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Design and optimization of a trajectory for Moon departure Near Earth Asteroid exploration

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The lunar probe often has some remaining fuel on completing the predefined Moon exploration mission and may carry out some additional tasks from the Moon orbit using the fuel. The possibility for the lunar probe to escape from the Moon and the Earth is analyzed. Design and optimization of the trajectory from the Moon orbit to the Near Earth Asteroids (NEAs) using the spacecraft's residual fuel is studied. At first, the semi-major axis, inclinations and the phase relations with the Earth of all the numbered NEAs are investigated to preliminarily select the possible targets. Based on the Sun-centered two-body problem, the launch window and the asteroid candidates are determined by calculating the minimum delta-v for two-impulse rendezvous mission and one-impulse flyby mission, respectively. For a precise designed trajectory, a full ephemeris dynamical model, which includes gravities of the Sun, the planets and the Moon, is adopted by reading the JPL ephemeris. The departure time, arrival time, burning time duration and thrust angles are set as variables to be designed and optimized. The optimization problem is solved via the Particle Swarm Optimization (PSO) algorithm. Moreover, two feasible NEA flyby missions are presented.

trajectory design, full ephemeris model, Near Earth Asteroid exploration, Particle Swarm Optimization

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1 Introduction

Moon exploration has been attracting great attention in China since the Chang'E lunar exploration programs were implemented. The second unmanned lunar probe Chang'E-2, launched at the Xichang satellite launch site in Sichuan Province on October 1, 2010, is now orbiting the Moon. The success of Chang'E-2 in accomplishing its mission marks another great achievement after the country successfully launched its first lunar probe in 2007 [1].

At the end of the predefined lunar exploration mission, there is probably some fuel left for the lunar probe. By using the residual propellant, the lunar probe can escape the gravitational pull of the Moon or the Earth-Moon system and carry out some additional scientific or technological tasks, making good use of the lunar probe and getting more scientific and engineering returns.

In recent years, Near Earth Asteroids (NEAs) are of great interest to many scientists, because asteroids hold the key information about the formation and evolution of our solar system and the planets. Much research has been conducted on the design and optimization of the trajectory to near earth asteroids [2–4]. A number of unmanned missions have been deployed to asteroids, and more such missions are being planned. In March 2010, the Japanese asteroid explorer Hayabusa, which was launched on May 9, 2003, has returned to Earth and released a small capsule with asteroid Itokawa into the reentry orbit [5]. The European Space Agency has launched the spacecraft Rosetta, which conducted a flyby of the asteroid Lutetia in July 2010 successfully [6].

The most important constraint in planning deep space exploration missions is budget. Scientists and engineers

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hope that one mission can generate more scientific returns. Can the lunar probe with some remaining fuel turn to visit an asteroid after finishing the lunar exploration mission?

The spacecraft is in the Moon-centered circular working orbit when completing the scientific and engineering exploration mission of the Moon. Accelerated by the onboard rocket engine in an appropriate escaping opportunity, the spacecraft can escape the gravitational pull of the Moon and the Earth to enter the heliocentric orbit, on which it can fly to an asteroid.

This paper analyzes the possibility of departing from the Earth-Moon system by the lunar probe and the selection of suitable target asteroids. Trajectory design and optimization using the full ephemeris dynamical model are investigated. The design of a trajectory is divided into two steps: the first step is preliminary design based on the impulsive and two-body approximation, and then the initial value is substituted into the fidelity dynamical model to get the precise results. At the end of the paper, two feasible missions of Moon departure near earth asteroid flyby are presented.

This work is closely combined with the China lunar exploration program. With respect to past studies, this work addresses the approach to trajectory design for deep space exploration missions in a high fidelity dynamical model, which considers not only the gravitations of the Sun, the planets and the Moon but also the finite-thrust powered phases of finite duration, and intends to demonstrate possible alternatives for additional missions by the lunar probe.

2 Escaping from the Moon and the Earth

In order to escape the gravitational pull of the Moon, the spacecraft must travel along a hyperbolic trajectory relative to the Moon, arriving at its sphere of influence with a relative velocity v_∞ (hyperbolic excess velocity), greater than zero (as shown in Figure 1) [7]. On a parabolic trajectory, the probe will arrive at the sphere of influence with a relative speed of zero. In that case the lunar probe remains in the same orbit as the Moon.

