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Thermal entanglement in a mixed-spin Heisenberg XXZ model under a nonuniform external magnetic field

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The thermal entanglement in (1/2,1) mixed-spin Heisenberg XXZ model is investigated under an external nonuniform magnetic field. In the uniform magnetic field system, the critical magnetic field B_c and critical temperature T_c are increased by increasing the anisotropic parameter *k*. The degree of magnetic field *b* plays an important role in improving the critical temperature and enlarging the region of entanglement in the nonuniform magnetic field system.

entanglement, mixed-spin, negativity, quantum phase transition

Entanglement is a nonlocal correlation between quantum systems that does not exist classically. Entangled pairs of quantum systems remain strongly correlated even if they are well separated spatially; observing the state of one fixes with certainty the state of the other[1]. Due to so much fascinating feature and a central role in quantum information processing, entanglement has been noted in many fields of physics[2], such as quantum key distribution[3], quantum computation[4] and quantum teleportation[5]. An important emerging field is the quantum entanglement in solid state systems such as spin chains[6−9]. The 1D Heisenberg chain, a simple but realistic and extensively studied solid state system[10], has been used to construct a quantum computer and quantum dots[13]. For any real physical system, the temperature should be considered since in most cases the thermal fluctuation may suppress quantum effects[11]. Therefore, it is necessary to study the behaviors of thermal entanglement under external magnetic field conditions[12−14].

Since natural thermal entanglement in a one-dimensional Heisenberg model was introduced by Amesen et al.[10], it has attracted much attention. Lots of studies have been done, including affects of the uniform magnetic field and the nonuniform magnetic field on thermal entanglement[11,15]. However, only spin-halves systems were considered in the previous studies as there exists a good measure of entanglement of two spin-halves, the concurrence[16], which is applicable to an arbitrary state of two spin halves, such as pure state and mixed state. As for mixed spin or higher system, however, there are few studies on entanglement due to the lack of good operational entanglement measures. Although the measure of entanglement called negativity, which was introduced by Vedral[17], fails to quantify the entanglement of some entangled states in dimensions higher than six, however, for the systems of 2×2 and 2×3 considered here, the task of calculating the negativity is sufficient and necessary[18,19]. Therefore, the appearance of negativity plays an important role to study the entanglement of mixed-spin or higher spin systems. Now, some novel work on Heisenberg model has come out successively. Zhou et al. investigated the thermal entanglement of spin-1 atoms with nonlinear coupling in an optical lattice chain[20]. In spin-1 Heisenberg chain, there were some work in different circumstances[21]. Sun et al. in-

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vestigated entanglement in (1/2, 1) mixed-spin Heisenberg systems[22]. In a 6-Li atom system, quantum phase transition in the magnetic field and entanglement by use of von Neumann entropy are accurately calculated[23]. Under the uniform magnetic field and nonuniform magnetic field[24], the study on entanglement of mixed-spin systems revealed the differences with spin-halves systems. We notice that the thermal entanglement for a XXZ mixed-spin model in a nonuniform field has not been discussed. In the theoretical analysis, we can see that the analysis of such systems is meaningful and could shed new light which is different from other works.

1 Theoretical method and calculation

The Peres-Horodecki criterion[25] gives a qualitative way for judging whether the state is entangled or not, that is PPT criterion. The quantitative version of the criterion was developed by Vidal and Werner[19]. They presented a measure of entanglement called negativity that can be computed efficiently, and the negativity does not increase under local manipulations of the system. The negativity of a state ρ by definition is

$$N(\rho)=\sum_i |\mu_i|, \tag{1}$$

where μ_i is the negative eigenvalue of ρ^{T_1}, and T_1 denotes the partial transpose with respect to the first system. The negativity N is related to the trace norm of ρ^{T_1} via

$$N(\rho)=\frac{\left\|\rho^{T_1}\right\|_1-1}{2}, \tag{2}$$

where the trace norm of ρ^{T_1} is equal to the sum of the absolute values of the eigenvalues of ρ^{T_1}. In this paper, we use the concept of negativity to study entanglement in (1/2, 1) mixed-spin systems.

