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Teleportation of an arbitrary two-qudit state based on the non-maximally four-qudit cluster state

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Two different schemes are presented for quantum teleportation of an arbitrary two-qudit state using a non-maximally four-qudit cluster state as the quantum channel. The first scheme is based on the Bell-basis measurements and the receiver may probabilistically reconstruct the original state by performing proper transformation on her particles and an auxiliary two-level particle; the second scheme is based on the generalized Bell-basis measurements and the probability of successfully teleporting the unknown state depends on those measurements which are adjusted by Alice. A comparison of the two schemes shows that the latter has a smaller probability than that of the former and contrary to the former, the channel information and auxiliary qubit are not necessary for the receiver in the latter.

quantum teleportation, qudit cluster state, generalized Bell-basis

Quantum entanglement[1−3], which has no classical analog, is one of the most notable features of quantum mechanics. It supplies many interesting applications in the field of information and the quantum teleportation is one of them. Quantum teleportation, a method of indirectly transmitting quantum information via dual resources of classical communication and previously shared entanglement, is one of the most profound achievements of quantum information theory. Since Bennett et al.[4] proposed the first teleportation protocol, quantum teleportation has been intensively investigated[5−17].

GHZ and W class states are two kinds of important entangled states and they are always used as the quantum channel in quantum information. Recently, Briegel and Raussendorf[18] introduced a new class of N-qubit entangled state, cluster state[19,20], which was different from both GHZ class and W class of N-qubit states. They presented that the cluster state was much more entangled than the GHZ state in some respects and they were more difficultly destroyed by local operations.

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Consequently, this class state was introduced to implement the quantum computation and quantum information processing[15,16,21,22].

In a realistic scenario, noisy effects and decoherence decrease the entanglement of the channel (channels). In this scenario, the quantum channels may be non-maximally entangled states. In fact, there have already been some teleportation schemes by using non-maximally entangled states as the quantum channel[12−14]. So it is significative to investigate the teleportation scheme by using a non-maximally entangled cluster state as the quantum channel. In this paper, we present two different schemes for teleporting an arbitrary two-qudit state by using a non-maximally entangled cluster state as the quantum channel.

1 Teleportation of an arbitrary two-qudit state based on the non-maximally four-qudit cluster state

In this part, two schemes are presented for teleporting an arbitrary two-qudit state by using a non-maximally four-qudit cluster state as the quantum channel. Suppose Alice has two qudits in an unknown state:

$$|\phi\rangle_{12}=\sum_{i_1=0}^{d-1}\sum_{i_2=0}^{d-1}\alpha_{i_1i_2}|i_1i_2\rangle_{12}, \tag{1}$$

where 1 and 2 label Alice's two qudits and $\{\alpha_{i_1i_2}\}$ are the unknown complex coefficients which satisfy the normalization condition, $\sum_{i_1=0}^{d-1}\sum_{i_2=0}^{d-1}|\alpha_{i_1i_2}|^2=1$. And Alice wants to teleport this state to a distant receiver Bob. We consider that Alice shares an entangled four-qudit cluster state in advance as the quantum channel with Bob:

$$\begin{aligned}|\psi\rangle_{3456}&=D\sum_{j=0}^{d-1}\sum_{k=0}^{d-1}\mathrm{e}^{2\pi\mathrm{i}jk/d}d_{jk}|jj\rangle|kk\rangle_{3456},\\|\psi\rangle_{3456}&=\frac{1}{d}\sum_{j=0}^{d-1}\sum_{k=0}^{d-1}\mathrm{e}^{2\pi\mathrm{i}jk/d}c_{jk}|jj\rangle|kk\rangle_{3456},\end{aligned} \tag{2}$$

where $\{d_{jk}\}$ and $\{c_{jk}\}$ are complex coefficients, $D=1\Big/\sqrt{\sum_{j=0}^{d-1}\sum_{k=0}^{d-1}|d_{jk}|^2}$ is the normalization constant and $\{c_{jk}\}$ satisfies the condition:

