Science in China Series G: Physics, Mechanics & Astronomy

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Optimal configuration for a finite high-temperature source heat engine cycle with the complex heat transfer law

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The optimal configuration of a heat engine operating between a finite high-temperature source and an infinite low-temperature reservoir is derived by using finite time thermodynamics based on a complex heat transfer law, including Newtonian heat transfer law, linear phenomenological heat transfer law, radiative heat transfer law, Dulong-Petit heat transfer law, generalized convective heat transfer law and generalized radiative heat transfer law, $q \propto (\Delta T^n)$. In the engine model the only irreversibility of finite rate heat transfer is considered. The optimal relation between the power output and efficiency of the heat engine is also derived by using an equivalent temperature of the hot reservoir. The obtained results include those obtained in recent literature and can provide some theoretical guidance for the designs of practical engines.

finite time thermodynamics, finite heat capacity reservoir, heat engine cycle, optimal configuration, optimal performance, heat transfer law

In the analysis of finite-time thermodynamics[1−8], the basic thermodynamic model is the so-called "endoreversible Carnot engine", in which only the irreversibility of the finite-rate heat transfer is considered. A major objective of finite-time thermodynamics is to understand irreversible finite time processes and to establish the general, natural bounds upon the efficiency or maximum work-output for such processes. Another primary goal of finite-time thermodynamics is to establish general operating principles (e.g., the path that the system should follow for the maximum efficiency, work or chemical efficiency) for the system which serves as a model for real processes.

It is often the case in practice that the power is generated from heat, which is carried by a finite amount of material with finite heat capacity, rather than from the heat extracted from an isothermal, infinite reservoir. In the reversible (infinite-time) limit, the cycle, which extracts the maximum work from a finite heat source, is qualitatively different from the Carnot cycle, and its theoretical efficiency is considerably smaller[9].

For the endoreversible cycle, the research into the effect of having a finite heat-reservoir on the performance includes two aspects: The first is to determine the optimal performance of the given finite thermal-capacity cycles, such as the Carnot cycle[10,11], Rankine cycle[12] and Brayton cycle[13−15]. The optimization of the first aspect may be carried out with a fixed heat-input[10] or with a variable heat-input[11−14]. The second is to determine the optimal configuration of heat engines under given conditions. For example, the optimal configuration of an endoreversible constant-temperature heat reservoir heat engine is the Curzon-Ahlborn engine[15], and the optimal configuration of Newtonian law ($Q \propto \Delta(T)$) system variable-temperature heat reservoir heat engine

Received December 2, 2007; accepted December 5, 2008
doi: 10.1007/s11433-009-0074-5
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Supported by the Program for New Century Excellent Talents in University of China (Grant No. 20041006) and the Foundation for the Author of National Excellent Doctoral Dissertation of China (Grant No. 200136)

is a generalized Carnot heat engine[16,17] (in which the temperatures of the heat reservoirs and the working fluid change exponentially with time and the ratio of the temperatures of the working fluid to the heat reservoir is a constant). The optimal configuration of linear phenomenological law ($Q \propto \Delta(T^{-1})$) system variable-temperature heat reservoir heat engine is another generalized Carnot cycle[18] (in which the difference of reciprocal temperatures of the heat reservoirs and the working fluid is a constant). Xiong et al.[19] obtained the optimal performance of a variable-temperature heat-reservoir heat-engine with generalized radiative heat transfer law $Q \propto \Delta(T^n)$. Chen et al.[20] obtained the optimal configuration and performance of a variable-temperature heat-reservoir heat-engine with generalized convective heat transfer law $Q \propto (\Delta T)^m$. Chen et al.[21] further studied the optimal configuration and performance of a generalized Carnot cycle under the condition of mixed heat-resistances. The effects of heat leakage between the heat reservoirs on the optimal configuration and performance of variable-temperature heat-reservoir heat engine cycles with Newtonian and linear phenomenological heat transfer laws were studied[22,23]. The results of refs. [18—21] show that heat transfer laws have significant influences on the optimal configurations and performance of heat engine cycles, and a study on the effect of the heat transfer law on optimal configuration of cycles is necessary.