The Hamiltonian H for the two-spin anisotropic (1/2,1) mixed spin Heisenberg XXZ model in a nonuniform magnetic field is

$$\begin{aligned} H = & J(s_1^x S_2^x + s_1^y S_2^y + k s_1^z S_2^z) \\ & + (B+b)s_1^z + (B-b)S_2^z, \end{aligned} \tag{3}$$

where J represents the interaction between two spins, the parameter k ($-1\leqslant k\leqslant 1$) measures the anisotropy of the system and equals 0 for the isotropic XX model and ±1 for the XXX model, $0\leqslant B$ is restricted, and the magnetic fields on the two spins have been so parametrized that b controls the degree of inhomogeneity. s_1 and S_2 are operators of 1/2 spin and 1 spin respectively. We choose the following basis

$$\left\{\left|-\frac{1}{2},-1\right\rangle,\left|\frac{1}{2},0\right\rangle,\left|-\frac{1}{2},1\right\rangle,\left|\frac{1}{2},-1\right\rangle,\left|-\frac{1}{2},0\right\rangle,\left|\frac{1}{2},1\right\rangle\right\}, \tag{4}$$

where $|m,M\rangle$ is the eigenstate of s_z and S_z with corresponding eigenvalues given by m and M, respectively, and $J>0$ is the antiferromagnetic exchange interaction between spins.

For a system in equilibrium at temperature T, the state is $\rho(T)=Z^{-1}\exp\left(\frac{-H}{k_B T}\right)$, where $Z=\mathrm{Tr}\left[\exp\left(\frac{-H}{k_\mathrm{B} T}\right)\right]$ is the partition function and k_B is Boltzmann constant. As $\rho(T)$ also describes a thermal state, so the entanglement is called thermal entanglement. The corresponding eigenvalues and eigenvectors of H can be directly given as follows:

$$\begin{aligned} H\left|-\frac{1}{2},-1\right\rangle &= \left(\frac{k}{2}-\frac{3B}{2}+\frac{b}{2}\right)\left|-\frac{1}{2},-1\right\rangle, \\ H\left|\frac{1}{2},1\right\rangle &= \left(\frac{k}{2}+\frac{3B}{2}-\frac{b}{2}\right)\left|\frac{1}{2},1\right\rangle, \end{aligned} \tag{5}$$

$$H|\psi_{2,3}\rangle = E_{2,3}|\psi_{2,3}\rangle,\ H|\psi_{4,5}\rangle = E_{4,5}|\psi_{4,5}\rangle, \tag{6}$$

where

$$\begin{aligned} E_{2,3} &= -\frac{k}{4}+\frac{B}{2}-\frac{b}{2}\pm\frac{m}{4}, \\ E_{4,5} &= -\frac{k}{4}-\frac{B}{2}+\frac{b}{2}\pm\frac{n}{4}, \end{aligned} \tag{7}$$

$$\begin{aligned} m &= \sqrt{k^2+8kb+16b^2+8}, \\ n &= \sqrt{k^2-8kb+16b^2+8}, \end{aligned} \tag{8}$$

$$|\psi_{2,3}\rangle = \frac{1}{\sqrt{1+2(-\frac{k}{4}-b\pm\frac{m}{4})^2}} \times\left(\left|\frac{1}{2},0\right\rangle+\sqrt{2}\left(-\frac{k}{4}-b\pm\frac{m}{4}\right)\left|-\frac{1}{2},1\right\rangle\right), \tag{9}$$

$$|\psi_{4,5}\rangle = \frac{1}{\sqrt{1+2(-\frac{k}{4}+b\pm\frac{n}{4})^2}} \times\left(\left|-\frac{1}{2},0\right\rangle+\sqrt{2}\left(-\frac{k}{4}+b\pm\frac{m}{4}\right)\left|\frac{1}{2},-1\right\rangle\right), \tag{10}$$

when b=0, k=0, $|\psi_{2,3}\rangle$ and $|\psi_{4,5}\rangle$ are maximally entangled states. Thus, the density-matrix $\rho(T)$ for the thermal state in the standard basis can be written as

$$\rho(T)=\begin{pmatrix} a_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & a_{32} & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & a_{45} & 0 \\ 0 & 0 & 0 & a_{54} & a_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & a_{66} \end{pmatrix}, \quad (11)$$