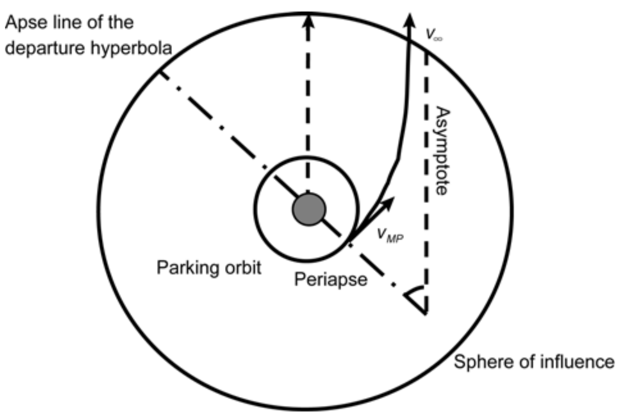

Figure 1 Departure hyperbola of a spacecraft.

The lunar probe is ordinarily in the Moon-centered circular orbit. Let $\boldsymbol{r}_M$ be the position vector relative to the Moon, and the magnitude of the circular orbit velocity can be written as

$$v_{MC} = \sqrt{\frac{\mu_M}{r_M}}, \tag{1}$$

where $r_M=|\boldsymbol{r}_M|$, the magnitude of $\boldsymbol{r}_M$, and μ_M is the gravitational constant of the Moon.

The radius of this round orbit equals the periapse radius r_{MP} of the departure hyperbola with respect to the Moon (Figure 1). Then the magnitude of the periapse speed v_{MP} is computed using the following equation

$$v_{MP} = \sqrt{v_\infty^2 + \frac{2\cdot\mu_M}{r_{MP}}} = \sqrt{v_\infty^2 + \frac{2\cdot\mu_M}{r_M}}, \tag{2}$$

which can be found from the two-body angular momentum equation. With eqs. (1) and (2), one can calculate the Δv required to put the lunar probe onto the Moon-centered hyperbolic departure trajectory,

$$\Delta v_P = v_{MP} - v_{MC} = \sqrt{v_\infty^2 + \frac{2\cdot\mu_M}{r_M}} - \sqrt{\frac{\mu_M}{r_M}}, \tag{3}$$

let $v_\infty=0$, and the minimum velocity increment to escape from the Moon is obtained

$$\Delta v_{\min} = \sqrt{\frac{2\cdot\mu_M}{r_M}} - \sqrt{\frac{\mu_M}{r_M}}. \tag{4}$$

And Δv, the magnitude of the velocity increment, is related to Δm, the mass of propellant consumed, by the formula

$$\frac{\Delta m}{m} = 1 - \exp\left(-\frac{\Delta v}{I_{SP} g_0}\right), \tag{5}$$

where m is the mass of the spacecraft before the engine burn, I_{SP} is the specific impulse of onboard engine, and g_0 is the sea-level standard acceleration of gravity. Then the maximum Δv provided by the residual fuel can be computed as

$$\Delta v_R = I_{SP} g_0 \ln\left(\frac{m}{m - \Delta m_R}\right), \tag{6}$$

where Δm_R is the total mass of the residual propellant.

From eqs. (4) and (6), the necessary conditions of departing from the Moon can be derived as follows:

$$\Delta v_R \geqslant \Delta v_{\min}. \tag{7}$$

That is to say

$$\Delta m_R \geqslant \Delta m_{M\min} = m\cdot\left(1 - \exp\left(-\frac{\left(\sqrt{2}-1\right)\sqrt{\mu_M / r_M}}{I_{SP} g_0}\right)\right), \tag{8}$$

where $\Delta m_{M\min}$ is the minimum mass of propellant required to leave the Moon.

In the geocentric frame of reference, $\boldsymbol{r}_E$ and $\boldsymbol{v}_E$, the position and velocity vectors of the lunar probe relative to the Earth, can be written respectively as

$$\begin{aligned}\boldsymbol{r}_E &= \boldsymbol{r}_M + \boldsymbol{r}_{ME},\\ \boldsymbol{v}_E &= \boldsymbol{v}_M + \boldsymbol{v}_{ME},\end{aligned} \tag{9}$$

where $\boldsymbol{r}_{ME}$ denotes the position vector pointing from the Earth to the Moon and $\boldsymbol{v}_{ME}$ is the velocity vector of the Moon relative to the Earth. With a similar analysis to the case of Moon departure, $\Delta m_{M\min}$, the minimum amount of fuel for the spacecraft departing from the Earth, is calculated by the following equation

$$\Delta m_{E\min} = m\cdot\left(1-\exp\left(-\frac{\sqrt{2\cdot\mu_E / r_E} - v_E}{I_{SP} g_0}\right)\right), \tag{10}$$

where r_E and v_E are the magnitudes of $\boldsymbol{r}_E$ and $\boldsymbol{v}_E$.

Evidently it is easy for a lunar probe departing from the Moon to escape the gravitational pull of the Earth, since the speed of the Moon is added to the velocity of the spacecraft in the geocentric inertial frame of reference attached to the Earth.

3 Coordinate system and transformation

3.1 Reference frame definitions

A variety of different reference frames are used in the investigation of trajectory design for asteroid exploration from the Moon. Their definitions are detailed here for clarity, as shown in Figure 2.

