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Decentralized output-feedback control of triangular large-scale nonlinear impulsive systems with time-varying delays: a gain scaling approach

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Abstract In this study, we investigate the decentralized output-feedback control problem for a class of triangular large-scale nonlinear impulsive systems (TLSNISs) with time-varying delays. Unlike existing design approaches in impulsive systems, a gain scaling approach is proposed for the first time to counteract the structural uncertainties of interconnected nonlinearities. Specifically, by fully exploiting the static gain, novel delay-independent impulsive observers are delicately constructed to estimate the unavailable states. Furthermore, the undesirable effects of time-varying delays and impulsive disturbances are eliminated using the comparison principle and average impulsive interval technique. The designed gain-scaling-based decentralized output-feedback controllers have concise linear-like forms and are independent of time delays. Moreover, by strengthening the gain scaling mechanism, we further develop an improved control scheme that endows the controllers with the capability to tolerate unknown external disturbances, thus improving its robustness. It is shown that the system states converge exponentially to the origin in the disturbance-free case with the designed controllers (or to an adjustable neighborhood of the origin in the presence of disturbance). Finally, two examples, including an engineering system design example, are provided to demonstrate the effectiveness of the designed controllers for both lower TLSNISs and upper TLSNISs.

Keywords triangular large-scale systems, nonlinear impulsive systems, time-varying delay, output feedback, gain scaling approach

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1 Introduction

Over the past few decades, impulsive systems have gained much attention due to their widespread applications in electronic communication, satellite orbital transmission, and chemical industries; see [1–3]. Impulsive systems, an important class of hybrid systems, consist of three parts: the continuous-time dynamics, which is described by differential equations and exhibits the evolution of system states between impulses; the discrete-time dynamics, which is represented by difference equations and describes the abrupt changes in system states at impulse moments; and an impulsive law, which determines when those impulses occur [4].

The impulsive disturbance problem is known to be a widely studied area in the field of impulsive systems [5–7]. It is essentially a control problem in which control systems without impulse effects have certain performance and yet maintain corresponding performance even when subjected to abrupt disturbances [7]. Impulsive disturbances are encountered in a wide variety of fields, including biology, electricity, and mechanics. For example, in biological neural networks, electrical stimuli transmitted through receptors can be regarded as impulsive disturbances [7]. Impulsive disturbances are a class of discontinuous disturbances that can lead to some complex dynamical behaviors, including instabilities and oscillations. Up to now, valuable results have been obtained by scholars in the field of impulsive systems with impulsive disturbances; see [5–9]. Remarkably, most existing results were restricted to state-feedback control

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strategy obtained within the framework of a single system. In fact, nearly all industrial systems are nonlinear and contain coupled interconnected subsystems, such as power systems [10] and transportation networks [11]. Moreover, the states of such systems may not be fully available due to physical constraints or implementation costs, which suggests that there may be significant challenges for state-feedback control strategies in practical systems. In recent years, triangular large-scale nonlinear systems, which consist of multiple interconnected subsystems, have been attracting significant attention from researchers [12]. However, when impulsive disturbances are involved, existing results on triangular large-scale nonlinear systems with continuous-time dynamics, such as [10–13], are invalid due to the discontinuity of system states. Consequently, the research on triangular large-scale nonlinear impulsive systems (TLSNISs) is still not performed until now, not to mention in the case of output-feedback control.

Time delays exist in many practical systems, and are an important factor causing performance deterioration and even instability; see [14–16]. Therefore, it is crucial to address these undesirable effects. Recently, many excellent results have been reported for systems containing time delays; see [17–19]. From these results, we can see that the Lyapunov-Krasovskii functional (LKF) is a key tool for analyzing time-delay systems. However, as mentioned in [20, 21], constructing an LKF is not just a simple extension of the delay-free case. In particular, a strict restriction needs to be imposed on the type of time delay, i.e., slow time delay $\tau(t)$ with $\dot{\tau}(t) < 1$, in the LKF design framework, which makes it very difficult, and even impossible, to solve fast time delay problems using the LKF tool. Therefore, it is extremely challenging to study TLSNISs with more general time delays.

The gain scaling approach is known to be an effective tool for control problems of nonlinear systems with triangular structures [22–24]. The controller construction problem can be transformed into a gain design problem by introducing a suitable scaling gain in the state transformation [25]. When the system uncertainties arising from system parameters and/or system nonlinearities are relatively mild, such as a known constant growth rate, a static/constant gain [26] or a bounded time-varying gain [27] is usually enough to achieve the control objective. The gain scaling approach can avoid the complex design procedure involved in an iterative design approach and can construct a controller with a concise form [28]. Hence, the gain scaling approach is favored by many scholars due to its unique advantages, especially in the realm of output-feedback control.

To the best of our knowledge, no prior work has been devoted to studying the decentralized output-feedback control problem for TLSNISs with time-varying delays. Moreover, many existing results in impulsive systems are obtained based on the linear matrix inequality (LMI) approach without incorporating the gain scaling approach into the design framework. It should be noted that many existing LMI-based results are invalid when serious structural uncertainties exist in system nonlinearities. Motivated by these considerations, our objective is to apply the gain scaling approach to study the decentralized output-feedback control problem for a class of TLSNISs with time-varying delays. The main contributions of this paper are summarized as follows.

- In terms of system model: The novelty of the system model can be distilled into the following three points. (i) The most significant is that impulsive disturbances have been considered for the first time in the framework of triangular large-scale nonlinear systems, which is significantly more challenging than it is for either single impulsive systems [7, 29] or triangular systems with continuous-time dynamics [11, 12, 30] due to the effect of interconnected subsystems and discrete-time dynamics. (ii) Compared with studies by [7, 11, 12, 30], a more general type of time-varying delay is considered in this paper. The boundaries of the time delays are allowed to be unknown, which means that the system dynamics can involve unmeasurable time delays. Additionally, the derivative restrictions on time delays are removed, which indicates that either slow or fast time delays can be contained in system dynamics. (iii) Disturbance evaluations are carried out for lower TLSNISs and upper TLSNISs, respectively, which is more in line with practical requirements. More detailed comparisons can be found in Table 1.
- In terms of design approach: In contrast to the LMI-based design approach for impulsive systems in existing studies [7, 29], a gain scaling approach is expected to compensate for the structural uncertainties of interconnected nonlinearities. Moreover, compared with iterative design approaches, including the backstepping design approach [31,32] and forwarding and saturation design approach [33,34], the proposed design approach can simplify design processes and reduce complex calculations. Crucially, the gain scaling approach provides a new design perspective for lower TLSNISs and upper TLSNISs, respectively.
- In terms of control strategy: Unlike the state-feedback control strategies in [7, 30], two decentralized output-feedback control strategies are designed. Particularly, two novel delay-independent impulsive observers are constructed to estimate the unavailable states. Moreover, the designed decentralized output-

feedback controllers have concise linear-like forms and are independent of time delays.

The remainder of the paper is organized as follows. Preliminaries and problem formulation are described in Section 2. The decentralized output-feedback controllers of the lower and upper TLSNISs are designed in Sections 3 and 4, respectively. In Section 5, two examples are given to demonstrate the effectiveness of the designed controllers. Finally, the conclusion is presented in Section 6.