with the partition function and the matrix elements given by

$$Z=2\left(\mathrm{e}^{-\frac{k}{2T}}\cosh\left(\frac{3B-b}{2T}\right)+\mathrm{e}^{-\frac{-k+2B-2b}{4T}}\cosh\left(\frac{m}{4T}\right)\right.$$
$$\left.+\mathrm{e}^{-\frac{-k-2B-2b}{4T}}\cosh\left(\frac{n}{4T}\right)\right), \quad (12)$$

$$a_{11}=\mathrm{e}^{-\frac{k-3B+b}{2T}}/Z, \quad (13)$$

$$a_{22}=\frac{1}{Z}\left(\frac{\mathrm{e}^{-E_2/T}}{1+2\left(-\frac{k}{4}-b+\frac{m}{4}\right)^2}+\frac{\mathrm{e}^{-E_3/T}}{1+2\left(-\frac{k}{4}-b-\frac{m}{4}\right)^2}\right), \quad (14)$$

$$a_{33}=\frac{1}{Z}\left(\frac{2\mathrm{e}^{-E_2/T}\left(-\frac{k}{4}-b+\frac{m}{4}\right)^2}{1+2\left(-\frac{k}{4}-b+\frac{m}{4}\right)^2}\right.$$
$$\left.+\frac{2\mathrm{e}^{-E_3/T}\left(-\frac{k}{4}-b-\frac{m}{4}\right)^2}{1+2\left(-\frac{k}{4}-b-\frac{m}{4}\right)^2}\right), \quad (15)$$

$$a_{23}=a_{32}=\frac{1}{Z}\left(\frac{\sqrt{2}\mathrm{e}^{-E_2/T}\left(-\frac{k}{4}-b+\frac{m}{4}\right)}{1+2\left(-\frac{k}{4}-b+\frac{m}{4}\right)^2}\right.$$
$$\left.+\frac{\sqrt{2}\mathrm{e}^{-E_3/T}\left(-\frac{k}{4}-b-\frac{m}{4}\right)}{1+2\left(-\frac{k}{4}-b-\frac{m}{4}\right)^2}\right), \quad (16)$$

$$a_{44}=\frac{1}{Z}\left(\frac{2\mathrm{e}^{-E_4/T}\left(-\frac{k}{4}-b+\frac{n}{4}\right)^2}{1+2\left(-\frac{k}{4}-b+\frac{n}{4}\right)^2}\right.$$
$$\left.+\frac{2\mathrm{e}^{-E_5/T}\left(-\frac{k}{4}-b-\frac{n}{4}\right)^2}{1+2\left(-\frac{k}{4}-b-\frac{n}{4}\right)^2}\right), \quad (17)$$

$$a_{55}=\frac{1}{Z}\left(\frac{\mathrm{e}^{-E_4/T}}{1+2\left(-\frac{k}{4}-b+\frac{n}{4}\right)^2}\right.$$
$$\left.+\frac{\mathrm{e}^{-E_5/T}}{1+2\left(-\frac{k}{4}-b-\frac{n}{4}\right)^2}\right), \quad (18)$$

$$a_{45}=a_{54}=\frac{1}{Z}\left(\frac{\sqrt{2}\mathrm{e}^{-E_4/T}\left(-\frac{k}{4}-b+\frac{n}{4}\right)}{1+2\left(-\frac{k}{4}-b+\frac{n}{4}\right)^2}\right.$$
$$\left.+\frac{\sqrt{2}\mathrm{e}^{-E_5/T}\left(-\frac{k}{4}-b-\frac{n}{4}\right)}{1+2\left(-\frac{k}{4}-b-\frac{n}{4}\right)^2}\right), \quad (19)$$

$$a_{66}=\mathrm{e}^{-\frac{k+3B-b}{2T}}/Z. \quad (20)$$

After the partial transpose with respect to the first spin-halfsubsystem, we can get ρ^{T_1}

$$\rho(T)=\begin{pmatrix} a_{11} & a_{45} & 0 & 0 & 0 & 0 \\ a_{45} & a_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & a_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & a_{55} & a_{32} \\ 0 & 0 & 0 & 0 & a_{23} & a_{66} \end{pmatrix}. \quad (21)$$

The negativity is thus given by

$$N=\frac{1}{2}\max[0,\sqrt{(a_{11}-a_{22})^2+4a_{45}^2}-a_{11}-a_{22}]$$
$$+\frac{1}{2}\max[0,\sqrt{(a_{55}-a_{66})^2+4a_{23}^2}-a_{55}-a_{66}]. \quad (22)$$

From eqs. (12)—(20) and (22), we can obtain the negativity of the two-spin (1/2,1) mixed-spin system. In the following calculations, J is chosen as the energy unit.