One of the aims of finite time thermodynamics is to pursue generalized rules and results. This paper will extend the previous work by using a complex heat transfer law, including Newtonian heat transfer law, linear phenomenological heat transfer law, radiative heat transfer law, Dulong-Petit heat transfer law, generalized convective heat transfer law and generalized radiative heat transfer law, $Q \propto (\Delta T^n)^m$, in the heat transfer processes between the engine and its surroundings to find the optimal configuration of the variable-temperature heat-reservoir heat engine cycles. In the engine model the only irreversibility of finite rate heat transfer is considered. The optimal relation between the power output and efficiency of the heat engine is also derived by using an equivalent temperature of the hot reservoir. The optimal performance of both Carnot heat engine[24] and Carnot heat pump[25] with infinite thermal-capacity (constant-temperature) heat reservoirs with the same heat transfer law was derived by Chen et al.[24] and Li et al.[25]. The obtained results include those obtained in recent literatures and can provide some theoretical guidance for the designs of practical engines.

1 The heat engine model

The generalized engine-model and its surroundings to be considered in this paper are shown in Figure 1. The following assumptions are made for this model. The system adapted is a working fluid alternately connected to a heat source with finite heat capacity and to a heat sink with infinite heat-capacity. The heat engine operates in a cyclic fashion with a fixed time τ allotted for each cycle. The high-temperature heat-source is assumed to have a constant heat-capacity C, its temperature is given by $T_x(t)$, and its initial temperature is given by T_H. The low-temperature heat-sink is assumed, for simplicity, to be infinite in size and therefore it has a fixed temperature, T_L. The heat transfer between heat source, heat sink and working fluid obeys a complex law, including Newtonian heat transfer law, linear phenomenological heat transfer law, radiative heat transfer law, Dulong-Petit heat transfer law, generalized convective heat transfer law and generalized radiative heat transfer law, $Q \propto (\Delta T^n)^m$. The absorbed and released heats of the working fluid are Q_1 and Q_2, respectively.

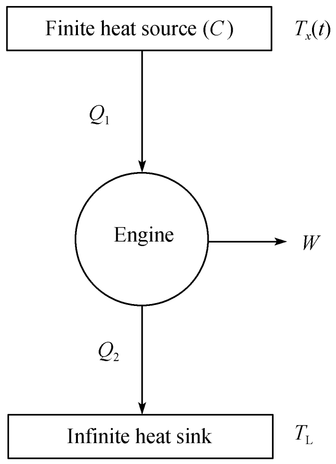

Figure 1 Model of the heat engine.

The two steps in the cycle during which the working fluid is disconnected from one reservoir and connected to another are taken to be reversibly adiabatic. It is assumed that these steps occur instantaneously, which implies that the temperature of the working fluid changes discontinuously.

2 The optimal configuration

Consider that the heat transfer between the heat engine and its surroundings follows a complex law $Q \propto (\Delta T^n)^m$. Then

$$Q_1 = \int_0^\tau \alpha(t)[T_x^n(t) - T^n(t)]^m \mathrm{d}t, \tag{1}$$

$$Q_2 = \int_0^\tau \beta(t)[T^n(t) - T_\mathrm{L}^n]^m \mathrm{d}t, \tag{2}$$

where $\alpha(t)$ and $\beta(t)$ are heat conductivities between the heat source, heat sink and working fluid, respectively. The heat conductivity is the product of the overall heat transfer coefficient and the corresponding heat transfer surface area of the heat exchanger. It is assume that at $t=0$ the working fluid is in contact with the high-temperature heat source and is separated from the low-temperature heat sink by an adiabatic boundary. At a later time $t_1(0<t_1<\tau)$, the contact with the heat source is broken and the working fluid is placed in contact with the heat sink. Therefore, one has the following relationships

$$\alpha(t) = \begin{cases} \alpha, & 0 \leqslant t < t_1, \\ 0, & t_1 \leqslant t < \tau, \end{cases} \tag{3}$$

$$\beta(t) = \begin{cases} 0, & 0 \leqslant t < t_1, \\ \beta, & t_1 \leqslant t < \tau, \end{cases} \tag{4}$$

where α and β are constants.

From the first law of thermodynamics, the work output from the cycle is

$$W = \int_0^\tau \{\alpha(t)[T_x^n(t) - T^n(t)]^m - \beta(t)[T^n(t) - T_\mathrm{L}^n]^m\}\mathrm{d}t. \tag{5}$$

From the second law of thermodynamics, the entropy change of the working fluid per cycle is

$$\Delta S = \int_0^\tau \frac{1}{T(t)}\{\alpha(t)[T_x^n(t) - T^n(t)]^m - \beta(t) \times [T^n(t) - T_\mathrm{L}^n]^m\}\mathrm{d}t = 0. \tag{6}$$

Furthermore, since the heat capacity of the hot source is assumed to be constant, one has

$$\mathrm{d}Q_1 = -C\mathrm{d}T_x(t). \tag{7}$$

Substituting eq. (1) into (7) yields

$$C\dot{T}_x(t) + \alpha(t)[T_x^n(t) - T^n(t)]^m = 0, \tag{8}$$

where $\dot{T}_x(t) = \mathrm{d}T_x(t)/\mathrm{d}t$.