Notations. Let $\mathbb{Z}^+$, $\mathbb{R}$, $\mathbb{R}^+$, and $\mathbb{R}^{n_k}$ denote the set of positive integer numbers, the set of real numbers, the set of non-negative real numbers, and the real n_k-dimensional vector space, respectively. $\mathbf{0}$ and I are used to represent the zero column vector and identity matrix with appropriate dimensions, respectively. A^{T} denotes the transpose of the matrix A. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ denote the minimum and maximum eigenvalues of the matrix A, respectively. For any interval $S_1 \subseteq \mathbb{R}$ and any set $S_2 \subseteq \mathbb{R}^i$, $1 \leqslant i \leqslant 2n_k$, we put $\mathrm{PC}(S_1, S_2) = \{\varphi\colon\ S_1 \to S_2$ is continuous everywhere except at a finite number of points t, at which $\varphi(t^-)$, $\varphi(t^+)$ exist and $\varphi(t^+) = \varphi(t)\}$. For a, $b \in \mathbb{R}$, $a < b$, let $\mathrm{PC}([a,b],\mathbb{R}^i)$ denote the set of the piecewise right continuous function φ: $[a,b] \to \mathbb{R}^i$ with the norm defined by $\|\varphi\|_{[a,b]} = \sup_{a\leqslant t\leqslant b}\|\varphi(t)\|$. For given $\bar{\tau} > 0$, let $\mathbb{PC}^i_{\bar{\tau}} = \mathrm{PC}([t_0 - \bar{\tau}, t_0], \mathbb{R}^i)$ and $\|\varphi\|_{\bar{\tau}} = \|\varphi\|_{[t_0-\bar{\tau},t_0]}$.

2 Preliminaries and problem formulation

In this section, preliminaries and problem formulation are described in detail.

2.1 Preliminaries

Definition 1 ([5])**.** The average impulsive interval of the impulse time sequence $\{t_m\}_{m\in\mathbb{Z}^+}$ is said to be not less than ϑ, if there exist positive constants N_0 and ϑ, such that, for $t_0 \leqslant s \leqslant t$,

$$N(t,s) \leqslant \frac{t-s}{\vartheta} + N_0,$$

where $N(t,s)$ is the number of impulses of the time sequence $\{t_m\}_{m\in\mathbb{Z}^+}$ occurring on the interval $(s,t]$, ϑ is the average impulsive interval constant, and N_0 is the elasticity number.

Lemma 1 ([8])**.** Let functions $\phi_1(t)$, $\phi_2(t) \in \mathrm{PC}([t_0 - \bar{\tau}, +\infty), \mathbb{R}^+)$, and $h(t) \in \mathrm{PC}([t_0, +\infty), \mathbb{R}^+)$. If there exist $\bar{\sigma}_1 \in \mathbb{R}$, $\bar{\sigma}_2 \in \mathbb{R}^+$, and $\bar{\sigma}_3 \in \mathbb{R}^+$, such that

$$\begin{cases} D^+\phi_1(t) \leqslant \bar{\sigma}_1\phi_1(t) + \bar{\sigma}_2\phi_1(t-\tau(t)) + h(t),\ t \neq t_m,\ t \geqslant t_0, \\ \phi_1(t) \leqslant \bar{\sigma}_3\phi_1(t^-),\ t = t_m,\ m \in \mathbb{Z}^+, \end{cases}$$

and

$$\begin{cases} D^+\phi_2(t) > \bar{\sigma}_1\phi_2(t) + \bar{\sigma}_2\phi_2(t-\tau(t)) + h(t),\ t \neq t_m,\ t \geqslant t_0, \\ \phi_2(t) \geqslant \bar{\sigma}_3\phi_2(t^-),\ t = t_m,\ m \in \mathbb{Z}^+, \end{cases}$$

then $\phi_1(t) \leqslant \phi_2(t)$ for $t_0 - \bar{\tau} \leqslant t \leqslant t_0$ implies that $\phi_1(t) \leqslant \phi_2(t)$ for $t \geqslant t_0$.

Lemma 2 ([8])**.** Assume that there exist functions $\phi(t) \in \mathrm{PC}([t_0-\bar{\tau}, +\infty), \mathbb{R}^+)$, $h(t) \in \mathrm{PC}([t_0, +\infty), \mathbb{R}^+)$, constants $\bar{\sigma}_1 \in \mathbb{R}$, $\bar{\sigma}_2 \in \mathbb{R}^+$, and $\bar{\sigma}_3 \in \mathbb{R}^+$, such that

$$\begin{cases} D^+\phi(t) \leqslant \bar{\sigma}_1\phi(t) + \bar{\sigma}_2\phi(t-\tau(t)) + h(t),\ t \neq t_m,\ t \geqslant t_0, \\ \phi(t) \leqslant \bar{\sigma}_3\phi(t^-),\ t = t_m,\ m \in \mathbb{Z}^+. \end{cases}$$

It then holds that, for $t \geqslant t_0$,

$$\phi(t) \leqslant \phi(t_0)\bar{\sigma}_3^{N(t,t_0)}\mathrm{e}^{\bar{\sigma}_1(t-t_0)} + \int_{t_0}^{t} \bar{\sigma}_3^{N(t,s)}\mathrm{e}^{\bar{\sigma}_1(t-s)}(\bar{\sigma}_2\phi(s-\tau(s)) + h(s))\mathrm{d}s.$$

Lemma 3 ([12,22])**.** There exist real numbers $a_{k,i}$, $b_{k,i}$, $i = 1, 2, \ldots, n_k$, and positive definite matrices P_k, $k = 1, 2, \ldots, \mathcal{N}$, satisfying the following inequalities:

$$P_k\Xi_k + \Xi_k^{\mathrm{T}}P_k \leqslant -I_{2n_k},\ k = 1, 2, \ldots, \mathcal{N},$$

where $\mathcal{N} \in \mathbb{Z}^+$ denotes the number of matrices, $\Xi_k = \left[\begin{smallmatrix} B_k & a_k\bar{\Gamma}_k^{\mathrm{T}} \\ 0_{n_k\times n_k} & A_k \end{smallmatrix}\right]$, $A_k = \Lambda_k - a_k\bar{\Gamma}_k^{\mathrm{T}}$, $B_k = \Lambda_k - \underline{\Gamma}_k b_k^{\mathrm{T}}$, $\Lambda_k = \left[\begin{smallmatrix} \mathbf{0}_{n_k-1} & I_{(n_k-1)\times(n_k-1)} \\ 0 & \mathbf{0}_{n_k-1}^{\mathrm{T}} \end{smallmatrix}\right]$, $a_k = [a_{k,1}, a_{k,2}, \ldots, a_{k,n_k}]^{\mathrm{T}}$, $b_k = [b_{k,1}, b_{k,2}, \ldots, b_{k,n_k}]^{\mathrm{T}}$, $\bar{\Gamma}_k = [1, 0, \ldots, 0]^{\mathrm{T}} \in \mathbb{R}^{n_k}$, and $\underline{\Gamma}_k = [0, \ldots, 0, 1]^{\mathrm{T}} \in \mathbb{R}^{n_k}$, $k = 1, 2, \ldots, \mathcal{N}$.

2.2 Problem formulation

Consider the following large-scale nonlinear impulsive system with time-varying delays:

$$\begin{cases}\begin{cases}\dot{x}_{k,i}(t)=x_{k,i+1}(t)+f_{k,i}(t,x(t),u(t),x(t-\tau(t)),u(t-\tau(t))),\ i=1,2,\ldots,n_k-1,\\ \dot{x}_{k,n_k}(t)=u_k(t)+f_{k,n_k}(t,x(t),u(t),x(t-\tau(t)),u(t-\tau(t))),\\ y_k(t)=x_{k,1}(t),\ t\neq t_m,\ t\geqslant t_0,\end{cases}\\ x_{k,j}(t)=g_{k,j}(t^-,x(t^-)),\ j=1,2,\ldots,n_k,\ t=t_m,\ m\in\mathbb{Z}^+,\\ x_k(t)=\varsigma_k(t)\in\mathbb{PC}^{n_k}_{\bar{\tau}},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases} \tag{1}$$

