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Perfect teleportation of an arbitrary three-qubit state with the highly entangled six-qubit genuine state

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The perfect teleportation of an arbitrary three-qubit state with the highly entangled six-qubit genuine state introduced by Borras et al. (*J. Phys. A: Math. Theor.* 40 (2007) 13407) is studied. Some appropriate measuring bases the sender can take and the corresponding unitary operations the receiver should execute in terms of the sender's measurement outcome are explicitly given. The flexibility between the measurement difficulty and the reconstruction difficulty is shown. Moreover, discussions and comparisons between our scheme and the recent incomplete scheme (Choudhury et al, *J. Phys. A: Math. Theor.* 42 (2009) 115303) are made.

quantum teleportation, highly entangled six-qubit genuine state, measuring bases, reconstruction operation, flexibility

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1 Introduction

Quantum teleportation (QT) was initially invented by Bennett et al. [1] in 1993. It provides a novel way for transferring an arbitrary quantum state between two sites which could be remotely separated. Specifically, in Bennett et al.'s scheme, with the aid of Bell state shared between the sender and the receiver, an arbitrary unknown quantum state can be teleported from the sender's site to the receiver's place without physically transmitting any particles between them. During the whole process, the sender only need perform a Bell-state measurement (BSM) and publish the measurement outcome. With the sender's classical message the receiver then executes an appropriate unitary operation to reconstruct the original state. Moreover, Bennett et al. [1] also generalized the scheme to treat an unknown qubit state using maximally entangled state in $d \times d$ dimensional Hilbert space. Their work shows in essence the interchangeability of different resources in quantum mechanics. After Bennett et al.'s pioneering work [1], QT has attracted much attention and many schemes in both theoretical and experimental aspects have been presented with various entangled states [1–34]. In 2007, a new quantum state (namely, the genuinely entangled six-qubit state), was introduced by Borras et al. [35]. This state demonstrates many novelties. It exhibits fancy genuine entanglement according to many measures and satisfies the monogamy inequality given by ref. [36]. Its reduced single-, two- and three-qubit density matrices are all completely mixed and no other six-qubit pure state is found to evolve to a mixed state with a higher amount of entanglement [37]. It has turned out to be an important resource in quantum information processing. In 2009, Choudhury et al. [38] investigated its usefulness in teleporting an arbitrary three-qubit state. Nonetheless, the investigation is not thorough, for the authors only gave a simple example which represents only a part of the total decomposition. Apparently, the QT scheme is practically incomplete and thus infeasible. Hence the demonstration is insufficient to show the usefulness and the resultant conclusion is unconvincing though it is conclusively right. In this paper we will further explore the perfect teleportation of an arbitrary three-qubit state by using the genuinely entangled six-qubit state introduced by Borras et al. [35]. We will explicitly give the different measuring bases which the sender can use and the proper unitary operation which the receiver should perform in terms of the sender's measurement result. Subsequently, we will reveal the operation difficulty flexibil-

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ity by using the relation between the measurement and the reconstruction. Besides, we will compare our scheme with a recent incomplete one [38].

The organization of the rest of this paper is as follows. In sec. 2, we will amply depict our QT scheme for teleporting an arbitrary three-qubit state by using the genuinely entangled six-qubit state as quantum channel between two parties. The emphases are placed on the sender's measuring bases and the receiver's unitary operation. In sec. 3 we make brief discussion and simply compare our scheme with the incomplete one. At last, a concise summary is given in sec. 4.