$$\frac{1}{d^2}\sum_{j=0}^{d-1}\sum_{k=0}^{d-1}|c_{jk}|^2=1. \tag{3}$$

Comparing the two equations in eq. (2), one obtains $c_{jk}=dDd_{jk}$. Particles 3 and 5 belong to the sender Alice, and particles 4 and 6 belong to the receiver Bob. Subsequently, the initial state of the system is written as

$$\begin{aligned}|\Psi\rangle_S&=|\phi\rangle_{12}\otimes|\psi\rangle_{3456}=\frac{1}{d}\sum_{i_1=0}^{d-1}\sum_{i_2=0}^{d-1}\sum_{j=0}^{d-1}\sum_{k=0}^{d-1}\alpha_{i_1i_2}c_{jk}\mathrm{e}^{2\pi\mathrm{i}jk/d}|i_1i_2\rangle_{12}|jj\rangle_{34}|kk\rangle_{56}\\&=D\sum_{i_1=0}^{d-1}\sum_{i_2=0}^{d-1}\sum_{j=0}^{d-1}\sum_{k=0}^{d-1}\alpha_{i_1i_2}d_{jk}\mathrm{e}^{2\pi\mathrm{i}jk/d}|i_1i_2\rangle_{12}|jj\rangle_{34}|kk\rangle_{56}.\end{aligned} \tag{4}$$

First, we describe the first teleportation scheme. In order to teleport the unknown state, Alice first makes the following Bell-basis measurements on her qudit pairs {1, 3} and {2, 5}:

$$|\Phi^{xy}\rangle=\frac{1}{\sqrt{d}}\sum_{k=0}^{d-1}\mathrm{e}^{2\pi\mathrm{i}kx/d}|k\rangle\otimes|k\oplus y\rangle, \tag{5}$$

where $k \oplus y$ means the sum of modulo d of k and y; the indices x and y take integer values between 0 and $d-1$. For the outcomes $\left|\Phi^{x_1 y_1}\right\rangle_{13}$ and $\left|\Phi^{x_2 y_2}\right\rangle_{25}$, the state of qudits 4 and 6 collapses into the following state:

$$|\psi\rangle_{46} = \frac{1}{d^2}\sum_{k_1,k_2=0}^{d-1} c_{k_1\oplus y_1,k_2\oplus y_2}\alpha_{k_1k_2}\mathrm{e}^{2\pi\mathrm{i}((k_1\oplus y_1)(k_2\oplus y_2)-k_1x_1-k_2x_2)/d}|k_1\oplus y_1\rangle_4|k_2\oplus y_2\rangle_6. \tag{6}$$

Then Alice informs Bob of her measurement results. After knowing the classical information from Alice, Bob should perform a proper local unitary operation to reconstruct the teleported state. For the outcomes $\left|\Phi^{x_1 y_1}\right\rangle_{13}$ and $\left|\Phi^{x_2 y_2}\right\rangle_{25}$, the corresponding local unitary operation is

$$U_{46} = \sum_{k_1=0}^{d-1}\sum_{k_2=0}^{d-1}\mathrm{e}^{2\pi\mathrm{i}(k_1x_1+k_2x_2-(k_1\oplus y_1)(k_2\oplus y_2))/d}\left(|k_1\rangle\langle k_1\oplus y_1|\right)\left(|k_2\rangle\langle k_2\oplus y_2|\right). \tag{7}$$

Consequently, the state of the receiver becomes

$$|\psi\rangle_{46} = \frac{1}{d^2}\sum_{k_1,k_2=0}^{d-1} c_{k_1\oplus y_1,k_2\oplus y_2}\alpha_{k_1k_2}|k_1\rangle_4|k_2\rangle_6. \tag{8}$$