where $x_k(t)=[x_{k,1}(t),x_{k,2}(t),\ldots,x_{k,n_k}(t)]^{\mathrm{T}}\in\mathbb{R}^{n_k}$ $(n_k\geqslant 2)$, $u_k(t)\in\mathbb{R}$, and $y_k(t)\in\mathbb{R}$ are the state, input, and output of the kth subsystem; $x(t)=[x_1^{\mathrm{T}}(t),x_2^{\mathrm{T}}(t),\ldots,x_N^{\mathrm{T}}(t)]^{\mathrm{T}}\in\mathbb{R}^{n_1+n_2+\cdots+n_N}$; $u(t)=[u_1(t),u_2(t),\ldots,u_N(t)]^{\mathrm{T}}\in\mathbb{R}^N$; N denotes the number of subsystems; t_0 is the initial moment; $\dot{x}_{k,i}(t)$ is the upper right-hand derivative of $x_{k,i}(t)$; $\tau(t)$ denotes the time-varying delay; $f_{k,i}(\cdot)$: $\mathbb{R}^+\times\mathbb{R}^{n_1+n_2+\cdots+n_N}\times\mathbb{R}^N\times\mathbb{R}^{n_1+n_2+\cdots+n_N}\times\mathbb{R}^N\rightarrow\mathbb{R}$, $i=1,2,\ldots,n_k$ are called unknown nonlinear interconnected functions between the kth subsystem and other subsystems, and $g_{k,i}(\cdot)$: $\mathbb{R}^+\times\mathbb{R}^{n_1+n_2+\cdots+n_N}\rightarrow\mathbb{R}$, $i=1,2,\ldots,n_k$ are called impulsive jump functions. To prevent the occurrence of accumulation points, such as Zeno phenomenon, the impulse time sequence $\{t_m\}_{m\in\mathbb{Z}^+}$ is considered to satisfy $0\leqslant t_0<t_1<\cdots<t_m\rightarrow+\infty$, as $m\rightarrow+\infty$. Throughout this paper, we assume that the solutions of all dynamical systems are right continuous, i.e., $x(t)=x(t^+)=\lim_{\epsilon\rightarrow 0^+}x(t+\epsilon)$. Next, we make the following mild assumptions for system (1).

Assumption 1. The time-varying delay $\tau(t)$ is unknown, and there exists an unknown nonnegative constant $\bar{\tau}$, such that

$$0\leqslant\tau(t)\leqslant\bar{\tau}.$$

Assumption 2. For nonlinear impulsive jump functions $g_{k,i}(\cdot)$, $i=1,2,\ldots,n_k$, the structure is known, $g_{k,i}(t,\mathbf{0})\equiv 0$, and there exist known constants $\mu_k\geqslant 1$, $k=1,2,\ldots,N$, such that, for all $\hbar(t)$ and $\hat{\hbar}(t)$,

$$|g_{k,i}(t,\hbar(t))-g_{k,i}(t,\hat{\hbar}(t))|\leqslant\mu_k|\hbar_{k,i}(t)-\hat{\hbar}_{k,i}(t)|,$$

where $\hbar(t)=[\hbar_{1,1}(t),\ldots,\hbar_{k,i}(t),\ldots,\hbar_{N,n_N}(t)]^{\mathrm{T}}$ and $\hat{\hbar}(t)=[\hat{\hbar}_{1,1}(t),\ldots,\hat{\hbar}_{k,i}(t),\ldots,\hat{\hbar}_{N,n_N}(t)]^{\mathrm{T}}$.

Assumption 3. The nonlinear interconnected functions $f_{k,i}(\cdot)$, $i=1,2,\ldots,n_k$, $k=1,2,\ldots,N$ satisfy either (i) or (ii).

(i) For $i=1,2,\ldots,n_k$, there exist known nonnegative constants θ_k, $k=1,2,\ldots,N$, such that

$$|f_{k,i}(\cdot)|\leqslant\theta_k\sum_{p=1}^{N}\left(\sum_{q=1}^{\min\{n_p,i\}}\left(|x_{p,q}(t)|+|x_{p,q}(t-\tau(t))|\right)\right).$$

(ii) For $i=1,2,\ldots,n_k$, there exist known nonnegative constants $\bar{\theta}_k$, $k=1,2,\ldots,N$, such that

$$|f_{k,i}(\cdot)|\leqslant\bar{\theta}_k\sum_{p=1}^{N}\left(\sum_{q=\max\{i+2+n_p-n_k,1\}}^{n_p+1}\left(|x_{p,q}(t)|+|x_{p,q}(t-\tau(t))|\right)+|u_p(t)|+|u_p(t-\tau(t))|\right),$$

where $f_{k,n_k}(\cdot)=0$ and $x_{k,n_k+1}(t)=x_{k,n_k+1}(t-\tau(t))=0$.

Remark 1. Assumption 1 allows the boundary of the time delay to be unknown and has no restriction on the derivative of time delay, which is weaker than assumptions in [7, 11, 12, 30]. It can be seen that the type of time delay involved in the system dynamics can be unmeasurable, slow or fast. Notably, due to the existence of impulsive disturbances, Assumption 2, as one of the most significant highlights, makes the considered system essentially different from systems with continuous-time dynamics [11,12,30]. Moreover, compared with [7,29], Assumption 2 allows impulsive disturbances to be nonlinear rather than merely linear.

Remark 2. In Assumption 3, the restrictions on the state types of nonlinear interconnected functions are different. Generally speaking, system (1) satisfying (i) belongs to a class of lower TLSNISs, while system

Table 1 Comparisons between our work and existing relevant studies

Ref.	Subsystem number	Impulsive disturbance	Time delay	External disturbance	Structure of nonlinearities	Design approach	Control strategy
[7]	1	Linear	No	Matched	Known, $f_{k,i}=0$	LMI-based approach	State feedback
[30]	1	No	No	Matched	Unknown	Gain scaling approach	State feedback
[29]	1	Linear	Slow/ fast	Matched & unmatched	Known	LMI-based approach	Output feedback
[11]	$N \geqslant 1$	No	Slow	No	Unknown	Gain scaling approach	Output feedback
[12]	$N \geqslant 1$	No	Slow	No	Unknown	Gain scaling approach	Output feedback
This paper	$N \geqslant 1$	Nonlinear	Slow/ fast	Matched & unmatched	Unknown	Gain scaling approach	Output feedback

(1) satisfying (ii) belongs to a class of upper TLSNISs. Both classes of systems have extensive research backgrounds in fields such as mechanics and electricity. For example, referring to [35], the coupled inverted pendulums can be modeled as a class of lower TLSNISs due to the abrupt changes in angle and angular velocity arising from the irregular spring contraction; the liquid level control resonant circuit systems can be modeled as a class of upper TLSNISs in view of the persistent impulsive disturbances generated by frequent trigger switching and unknown ground network factors. Due to the different restrictions on nonlinear interconnected functions, the impulsive observers constructed for lower TLSNISs will not apply to upper TLSNISs. Therefore, it brings great challenges to accurately estimate the unavailable states of lower TLSNISs and upper TLSNISs, respectively. In Table 1, we present some comparisons of our work and relevant studies. Up to now, no prior work has considered the control design for system (1) satisfying Assumptions 1–3.

Control objective. This paper aims to design the decentralized output-feedback controllers, such that all the signals of the resulting closed-loop impulsive system are globally bounded, and system states $x_{k,i}(t)$, $i=1,2,\ldots,n_k$, $k=1,2,\ldots,N$ converge exponentially to the origin in the disturbance-free case (or to an adjustable neighborhood of the origin in the presence of disturbance).

3 Decentralized output-feedback control of lower TLSNISs

In this section, we design the decentralized output-feedback controllers for system (1) under Assumptions 1, 2, and 3(i).