Our problem now is to determine the optimal configuration of the model cycle in which the maximum work output is obtained under a given cycle duration τ. Using eqs. (5), (6) and (8), one has the modified Lagrangian:

$$\begin{aligned} L = {} & \alpha(t)[T_x^n(t) - T^n(t)]^m - \beta(t)[T^n(t) - T_\mathrm{L}^n]^m \\ & + \lambda\{\alpha(t)[T_x^n(t) - T^n(t)]^m - \beta(t)[T^n(t) - T_\mathrm{L}^n]^m\}/T(t) \\ & + \mu(t)\{C\dot{T}_x(t) + \alpha(t)[T_x^n(t) - T^n(t)]^m\}, \end{aligned} \tag{9}$$

where λ is the Lagrangian constant, and $\mu(t)$ is a function of time. The path for the working fluid which results in the maximum work for a given time interval $\{0,\tau\}$ may now be obtained from the solution of the Euler-Lagrange equations. The Euler-Lagrange equations are given by

$$\frac{\partial L}{\partial T(t)} - \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial L}{\partial \dot{T}(t)}\right] = 0, \tag{10}$$

$$\frac{\partial L}{\partial T_x(t)} - \frac{\mathrm{d}}{\mathrm{d}t}\left[\frac{\partial L}{\partial \dot{T}_x(t)}\right] = 0. \tag{11}$$

For $0 \leqslant t < t_1$, substituting eqs. (3), (4) and (9) into (10) and (11), respectively, yields

$$mn\alpha T_x^{n-1}(t)\{\lambda + [\mu(t)+1]T(t)\}[T_x^n(t) - T^n(t)]^{m-1} = CT(t)\dot{\mu}(t), \tag{12}$$

$$\lambda[T_x^n(t) - T^n(t)] + mnT^n(t)\{\lambda + [\mu(t)+1]T(t)\} = 0. \tag{13}$$

From eq. (13) one may obtain

$$\mu(t) = -\frac{\lambda}{T(t)} - \frac{\lambda[T_x^n(t) - T^n(t)]}{mnT^{n+1}(t)} - 1. \tag{14}$$

The derivative of eq. (14) with respect to t is

$$\dot{\mu}(t) = \frac{\lambda[-nT(t)T_x^n(t)\dot{T}_x(t) + (n+1)\dot{T}(t)T_x^{n+1}(t) + (mn-1)T^n(t)\dot{T}(t)T_x(t)]}{mnT^{n+2}(t)T_x(t)}. \tag{15}$$

Substituting eqs. (8), (14), and (15) into (12) yields

$$\begin{aligned} & n(m+1)T(t)[T_x^{n-1}(t)\dot{T}_x(t) - T^{n-1}(t)\dot{T}(t)] \\ & - (n+1)\dot{T}(t)[T_x^n(t) - T^n(t)] = 0. \end{aligned} \tag{16}$$

The solution of eq. (16) is

$$[T_x^n(t) - T^n(t)]T(t)^{-(n+1)/(m+1)} = a(mn), \quad 0 \leqslant t < t_1, \tag{17}$$

where $a(mn)$ is a constant dependent on mn.

Using the same way of calculation in the case of $0 \leqslant t < t_1$, one may obtain the relation of $T(t)$ and T_L for $t_1 \leqslant t < \tau$:

$$\lambda T_L^n + \lambda(mn-1)T^n(t) + mnT^{n+1}(t) = 0, \quad t_1 \leqslant t \leqslant \tau, \tag{18}$$

Eqs. (17) and (18) are the major results of this paper. They determine the relation between the temperatures of heat reservoirs and the working fluid. The heat reservoir temperature $T_x(t)$ and the working fluid temperature may be obtained by using eqs. (8), (17) and (18), i.e., the optimal configuration of the heat engine cycles.

3 The effects of heat transfer laws

When $n=1$, the heat transfer law becomes the generalized convective heat transfer law, and then eqs. (17) and (18) become

$$[T_x(t)-T(t)]T(t)^{-2/(m+1)}=a(m),\quad 0\leqslant t<t_1,\quad (19)$$

$$m-\frac{\lambda}{T^2(t)}[T(t)-T_{\rm L}]+\frac{\lambda}{T(t)}m=0,\quad t_1\leqslant t<\tau,\quad (20)$$

where $a(m)$ is a constant dependent on m. They are the same results as those obtained in ref. [20].