2 Results and discussion

We first consider the case of the uniform magnetic field, that is to say, b=0. When k=1, the system is the XXX Heisenberg model, which is analyzed in ref. [22]. Negativity and the critical temperature T_c can be written as

$$N=\frac{1}{3}\max\left[0,\frac{e^{1/T}-4e^{-1/(2T)}}{e^{1/T}+2e^{-1/(2T)}}\right],\qquad (23)$$
$$T_c=3/(4\ln 2)\approx 1.082.$$

From the eq. (23), we can easily see T_c is independent of the magnetic field. In the mixed-spin (1/2, 1) XX Heisenberg model, we have obtained similar conclusion. So we can say T_c is independent of the magnetic field for mixed-spin Heisenberg model.

For the different anisotropic parameters, plottings of the negativity as a function of both T and B are given in Figures 1, 2 and 3. The results for the three cases have similar features: negativity is not maximal when T=0 and B=0; when B is increasing, Negativity rapidly becomes maximal value. That is because, at T=0, when B=0, the ground state is two-fold degenerate; when B>0, the degenerate vanishes. With the anisotropic parameter increasing, the critical temperature becomes bigger; for example, k=0, T_c=0.670; k=1, T_c=1.082. This means that anisotropic parameters can suppress the effects of thermal fluctuation on entanglement.

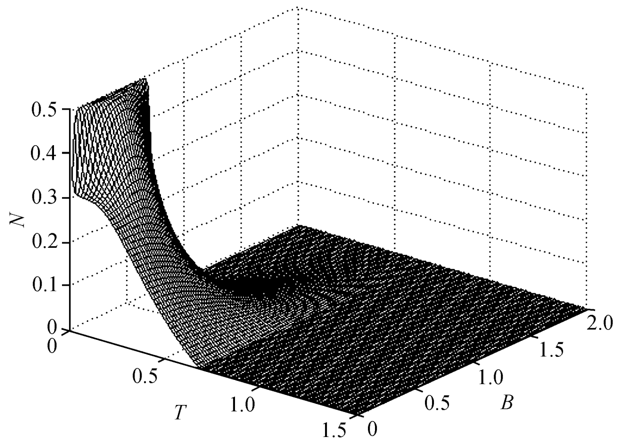

Figure 1 The negativities versus temperature T and magnetic field B with k=0.

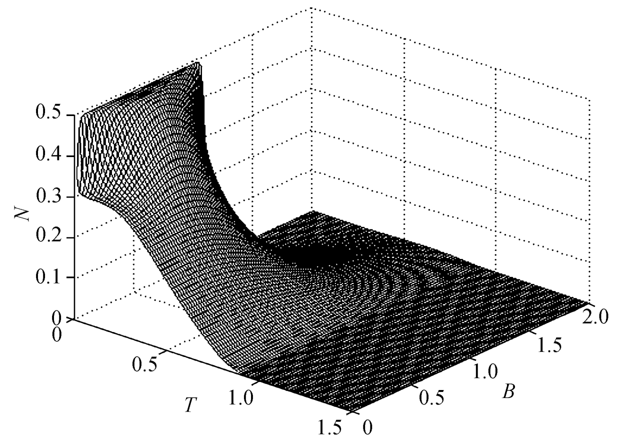

Figure 2 The negativities versus temperature T and magnetic field B with k=0.5.