3.1 Decentralized output-feedback controllers design of lower TLSNISs

First, to estimate unavailable states of system (1), a novel delay-independent high-gain impulsive observer is constructed for the kth subsystem as follows:

$$\begin{cases} \begin{cases} \dot{\hat{x}}_{k,i}(t) = \hat{x}_{k,i+1}(t) + a_{k,i} r^i (y_k(t) - \hat{x}_{k,1}(t)),\ i=1,2,\ldots,n_k-1, \\ \dot{\hat{x}}_{k,n_k}(t) = u_k(t) + a_{k,n_k} r^{n_k} (y_k(t) - \hat{x}_{k,1}(t)),\ t \neq t_m,\ t \geqslant t_0, \end{cases} \\ \hat{x}_{k,j}(t) = g_{k,j}(t^-, \hat{x}(t^-)),\ j=1,2,\ldots,n_k,\ t=t_m,\ m\in\mathbb{Z}^+, \\ \hat{x}_k(t) = \hat{\varsigma}_k \in \mathbb{R}^{n_k},\ t\in[t_0-\bar{\tau}, t_0],\ k=1,2,\ldots,N, \end{cases} \tag{2}$$

where $a_{k,i}$, $i=1,2,\ldots,n_k$ are constants determined in Lemma 3, $\hat{x}_k(t) = [\hat{x}_{k,1}(t), \hat{x}_{k,2}(t), \ldots, \hat{x}_{k,n_k}(t)]^{\mathrm{T}}$, $\hat{\varsigma}_k = [\hat{\varsigma}_{k,1}, \hat{\varsigma}_{k,2}, \ldots, \hat{\varsigma}_{k,n_k}]^{\mathrm{T}}$, $k=1,2,\ldots,N$, $\hat{x}(t) = \left[\hat{x}_1^{\mathrm{T}}(t), \hat{x}_2^{\mathrm{T}}(t), \ldots, \hat{x}_N^{\mathrm{T}}(t)\right]^{\mathrm{T}}$, and r is a static gain to be determined later.

Then, the following state transformations are introduced, for $t \geqslant t_0 - \bar{\tau}$,

$$\eta_{k,i}(t) = \frac{\hat{x}_{k,i}(t)}{r^i},\ e_{k,i}(t) = \frac{x_{k,i}(t) - \hat{x}_{k,i}(t)}{r^i},\ i=1,2,\ldots,n_k,\ k=1,2,\ldots,N. \tag{3}$$

Next, the decentralized output-feedback controllers are designed as follows, for $t \geqslant t_0$,

$$u_k(t) = -r^{n_k+1} \sum_{i=1}^{n_k} b_{k,i} \eta_{k,i}(t),\ k=1,2,\ldots,N, \tag{4}$$

where $b_{k,i},\ i=1,2,\ldots,n_k,\ k=1,2,\ldots,N$ are constants determined in Lemma 3.

By (2)–(4), we have

$$\begin{cases}\begin{cases}\dot{\eta}_{k,i}(t)=r\eta_{k,i+1}(t)+ra_{k,i}e_{k,1}(t),\ i=1,2,\ldots,n_k-1,\\ \dot{\eta}_{k,n_k}(t)=-r\sum_{j=1}^{n_k}b_{k,j}\eta_{k,j}(t)+ra_{k,n_k}e_{k,1}(t),\ t\neq t_m,\ t\geqslant t_0,\end{cases}\\ \eta_{k,j}(t)=\frac{1}{r^j}g_{k,j}(t^-,\hat{x}(t^-)),\ j=1,2,\ldots,n_k,\ t=t_m,\ m\in\mathbb{Z}^+,\\ \eta_k(t)=\hat{\chi}_k\in\mathbb{R}^{n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases}\tag{5}$$

where $\eta_k(t)=[\eta_{k,1}(t),\eta_{k,2}(t),\ldots,\eta_{k,n_k}(t)]^{\mathrm{T}}$ and $\hat{\chi}_k=[\frac{1}{r}\hat{\varsigma}_{k,1},\frac{1}{r^2}\hat{\varsigma}_{k,2},\ldots,\frac{1}{r^{n_k}}\hat{\varsigma}_{k,n_k}]^{\mathrm{T}}$, $k=1,2,\ldots,N$.

Based on (5), the following matrix form is obtained:

$$\begin{cases}\dot{\eta}_k(t)=rB_k\eta_k(t)+ra_ke_{k,1}(t),\ t\neq t_m,\ t\geqslant t_0,\\ \eta_{k,j}(t)=\frac{1}{r^j}g_{k,j}(t^-,\hat{x}(t^-)),\ j=1,2,\ldots,n_k,\ t=t_m,\ m\in\mathbb{Z}^+,\\ \eta_k(t)=\hat{\chi}_k\in\mathbb{R}^{n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases}\tag{6}$$

where B_k and a_k, $k=1,2,\ldots,N$ are given in Lemma 3.

Moreover, it follows from (1)–(3) that

$$\begin{cases}\begin{cases}\dot{e}_{k,i}(t)=re_{k,i+1}(t)-ra_{k,i}e_{k,1}(t)+\frac{1}{r^i}f_{k,i}(t,t-\tau(t)),\ i=1,2,\ldots,n_k-1,\\ \dot{e}_{k,n_k}(t)=-ra_{k,n_k}e_{k,1}(t)+\frac{1}{r^{n_k}}f_{k,n_k}(t,t-\tau(t)),\ t\neq t_m,\ t\geqslant t_0,\end{cases}\\ e_{k,j}(t)=\frac{1}{r^j}\big(g_{k,j}(t^-,x(t^-))-g_{k,j}(t^-,\hat{x}(t^-))\big),\ j=1,2,\ldots,n_k,\ t=t_m,\ m\in\mathbb{Z}^+,\\ e_k(t)=\bar{\chi}_k(t)\in\mathbb{PC}_{\bar{\tau}}^{n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases}\tag{7}$$

where $e_k(t)=[e_{k,1}(t),e_{k,2}(t),\ldots,e_{k,n_k}(t)]^{\mathrm{T}}$ and $\bar{\chi}_k(t)=[\frac{1}{r}(\varsigma_{k,1}(t)-\hat{\varsigma}_{k,1}),\frac{1}{r^2}(\varsigma_{k,2}(t)-\hat{\varsigma}_{k,2}),\ldots,\frac{1}{r^{n_k}}(\varsigma_{k,n_k}(t)-\hat{\varsigma}_{k,n_k})]^{\mathrm{T}}$, $k=1,2,\ldots,N$.

Then, by (7), the following matrix form is derived:

$$\begin{cases}\dot{e}_k(t)=rA_ke_k(t)+\bar{f}_k(t,t-\tau(t)),\ t\neq t_m,\ t\geqslant t_0,\\ e_{k,j}(t)=\frac{1}{r^j}\big(g_{k,j}(t^-,x(t^-))-g_{k,j}(t^-,\hat{x}(t^-))\big),\ j=1,2,\ldots,n_k,\ t=t_m,\ m\in\mathbb{Z}^+,\\ e_k(t)=\bar{\chi}_k(t)\in\mathbb{PC}_{\bar{\tau}}^{n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases}\tag{8}$$

where A_k is given in Lemma 3, and $\bar{f}_k(t,t-\tau(t))=\big[\frac{1}{r}f_{k,1}(t,t-\tau(t)),\frac{1}{r^2}f_{k,2}(t,t-\tau(t)),\ldots,\frac{1}{r^{n_k}}f_{k,n_k}(t,t-\tau(t))\big]^{\mathrm{T}}$, $k=1,2,\ldots,N$.

