signals. The radio frequency signals are converted into intermediate frequency signals after they pass through the down-converter and the band-pass filter at a bandwidth of 100–500 MHz. The bandwidth

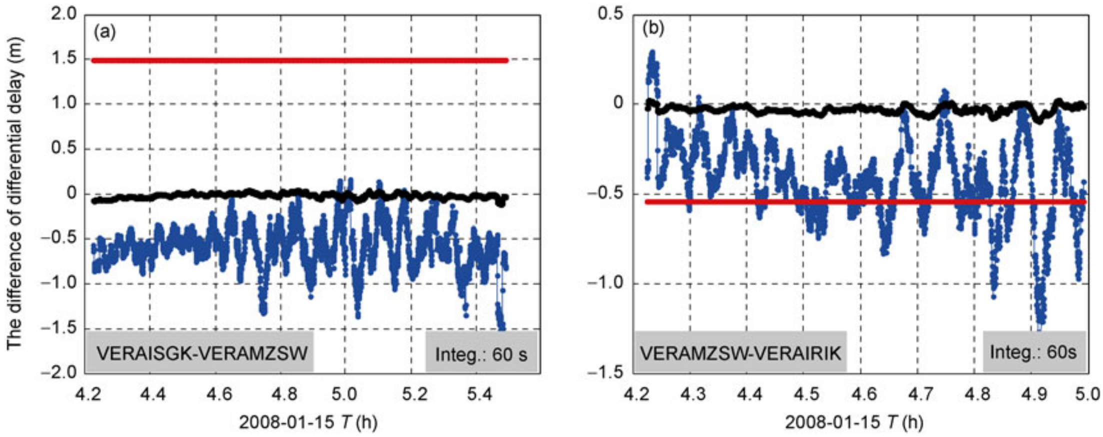

Figure 3 (Color online) Differential group delay and biased differential phase delay were compared with differential phase delay at baseline VERAISGK-VERAMZSW and VERAMZSW-VERAIRIK. (a) The differential group delay of f1–f2/f1–f3 and the biased differential phase delay of f1 minus the differential phase delay of f1 at baseline VERAISGK-VERAMZSW. The mean value for the difference of f1–f2 is –0.582 m and the standard deviation is 0.251 m. The mean value for the difference of f1–f3 is –0.021 m and the standard deviation is 0.027 m. The bias of the biased differential phase delay of f1 is 1.491 m. (b) The differential group delay of f1–f2/f1–f3 and the biased differential phase delay of f1 minus the differential phase delay of f1 at baseline VERAMZSW- VERAIRIK. The mean value for the difference of f1–f2 is –0.408 m and the standard deviation is 0.233 m. The mean value for the difference of f1–f3 is –0.029 m and the standard deviation is 0.020 m. The bias of the biased differential phase delay of f1 is –0.542 m. The integral period is 60 s. The data rate is 1 point per second.

of the filter is wide. The bandwidth of the signals is narrow and can be incorporated in the middle of the bandwidth of the filter. The influence of the filter on the frequency-phase characteristics of the intermediate frequency is very small. The intermediate frequency signals are converted into signals with a bandwidth of 0–2 MHz after they pass through the down-converter and the low-pass filter. The signals with a bandwidth of 2 MHz are converted to low frequency signals with a bandwidth of 0–100 kHz after they pass through the down-converter and the low-pass filter. The low frequency signals are then sampled and recorded. For the low frequency signals, the low-pass filter enables only the beginning of the signals of 2 MHz to pass through. The nonlinear frequency-phase characteristics of the filter in the head influences the phase of the low frequency signals. The phase fluctuation in the bandwidth of 100 kHz can reach approximately 2 degrees [12]. The length of the observation in Figure 3(b) is shorter than that in Figure 3(a). This discrepancy is caused not by the tracking process, but by the interrupted phase. The interruption in phase is related to the characteristics of the instruments. The feeder of the receiving antenna of VEAR is composed of a screw vibrator array. Many electrical devices are found in the feeder cabin; these may produce disturbance. The disturbance signals may be as strong as the received signals, which can be seen from the frequency spectrum of the receiving signals [11]. The frequency of the received signals continuously shifts because of the Doppler effects. When the frequency of the received signals approaches the frequency of the disturbance signals, distinguishing the received signals from the recorded data will be extremely difficult. At this time, the phase of the received signals might be interrupted. This kind of influence should be avoided when designing the measurement systems of the lunar rover and the lander.