(1) J2000 geocentric inertial frame Earth-*XYZ*

The origin of this coordinate system is the center of the Earth, and the unit vector *X* is directed toward the J2000 vernal equinox, the *X*-*Y* plane is the Earth's mean equatorial plane, and the right-handed axis set is completed by the *Z* axis. The position and velocity vectors of the Moon and the Sun are given in this frame of reference by reading the JPL planetary ephemeris.

(2) J2000 Moon-centered inertial frame Moon-*X′Y′Z′*

The origin is the center of the Moon, and the *X′*, *Y′* and *Z′* axis are parallel to the *X*, *Y* and *Z* axis of the J2000 geocentric inertial frame. The motion of the lunar probe working in the Moon-centered circular orbit is described in this frame.

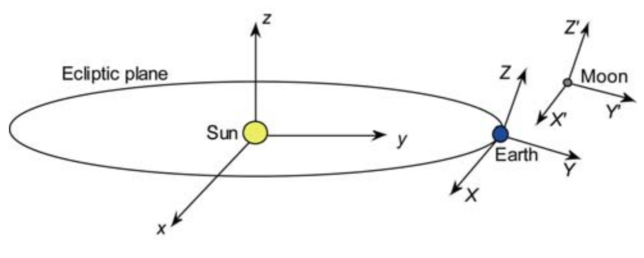

Figure 2 The definitions of different coordinate systems.

(3) J2000 heliocentric ecliptic inertial frame Sun-*xyz*

Considering the heliocentric interplanetary transfer of the spacecraft to asteroid, the J2000 heliocentric ecliptic inertial frame is used. The origin is the Sun, the *x* axis points in the J2000 vernal equinox direction, the *x*-*y* plane is the J2000 mean ecliptic plane, and the *z* axis coincides with the norm to the *x*-*y* plane.

3.2 Coordinate transformation

The transformation from the equatorial frame into the ecliptic frame can be accomplished by one rotation, which is around the *X*(*x*) axis through the J2000 mean obliquity of the ecliptic. If the components of the state vector in the equatorial frame are $\boldsymbol{x}$, the components of the same vector in the ecliptic frame are found by carrying out the matrix multiplications

$$\boldsymbol{x}' = M_x(\varepsilon)\cdot\boldsymbol{x} = \begin{bmatrix}1 & 0 & 0\\ 0 & \cos\varepsilon & \sin\varepsilon\\ 0 & -\sin\varepsilon & \cos\varepsilon\end{bmatrix}\cdot\boldsymbol{x}, \tag{11}$$

where M_x is the transformation matrix around the *X*(*x*) axis. ε is the J2000 mean obliquity of the ecliptic.

4 Selection of mission targets

Up to now, more than 7000 near earth asteroids have been found. However, only the near earth asteroid with the sequential number should be treated as possible targets of exploration mission for the reason that a near earth asteroid receives the sequential number after its orbit is accurately determined. The orbits of unnumbered asteroids are not (yet) precisely known. By the end of December 2010, there are 1054 numbered near earth asteroids, which are the concern in this work.

The primary consideration in selecting the object for exploration is accessibility, or the minimum total amount of propellant required for a rendezvous or flyby mission. In addition, some practical engineering factors should be taken into account, such as the scientific significance of the asteroids, the tracking and control conditions and the flight time from the Moon to the asteroid. And the diameters of the target asteroids should not be too small so that they are more likely to be found by the spacecraft in deep space.

Given that the distance between the Moon and the Earth is much smaller than the distance between the two and the Sun, only the heliocentric Earth (Earth-Moon system, more precisely) to asteroid transfer orbit should be studied in the stage of targets selection. And the spacecraft is subjected only to the central force of the Sun's gravity.

At first, orbital parameters, including the semi-major axis, inclinations and the phase relations with the Earth-Moon system, are examined for all the numbered asteroids known, which is called preliminary selection in this paper. And then each asteroid's minimum Δv for exploration missions (rendezvous or flyby) is calculated to determine the potential candidate asteroids through the Particle Swarm Optimization (PSO) algorithm.

4.1 Preliminary selection

To narrow the searching scope from the total population of numbered asteroids, one should investigate the semi-major axis, inclinations and the phases of the 1054 numbered asteroids.

(1) Semi-major axis and inclination

Generally speaking, the semi-major axis and inclination of the target asteroids should be less than 2 AU and 25 deg, respectively. Otherwise the transfer energy needed will be too large for the spacecraft to reach the asteroid.

(2) Phase

When a spacecraft flies from the Earth-Moon system to an asteroid, the certain phase relation should be satisfied to avoid large fuel consumption.