Let $\xi_k(t)=[\eta_k^{\mathrm{T}}(t),e_k^{\mathrm{T}}(t)]^{\mathrm{T}}$, $k=1,2,\ldots,N$. Combining (6) and (8), the closed-loop impulsive system can be represented in the following compact form:

$$\begin{cases}\dot{\xi}_k(t)=r\Xi_k\xi_k(t)+\bar{F}_k(t,t-\tau(t)),\ t\neq t_m,\ t\geqslant t_0,\\ \xi_k(t)=G_k(t^-),\ t=t_m,\ m\in\mathbb{Z}^+,\\ \xi_k(t)=\zeta_k(t)\in\mathbb{PC}_{\bar{\tau}}^{2n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases}\tag{9}$$

where Ξ_k is given in Lemma 3, $\bar{F}_k(t,t-\tau(t))=\big[\mathbf{0}_{n_k}^{\mathrm{T}},\bar{f}_k^{\mathrm{T}}(t,t-\tau(t))\big]^{\mathrm{T}}$, $G_k(t^-)=\big[\frac{1}{r}g_{k,1}(t^-,\hat{x}(t^-)),\frac{1}{r^2}g_{k,2}(t^-,\hat{x}(t^-)),\ldots,\frac{1}{r^{n_k}}g_{k,n_k}(t^-,\hat{x}(t^-)),\frac{1}{r}\big(g_{k,1}(t^-,x(t^-))-g_{k,1}(t^-,\hat{x}(t^-))\big),\frac{1}{r^2}\big(g_{k,2}(t^-,x(t^-))-g_{k,2}(t^-,\hat{x}(t^-))\big),\ldots,\frac{1}{r^{n_k}}\big(g_{k,n_k}(t^-,x(t^-))-g_{k,n_k}(t^-,\hat{x}(t^-))\big)\big]^{\mathrm{T}}$, and $\zeta_k(t)=\big[\hat{\chi}_k^{\mathrm{T}},\bar{\chi}_k^{\mathrm{T}}(t)\big]^{\mathrm{T}}$, $k=1,2,\ldots,N$.

3.2 Main results of lower TLSNISs

Under the framework of lower TLSNISs, we formally state the main results in the following theorem.

Theorem 1. Consider system (1) under Assumptions 1, 2, and 3(i). If the static gain r and constants ϑ, N_0 satisfy the following conditions:

$$r\geqslant\max\left\{\max_{k=1,\ldots,N}\{c_{1,k}\}+\max_{k=1,\ldots,N}\{\lambda_{\max}(P_k)\}\big(\sigma_2\mu^{2N_0}+2\Delta_1\ln\mu\big),1\right\},\tag{10}$$

$$\vartheta > \frac{2\ln\mu}{\sigma_1 - \sigma_2\mu^{2N_0}} > 0, \tag{11}$$

where $c_{1,k}$, σ_1, σ_2, μ are positive constants to be determined later, Δ_1 is an adjustable positive constant, and P_k, $k = 1, 2, \ldots, N$ are positive definite matrices satisfying Lemma 3, then by decentralized output-feedback controllers (4), all the signals of the resulting closed-loop impulsive system are globally bounded and system states $x_{k,i}(t)$, $i = 1, 2, \ldots, n_k$, $k = 1, 2, \ldots, N$ converge exponentially to the origin.

Proof. Choose a Lyapunov function candidate, for $t \geqslant t_0$,

$$V(t) = \sum_{k=1}^{N} \xi_k^{\mathrm{T}}(t) P_k \xi_k(t), \tag{12}$$

where the positive definite matrices P_k, $k = 1, 2, \ldots, N$ are determined in Lemma 3.

Now, for $t \neq t_m$, $m \in \mathbb{Z}^+$, taking the time derivative of (12) along the trajectory of system (9), one has

$$D^+V(t) \leqslant \sum_{k=1}^{N} \Big(- r\|\xi_k(t)\|^2 + 2\xi_k^{\mathrm{T}}(t) P_k \bar{F}_k(t, t-\tau(t)) \Big). \tag{13}$$

For further analysis, the last term on the right-hand side of (13) needs to be estimated. Firstly, based on Assumption 3(i) and (3), it follows that

$$\begin{aligned} \left| \frac{f_{k,i}(t, t-\tau(t))}{r^i} \right| &\leqslant \theta_k \sum_{p=1}^{N} \left(\sum_{q=1}^{\min\{n_p, i\}} \big(|\eta_{p,q}(t)| + |e_{p,q}(t)| + |\eta_{p,q}(t-\tau(t))| + |e_{p,q}(t-\tau(t))| \big) \right) \\ &\leqslant 2\theta_k \sum_{p=1}^{N} \sqrt{n_p} \left(\|\xi_p(t)\| + \|\xi_p(t-\tau(t))\| \right). \end{aligned} \tag{14}$$

Then, by (14) and Young's inequality, there exist known positive constants $c_{1,k}$ and $c_{2,k}$ such that

$$\sum_{k=1}^{N} 2\xi_k^{\mathrm{T}}(t) P_k \bar{F}_k(t, t-\tau(t)) \leqslant \sum_{k=1}^{N} \big(c_{1,k}\|\xi_k(t)\|^2 + c_{2,k}\|\xi_k(t-\tau(t))\|^2 \big). \tag{15}$$

Substituting (15) into (13), and noting (10), for $t \neq t_m$, $m \in \mathbb{Z}^+$, one can obtain

$$\begin{aligned} D^+V(t) &\leqslant \sum_{k=1}^{N} \Big(- \big(r - c_{1,k}\big)\|\xi_k(t)\|^2 + c_{2,k}\|\xi_k(t-\tau(t))\|^2 \Big) \\ &\leqslant - \sum_{k=1}^{N} \max_{k=1,\ldots,N}\{\lambda_{\max}(P_k)\}\big(\sigma_2\mu^{2N_0} + 2\Delta_1 \ln\mu\big)\|\xi_k(t)\|^2 \\ &\quad + \sum_{k=1}^{N} c_{2,k}\|\xi_k(t-\tau(t))\|^2 \\ &\leqslant -\sigma_1 V(t) + \sigma_2 V(t-\tau(t)), \end{aligned} \tag{16}$$

where $\sigma_1 = \sigma_2\mu^{2N_0} + 2\Delta_1 \ln\mu$ and $\sigma_2 = \frac{\max_{k=1,\ldots,N}\{c_{2,k}\}}{\min_{k=1,\ldots,N}\{\lambda_{\min}(P_k)\}}$.

For $t = t_m$, $m \in \mathbb{Z}^+$, by Assumption 2, (3), and (9), we deduce

$$\begin{aligned} V(t_m) &= \sum_{k=1}^{N} \xi_k^{\mathrm{T}}(t_m) P_k \xi_k(t_m) = \sum_{k=1}^{N} (G_k(t_m^-))^{\mathrm{T}} P_k (G_k(t_m^-)) \\ &\leqslant \sum_{k=1}^{N} \lambda_{\max}(P_k)\|G_k(t_m^-)\|^2 \leqslant \sum_{k=1}^{N} \lambda_{\max}(P_k)\mu_k^2\|\xi_k(t_m^-)\|^2 \\ &\leqslant \max_{k=1,2,\ldots,N} \left\{ \frac{\mu_k^2 \lambda_{\max}(P_k)}{\lambda_{\min}(P_k)} \right\} \sum_{k=1}^{N} \xi_k^{\mathrm{T}}(t_m^-) P_k \xi_k(t_m^-) \end{aligned}$$

$$\leqslant \mu^2 V(t_m^-), \tag{17}$$

where $\mu = \max_{k=1,2,\ldots,N}\{\mu_k \sqrt{\frac{\lambda_{\max}(P_k)}{\lambda_{\min}(P_k)}}\}$.