2.3 Error analysis of the bias from the modified biased differential phase delay

The same-beam VLBI observation between the lunar rover and the lander differs from that in SELENE in several aspects. All these attributes are beneficial in improving the accuracy of same-beam VLBI observation. The rover is located near the lander. The maximum separation distance between the rover and the lander is only several kilometers. The separation angle from the two explorers to the receiving antenna on the ground is very small. For example, the separation angle is less than 0.001 degrees when the distance on the lunar surface is 5 km. The phase error influenced by the phase pattern of the receiving antenna and the transmission media may decrease to several millimeters, which is highly beneficial for the same-beam VLBI observation. In determining the relative position of the lunar rover and the lander, the model value is computed based on the determined position of the lander. The differential residual delay directly reflects the relative position of the lunar rover. The receiver in the China VLBI network already uses the DBBC [13]. Compared with the frequency-phase characteristics of the ABBC, those of the DBBC are more stable, and the nonlinear variation of the band-pass is very small. The nonlinear fluctuation can be decreased further by correction. The phase fluctuation for different frequencies in the same channel decreases and the system error of the differential delay becomes very small. Figure 4 shows the new characteristics of the frequency-phases among the channels. The effects of the band-bass correction are also shown in the figure.

Figure 4 illustrates that data from channel 1 are taken as reference. Data from channel 5 are correlated with data from channel 1. Channels 1–4 come from one DBBC, and channels 5–8 come from another DBBC. Five percent of the data at the head and the tail of each channel are removed before the analysis to decrease the nonlinear influence. The rms of the residual correlation phase after band-pass correction ranges from 0.3 to 0.5 degrees. The rms increases when the bandwidth of the channel increases. The residual correlation phase shows white noise characteristics. If the integral period is increased, the error decreases. These characteristics are beneficial for decreasing the system error of the differential group delay.

3 Principle and examples of designing radio sources on board the lunar rover and the lander

The same-beam VLBI observation is used to determine the relative position of the lunar rover. The principle for designing the same-beam VLBI observation is that the frequencies from the rover and the lander should be recorded at the same channel of the receiver. In such conditions, the influences from the transmission media and the instruments are cancelled by differencing the phase of the two signals. Hence, the accuracy of the differential delay is extremely high. A differential phase delay with an error of 1 ps can be obtained using specially designed multi-frequency radio sources [4]. Suppose that the lander can transmit the DOR signals and the data transmission signals, whereas the rover can transmit only the data transmission signals. According to the aforementioned principle, the on board radio sources are specially designed to achieve highly accurate same-beam VLBI observations. The examples of radio sources can be referenced in future lunar explorations.

Figure 5(a) shows that the data transmission signals with the bandwidth of 1.0 MHz are placed in the scope between the range tone and the first tone of the DOR at 3.8 MHz relative to the carrier of the lander. If the lander simultaneously transmits the carrier, as well as the range and first tones of DOR, the receiver of the VLBI system can record these lander signals together with the data transmission signals of the rover with a bandwidth of 8 MHz. The group delay of the lander and the rover can be obtained from the recorded data. The differential group delay can be obtained from the two group delays. The modified biased differential phase delay can be determined from the biased differential phase delay and the differential group delay. Figure 5(b)