Suppose $|c_{p_1p_2}|^2 = \min\{|c_{k_1k_2}|^2,\ k_1, k_2 = 0,\ 1,\ 2,\ \ldots\ d-1\}$. In order to extract information of the original state $|\phi\rangle$ from $|\psi\rangle_{46}$ in eq. (7) probabilistically, Bob performs a general evolution U on particles 4, 6 and an auxiliary qubit whose original state is $|0\rangle_{\text{aux}}$. In detail, on the basis $|0\rangle_4|0\rangle_6|0\rangle_{\text{aux}}$, $|0\rangle_4|0\rangle_6|0\rangle_{\text{aux}}$, $\ldots|d-1\rangle_4|d-1\rangle_6|0\rangle_{\text{aux}}$, $|0\rangle_4|0\rangle_6|0\rangle_{\text{aux}}$, $|0\rangle_4|0\rangle_6|0\rangle_{\text{aux}}$, $\ldots$ $|d-1\rangle_4|d-1\rangle_6|1\rangle_{\text{aux}}$, the collective unitary transformation U is chosen as

$$U = \begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix}, \tag{9}$$

where A_1 and A_2 are $d^2 \times d^2$ matrices which are expressed as

$$\begin{aligned} &A_1 = \mathrm{diag}(a_{00},\ a_{01},\ \ldots\ a_{0d-1},\ a_{10},\ \ldots\ a_{d-1d-1}),\\ &A_2 = \mathrm{diag}\left(\sqrt{1-|a_{00}|^2},\ \sqrt{1-|a_{01}|^2},\ \ldots\ \sqrt{1-|a_{0d-1}|^2},\ \sqrt{1-|a_{10}|^2},\ \ldots\ \sqrt{1-|a_{d-1d-1}|^2}\right),\\ &a_{ij} = \frac{c_{p_1p_2}}{c_{(y_1\oplus i)(y_2\oplus j)}},\quad i, j = 0,\ 1,\ \ldots\ d-1. \end{aligned} \tag{10}$$

Then the state of the particles in the hands of Bob becomes

$$\begin{aligned} &U|\psi\rangle_{46}|0\rangle_{\text{aux}}\\ &= \frac{1}{d^2}\sum_{k_1,k_2=0}^{d-1} c_{k_1\oplus y_1,k_2\oplus y_2}\alpha_{k_1k_2}|k_1\rangle_4|k_2\rangle_6\left(\frac{c_{p_1p_2}}{c_{k_1\oplus y_1,k_2\oplus y_2}}|0\rangle_{\text{aux}} + \sqrt{1-\left|\frac{c_{p_1p_2}}{c_{k_1\oplus y_1,k_2\oplus y_2}}\right|^2}|1\rangle_{\text{aux}}\right). \end{aligned} \tag{11}$$

Bob measures his auxiliary particle after performing the unitary transformation U. The teleportation succeeds if the measurement result is $|0\rangle_{\text{aux}}$; otherwise the teleportation fails, and the information of the original state disappears.

The maximal probability for extracting the unknown state with the fidelity unit from the state

$|\psi\rangle_{46}$ is $P_{1\max} = d^4 \times \left|\frac{c_{p_1p_2}}{d^2}\right|^2 = |c_{p_1p_2}|^2$, i.e., the square of the minimal coefficient.

Now we will describe the second scheme. We introduce a set of generalized entangled Bell state as follows:

$$\left|\Phi^{xy}\right\rangle = D^{xy}\sum_{k=0}^{d-1} \mathrm{e}^{2\pi \mathrm{i}kx/d} d_k^{xy} \left|k\right\rangle \otimes \left|k \oplus y\right\rangle, \tag{12}$$

where $x, y \in \{0, 1, \ldots d-1\}$ and $D^{xy} = 1\Big/\sqrt{\sum_{k=0}^{d-1} |d_k^{xy}|^2}$ which is the normalization constant. The coefficients $\{d_k^{xy}\}$ should satisfy the following orthonormal condition:

$$D^{x_1y}D^{x_2y}\sum_{k=0}^{d-1} d_k^{*x_1y} d_k^{x_2y} \mathrm{e}^{2\pi \mathrm{i}k(x_2-x_1)/d} = \delta^{x_1x_2}. \tag{13}$$