(i) If $m=1$ further, eqs. (19) and (20) are the results of the heat engine cycle with Newtonian heat transfer law[16,17]. Combining eqs. (8), (19) with (20) gives

$$T_x(t)=T(t)/u,\quad 0\leqslant t<t_1,\quad (21)$$

$$T(t)=\begin{cases}uT_{\rm H}\exp[\alpha t(u-1)/C], & 0\leqslant t<t_1,\\ vT_{\rm L}, & t_1\leqslant t<\tau,\end{cases}\quad (22)$$

where u and v are two constants. Eqs. (21) and (22) are the same results of refs. [16,17]. They indicate that the temperatures of the heat source and working fluid decrease exponentially with time in the time interval $\{0,t_1\}$, and the ratio of the temperatures of the working fluid to heat source is a constant.

(ii) If $m=1.25$, they are the results of the heat engine cycle with Dulong-Petit heat transfer law[26]. In this case, the heat releasing process is still a constant temperature process. The varying laws of $T_x(t)$ and $T(t)$ in the heat absorbing process become complicate and follow the relations below:

$$[T_x(t)-T(t)]T(t)^{-8/9}=a_1,\quad (23)$$

$$C\dot{T}_x(t)=-\alpha[T_X(t)-T(t)]^{5/4},\quad (24)$$

where a_1 is a constant.

When $m=1$, the heat transfer law becomes the generalized radiative heat transfer law. Eqs. (17) and (18) become

$$[T_x^n(t)-T^n(t)]T(t)^{-(n+1)/2}=a(n),\quad 0\leqslant t<t_1,\quad (25)$$

$$\lambda T_{\rm L}^n+\lambda(n-1)T^n(t)+nT^{n+1}(t)=0,\quad t_1\leqslant t<\tau,\quad (26)$$

where $a(n)$ is a constant dependent on n. Eqs. (25) and (26) are the same results of ref. [19].

(i) If $n=1$ further, eqs. (25) and (26) are the results of the heat engine cycle with Newtown's heat transfer law, i.e., eqs. (21) and (22), which are the same results of refs. [16,17].

(ii) If $n=4$, eqs. (25) and (26) are the results of the heat engine cycle with radiative heat transfer law[19]. The temperatures of heat reservoirs and the working fluid are complicate and follow the relations below:

$$\begin{cases}[T_x^4(t)-T^4(t)]T(t)^{-5/2}=a_2,\\ C\dot{T}_x(t)=-\alpha[T_x^4(t)-T^4(t)],\end{cases}\quad 0\leqslant t<t_1,\quad (27)$$

$$\lambda T_{\rm L}^4+3\lambda T^4(t)+4T^5(t)=0,\quad t_1\leqslant t<\tau,\quad (28)$$

where a_2 is a constant.

(iii) If $n=-1$, eqs. (25) and (26) are the results of the heat engine cycle with linear phenomenological heat transfer law[18,19]. Combining eqs. (8), (25) and (26) gives

$$T(t)=\begin{cases}\dfrac{T_{\rm H}-(\alpha a/C)t}{a[T_{\rm H}-(\alpha a/C)t]+1}, & 0\leqslant t<t_1,\\ \dfrac{T_{\rm L}}{1-bT_{\rm L}}, & t_1\leqslant t<\tau,\end{cases}\quad (29)$$

where a and b are two constants. Eq. (29) is the same result as that obtained in refs. [18,19].

4 The fundamental optimal relation

Combining the change in the entropy of the working fluid heat absorbing process:

$$\mathrm{d}S_x=C\ln(1-Q_1/CT_{\rm H}),\quad (30)$$

with the condition of internal reversibility, one can introduce an equivalent temperature of the hot reservoir:

$$\overset{*}{T}_{\rm H}=-\frac{Q_1}{\mathrm{d}S_x}=-\frac{Q_1}{C\ln(1-Q_1/CT_{\rm H})},\quad (31)$$

and an equivalent temperature of the working fluid at the heat absorbing process:

$$\overset{*}{T}_1=T_2Q_1/Q_2,\quad (32)$$

where T_2 is the temperature of the working fluid at the heat releasing process. Therefore, one may derive

$$Q_1=\alpha(\overset{*}{T}{}_{\rm H}^n-\overset{*}{T}{}_1^n)^m t_1,\quad (33)$$

$$Q_2=\beta(T_2^n-T_{\rm L}^n)^m(\tau-t_1),\quad (34)$$

$$\eta=1-T_2\Big/\overset{*}{T}_1,\quad (35)$$

where η is the efficiency of the heat engine cycle.