The phases of the Earth-Moon system and the asteroid in the ecliptic plane with respect to equinox can be given approximately as

$$\begin{aligned} \theta_E &\approx \Omega_E + \omega_E + M_E, \\ \theta_{AS} &\approx \Omega_{AS} + \omega_{AS} + M_{AS}, \end{aligned} \tag{12}$$

where Ω_E, ω_E, M_E and Ω_{AS}, ω_{AS}, M_{AS} denote the longitude of ascending node, argument of perigee, mean anomaly of the Earth-Moon system and the asteroid, respectively. The angular velocity of the spacecraft which flies from the Earth-Moon system to the asteroid can also be approximated as

$$n \approx \left(\sqrt{\mu_S / a_E^3} + \sqrt{\mu_S / a_{AS}^3}\right)/2, \tag{13}$$

where the semi-major axis of the Earth-Moon system and the asteroid are denoted by a_E and a_S, respectively. μ_S is the gravitational constant of the Sun while the angular velocity of the asteroid holds the form

$$n_{AS} = \sqrt{\mu_S / a_{AS}^3}. \tag{14}$$

Let t denote the transfer time of cruise ellipse relative to the Sun, and we have

$$\theta_E + n \cdot t = \theta_{AS} + n_{AS} \cdot t. \tag{15}$$

If the asteroid rendezvous or flyby mission has to be completed in one orbit period, the phase relation between the Earth-Moon system and the asteroid can be obtained as follows:

$$\begin{cases} \theta_E < \theta_{AS}, \text{ if } a_E < a_{AS}, \\ \theta_E > \theta_{AS}, \text{ if } a_E > a_{AS}. \end{cases} \tag{16}$$

4.2 Candidate determination with the PSO algorithm

In most cases, there are only hundreds of asteroids which not only satisfy the certain phase relations but also have relatively small semi-major axis and inclinations. To determine whether an asteroid is one of the candidates or not, we use the Particle Swarm (PSO) Algorithm to minimize the velocity increment for the transfer trajectory from the Earth-Moon system to an asteroid.

The PSO first presented by Eberhart and Kennedy [8] in the mid 1990s is an evolutionary search algorithm. This algorithm does not require gradient information of the objective function to be optimized. PSO has been successfully applied in trajectory optimization problems [9,10].

PSO searches for an optimal solution to an optimization problem by mimicking the unpredictable motion of bird flocks looking for food, which take advantage of the mechanism of information sharing that affects the overall behavior of a swarm. Each particle represents a possible solution to the problem. The position of each particle is updated toward its own known optimal position and the globally optimal position of entire particles. At the end of the optimization process, the best particle is selected. For the iteration process of PSO algorithm, please refer to [9].

Let t_D and t_A, the launch time from the Earth-Moon system and the arrival time to the asteroid respectively, be the optimization variables. We can compute the position and velocity vectors of the Earth-Moon system and the asteroid. And then the departure and arrival velocity vectors can be obtained by solving the Lambert problem. For the rendezvous mission, the performance index is expressed as

$$J = \|\boldsymbol{v}_1 - \boldsymbol{v}_E\| + \|\boldsymbol{v}_2 - \boldsymbol{v}_{AS}\|, \tag{17}$$

where $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$ are the velocity vectors of a spacecraft at departure and arrival, $\boldsymbol{v}_E$ and $\boldsymbol{v}_{AS}$ respectively represent the velocity vectors of the Earth and the asteroid at t_D and t_A. And for the flyby mission, J becomes

$$J = \|\boldsymbol{v}_1 - \boldsymbol{v}_E\|. \tag{18}$$

That is to say, only the departure velocity increment is to be discussed for the flyby mission. However, the relative speed between the spacecraft and the asteroid shouldn't be very large when the spacecraft arrives at the asteroid, otherwise, it will be too difficult for the camera onboard to track the target asteroid.

4.3 Searching results

Having examined the orbital parameters of all the known numbered near earth asteroids, we have found more than half of these asteroids are excluded, for their semi-major axis are larger than 2 AU or inclinations are greater than 25 deg. Figure 3 portrays the distribution of semi-major axis and inclinations of these asteroids. The horizontal lines are the upper limits of these two orbital elements.

The flyby mission is more achievable than rendezvous mission in engineering, since there are not a lot of remaining propellants for the lunar probe at the end of the predefined lunar mission. Determined by the PSO global optimization algorithm, the candidate numbered asteroids, of which the minimum Δv for the flyby mission is less than 1.5 km/s, are listed in Table 1 as well as their physical properties. The departure date from the Moon is chosen from April 1, 2011 to May 1, 2011 and the flight time of the mission is constrained within 300 days.