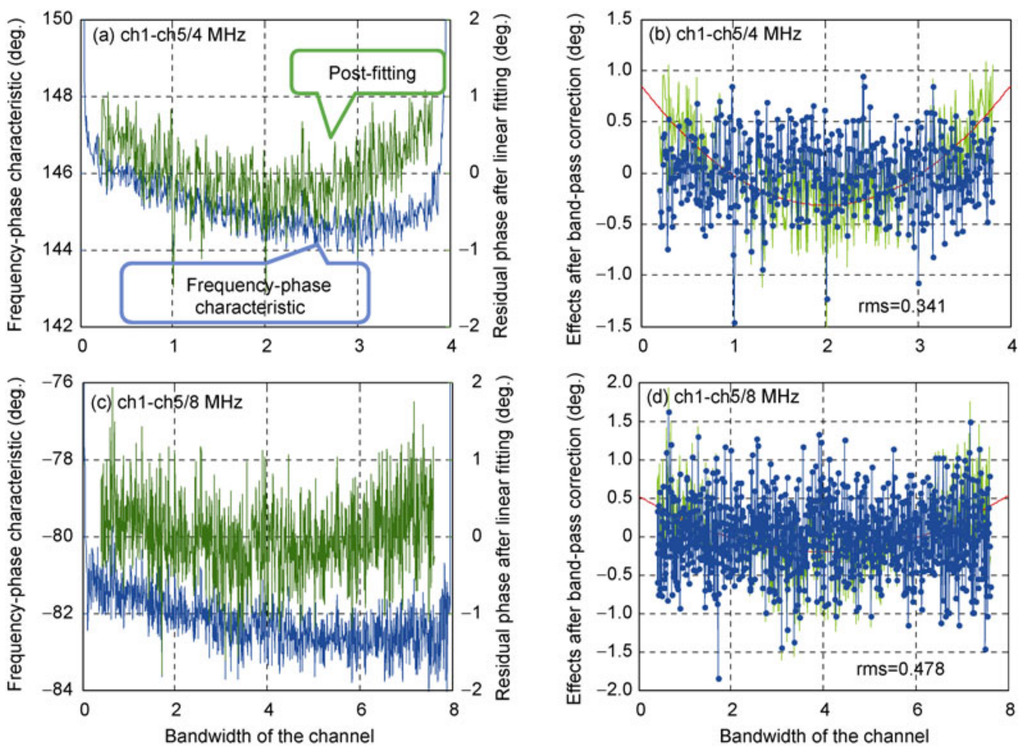

Figure 4 (Color online) Frequency-phase characteristic test among the DBBC receiving channels. The input signals are X-band signals. The bandwidth of the receiving channel is set as 4 MHz/8 MHz. (a) Correlation phase of channels 1 and 5 with a bandwidth of 4 MHz. Left *Y*-axis represents the phase. Right *Y*-axis represents the residual phase after linear fitting; (b) two order polynomial fitting to the residual phase of (a) used as the correction value. The rms is 0.341 degrees after fitting; (c) and (d) represent the correlation phase of channels 1 and 5 with a bandwidth of 8 MHz. The rms is 0.478 degrees after fitting. The integral period for the correlation phase is 1 second.

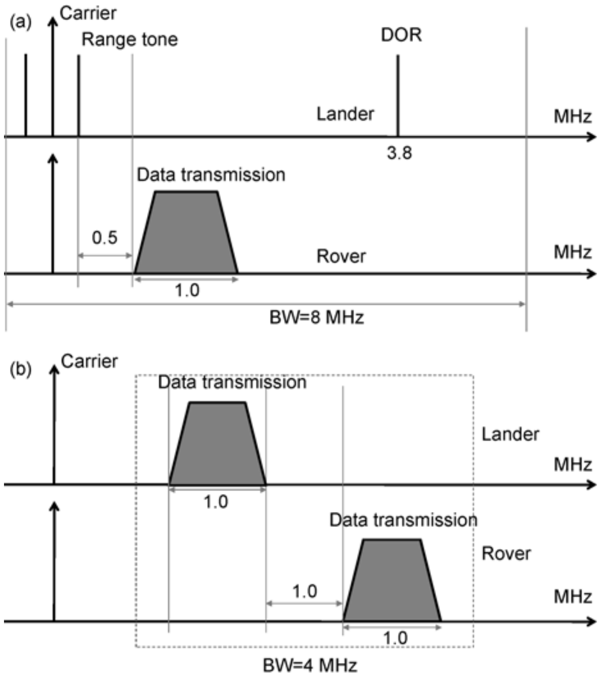

Figure 5 Examples of radio sources fit for the same-beam VLBI observation: (a) using the frequencies of the lander and data transmission signals of the rover in same-beam VLBI observation; (b) using the data transmission signals of the lander and the data transmission signals of the rover in the same-beam VLBI observation.