2 Perfect three-qubit-state QT with the genuine state

Now let us put forward the perfect teleportation of an arbitrary three-qubit state with the highly entangled six-qubit genuine state [35] as the quantum channel. Suppose Alice and Bob are the two involved parties in QT. They share in advance the genuine state which takes the form

$$
\begin{aligned}
&|\mathcal{G}\rangle_{a_1a_2a_3b_1b_2b_3}\\
&= \frac{1}{\sqrt{32}}[|000000\rangle + |111111\rangle + |000011\rangle\\
&+|111100\rangle + |000101\rangle + |111010\rangle + |000110\rangle\\
&+|111001\rangle + |001001\rangle + |110110\rangle + |001111\rangle\\
&+|110000\rangle + |010001\rangle + |101110\rangle + |010010\rangle\\
&+|101101\rangle + |011000\rangle + |100111\rangle + |011101\rangle\\
&+|100010\rangle - (|001010\rangle + |110101\rangle + |001100\rangle\\
&+|110011\rangle + |010100\rangle + |101011\rangle + |010111\rangle\\
&+|101000\rangle + |011011\rangle + |100100\rangle + |011110\rangle\\
&+|100001\rangle)]_{a_1a_2a_3b_1b_2b_3}, \qquad (1)
\end{aligned}
$$

where the qubit triplet (a_1, a_2, a_3) is at Alice's site while the qubit triplet (b_1, b_2, b_3) is in Bob's hand. Incidentally, one can easily work out that the quantum entanglement shared between the two parties is three ebits. Evidently the state can be used as the quantum channel for perfectly teleporting an arbitrary three-qubit state. Nonetheless, it is still intriguing to ask how to realize the QT in specific ways and to understand how different ways are inherently related. For instance, which measuring bases can Alice use? What is the corresponding reconstruction operation Bob should execute with respect to Alice's classical message on her measurement? Is there flexibility between the measurement difficulty and reconstruction difficulty? To address these questions conclusively, we continue to depict amply our QT scheme. Assume that Alice wants to send her quantum information to Bob without directly transmitting any qubit(s). The quantum information initially inhabits her qubit triplet (x_1, x_2, x_3) and can be expressed as

$$|\mathcal{U}\rangle_{x_1x_2x_3} = \sum_{i=0}^{1}\sum_{j=0}^{1}\sum_{k=0}^{1}\xi_{ijk}|ijk\rangle_{x_1x_2x_3}. \qquad (2)$$

In this case, the total joint state of the nine-qubit system consisting of the quantum information and the channel is

$$|\mathcal{T}\rangle_{x_1x_2x_3a_1a_2a_3b_1b_2b_3} \equiv |\mathcal{U}\rangle_{x_1x_2x_3}|\mathcal{G}\rangle_{a_1a_2a_3b_1b_2b_3}. \qquad (3)$$

Towards the goal of teleporation, first Alice should choose a set of proper measuring bases to measure her six qubits. The following sets of orthogonal and complete bases we defined are appropriate,

$$
\begin{aligned}
&\{|\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle_{a_1a_2a_3x_1x_2x_3}\\
&\equiv \Upsilon_{x_1x_2x_3}\sigma_{x_1}^{(i_1j_1)}\sigma_{x_2}^{(i_2j_2)}\sigma_{x_3}^{(i_3j_3)}|\phi^+\rangle_{a_1x_1}\\
&\times|\phi^+\rangle_{a_2x_2}|\phi^+\rangle_{a_3x_3},\ \ i's, j's = 0, 1\}, \qquad (4)
\end{aligned}
$$

where Υ is an arbitrary real unitary operator, $\sigma^{(00)} = |0\rangle\langle 0| + |1\rangle\langle 1|$, $\sigma^{(01)} = |0\rangle\langle 1| + |1\rangle\langle 0|$, $\sigma^{(10)} = |0\rangle\langle 0| - |1\rangle\langle 1|$, $\sigma^{(11)} = |0\rangle\langle 1| - |1\rangle\langle 0|$, and $|\phi^+\rangle = (|00\rangle + |11\rangle)/\sqrt{2}$. Alternatively, in terms of representation transformation the above measuring bases can be rewritten as