From (16) and (17), we construct the following comparison system:

$$\begin{cases} \dot{\varepsilon}(t) = -\sigma_1\varepsilon(t) + \sigma_2\varepsilon(t-\tau(t)) + \omega, \ t \neq t_m, \\ \varepsilon(t) = \mu^2\varepsilon(t^-), \ t = t_m, \ m \in \mathbb{Z}^+, \\ \varepsilon(t) = \sum_{k=1}^N \lambda_{\max}(P_k)\|\zeta_k(t)\|^2, \ t \in [t_0-\bar{\tau}, t_0], \end{cases} \tag{18}$$

where $\dot{\varepsilon}(t)$ is the upper right-hand derivative of $\varepsilon(t)$. Suppose that $\varepsilon_\omega(t)$ is the corresponding maximal solution of system (18) for any given $\omega > 0$. It follows from Lemma 1 that $V(t) \leqslant \varepsilon_\omega(t)$ for $t \geqslant t_0$.

Then, from Lemma 2, it can be inferred that, for $t \geqslant t_0$,

$$\varepsilon_\omega(t) \leqslant \varepsilon_\omega(t_0)\mu^{2N(t,t_0)}\mathrm{e}^{-\sigma_1(t-t_0)} + \int_{t_0}^t \mu^{2N(t,s)}\mathrm{e}^{-\sigma_1(t-s)}(\sigma_2\varepsilon_\omega(s-\tau(s)) + \omega)\mathrm{d}s. \tag{19}$$

According to Definition 1, one gets

$$\begin{aligned} \mu^{2N(t,s)}\mathrm{e}^{-\sigma_1(t-s)} &\leqslant \mu^{2N_0 + 2\frac{t-s}{\vartheta}}\mathrm{e}^{-\sigma_1(t-s)} \\ &\leqslant \mu^{2N_0}\mathrm{e}^{2\frac{t-s}{\vartheta}\ln\mu}\mathrm{e}^{-\sigma_1(t-s)} \\ &\leqslant \delta\mathrm{e}^{-\sigma_3(t-s)}, \end{aligned} \tag{20}$$

where $\delta = \mu^{2N_0}$ and $\sigma_3 = \sigma_1 - \frac{2\ln\mu}{\vartheta} > 0$.

Then, substituting (20) into (19) can straightforwardly yield that

$$\begin{aligned} \varepsilon_\omega(t) &\leqslant \delta\varepsilon_\omega(t_0)\mathrm{e}^{-\sigma_3(t-t_0)} + \int_{t_0}^t \delta\mathrm{e}^{-\sigma_3(t-s)}(\sigma_2\varepsilon_\omega(s-\tau(s)) + \omega)\mathrm{d}s \\ &\leqslant \alpha\mathrm{e}^{-\sigma_3(t-t_0)} + \int_{t_0}^t \delta\mathrm{e}^{-\sigma_3(t-s)}(\sigma_2\varepsilon_\omega(s-\tau(s)) + \omega)\mathrm{d}s, \end{aligned} \tag{21}$$

where $\alpha = \delta\sum_{k=1}^N \lambda_{\max}(P_k)\|\zeta_k(t)\|_{\bar{\tau}}^2$.

Let

$$\rho(s) = \sigma_2\delta\mathrm{e}^{\bar{\tau}s} + s - \sigma_3. \tag{22}$$

By (11) and (22), one deduces that $\rho(0) = \sigma_2\delta - \sigma_3 = \sigma_2\mu^{2N_0} - \sigma_1 + \frac{2\ln\mu}{\vartheta} < 0$. Moreover, it is easy to get that $\rho(+\infty) = +\infty$ and $\dot{\rho}(s) = \sigma_2\delta\bar{\tau}\mathrm{e}^{\bar{\tau}s} + 1 > 0$. Hence, there exists a unique positive constant σ, such that $\sigma_2\delta\mathrm{e}^{\bar{\tau}\sigma} + \sigma - \sigma_3 = 0$.

Next, we present an important proposition as the basis for the proof of Theorem 1, which states the intrinsic property of the solution of comparison system (18).

Proposition 1. For the solution of comparison system (18), the following inequality holds:

$$\varepsilon_\omega(t) < \alpha\mathrm{e}^{-\sigma(t-t_0)} + \frac{\omega\delta}{\sigma_3 - \sigma_2\delta}, \ t \geqslant t_0 - \bar{\tau}. \tag{23}$$

Proof. Firstly, we define $\Phi(t) = \alpha\mathrm{e}^{-\sigma(t-t_0)} + \frac{\omega\delta}{\sigma_3-\sigma_2\delta}$, $t \geqslant t_0 - \bar{\tau}$. For $t \in [t_0-\bar{\tau}, t_0]$, noting $\delta > 1$, it follows that

$$\begin{aligned} \varepsilon_\omega(t) &= \sum_{k=1}^N \lambda_{\max}(P_k)\|\zeta_k(t)\|^2 \\ &< \delta\sum_{k=1}^N \lambda_{\max}(P_k)\|\zeta_k(t)\|_{\bar{\tau}}^2 \\ &= \alpha \\ &< \Phi(t). \end{aligned} \tag{24}$$

For $t > t_0$, suppose that Proposition 1 does not hold. Then, we define $t^* = \inf\{t > t_0 : \varepsilon_\omega(t) \geqslant \Phi(t)\}$. If t^* is not an impulse moment, it holds that $\varepsilon_\omega(t^*) = \Phi(t^*)$. Otherwise, it then holds that $\varepsilon_\omega(t^*) \geqslant \Phi(t^*)$. Hence, we have

$$\begin{cases} \varepsilon_\omega(t) < \Phi(t), \ t \in [t_0 - \bar{\tau}, t^*), \\ \varepsilon_\omega(t^*) \geqslant \Phi(t^*). \end{cases} \tag{25}$$

By (21), (23), and (25), we can derive that

$$\begin{aligned} \varepsilon_\omega(t^*) &\leqslant \alpha \mathrm{e}^{-\sigma_3(t^*-t_0)} + \int_{t_0}^{t^*} \delta \mathrm{e}^{-\sigma_3(t^*-s)} \big(\sigma_2 \varepsilon_\omega(s-\tau(s)) + \omega\big) \mathrm{d}s \\ &\leqslant \alpha \mathrm{e}^{-\sigma_3(t^*-t_0)} + \int_{t_0}^{t^*} \delta \mathrm{e}^{-\sigma_3(t^*-s)} \left(\sigma_2 \alpha \mathrm{e}^{-\sigma(s-\tau(s)-t_0)} + \frac{\sigma_2 \omega \delta}{\sigma_3 - \sigma_2 \delta} + \omega\right) \mathrm{d}s. \end{aligned} \tag{26}$$

From (26), and noting $\sigma_2 \delta \mathrm{e}^{\bar{\tau}\sigma} + \sigma - \sigma_3 = 0$, it can be inferred that

$$\begin{aligned} &\int_{t_0}^{t^*} \delta \mathrm{e}^{-\sigma_3(t^*-s)} \left(\sigma_2 \alpha \mathrm{e}^{-\sigma(s-\tau(s)-t_0)} + \frac{\sigma_2 \omega \delta}{\sigma_3 - \sigma_2 \delta} + \omega\right) \mathrm{d}s \\ &\leqslant \alpha \sigma_2 \delta \mathrm{e}^{\sigma(\bar{\tau}+t_0) - \sigma_3 t^*} \int_{t_0}^{t^*} \mathrm{e}^{(\sigma_3 - \sigma)s} \mathrm{d}s + \delta \left(\frac{\sigma_2 \omega \delta}{\sigma_3 - \sigma_2 \delta} + \omega\right) \mathrm{e}^{-\sigma_3 t^*} \int_{t_0}^{t^*} \mathrm{e}^{\sigma_3 s} \mathrm{d}s \\ &= \frac{\alpha \sigma_2 \delta \mathrm{e}^{\bar{\tau}\sigma}}{\sigma_3 - \sigma} \big(\mathrm{e}^{-\sigma(t^*-t_0)} - \mathrm{e}^{-\sigma_3(t^*-t_0)}\big) + \frac{\omega \delta}{\sigma_3 - \sigma_2 \delta} \big(1 - \mathrm{e}^{-\sigma_3(t^*-t_0)}\big) \\ &< \alpha \mathrm{e}^{-\sigma(t^*-t_0)} - \alpha \mathrm{e}^{-\sigma_3(t^*-t_0)} + \frac{\omega \delta}{\sigma_3 - \sigma_2 \delta}. \end{aligned} \tag{27}$$

Substituting (27) into (26), we have

$$\varepsilon_\omega(t^*) < \alpha \mathrm{e}^{-\sigma(t^*-t_0)} + \frac{\omega \delta}{\sigma_3 - \sigma_2 \delta} = \Phi(t^*). \tag{28}$$

From this, the inequality (28) means that the hypothesis does not hold. The contradiction argument appears. Thus, we can conclude that the inequality (23) holds for $t \geqslant t_0 - \bar{\tau}$. This completes the proof of Proposition 1.