Thus the state of the system is rewritten as

$$\left|\Psi\right\rangle_s = \frac{1}{d} D \sum_{x_1,y_1=0}^{d-1}\sum_{x_2,y_2=0}^{d-1} \left|\Phi^{x_1y_1}\right\rangle_{13} \left|\Phi^{x_2y_2}\right\rangle_{25} D^{x_1y_1}D^{x_2y_2}$$
$$\times\left(\sum_{k_1,k_2=0}^{d-1} \mathrm{d}_{k_1}^{*x_1y_1}\mathrm{d}_{k_2}^{*x_2y_2}\mathrm{d}_{k_1\oplus y_1,k_2\oplus y_2}\alpha_{k_1k_2}\mathrm{e}^{2\pi\mathrm{i}((k_1\oplus y_1)(k_2\oplus y_2)-k_1x_1-k_2x_2)/d}\left|k_1\oplus y_1\right\rangle_4\left|k_2\oplus y_2\right\rangle_6\right). \tag{14}$$

Now Alice performs the generalized Bell-basis $\left\{\left|\Phi^{xy}\right\rangle\right\}$ measurements on her qudit pairs $\{1, 3\}$ and $\{2, 5\}$. For the outcomes $\left|\Phi^{x_1y_1}\right\rangle_{13}$ and $\left|\Phi^{x_2y_2}\right\rangle_{25}$, the state of qudits 4 and 6 collapses into the following state:

$$\left|\Psi\right\rangle_{46} = \frac{1}{d} DD^{x_1y_1}D^{x_2y_2}\sum_{k_1,k_2=0}^{d-1} d_{k_1}^{*x_1y_1} d_{k_2}^{*x_2y_2} d_{k_1\oplus y_1,k_2\oplus y_2}\alpha_{k_1k_2}$$
$$\times\, \mathrm{e}^{2\pi\mathrm{i}((k_1\oplus y_1)(k_2\oplus y_2)-k_1x_1-k_2x_2)/d}\left|k_1\oplus y_1\right\rangle_4\left|k_2\oplus y_2\right\rangle_6. \tag{15}$$

Then Alice informs Bob of her measurement results. After knowing the results from Alice, Bob reconstructs the unknown state on particles 4 and 5 by performing proper local unitary operation. For the outcomes $\left|\Phi^{x_1y_1}\right\rangle_{13}$ and $\left|\Phi^{x_2y_2}\right\rangle_{25}$, the corresponding local unitary operation is U_{46} in eq. (7). Consequently, the state of the receiver becomes

$$\left|\Psi\right\rangle_{46} = \frac{1}{d} DD^{x_1y_1}D^{x_2y_2}\sum_{k_1,k_2=0}^{d-1} d_{k_1\oplus y_1}^{*x_1y_1} d_{k_2\oplus y_2}^{*x_2y_2} d_{k_1\oplus y_1,k_2\oplus y_2}\alpha_{k_1k_2}\left|k_1\right\rangle_4\left|k_2\right\rangle_6. \tag{16}$$

From eq. (16), we find that the measuring basis parameters $\{d_{k_1}^{x_1y_1}, d_{k_2}^{x_2y_2}\}$ are freely manipulated by Alice. By properly adjusting them, Alice achieves a unity fidelity protocol with a probability. Obviously, for the outcomes $\left|\Phi^{x_1y_1}\right\rangle_{13}$ and $\left|\Phi^{x_2y_2}\right\rangle_{25}$, the protocol will be successful with a probability when

$$\{d_{k_1}^{x_1y_1}d_{k_2}^{x_2y_2}\} = \left\{\frac{1}{d^*_{k_1\oplus y_1,k_2\oplus y_2}}\right\}. \tag{17}$$