shows that the data transmission signals of the rover with a bandwidth of 1.0 MHz are placed beside the data transmission signals of the lander. The receiver of the VLBI system can simultaneously record the data transmission signals of both the lander and the rover with a bandwidth of 4 MHz. The modified biased differential phase delay can be obtained after determining the group delay of the lander and the rover. In the two cases, the signals of the rover are designed to be as proximate to one of the signals of the lander as possible to simultaneously receive and record these signals from the two explorers in one narrow channel. Because the signals from the two explorers are received in the same channel, the difference from different channels has no influence on the phase of the signals. The influence from the channel is cancelled by differencing the phase of the signals from the two explorers. The frequencies from the two explorers are near each other; thus, the bandwidth of the receiving channel is narrow, and the influence of the frequency-phase characteristics of the channel can be decreased to a very low level. This level can be decreased further by enlarging the integral period. The test data of the DBBC show that the influence of the frequency-phase characteristics is approximately 0.3–0.5 degrees with a bandwidth of 8 MHz when the integral period is 1 second.

The design of the radio sources is highly important in achieving effective same-beam VLBI observation. If the frequencies from the two explorers are far from each other, the bandwidth of the channel has to be very large to simultaneously record these signals in one channel, indicating that the volume of the sample data is extremely large. This phenomenon makes cancelling the influences from the instruments and the transmission media difficult. The error of the observables may increase.

4 Simulation results of the differential delay between the lunar rover and the lander

Determining the relative position of the lunar rover necessitates the sensitivity of the observables to the relative position. The relationship between the differential delay and the relative position of the rover and the lander in different conditions is analyzed through simulations.

The conditions needed for the simulation include ground stations selection, objects setting, and the observation period of interest. The four selected China ground stations are the Shanghai (SH), Beijing (BJ), Kunming (KM), and Urumqi (UR) stations. The four stations can construct six baselines. Two of them are selected in the simulation analyses, the BJ-KM baseline, which ranges from north to south and the SH-UR baseline, which ranges from west to east. The two baselines are almost orthogonal in direction.

The lander is set on a fixed place A of the lunar surface, which is the center of the near-side on the lunar surface facing the earth. It can be defined as the point of intersection between the *X*-axis of the lunar body-fixed coordinate and the lunar surface. The separation distances between the lunar rover and the lander are set as 100 m, 1 km, and 10 km. The directions of the rover relative to the lander are considered east and north. The minimum elevation of the ground station is 10 degrees. The simulation observation period is one and a half days starting from 1 October 2015. The software used for the simulation is the same as that used in CE-1 satellite orbit determination [14].

The following conclusions can be drawn from the simulation results: (1) The differential delays along the west-east and north-south directions are clearly different, indicating that the observables are sensitive to the direction of the relative position. (2) The differential delay varies linearly with the separation distance in the same direction, implying that the observables are sensitive to the separation distance. (3) The differential delay varies in the scope of several tens of centimeters in the entire observation period when the separation distance is 100 m. The error of the differential delay should be less than the centimeters level to resolve the 10 m level changes in the relative position. Achieving such accurate observation using the traditional differential group delay is extremely difficult.

The conditions in which the rover is stationary to the lander have been discussed above. The moving conditions are analyzed as follows. Suppose that the lander stays in place A and the rover is moving relative to the lander. Two cases are discussed. One is that the rover moves from place