In Table 1, D is the diameter of the asteroid. J is the velocity increment defined by equation (18). dvf denotes the arrival velocity magnitude of the spacecraft relative to the asteroid. d represents the distance between the spacecraft and the asteroid at flyby time. a, e and i denote the semi-major axis, eccentricity and inclination of the near earth asteroid.

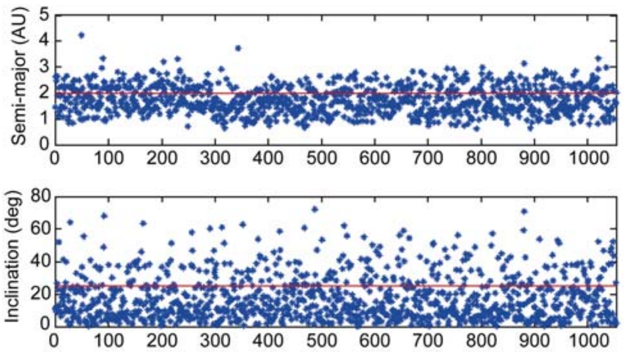

Figure 3 Distribution of the semi-major axis and the inclination for all the numbered near earth asteroids.

5 Precise trajectory design

5.1 The full ephemeris dynamical model

The asteroid candidates and the general launch opportunities have been found within the framework of a two-body problem, which, however, is not consistent with the fidelity full ephemeris dynamical model. Therefore the design and optimization of trajectory using full ephemeris dynamical model should be introduced to get more practical results. The full ephemeris dynamical model is applied with DE405 files for the position and velocity of each celestial body.

At the starting of the asteroid flyby mission, the orbital elements of the lunar probe are given in the Moon-centered inertia frame of reference. It is easy to compute the components of its position and velocity vectors relative to the Moon in the same frame. So the position and velocity of the lunar probe in the heliocentric ecliptic coordinate system, $\boldsymbol{r}_0$ and $\boldsymbol{v}_0$, are derived respectively as

$$\begin{aligned} \boldsymbol{r}_0 &= M_x(\varepsilon)\cdot(\boldsymbol{r}_M+\boldsymbol{r}_{MS}), \\ \boldsymbol{v}_0 &= M_x(\varepsilon)\cdot(\boldsymbol{v}_M+\boldsymbol{v}_{MS}), \end{aligned} \tag{19}$$

where $\boldsymbol{r}_M$, and $\boldsymbol{v}_M$ represent the position and velocity vectors of the lunar probe relative to the Moon. $\boldsymbol{r}_{MS}$ and $\boldsymbol{v}_{MS}$ are the position and velocity vectors of the Moon relative to the Sun in the J2000 equatorial frame system, which can be obtained by reading the JPL ephemeris. M_x is the rotation matrix round the x axis.

Table 1 The asteroid candidates for the flyby mission departing from the Moon from April 1, 2011 to May 1, 2011 a)

Name	J (km/s)	D (m)	t_D (MJD)	t_A (MJD)	dvf (km/s)	d (10^4 km)	a (AU)	e	i (deg)
190491	0.686642	170–380	55686.0	55846.8	6.353131	2220	1.3174	0.2336	5.2859
154590	0.765428	130–300	55656.0	55921.1	11.129394	5501	1.1054	0.4023	1.4129
162421	0.904988	700	55686.0	55974.6	11.454223	712	0.9469	0.1235	22.3223
162422	0.948826	400	55968.5	55733.1	15.163958	5820	1.2077	0.5311	1.3931
7753	1.007546	1000	55656.0	55941.7	15.564588	9863	1.4673	0.4817	3.1243
194268	1.024897	700–1500	55686.0	55948.1	29.521341	6315	1.4532	0.7871	5.4292
250680	1.068862	340–770	55686.0	55863.8	12.431758	3431	0.8933	0.3646	9.4493
2062	1.173890	900	55685.9	55976.4	9.148468	11160	0.9665	0.1826	18.9330
175706	1.200124	1600	55656.0	55885.3	10.749490	1518	1.0544	0.3499	1.9901
65679	1.221099	710	55656.0	55885.8	10.302283	7822	0.9152	0.2648	1.2919
162142	1.257182	600	55656.0	55913.7	10.262373	11127	0.8757	0.3180	21.8034
85990	1.300647	400	55656.0	55909.9	9.013932	4684	1.0077	0.3112	5.3134
164294	1.321818	250–560	55656.0	55850.1	10.341891	6612	0.6176	0.4543	2.9536
7341	1.338554	1400	55683.6	55961.4	8.562595	1201	1.8418	0.5067	5.4231
209215	1.341620	40–90	55686.0	55982.3	4.891349	4584	0.9894	0.1210	2.5620
101955	1.423037	200	55656.0	55820.1	7.191871	2671	1.1263	0.2037	6.0350

a) Parameters are referenced from NEO Information Services of Pisa University and the JPL Near Earth Object Program.