$$
\begin{aligned}
&\{|\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle_{a_1a_2a_3x_1x_2x_3}\\
&\equiv \sigma_{x_1}^{(i_1j_1)}\sigma_{x_2}^{(i_2j_2)}\sigma_{x_3}^{(i_3j_3)}\Upsilon'_{x_1x_2x_3}|\phi^+\rangle_{a_1x_1}\\
&\times|\phi^+\rangle_{a_2x_2}|\phi^+\rangle_{a_3x_3},\ \ i's, j's = 0, 1\}, \qquad (5)
\end{aligned}
$$

where Υ' is a resultant unitary operator form Υ. After the measurement, Alice informs Bob of the set of her measuring bases and her measurement result via a public classical channel. Additionally, Alice and Bob agree in priori that, the six classical bits (cbits) "$i_1j_1i_2j_2i_3j_3$" correspond to Alice's measurement outcome $|\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle$. Evidently, Alice's measurement leads to the following collapse

$$
\begin{aligned}
&|\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle_{a_1a_2a_3x_1x_2x_3\ a_1a_2a_3x_1x_2x_3}\\
&\times\langle\mathcal{D}_{i_1j_1i_2j_2i_3j_3}|\mathcal{T}\rangle_{x_1x_2x_3a_1a_2a_3b_1b_2b_3}\\
&= |\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle_{a_1a_2a_3x_1x_2x_3\ a_3x_3}\\
&\times\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+|\sigma_{x_3}^{(i_3j_3)\dagger}\sigma_{x_2}^{(i_2j_2)\dagger}\sigma_{x_1}^{(i_1j_1)\dagger}\Upsilon^\dagger_{x_1x_2x_3}\\
&\times|\mathcal{U}\rangle_{x_1x_2x_3}|\mathcal{G}\rangle_{a_1a_2a_3b_1b_2b_3} \qquad (6)\\
&= |\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle_{a_1a_2a_3x_1x_2x_3\ a_3x_3}\\
&\times\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+|\Upsilon'^\dagger_{x_1x_2x_3}\sigma_{x_3}^{(i_3j_3)\dagger}\sigma_{x_2}^{(i_2j_2)\dagger}\sigma_{x_1}^{(i_1j_1)\dagger}\\
&\times|\mathcal{U}\rangle_{x_1x_2x_3}|\mathcal{G}\rangle_{a_1a_2a_3b_1b_2b_3}. \qquad (7)
\end{aligned}
$$

Complimentarily but importantly, the genuinely entangled six-qubit state given as quantum channel can be rewritten as

$$|\mathcal{G}\rangle_{a_1a_2a_3b_1b_2b_3} = \Omega_{a_1a_2a_3}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{a_3b_3}, \qquad (8)$$

where the operator Ω under the ordering bases $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$ takes the form of

$$\Omega = \frac{1}{2}\begin{pmatrix} 1 & 0 & 0 & 1 & 0 & 1 & 1 & 0\\ 0 & 1 & -1 & 0 & -1 & 0 & 0 & 1\\ 0 & 1 & 1 & 0 & -1 & 0 & 0 & -1\\ 1 & 0 & 0 & -1 & 0 & 1 & -1 & 0\\ 0 & -1 & 1 & 0 & -1 & 0 & 0 & 1\\ -1 & 0 & 0 & -1 & 0 & 1 & 1 & 0\\ 1 & 0 & 0 & -1 & 0 & -1 & 1 & 0\\ 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{pmatrix}. \qquad (9)$$