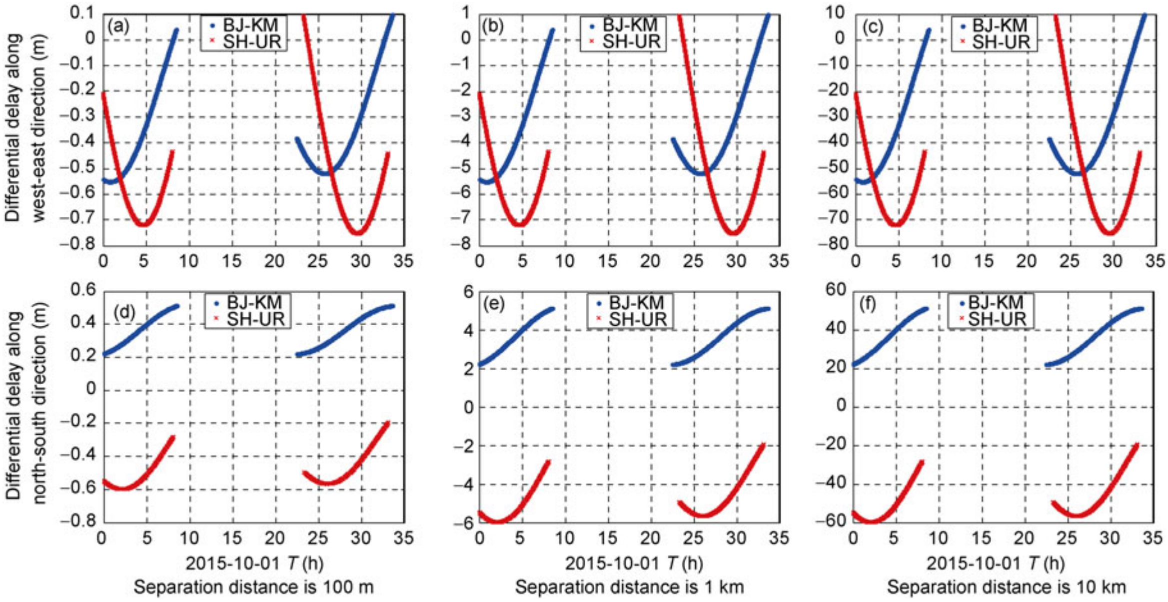

Figure 6 (Color online) Simulation results of the differential delay with different relative positions for the rover and the lander on the lunar surface. (a) The separation distance is 100 m in the west-east direction; (b) the separation distance is 1 km in the west-east direction; (c) the separation distance is 10 km in the west-east direction; (d) the separation distance is 100 m in the north-south direction; (e) the separation distance is 1 km in the north-south direction; (f) the separation distance is 10 km in the north-south direction. In the figure, ' •' denotes data from the BJ-KM baseline, '×' denotes data from the SH-UR baseline.

B2 (which is located at the east of A with a separation distance of 1 km) to place B3 (which is located at the east of A with a separation distance of 10 km) with an unchangeable speed of 6 km per day. The other case is that the rover moves from place C2 (which is located at the north of A with a separation distance of 1 km) to place C3 (which is located at the north of A with a separation distance of 10 km) with an unchangeable speed of 6 km per day. The simulation results of the differential delays are shown in Figure 6.

Figure 7 shows that the rover moves from the place with a separation distance of 1 km, so that the curves initially overlap with that of the fixed separation distance of 1 km. The rover reaches the place with a separation distance of 10 km, so that the curves overlap with that of the fixed separation distance of 10 km at the end (The overlap is not evident because the ending area cannot be observed.). Figure 7 also shows that the curves simultaneously move across the *X*-axis, a behavior that reflects the special relative position between the two explorers and the two ground stations.

The case in which the lander stays in the center of the lunar nearside is analyzed. In this case, the differential delay observables are most sensitive to the relative position of the rover and the lander. For more general cases, the landing area of the lander can be assumed to have a longitude λ, which varies from –90 to 90 degrees, and a latitude ϕ. Given that the VLBI observables are sensitive to the direction in the plane-of-sky, the VLBI observables are also sensitive to the projection of the relative position in the plane-of-sky. The sensitivity of the VLBI observables to the relative position decreases when the landing point is far from the center of the lunar near-side. This decrease can be described by the factor $\cos\phi\cdot\cos\lambda$. This phenomenon is particularly true near the rim of the lunar near-side, because the VLBI observables show hardly any information on the relative position.