The solar system accurate ephemeris model, including the gravitation of the Sun, the Planets and the Moon as well as the thrust of an onboard rocket engine, is used to propagate the orbit for the spacecraft to depart from the Moon orbit and fly to the asteroid. Preliminary trajectory analyses for space missions are often based on an impulsive approximation in which the finite duration of powered phases and the gravitational losses are neglected. But the finite-thrust trajectories must be considered in the precise practical work. In the J2000 heliocentric ecliptic reference frame, the dynamic equation could be written as follows:

$$\begin{aligned}\ddot{\boldsymbol{r}} &= -\frac{\mu_S}{r^3}\boldsymbol{r} + \boldsymbol{f}_M + \boldsymbol{f}_E + \boldsymbol{f}_P + \chi\frac{T}{m}\boldsymbol{n},\\ \dot{m} &= -\frac{T}{I_{SP}g_0},\end{aligned} \tag{20}$$

where $\boldsymbol{r}$ and $\boldsymbol{v}$ are the position and velocity vectors of the spacecraft, and $r=\|\boldsymbol{r}\|$, the magnitude of $\boldsymbol{r}$. The instantaneous mass of probe is represented by m. T is the thrust of an onboard engine and $\boldsymbol{n}$ is the unit vector of thrust direction. χ is the switch flag (χ=1 if the engine is on, while χ=0 if the engine is off). $\boldsymbol{f}_M$, $\boldsymbol{f}_E$ and $\boldsymbol{f}_P$ are the gravity of the Moon, the Earth and other Planets of the solar system, giving the following respective expressions

$$\boldsymbol{f}_M = -\mu_M\left(\frac{1}{d_{XM}^3}\boldsymbol{d}_{XM} + \frac{1}{x_{MS}^3}\boldsymbol{x}_{MS}\right), \tag{21}$$

$$\boldsymbol{f}_E = -\mu_E\left(\frac{1}{d_{XE}^3}\boldsymbol{d}_{XE} + \frac{1}{x_{ES}^3}\boldsymbol{x}_{ES}\right), \tag{22}$$

$$\boldsymbol{f}_P = \sum\left(-\mu_P\left(\frac{1}{d_{XP}^3}\boldsymbol{d}_{XP} + \frac{1}{x_{PS}^3}\boldsymbol{x}_{PS}\right)\right), \tag{23}$$

$\boldsymbol{d}_{XM}$, $\boldsymbol{d}_{XE}$ and $\boldsymbol{d}_{XP}$ denote the position vectors of the probe relative to the Moon, the Earth and the Planets. $\boldsymbol{x}_{MS}$, $\boldsymbol{x}_{ES}$ and $\boldsymbol{x}_{PS}$ are the position vectors pointing from the Sun to the Moon, the Earth and the Planets. μ_P (P=1,2,…) is the gravitational constant of each planet, respectively.

5.2 Precise trajectory design and optimization

There are 5 variables to be designed and optimized, which are determined as follows:

(1) t_0 and t_f

t_0 and t_f represent the Moon departure launch time and the asteroid flyby time, which are constrained near t_D and t_A as mentioned in sect. 4.2.

(2) dt

dt is the thrusting time duration when the spacecraft departs from the Moon. The search range of dt is defined as (0, $dt_{\max}$), where $dt_{\max}$ is the maximum thrusting time, which can be found by integrating the second expression of eq. (20)

$$dt_{\max} = \frac{\Delta m_R I_{SP} g_0}{T}. \tag{24}$$

(3) α and δ

α and δ denote the longitude angle and latitude angle of the thrust vector in the ecliptic reference frame, as shown in Figure 4. These two angle variables are constrained to the domain [0, 2π] and [−π/2, π/2], respectively. Then the unit vector of thrust direction $\boldsymbol{n}$ can be related to α and δ by the equation

$$\boldsymbol{n} = \begin{bmatrix}\cos\alpha\cos\delta\\ \sin\alpha\cos\delta\\ \sin\delta\end{bmatrix}. \tag{25}$$

And considering the practical engineering, the propulsive thrust is in a constant direction from the burn-begin time to burn-end time. For the sake of accomplishing a flyby mission with an asteroid and making the minimum fuel consumption, the objective function for the problem of interest is

$$J = -m(t_f) + k\cdot\left\|\boldsymbol{r}(t_f) - \boldsymbol{r}_{AS}(t_f)\right\|^2, \tag{26}$$

which is composed of two parts: the first part is the final mass of spacecraft, and the second part is the position error between the spacecraft and the asteroid at flyby time as a quadratic penalty with the penalty factor k to be experimented on.

6 Feasible flyby missions

Two possible flyby missions recommended for the lunar probe departing from the Moon are demonstrated in this section. The near earth asteroids 190491 and 162421 are selected as targets of the flyby mission.