With Proposition 1 established, one can straightforwardly obtain that

$$\min_{k=1,2,\ldots,N}\{\lambda_{\min}(P_k)\} \sum_{k=1}^{N} \|\xi_k(t)\|^2 \leqslant V(t) \leqslant \varepsilon_\omega(t) \leqslant \Phi(t), \ t \geqslant t_0. \tag{29}$$

By (29), and letting $\omega \to 0^+$, we have

$$\sum_{k=1}^{N} \|\xi_k(t)\|^2 \leqslant \bar{c} \mathrm{e}^{-\sigma(t-t_0)}, \tag{30}$$

where $\bar{c} = \frac{\alpha}{\min_{k=1,2,\ldots,N}\{\lambda_{\min}(P_k)\}}$.

Then, it follows from (3) and (30) that

$$|\hat{x}_{k,i}(t)| \leqslant \sqrt{\bar{c}} r^{n_k} \mathrm{e}^{-\frac{\sigma}{2}(t-t_0)}, \ \ |x_{k,i}(t) - \hat{x}_{k,i}(t)| \leqslant \sqrt{\bar{c}} r^{n_k} \mathrm{e}^{-\frac{\sigma}{2}(t-t_0)}. \tag{31}$$

By (31), we obtain

$$|x_{k,i}(t)| \leqslant 2\sqrt{\bar{c}} r^{n_k} \mathrm{e}^{-\frac{\sigma}{2}(t-t_0)}. \tag{32}$$

Therefore, we can conclude that all the signals of the resulting closed-loop impulsive system are globally bounded and system states $x_{k,i}(t)$, $i = 1, 2, \ldots, n_k$, $k = 1, 2, \ldots, N$ converge exponentially to the origin. This completes the proof of Theorem 1.

3.3 Disturbance evaluation of lower TLSNISs

In the above results of Section 3, the gain scaling approach is used to design the decentralized output-feedback controllers, and the superiority of this scheme in resisting time-varying delays and impulsive disturbances is well confirmed. However, since practical systems are often subject to external disturbances, it is a valuable discussion to investigate whether the controllers (or improved ones) can tolerate such disturbances. In this subsection, we consider the system (1) subject to unknown external disturbances, i.e.,

$$\begin{cases}\begin{cases}\dot{x}_{k,i}(t)=x_{k,i+1}(t)+f_{k,i}(t,x(t),u(t),x(t-\tau(t)),u(t-\tau(t)))+d_{k,i}(t),\ i=1,2,\ldots,n_k-1,\\ \dot{x}_{k,n_k}(t)=u_k(t)+f_{k,n_k}(t,x(t),u(t),x(t-\tau(t)),u(t-\tau(t)))+d_{k,n_k}(t),\\ y_k(t)=x_{k,1}(t),\ t\neq t_m,\ t\geqslant t_0,\end{cases}\\ x_{k,j}(t)=g_{k,j}(t^-,x(t^-)),\ j=1,2,\ldots,n_k,\ t=t_m,\ m\in\mathbb{Z}^+,\\ x_k(t)=\varsigma_k(t)\in\mathbb{PC}_{\bar{\tau}}^{n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases}\tag{33}$$

where $d_{k,i}(t),\ i=1,2,\ldots,n_k,\ k=1,2,\ldots,N$ are unknown external disturbances, and other system variables are the same as those shown in system (1) while satisfying Assumptions 1, 2, and 3(i). Similar to the work in [36], we make the following assumption for the external additive disturbances.

Assumption 4. For $i=1,2,\ldots,n_k$, there exist unknown nonnegative constants $\bar{d}_k,\ k=1,2,\ldots,N$, such that

$$|d_{k,i}(t)|\leqslant\bar{d}_k.$$

Similar to Subsection 3.1, with the help of observers (2), state transformations (3), and controllers (4), the closed-loop impulsive system can be written as follows:

$$\begin{cases}\dot{\xi}_k(t)=r\Xi_k\xi_k(t)+\bar{F}_k(t,t-\tau(t))+\Psi_k(t),\ t\neq t_m,\ t\geqslant t_0,\\ \xi_k(t)=G_k(t^-),\ t=t_m,\ m\in\mathbb{Z}^+,\\ \xi_k(t)=\zeta_k(t)\in\mathbb{PC}_{\bar{\tau}}^{2n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases}\tag{34}$$

where $\Psi_k(t)=[\mathbf{0}_{n_k}^{\mathrm{T}},d_k^{\mathrm{T}}(t)]^{\mathrm{T}}$, $d_k(t)=\left[\frac{1}{r}d_{k,1}(t),\frac{1}{r^2}d_{k,2}(t),\ldots,\frac{1}{r^{n_k}}d_{k,n_k}(t)\right]^{\mathrm{T}}$, $k=1,2,\ldots,N$, and other system variables are the same as those in system (9).

The main results of lower TLSNISs in the disturbance case are summarized in the following corollary.

Corollary 1. Consider system (33) under Assumptions 1, 2, 3(i), and 4. If the static gain r and constants ϑ, N_0 satisfy condition (11) and the following condition:

$$r\geqslant\max\left\{\max_{k=1,\ldots,N}\{c_{1,k}\}+\max_{k=1,\ldots,N}\{\lambda_{\max}(P_k)\}\left(\sigma_2\mu^{2N_0}+2\Delta_1\ln\mu\right)+\Delta_2,1\right\},\tag{35}$$

where Δ_2 is an adjustable positive constant and other parameters are the same as those in (15), (16), and (17), then by decentralized output-feedback controllers (4), all the signals of the resulting closed-loop impulsive system are globally bounded, and system states $x_{k,i}(t),\ i=1,2,\ldots,n_k,\ k=1,2,\ldots,N$, converge to an adjustable neighborhood of the origin.

Proof. Similar to Subsection 3.2, for $t\neq t_m,\ m\in\mathbb{Z}^+$, taking the time derivative of (12) along the trajectory of system (34), and using (35) as well as Young's inequality, one has

$$\begin{aligned}D^+V(t)&\leqslant\sum_{k=1}^{N}\left(-(r-c_{1,k}-\Delta_2)\|\xi_k(t)\|^2+c_{2,k}\|\xi_k(t-\tau(t))\|^2+\frac{n_k\bar{d}_k^2}{\Delta_2}\|P_k\|^2\right)\\ &\leqslant-\sum_{k=1}^{N}\max_{k=1,\ldots,N}\{\lambda_{\max}(P_k)\}\left(\sigma_2\mu^{2N_0}+2\Delta_1\ln\mu\right)\|\xi_k(t)\|^2\\ &\quad+\sum_{k=1}^{N}c_{2,k}\|\xi_k(t-\tau(t))\|^2+\sum_{k=1}^{N}\frac{n_k\bar{d}_k^2}{\Delta_2}\|P_k\|^2\\ &\leqslant-\sigma_1V(t)+\sigma_2V(t-\tau(t))+\bar{\Psi},\end{aligned}\tag{36}$$

where $\bar{\Psi}=\sum_{k=1}^{N}\frac{n_k\bar{d}_k^2}{\Delta_2}\|P_k\|^2$, and $c_{1,k}$, $c_{2,k}$, σ_1, σ_2, μ are the same as those in (15)–(17).