Combining eqs. (31)—(35) gives the power output of the engine as

$$P = \alpha\eta \left\{ \frac{\alpha(1-\eta)}{\beta[(1-\eta)^n \overset{*}{T}{}_1^n - T_L^n]^m} + \frac{1}{(\overset{*}{T}{}_H^n - \overset{*}{T}{}_1^n)^m} \right\}^{-1}. \quad (36)$$

Taking the derivative of P with respective to $\overset{*}{T}_1$ and setting it equal to zero yield

$$\overset{*}{T}_1 = \left[\frac{(\alpha/\beta)^{1/(m+1)} \overset{*}{T}{}_H^n (1-\eta)^{(n+1)/(m+1)} + T_L^n}{(1-\eta)^n + (\alpha/\beta)^{1/(m+1)} (1-\eta)^{(n+1)/(m+1)}} \right]^{1/n}. \quad (37)$$

Substituting eq. (37) into (36) yields

$$P = \frac{\alpha\eta[\overset{*}{T}{}_H^n - T_L^n/(1-\eta)^n]^m}{\{1+[(\alpha/\beta)(1-\eta)^{1-mn}]^{1/(m+1)}\}^{m+1}}. \quad (38)$$

Eq. (38) is another major result of this paper. It determines the optimal relation between the power output and efficiency for the fixed heat input Q_1. Since $\overset{*}{T}_H$ in eqs. (37) and (38) is a function of Q_1, it is independent of Q_1 only if C approaches infinite. If C approaches infinite, eq. (38) becomes the fundamental optimal relation between the power output and efficiency of an endoreversible Carnot heat engine coupled to infinite thermal capacity (constant-temperature) heat reservoirs with a complex law $Q \propto (\Delta T^n)^m$[24].

The relations between the optimal power and efficiency with different values of m and n are shown in Figure 2 for fixed $Q_1 = 1000$ kJ with $\alpha/\beta = 1$, $T_H = 800$ K, $T_L = 300$ K, and $C = 10$ kJ/(kg·K). One will see that the power versus efficiency curve of the generalized endoreversible Carnot engine is a convex one, there is a maximum power output, and the larger the value of mn, the smaller the efficiency at $P = P_{max}$ point is when $n > 0$, and the larger the absolute value of mn, the smaller the efficiency at $P = P_{max}$ point is when $n < 0$.

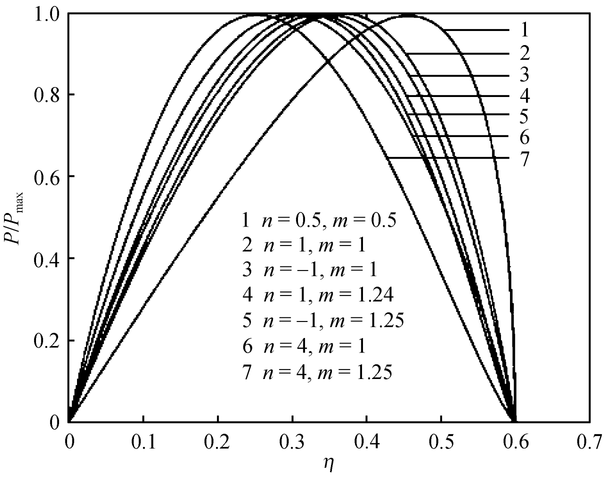

Figure 2 The relation between P/P_{max} and η with different values of m and n.

5 Conclusions

The optimal configuration and optimal performance of a heat engine operating between a finite high-temperature source and an infinite low-temperature reservoir is studied. In the engine model the only irreversibility of finite rate heat transfer is considered. The heat transfer obeys a complex heat transfer law, including Newtonian heat transfer law, linear phenomenological heat transfer law, radiative heat transfer law, Dulong-Petit heat transfer law, generalized convective heat transfer law and generalized radiative heat transfer law, $Q \propto \Delta(T^n)^m$. The power versus efficiency curve of the generalized endoreversible Carnot engine is a convex one, there is a maximum power output, and the bigger the value of mn, the smaller the efficiency at $P = P_{max}$ point is when $n > 0$; the bigger the absolute value of mn, the smaller the efficiency at $P = P_{max}$ point is when $n < 0$. The obtained results include those obtained in many literatures and can provide some theoretical guidance for the designs of practical engines.

The authors would like to thank the reviewers for their careful, unbiased and constructive suggestions, which led to this revised manuscript.

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