5 Method for determining the relative position of the rover

The basic process of lunar surface survey begins from the necessary preparations after landing. The rover is released from the lander and reaches one or more specified regions along the planned path. Communication between the rover, lander and stations continues. The weighted least squares method can be used to determine the relative position of the rover using the same-beam VLBI observation. The Taylor expansion of the estimated parameters at the prior value is used to develop the linear condition equation. The estimated values are obtained from the iteration. The condition equation can be simply expressed as $Px=y-b$, where P is the Jacobi matrix consisting of partial derivatives, x is the solved-for parameter vector, y is the residual vector, and b is the biases vector of the observables. For the differential delay, b is the bias of the modified biased differential phase delay. To establish condition equations, some other technologies including the coordinate system definition, estimated parameter selection, prior value determination, and the method of solving the bias also need to be studied.

The local Cartesian coordinate system is developed to describe the relative position between the lunar rover and the lander. The origin is the geometric center of the landing

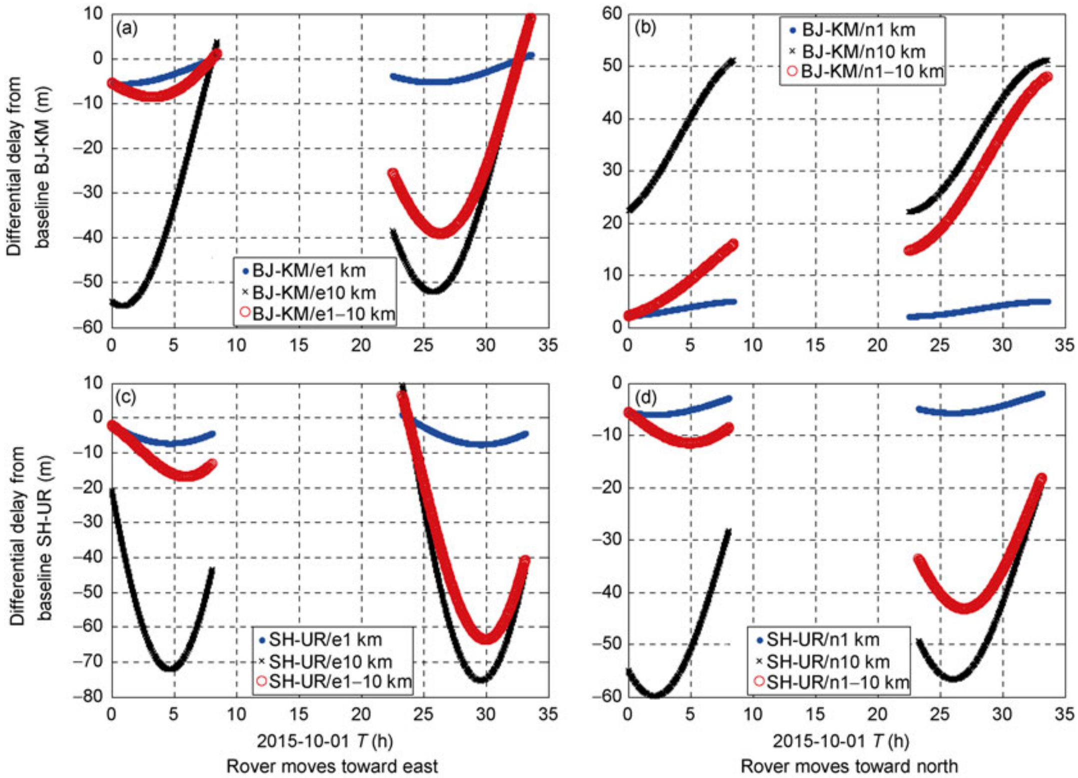

Figure 7 (Color online) Simulation results of the differential delay when the rover moves relative to the lander. (a) For baseline BJ-KM, the rover moves from west to east. (b) For baseline BJ-KM, the rover moves from south to north. (c) For baseline SH-UR, the rover moves from west to east. (d) For baseline SH-UR, the rover moves from south to north. In the figure, '·' denotes the differential delay with the fixed separation distance of 1 km; '×' denotes the differential delay with the fixed separation distance of 10 km; '○' denotes the differential delay when the separation distance varies from 1 to 10 km.