We note that those parameters must also satisfy the orthonormal condition in eq. (13). Combining the two conditions, we get

$$\sum_{k_1=0}^{d-1}\sum_{k_2=0}^{d-1}\frac{1}{|d_{k_1\oplus y_1,k_2\oplus y_2}|^2}\frac{1}{\sum_{j_1=0}^{d-1}\sum_{j_2=0}^{d-1}\frac{1}{|d_{j_1\oplus y_1,j_2\oplus y_2}|^2}}\mathrm{e}^{2\pi\mathrm{i}(k_1x_{12}+k_2x_{22}-k_1x_{11}-k_2x_{21})}=\delta^{x_{11}x_{12}}\delta^{x_{21}x_{22}}. \quad (18)$$

This equation shows us that the two conditions are satisfied simultaneously only when $x_{11}=x_{12}$ and $x_{21}=x_{22}$ if the channel is non-maximal. That is to say, the measurement results, which can lead to successful protocol, are at most d^2 out of the total d^4 possible results of Alice. So we explicitly calculate the maximal probability of success in teleporting an unknown single-qudit state:

$$P_{2\max}=d^2\,|DD^{x_1y_1}D^{x_2y_2}|^2=\frac{|D|^2\,d^2}{\sum_{k_1=0}^{d-1}\sum_{k_2=0}^{d-1}\left(\frac{1}{|d_{k_1k_2}|^2}\right)}=\frac{1}{\sum_{k_1=0}^{d-1}\sum_{k_2=0}^{d-1}\left(\frac{1}{|c_{k_1k_2}|^2}\right)}. \quad (19)$$

In other words, the successful probability is dependent on the Bell-basis measurement chosen by Alice. By adjusting the parameters $\left\{d_{k_1}^{x_1y_1},\, d_{k_2}^{x_2y_2}\right\}$, she controls the successful probability to get the value of $n^2\,|DD^{x_1y_1}D^{x_2y_2}|^2$, where n takes integer values between 0 and d.

2 Discussion and conclusions

Consider the successful probability in the two schemes when the channel is a maximal one. In the first scheme, the maximal probability $P_{1\max}$ for extracting the unknown state with the fidelity unit from the state $|\psi\rangle_{46}$ is $|c_{p_1p_2}|^2$, where $c_{p_1p_2}$ is the smallest one among the coefficients $\{c_{kj}\}$. So for the maximal entangled channel, i.e., $c_{kj}=1$ for all k and j values from 0 to $d-1$, the total successful probability reaches one. And in the second scheme, the conditions in eqs. (13) and (17) will be satisfied without any restriction in this case. That is to say, any result of Alice can lead to successful protocol, i.e., the successful probability will reach one.

Comparing the two probabilities $P_{1\max}$ and $P_{2\max}$, we find

$$\frac{P_{1\max}}{P_{2\max}}=|c_{p_1p_2}|^2\sum_{k_1=0}^{d-1}\sum_{k_2=0}^{d-1}\left(\frac{1}{|c_{k_1k_2}|^2}\right)=\sum_{k_1=0}^{d-1}\sum_{k_2=0}^{d-1}\left(\left|\frac{c_{p_1p_2}}{c_{k_1k_2}}\right|^2\right)>1. \quad (20)$$

This equation implies that the successful probability in the first scheme is larger than that of the second one. However, the latter scheme also has its merits: The receiver in this scheme need not know the channel information and also need not introduce an auxiliary qubit to extract the unknown state; moreover, the successful probability is dependent on the Bell-basis measurements chosen by Alice.

In summary, two different schemes are presented for quantum teleportation of an arbitrary two-qudit state by using a non-maximally four-qudit cluster state as the quantum channel. In the first scheme, the receiver probabilistically extracts the information of the originally unknown state by performing a general evolution on her particles and an auxiliary two-level particle. And in the second scheme, the sender Alice should introduce a generalized Bell-basis state as the joint measurement basis and the probability of successfully teleporting an unknown state is dependent on the joint measurements which are adjusted by Alice. By comparing the two schemes, we show that the latter has a smaller probability than that of the former. And contrary to the former, the channel information and auxiliary qubit are not necessary for the receiver in the latter.

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