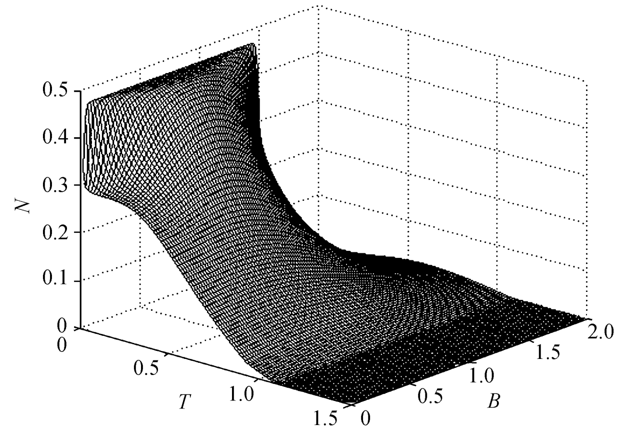

Figure 3 The negativities versus temperature T and magnetic field B with k=1.

In Figures 1, 2 and 3, we can also see that the maximal entanglement is decreasing with the anisotropic parameters increasing. That is because the ground state is decided by anisotropy parameters and magnetic field at low temperature[24]. For $B>3k/4+\sqrt{k^2+8}/4$, the ground state is $|-1/2,-1\rangle$, entanglement does not exist. For $B<3k/4+\sqrt{k^2+8}/4$, the ground state is

$$|\psi_0\rangle=\frac{1}{\sqrt{1+\frac{1}{8}\left(k+\sqrt{k^2+8}\right)}}\times\left(-\frac{\sqrt{2}}{4}(k+\sqrt{k^2+8})\left|\frac{1}{2},-1\right\rangle+\left|-\frac{1}{2},0\right\rangle\right).$$

This is a Schmidt model. Negativity can be written as $N=\frac{\sqrt{2}}{4}\frac{k+\sqrt{k^2+8}}{1+(1/8)(k+\sqrt{k^2+8})^2}$. It can easily be seen from formula above that negativity is only related to anisotropy parameter and decreases with increasing k. From the discussion above, entanglement drops suddenly and vanishes from the maximal constant value at $B=\frac{3}{4}k+\frac{\sqrt{k^2+8}}{4}$, which is called the critical magnetic field B_c, at which the quantum phase transition occurs[26]. Obviously, B_c is increased with the increasing k. The anisotropy parameter k causes a shift in the locations of the phase transitions. Namely, the presence of positive k increases the region over which the negativity attains its maximum value. This is different from (1/2, 1) that of the mixed-spin Heisenberg XY model[24]. This result means that in larger region of B and T, we can obtain stronger entanglement.

We now turn to the discussion of nonuniform magnetic field case. In order to compare with the case of uniform magnetic field, plottings of the negativity as a function of both T and B are also given in Figures 4, 5 and 6 with the corresponding anisotropy parameters. Some features display apparently through comparing with uniform magnetic field case.

(i) At low temperature, here, we consider T=0, the negativity is constant under nonuniform magnetic field with increasing B from zero, while the value starts from a small value then to the constant maximum under uniform magnetic field. The reason lies in that the presence of b destroys the degenerate state. In the system of nonuniform magnetic field, with increasing B, negativity

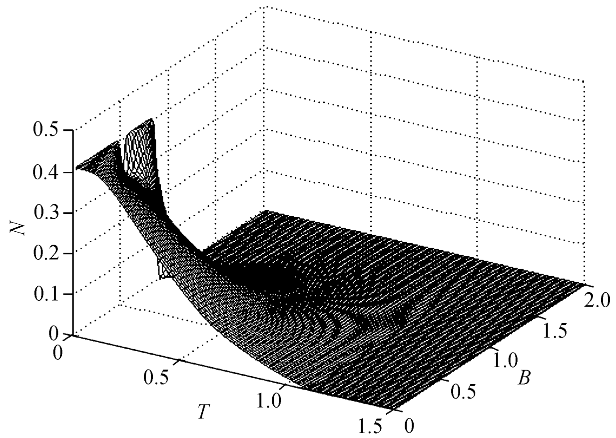

Figure 4 The negativities versus temperature T and magnetic field B with b=0.5 and k=0.