Consequently, after Alice's measurement the state in Bob's qubit triplet (b_1, b_2, b_3) can be expressed as either of the following two forms

$$\begin{aligned}&{}_{a_3x_3}\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+|\sigma_{a_3}^{(i_3j_3)\dagger}\sigma_{a_2}^{(i_2j_2)\dagger}\sigma_{a_1}^{(i_1j_1)\dagger}\Upsilon_{a_1a_2a_3}^\dagger\\&\times|\mathcal{U}\rangle_{x_1x_2x_3}|\mathcal{G}\rangle_{a_1a_2a_3b_1b_2b_3}\\&={}_{a_3x_3}\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+|\mathcal{U}\rangle_{x_1x_2x_3}\sigma_{a_3}^{(i_3j_3)\dagger}\sigma_{a_2}^{(i_2j_2)\dagger}\sigma_{a_1}^{(i_1j_1)\dagger}\\&\times\Upsilon_{a_1a_2a_3}^\dagger\Omega_{a_1a_2a_3}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{a_3b_3},\end{aligned}\tag{10}$$

$$\begin{aligned}&{}_{a_3x_3}\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+|\Upsilon'^\dagger_{a_1a_2a_3}\sigma_{a_3}^{(i_3j_3)\dagger}\sigma_{a_2}^{(i_2j_2)\dagger}\sigma_{a_1}^{(i_1j_1)\dagger}\\&\times|\mathcal{U}\rangle_{x_1x_2x_3}|\mathcal{G}\rangle_{a_1a_2a_3b_1b_2b_3}\\&={}_{a_3x_3}\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+|\mathcal{U}\rangle_{x_1x_2x_3}\Upsilon'^\dagger_{a_1a_2a_3}\sigma_{a_3}^{(i_3j_3)\dagger}\\&\times\sigma_{a_2}^{(i_2j_2)\dagger}\sigma_{a_1}^{(i_1j_1)\dagger}\Omega_{a_1a_2a_3}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{a_3b_3}.\end{aligned}\tag{11}$$

One can easily prove a crucial property, i.e.,

$$\begin{aligned}&\Lambda_{a_1a_2a_3}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{a_3b_3}\\&=\Lambda^\dagger_{b_1b_2b_3}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{a_3b_3},\end{aligned}\tag{12}$$

where Λ is an arbitrary real unitary operator. Taking advantage of the crucial property, from eqs. (10) and (11) one can easily obtain

$$\begin{aligned}&{}_{a_3x_3}\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+|\mathcal{U}\rangle_{x_1x_2x_3}\sigma_{a_3}^{(i_3j_3)\dagger}\sigma_{a_2}^{(i_2j_2)\dagger}\sigma_{a_1}^{(i_1j_1)\dagger}\\&\times\Upsilon_{a_1a_2a_3}^\dagger\Omega_{a_1a_2a_3}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{a_3b_3}\\&=\Omega^\dagger_{b_1b_2b_3}\Upsilon_{b_1b_2b_3}\sigma_{b_1}^{(i_1j_1)}\sigma_{b_2}^{(i_2j_2)}\sigma_{b_3}^{(i_3j_3)}\ {}_{a_3x_3}\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+\\&\times|\mathcal{U}\rangle_{x_1x_2x_3}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{a_3b_3}\\&=\Omega^\dagger_{b_1b_2b_3}\Upsilon_{b_1b_2b_3}\sigma_{b_1}^{(i_1j_1)}\sigma_{b_2}^{(i_2j_2)}\sigma_{b_3}^{(i_3j_3)}|\mathcal{U}\rangle_{b_1b_2b_3},\end{aligned}\tag{13}$$

$$\begin{aligned}&{}_{a_3x_3}\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+|\mathcal{U}\rangle_{x_1x_2x_3}\Upsilon'^\dagger_{a_1a_2a_3}\sigma_{a_3}^{(i_3j_3)\dagger}\\&\times\sigma_{a_2}^{(i_2j_2)\dagger}\sigma_{a_1}^{(i_1j_1)\dagger}\Omega_{a_1a_2a_3}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{a_3b_3}\\&=\Omega^\dagger_{b_1b_2b_3}\sigma_{b_1}^{(i_1j_1)}\sigma_{b_2}^{(i_2j_2)}\sigma_{b_3}^{(i_3j_3)}\Upsilon'_{b_1b_2b_3}\ {}_{a_3x_3}\langle\phi^+|_{a_2x_2}\langle\phi^+|_{a_1x_1}\langle\phi^+\\&\times|\mathcal{U}\rangle_{x_1x_2x_3}|\phi^+\rangle_{a_1b_1}|\phi^+\rangle_{a_2b_2}|\phi^+\rangle_{a_3b_3}\\&=\Omega^\dagger_{b_1b_2b_3}\sigma_{b_1}^{(i_1j_1)}\sigma_{b_2}^{(i_2j_2)}\sigma_{b_3}^{(i_3j_3)}\Upsilon'_{b_1b_2b_3}|\mathcal{U}\rangle_{b_1b_2b_3}.\end{aligned}\tag{14}$$