location, and the three components are along the east, north and zenith. The position of the lander is directly described in this coordinate system. Choosing the positions of the rover in the local Cartesian coordinate system as the state variables is a convenient approach. However, estimating the coordinate parameters of the rover is difficult when only the same-beam VLBI observation is used because of the limitation of the observables. VLBI is a unique observation technique for measuring the relative position discussed in this paper. VLBI observation can provide strong constraints to the projection of the relative position in the plane-of-sky. However, it shows weak constraints along the range direction, raising the need for a third constraint. Given the flat surface around the landing location, the height constraint is used, which can be obtained from the digital lunar topographic model. Because the grid in the available lunar topographic models [15–17] is excessively large to enable direct use, the regional digital lunar topographic model from the CE-2 mission is considered. Thus, the estimated parameters are $[x,y]$ coordinates. The prior value of the relative position is known before the rover leaves the lander. After the rover leaves the lander, the prior value required in subsequent observations can be obtained by two methods. The first is to take the estimated parameters in the previous observation as the prior value of the later observations because the rover moves slowly and continuously. The other method is to obtain the prior value from the planned trace of the rover. The biases of the observables can typically be estimated together with the other parameters. If the observation periods are limited, the biases cannot be solved but can be derived from the statistics of the residuals. The process can be iterative to obtain the final biases. In determining the relative position of the rover, the biases should not exceed the system error of the differential group delay; these errors can be used as constraints when calculating the biases. The biases derived from different baselines may be different, but they should be closed that they can be used to verify the obtained biases.

Scheduling the track is an important problem in determining the relative position of the rover. If the rover is observed continuously during its movement, the trace of the rover can be obtained. The observations begin before the rover is released from the lander, and continue throughout the entire observation period. The rover remains at rest at the designed characteristic place, while the observation is in progress. During the continuous observation, only one bias exists in the differential delay. This bias can be calculated using the observations before the movement of the rover, and is used as prior value for the subsequent trace. The simplified model can be set up to describe the trace between two characteristic places, in which the planned trace and telemetry navigation information are considered. The model parameters are calculated using the observation data. Continuous observation for long periods may be difficult; hence, the observations are designed piecewise. The characteristic places are then selected. The rover remains at rest at the characteristic places to facilitate continuous observations. Accurate relative positions are separately obtained. The biases of the differential delay at different places vary. In addition, determining the relative position is difficult if the observations can be obtained only during the movement of the rover, because its trace cannot be accurately known. The courses mentioned are listed below.

(1) The lander tracking begins as soon as it lands on the Moon to acquire the position of the lander on the lunar surface.

(2) When the lunar rover works normally after it is released from the lander, the same-beam VLBI observation is initiated. The self-test of the rover can be arranged at the same time. This observation period can be used to calibrate the tracking system error and obtain the initial phase difference.

(3) The characteristic places, such as sampling places, photo-taking areas, detection sites, and so on, are designed in accordance with the mission of the rover if the observation periods are limited. The rover remains at rest at the characteristic places to conduct continuous observations, which allows for the acquisition of accurate relative positions.

(4) If the continuous observation cannot be achieved at the characteristic places, the communication and data transmission should be arranged in time to collect the observations. All the observations are used to calculate the positions of the characteristic places. The biases vary for different observations.

(5) If continuous observation of the characteristic places is achieved, the bias of differential delay is unique. The trace can be modeled to solve the history of the trace. For example, the multi-section linear model can be considered.