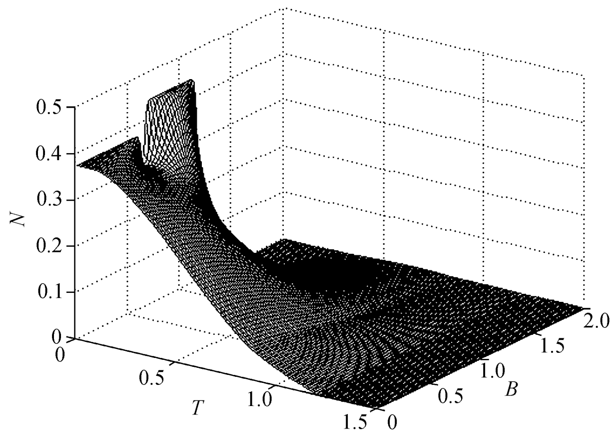

Figure 5 The negativities versus temperature T and magnetic field B with b=0.5 and k=0.5.

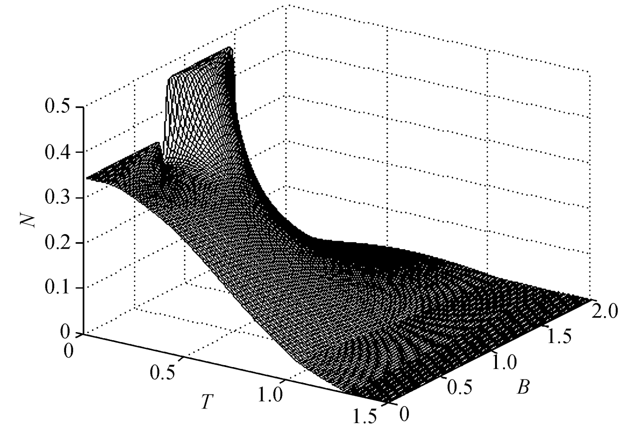

Figure 6 The negativities versus temperature T and magnetic field B with b=0.5 and k=1.

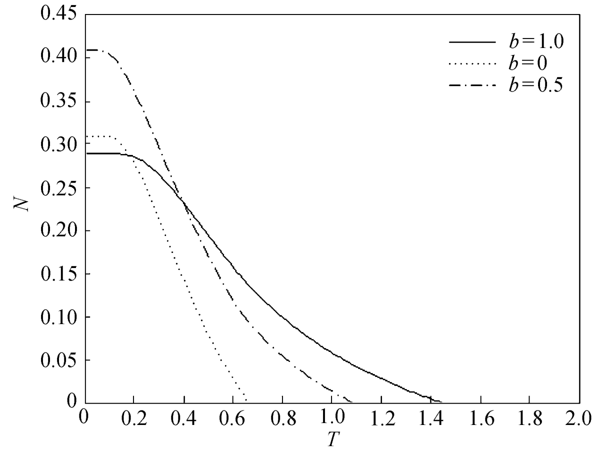

Figure 7 The negativities versus temperature with B=0 and k=0.

emerge thin split.

(ii) With increasing the anisotropic parameter, negativity decrease for the systems of uniform magnetic field and nonuniform magnetic field. However, negativity is enhanced when across magnetic field, where the split appears. The step-form change of entanglement should shed new light on the quantum information. We can also see that the critical temperature T_c can be increased under the nonuniform magnetic field in contrast to uniform magnetic field. For example, k=0, $T_c \approx 1.1$; k=1, $T_c \approx 1.3$. This means that entanglement can be obtained at high temperature by increasing the anisotropic parameter.

In Figure 7, the negativity versus temperature under the absolute nonuniform magnetic field is plotted, where B=0. It can be easily seen that the critical temperature is related to magnetic field, which is reverse with the result mentioned above. To a certain extent, entanglement and critical temperature can be enhanced under the nonuniform magnetic field in contrast to uniform magnetic field. This point implies that the nonuniform magnetic field may be a possible way to retain entanglement under finite temperature.

3 Conclusion

In conclusion, in this paper we study the effects of a nonuniform magnetic field on the thermal entanglement in a (1/2, 1) mixed-spin Heisenberg XXZ chain. Through analyzing the uniform magnetic field system, we find that with the increasing of *k*, the critical magnetic field B_c and critical temperature T_c are increased. This result means that in larger region of *B* and *T*, we can obtain stronger entanglement. In the nonuniform magnetic field system, the existence of *b* not only causes the emergence of thin split but also improves the critical temperature T_c. Therefore, the anisotropic parameter and the degree of magnetic field play important roles in improving the critical temperature and enlarging the region of entanglement.

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