This indicates that in terms of Alice's classical message Bob only needs to perform an appropriate unitary operation to recover the initial quantum information in his site. To be specific, if Alice gets the measurement result $|\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle_{a_1a_2a_3x_1x_2x_3}$, Bob's corresponding unitary operation is $\sigma_{b_1}^{(i_1j_1)\dagger}\sigma_{b_2}^{(i_2j_2)\dagger}\sigma_{b_3}^{(i_3j_3)\dagger}\Upsilon^\dagger_{b_1b_2b_3}\Omega_{b_1b_2b_3}$ or equivalently $\Upsilon'^\dagger_{b_1b_2b_3}\sigma_{b_1}^{(i_1j_1)\dagger}\sigma_{b_2}^{(i_2j_2)\dagger}\sigma_{b_3}^{(i_3j_3)\dagger}\Omega_{b_1b_2b_3}$. So far, we have detailed the teleportation scheme with the genuinely entangled six-qubit state [35] as quantum channel.

3 Discussion and comparisons

Now let us discuss the appropriate measuring bases and the corresponding reconstruction operations. From eqs. (4) and (5) one can see that, all the sets of the appropriate measuring bases Alice may use vary apparently with the unitary operator Υ. Incidentally, a set of measuring bases corresponds in essence to a set of state analyzers in reality. Hence in the passive mode that the quantum channel has already been the entangled state introduced by Borras et al. [35], Alice and Bob can employ any one set of the appropriate state analyzers available to achieve their teleportation. Obviously, Alice' measurement using different measuring bases will result in different collapses and accordingly Bob needs to perform corresponding operations on the collapsed state to reconstruct the original state. Intuitively, there should be a flexibility between the measurement difficulty and the reconstruction difficulty. One will clearly see this from the following three peculiar instantiations.

i) $\Upsilon = I$ with I being an identity operator:

In this case, the measuring bases Alice uses are composed of

$$\{\sigma_{a_1}^{(i_1j_1)}\sigma_{a_2}^{(i_2j_2)}\sigma_{a_3}^{(i_3j_3)}|\phi^+\rangle_{a_1x_1}|\phi^+\rangle_{a_2x_2}|\phi^+\rangle_{a_3x_3},\quad i's, j's=0,1\}.\tag{15}$$

Specifically, Alice performs three Bell-state measurements on her qubit pairs (a_1,x_1), (a_2,x_2) and (a_3,x_3), respectively. According to the outcome $\sigma_{a_1}^{(i_1j_1)}\sigma_{a_2}^{(i_2j_2)}\sigma_{a_3}^{(i_3j_3)}|\phi^+\rangle_{a_1x_1}\times|\phi^+\rangle_{a_2x_2}|\phi^+\rangle_{a_3x_3}$, Bob is required to execute the unitary operation $\Omega^\dagger_{b_1b_2b_3}\sigma_{b_1}^{(i_1j_1)}\sigma_{b_2}^{(i_2j_2)}\sigma_{b_3}^{(i_3j_3)}$ on his qubits to recover the original unknown state. Note that the unitary operator $\Omega_{b_1b_2b_3}$ defined in eq. (8) is a three-qubit joint operator.

ii) $\Upsilon = \Omega$.