6 Discussion and conclusion

In this article, the same-beam VLBI observation was used to determine the relative position of the rover. Considering the lunar rover enables only the transmission of data transmission signals; thus, a new method was proposed to determine the relative position of the rover with high accuracy through the biased differential phase delay. The method was confirmed using the same-beam VLBI observation data of SELENE. The designed radio sources for the rover and the lander, as well as the measured results of the DBBC of the receiver, show that a more accurate phase can be obtained using the same-beam VLBI observation between the lunar rover and the lander. The differential group delay is helpful in modifying the biased differential phase delay. Finally, a method for determining the relative position of the rover and a tracking strategy were introduced. Compared with the differential phase delay, the biased differential phase delay necessitates the determination of an additional bias in each continuous tracking period. The biases were solved together with the state parameters in determining the relative position. This solution does not directly affect the stability of the normal equation of the estimated parameters. Acquiring a relative position that is nearly as accurate as that from the differential phase delay observables is possible if the observations are reasonably scheduled. The relative position, with an error of 10 meters, can be determined using the differential phase delay observables [4]. Using the method proposed here, the relative position of the rover, with an error of several tens of meters, may be determined through the biased differential phase delay. A data processing approach was developed, whereas the software required and further analyses are reserved for future work.

This work was supported by the Hundred Talent Project(s) of Chinese Academy of Sciences, the National Natural Science Foundation of China (Grant Nos. 11073048 and 11073047), the Pujiang Project of Shanghai (Grant No. 10PJ1411700) and Shanghai Key Laboratory of Space Navigation and Position Techniques (Grant No. Y054262001).

1 Ouyang Z. Scientific objectives of Chinese lunar exploration and development strategy. Adv Earth Sci, 2004, 19(3): 355–357
2 http://scitech.people.com.cn/GB/6394791.html
3 http://space.cpst.net.cn/china/2009_05/243064422.html
4 Liu Q, Chen M, Xiong W, et al. Relative position determination of a Lunar Rover by using high-accuracy multi-frequency same-beam VLBI. Sci China-Phys Mech Astron, 2010, 53(3): 571–578
5 Li J L, Guo L, Qian Z H, et al. The application of the instantaneous states reduction to the orbital monitoring of pivotal arcs of the Chang'E-1 satellite. Sci China Ser G-Phys Mech Astron, 2009, 52: 1833–1841
6 Iwata T, Takahashi M, Namiki N, et al. Mission instruments for lunar gravity field measurements using SELENE sub-satellites. J Geod Soc Jpn, 2001, 47: 558–563
7 Kono Y, Hanada H, Ping J, et al. Precise positioning of spacecraft by multi-frequency VLBI. Earth Planets Space, 2003, 55: 581–5589
8 Goosens S, Matsumoto K, Liu Q, et al. Lunar gravity field determination using SELENE same-beam differential VLBI tracking data. J Geodesy, 2011, 85: 205–208
9 Kikuchi F, Liu Q, Hanada H, et al. Pico-second accuracy VLBI of the two sub-satellites of SELENE (KAGUYA) using multi-frequency and same beam methods. Radio Sci, 2009, 44: 1–7
10 Liu Q, Kikuchi F, Matsumoto K, et al. Same-beam VLBI observations of SELENE for improving lunar gravity field model. Radio Sci, 2010, 45: RS2004, doi: 10.1029/2009RS004203
11 Chen M, Liu Q. Calculation and analysis of same beam VLBI differential phase delay closure of lunar satellite (in Chinese). Progr Astron, 2010, 28: 415–423
12 Liu Q, Matsumoto K, Kikuchi F, et al. Same-beam differential VLBI technology using two satellites of the SELENE spacecraft. IEICE-JB, 2006, J89B-B: 602–617
13 Zhang X, Wei W, Xiang Y, et al. Progress of wideband VLBI digital system development in SHAO. In: Proc 5th IVS General Meeting, China, 2008. 381–385
14 Chen M, Tang G, Cao J, et, al. The precision orbit determination of CE-1 lunar satellite (in Chinese). Geomat Inform Sci Wuhan Univ, 2011, 36(2): 212–217
15 Araki H, Tazawa S, Noda H, et al. Lunar global shape and polar topography derived from Kaguya-LALT laser altimeter. Science, 2009, 323: 897–900
16 Ping J S, Huang Q, Yan J G, et al. Lunar topographic model CLTM-s01 from Chang'E-1 laser altimeter. Sci China-Phys Mech Astron, 2009, 52(7): 1105–1114
17 Li C L, Ren X, Liu J J, et al. Laser altimetry data of Chang'E-1 and the global lunar DEM model. Sci China-Earth Sci, 2010, 53(11): 1582–1593