6.1 Parameters

In the J2000 Moon-centered inertial coordinate system, the

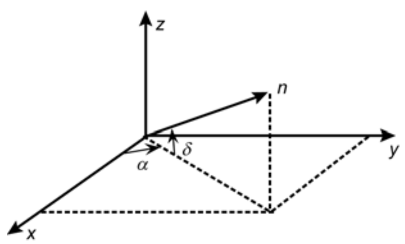

Figure 4 Angle variable definition of the thrust direction vector in the ecliptic reference frame.

orbit parameters of the lunar probe are listed in Table 2. The orbit epoch is 55656.0 MJD when the Moon exploration mission is completed by the spacecraft. Other useful parameters are portrayed in Table 3.

According to the parameters listed in Table 2, one can evaluate the possibility for the spacecraft to escape from the Moon. $\Delta m_{M\min}$, the minimum amount of fuel required for Moon departure, can be calculated as 367.37 kg. And the residual propellant is 600 kg, which is greater than $\Delta m_{M\min}$. So the residual fuel is enough for the spacecraft to escape from the Moon.

6.2 NEA 190491 flyby mission

190491 is an Apollo-class near earth asteroid which has an orbit cross that of the Earth. As seen from Table 1, the required delta-v for the asteroid 190491 flyby is minimal among the candidate asteroids in Table 1 and the flyby velocity of the spacecraft about the asteroid is relatively small. Some of 190491's orbital elements and physical information have been listed in Table 1.

Table 4 reports the results of design and optimization process. Figure 5(a) shows the heliocentric trajectory of the spacecraft flying to 190491 and Figure 5(b) portrays the departure trajectory relative to the Moon.

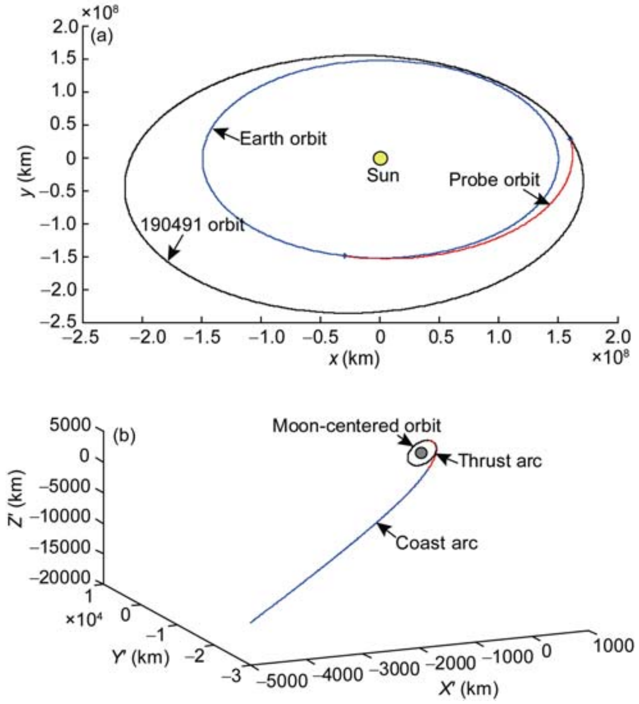

Figure 5 (a) 190491 flyby mission: heliocentric trajectory of the spacecraft; (b) 190491 flyby mission: the departure orbit of the spacecraft relative to the Moon.

6.3 NEA 162421 flyby mission

160421 is an Aten-class near earth asteroid. From Table 1, the distance between 160421 and the Earth at flyby time is the smallest, which will result in better tracking and control condition, while the Δv needed for the flyby mission with 162421 is relatively small. Some of the orbital elements and physical information of 162421 have been listed in Table 1.

Table 5 reports optimal variables of this optimization problem. The heliocentric trajectory of the spacecraft and the departure trajectory relative to the Moon are shown respectively in Figures 6(a) and 6(b).

Table 2 Orbital elements of the spacecraft relative to the Moon-centered frame of reference

A (km)	e	I (deg)	Ω (deg)	ω (deg)	M (deg)
1838	0.003	86	259	323	152

Table 3 Parameters for the Moon departure asteroid flyby mission

Parameter	Value	Unit
Initial mass	1800.0	kg
Dry mass	1200.0	kg
Thrust	500.0	N
$I_{SP}\times g_0$	3000.0	Ns/kg
$dt_{\max}$	3600.0	s

Table 4 Optimal solutions of the flyby mission for 190491

Variables	Value	Unit
Initial time t_0	55722.2040607922	MJD
Thrusting time dt	2798.07075412330	s
Final time t_f	55844.4200235855	MJD
Thrust pointing angle α	4.97430791414856	rad
Thrust pointing angle δ	−1.51550156705365	rad
iteration number	1254	–
Position error $\|\|\boldsymbol{r}(t_f)-\boldsymbol{r}_{AS}(t_f)\|\|$	0.743803409669524	km
Final mass	1333.65487431279	kg