In this situation, the measuring bases employed are

$$\begin{aligned}&\{|\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle_{a_1a_2a_3x_1x_2x_3}\\&\equiv\Omega_{a_1a_2a_3}\sigma_{a_1}^{(i_1j_1)}\sigma_{a_2}^{(i_2j_2)}\sigma_{a_3}^{(i_3j_3)}|\phi^+\rangle_{a_1x_1}|\phi^+\rangle_{a_2x_2}|\phi^+\rangle_{a_3x_3},\\&i's, j's=0,1\}.\end{aligned}\tag{16}$$

Obviously, $|\mathcal{D}_{000000}\rangle$ is the state $|\mathcal{G}\rangle$ expressed in eq. (1). This indicates that the measuring bases contain the state given as the quantum channel. After Alice's measurement, in light of her measurement result $\Omega_{a_1a_2a_3}\sigma_{a_1}^{(i_1j_1)}\sigma_{a_2}^{(i_2j_2)}\sigma_{a_3}^{(i_3j_3)}|\phi^+\rangle_{a_1x_1}|\phi^+\rangle_{a_2x_2}|\phi^+\rangle_{a_3x_3}$, Bob only needs to carry out the single-qubit operations $\sigma_{a_1}^{(i_1j_1)}\sigma_{a_2}^{(i_2j_2)}\sigma_{a_3}^{(i_3j_3)}$.

iii) $\Upsilon = \Gamma$, where Γ under the ordering bases {$|000\rangle$, $|001\rangle$, $|010\rangle$, $|011\rangle$, $|100\rangle$, $|101\rangle$, $|110\rangle$, $|111\rangle$} takes the following matrix form:

$$\Gamma=\frac{1}{2\sqrt{2}}\begin{pmatrix}1&-1&-1&1&-1&1&1&-1\\-1&-1&-1&-1&1&1&1&1\\-1&1&-1&1&-1&1&-1&1\\1&1&-1&-1&1&1&-1&-1\\1&-1&-1&1&1&-1&-1&1\\-1&-1&-1&-1&-1&-1&-1&-1\\-1&1&-1&1&1&-1&1&-1\\1&1&-1&-1&-1&-1&1&1\end{pmatrix}.\tag{17}$$

In this condition, the measuring bases consist of

$$\begin{aligned}&\{|\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle_{a_1a_2a_3x_1x_2x_3}\\&\equiv\Gamma_{a_1a_2a_3}\sigma_{a_1}^{(i_1j_1)}\sigma_{a_2}^{(i_2j_2)}\sigma_{a_3}^{(i_3j_3)}|\phi^+\rangle_{a_1x_1}|\phi^+\rangle_{a_2x_2}|\phi^+\rangle_{a_3x_3},\end{aligned}$$

$$i's, j's = 0, 1\}. \tag{18}$$

After a little complicated deductions, one can conclusively get the following relation:

$$\begin{aligned}|\mathcal{D}_{000000}\rangle_{a_1a_2a_3x_1x_2x_3} &= \Gamma_{a_1a_2a_3}|\phi^+\rangle_{a_1x_1}|\phi^+\rangle_{a_2x_2}|\phi^+\rangle_{a_3x_3}\\ &= \frac{1}{8}(|0\rangle_{a_1}\sigma_{x_1}^{(10)} + |1\rangle_{a_1})(|0\rangle_{x_1}\sigma_{a_2}^{(10)}\\ &\quad +|1\rangle_{x_1})(|0\rangle_{a_2}\sigma_{x_2}^{(10)} + |1\rangle_{a_2})\\ &\quad \times(|0\rangle_{x_2}\sigma_{a_3}^{(10)} + |1\rangle_{x_2})(|0\rangle_{a_3}\sigma_{x_3}^{(10)}\\ &\quad +|1\rangle_{a_3})(|0\rangle_{x_3} + |1\rangle_{x_3}).\end{aligned} \tag{19}$$