7 Conclusions

Against the background of China exploration program, the design and optimization of a trajectory for the Moon departure near earth asteroid flyby mission in full ephemeris

Table 5 Optimal solutions of the flyby mission for 162421

Variables	Value	Unit
Initial time t_0	55730.3413859409	MJD
Thrusting time dt	3518.48831701062	s
Final time t_f	55977.5965457486	MJD
Thrust pointing angle α	5.27864216408272	rad
Thrust pointing angle δ	1.30401885208435	rad
iteration number	4508	–
Position error $\|\|\boldsymbol{r}(t_f)-\boldsymbol{r}_{AS}(t_f)\|\|$	0.04290554447899	km
Final mass	1213.58528049823	kg

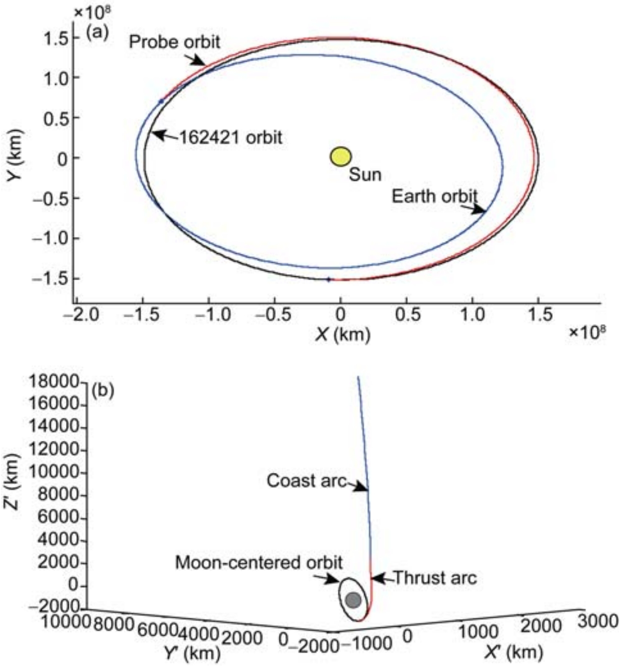

Figure 6 (a) 162421 flyby mission: heliocentric trajectory of the spacecraft; (b) 162421 flyby mission: the departure orbit of the spacecraft relative to the Moon.

dynamical model is investigated and two feasible schemes are recommended in this paper. This work presents a significant reference for the engineers designing the trajectory programming problem of Moon departure mission for lunar probes.

It is more complex to design the deep space exploration trajectory under the accurate full ephemeris dynamical model since the patched-conic technique failed. The numerical optimization algorithm can be employed to solve the problem of precise trajectory design.

Some problems should be solved before designing the trajectory of Moon departure asteroid exploration missions. Firstly, there must be an autonomous navigation device on the spacecraft flying to an asteroid. Also, midcourse correction maneuvers should be planned by taking the deviations of orbit determination and control into account.

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1 OUYANG Z Y. Science results of Chang'e-1 Lunar orbiter and mission goals of Chang'e-2. Spacecr Eng, 2010, 19(5): 1–6
2 Lau C O, Hulkower N D. Accessibility of Near Earth Asteroids. J Guid, 1987, 10(3): 225–232
3 Qiao D, Cui H T, Cui P Y. The design of transfer trajectory for Ivar asteroid exploration mission. Acta Astron, 2009, 65: 1553–1560
4 Perozzi E, Rossi A, Valsecchi G B. Basic targeting strategies for rendezvous and flyby missions to the Near Earth Asteroids. Planet Space Sci, 2001, 49(1): 3–22
5 Kawaguchi J, Yamada T, Kuninaka H. Hayabusa's reentry and recovery of its capsule. In: Proceedings of 61st International Astronautical Congress. Prague: International Astronautical Federation, 2010
6 Accomazzo A, Ferri P, Hubault A, et al. Rosetta visits Asteroid (21) Lutetia. In: Proceedings of 61st International Astronautical Congress. Prague: International Astronautical Federation, 2010
7 Curits H D. Orbital Mechanics for Engineering Student. Amsterdam Elsevier Butterworth Heinemann, 2005
8 Kennedy J, Eberhart R. Particle swarm optimization. In: Proceedings of IEEE International Conference on Neural Network. Perth, 1995. 1942–1948
9 Pontani M, Conway B A. Particle swarm optimization applied to space trajectories. J Guid Control Dyn, 2010, 33(5): 1429–1441
10 Venter G, Sobieszczanski-Sobieski J. Particle swarm optimization. AIAA J, 2003, 41(8): 1583–1589