It is worthy pointing out that the right side of eq. (19) is just the definition of a six-qubit one-dimensional cluster state [39]. This means that the measuring bases are eventually the orthogonal and complete set of six-qubit one-dimensional cluster states. When Alice measures $|\mathcal{D}_{i_1j_1i_2j_2i_3j_3}\rangle_{a_1a_2a_3x_1x_2x_3}$ and informs Bob of her result, then Bob is able to recover the original quantum state by performing the proper unitary operation $\sigma_{b_1}^{(i_1j_1)\dagger}\sigma_{b_2}^{(i_2j_2)\dagger}\sigma_{b_3}^{(i_3j_3)\dagger}\Gamma_{b_1b_2b_3}^{\dagger}\Omega_{b_1b_2b_3}$.

Thus far, we have accomplished the three peculiar instantiations, which confirms the existence of flexibility between the measurement difficulty and the reconstruction difficulty. In the first instantiation the measurement is the simplest, for Bell-state analyzers are enough for use, while the corresponding reconstruction is comparatively difficult. On the contrary, in the second instantiation the reconstruction is the easiest, for only single-qubit Pauli operations are sufficient, while the measurement is difficult. The third instantiation then exhibits further the applicability of different measuring bases and the flexibility between the measurement difficulty and reconstruction operation difficulty.

Now let us compare our scheme with Choudhury et al.'s scheme [38]. Their scheme is a three-qubit teleportation scheme by using the genuinely entangled six-qubit state as quantum channel, too. In their scheme, they only gave one measurement result, i.e., $\frac{1}{\sqrt{8}}\Sigma_{i_1,i_2,i_3=0}^{1}|i_1i_2i_3\rangle|i_1i_2i_3\rangle$. This state is actually equal to $|\phi^+\rangle|\phi^+\rangle|\phi^+\rangle$. The measurements in their scheme employ the Bell-state bases. After the measurement, Bob obtains the original state by performing an appropriate unitary operation on his qubits. In Choudhury et al.'s paper [38], the necessary reconstruction operations are not completely given so that their scheme is unfeasible or at least inconvenient. In this sense their scheme is incomplete. In our work, all possible appropriate measuring-basis sets are expressed in eq. (4). Obviously, they are not the only Bell-state bases. Hence the measuring bases are more general than Choudhury et al.'s. Moreover, corresponding to different measurement results with different measuring bases, the necessary reconstruction unitary operations are explicitly been given. Therefore our scheme is complete, more applicable and more feasible, for the involved two parties have more choices to achieve their teleportation under their practical conditions such as state analyzers available. When in Choudhury et al.'s paper [38] they said, "For instance, the cluster state cannot be used for teleporting a three-qubit state but $|\psi_6\rangle$, which is different from the GHZ and the cluster states under LOCC, can be used for this purpose", their declaration is wrong, for cluster states have been repeatedly explored in quantum information processing, including the six-qubit one as a quantum channel for teleporting an arbitrary three-qubit state. They can be easily seen from refs. [12,13,15,19–21,40–43].

At last, we want to point out a fact that preparing Bell states is much easier than preparing the highly entangled states in reality. Because of this, one doee not need to prepare such entangled states purposely for teleportation. Hence, the present scheme can only be a candidate scheme in essence. Alternatively, one can use the highly entangled states as quantum channel substitutes of Bell states in some urgent conditions.

4 Summary

To summarize, in this paper we have extensively studied the teleportation of an arbitrary three-qubit state with the highly entangled six-qubit genuine state [35] as the quantum channel. We have explicitly given the different measurement bases which the sender can use and the concrete unitary operation which the receiver should perform according to the sender's various kinds of bases and measurement results. The flexibility between the measurement difficulty and the reconstruction operation difficulty is exhibited and thus confirmed. A comparison between our scheme and Choudhury et al.'s [38] is made, shows that our work is complete, more applicable and more feasible.

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