The remaining proof of Corollary 1 is similar to the proof of Theorem 1, where ω in the comparison system (18) needs to be replaced by $\bar{\Psi}+\omega$, and thus is omitted here.

Remark 3. As pointed out in [35], the impulse as a controlling factor can promote system dynamics, and conversely, as a disturbance factor can destroy system dynamics. From the viewpoint of impulsive dynamics, when the intensity constants μ_k satisfy $\mu_k \geqslant 1$, $k=1,2,\ldots,N$, the considered impulses belong to a class of hybrid impulses, which can cover both positive and negative effects. Moreover, it can be seen from (10) and (17) that the values of the intensity constants μ_k, $k=1,2,\ldots,N$ can directly affect the value of the positive constant μ, which further affects the selection of the static gain r. Hence, the decentralized output-feedback controllers u_k, $k=1,2,\ldots,N$ can adapt to different intensity constants μ_k by adjusting the static gain r.

Remark 4. Unlike the LKF-based method, we use the comparison principle method to remove the strict restriction on $\dot{\tau}(t)$ and thereby deal with a more general class of time-varying delays. Moreover, applying the LKF-based method requires constructing a proper LKF as a prerequisite. In contrast, the comparison principle method only requires constructing a simple comparison system, which is more convenient in dealing with time delays.

4 Decentralized output-feedback control of upper TLSNISs

In this section, we extend the results of Section 3 to the upper TLSNISs. That is, we design the decentralized output-feedback controllers for system (1) under Assumptions 1, 2, and 3(ii).

4.1 Decentralized output-feedback controllers design of upper TLSNISs

For the kth subsystem, a novel delay-independent low-gain impulsive observer is constructed as follows:

$$\begin{cases}\begin{cases}\dot{\hat{x}}_{k,i}(t)=\hat{x}_{k,i+1}(t)+a_{k,i}r^{-i}(y_k(t)-\hat{x}_{k,1}(t)),\ i=1,2,\ldots,n_k-1,\\ \dot{\hat{x}}_{k,n_k}(t)=u_k(t)+a_{k,n_k}r^{-n_k}(y_k(t)-\hat{x}_{k,1}(t)),\ t\neq t_m,\ t\geqslant t_0,\end{cases}\\ \hat{x}_{k,j}(t)=g_{k,j}(t^-,\hat{x}(t^-)),\ j=1,2,\ldots,n_k,\ t=t_m,\ m\in\mathbb{Z}^+,\\ \hat{x}_k(t)=\hat{\varsigma}_k\in\mathbb{R}^{n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases} \tag{37}$$

where $a_{k,i}$, $i=1,2,\ldots,n_k$, $k=1,2,\ldots,N$ are constants determined in Lemma 3, and r is a static gain to be determined later.

Consider the following state transformations, for $t\geqslant t_0-\bar{\tau}$,

$$\eta_{k,i}(t)=\frac{\hat{x}_{k,i}(t)}{r^{n_k-i+1}},\ e_{k,i}(t)=\frac{x_{k,i}(t)-\hat{x}_{k,i}(t)}{r^{n_k-i+1}},\ i=1,2,\ldots,n_k,\ k=1,2,\ldots,N. \tag{38}$$

The decentralized output-feedback controllers are designed as follows, for $t\geqslant t_0$,

$$u_k(t)=-\sum_{i=1}^{n_k}b_{k,i}\eta_{k,i}(t),\ k=1,2,\ldots,N, \tag{39}$$

where $b_{k,i}$, $i=1,2,\ldots,n_k$, $k=1,2,\ldots,N$ are constants determined in Lemma 3.

By (1) and (37)–(39), the closed-loop impulsive system can be represented as follows:

$$\begin{cases}\dot{\xi}_k(t)=\frac{1}{r}\Xi_k\xi_k(t)+\bar{F}_k(t,t-\tau(t)),\ t\neq t_m,\ t\geqslant t_0,\\ \xi_k(t)=G_k(t^-),\ t=t_m,\ m\in\mathbb{Z}^+,\\ \xi_k(t)=\zeta_k(t)\in\mathbb{PC}_{\bar{\tau}}^{2n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N,\end{cases} \tag{40}$$

where $\xi_k(t)$ and Ξ_k are the same as those in system (9), $\bar{F}_k(t,t-\tau(t))=[\mathbf{0}_{n_k}^{\mathrm{T}},\bar{f}_k^{\mathrm{T}}(t,t-\tau(t))]^{\mathrm{T}}$, $\bar{f}_k(t,t-\tau(t))=[\frac{1}{r^{n_k}}f_{k,1}(t,t-\tau(t)),\ldots,\frac{1}{r^2}f_{k,n_k-1}(t,t-\tau(t)),0]^{\mathrm{T}}$, and $G_k(t^-)=[\frac{1}{r^{n_k}}g_{k,1}(t^-,\hat{x}(t^-)),\frac{1}{r^{n_k-1}}g_{k,2}(t^-,\hat{x}(t^-)),\ldots,\frac{1}{r}g_{k,n_k}(t^-,\hat{x}(t^-)),\frac{1}{r^{n_k}}(g_{k,1}(t^-,x(t^-))-g_{k,1}(t^-,\hat{x}(t^-))),\frac{1}{r^{n_k-1}}(g_{k,2}(t^-,x(t^-))-g_{k,2}(t^-,\hat{x}(t^-))),\ldots,\frac{1}{r}(g_{k,n_k}(t^-,x(t^-))-g_{k,n_k}(t^-,\hat{x}(t^-)))]^{\mathrm{T}}$, $k=1,2,\ldots,N$.

4.2 Main results of upper TLSNISs

Under the framework of upper TLSNISs, we formally state the main results in the following theorem.

Theorem 2. Consider system (1) under Assumptions 1, 2, and 3(ii). If the static gain r and constants ϑ, N_0 satisfy the following conditions:

$$r \geqslant \max\left\{\max_{k=1,\ldots,N}\{c_{1,k}^*\} + \max_{k=1,\ldots,N}\{\lambda_{\max}(P_k)\}\left(\bar{\sigma}_2^*\mu^{2N_0} + 2\Delta_1^*\ln\mu\right), 1\right\}, \tag{41}$$

$$\vartheta > \frac{2\ln\mu}{\sigma_1^* - \sigma_2^*\mu^{2N_0}} > 0, \tag{42}$$

where $c_{1,k}^*$, σ_1^*, σ_2^*, $\bar{\sigma}_2^*$, μ are positive constants to be determined later, Δ_1^* is an adjustable positive constant, and P_k, $k = 1, 2, \ldots, N$ are positive definite matrices satisfying Lemma 3, then by decentralized output-feedback controllers (39), all the signals of the resulting closed-loop impulsive system are globally bounded and system states $x_{k,i}(t)$, $i = 1, 2, \ldots, n_k$, $k = 1, 2, \ldots, N$ converge exponentially to the origin.

Proof. Choose a Lyapunov function candidate $V(t) = \sum_{k=1}^N \xi_k^{\mathrm{T}}(t)P_k\xi_k(t)$, where the positive definite matrices P_k, $k = 1, 2, \ldots, N$ are determined in Lemma 3. For $t \neq t_m$, $m \in \mathbb{Z}^+$, taking the time derivative of $V(t)$ along the trajectory of system (40), it follows that

$$D^+V(t) \leqslant \sum_{k=1}^N \left(-\frac{1}{r}\|\xi_k(t)\|^2 + 2\xi_k^{\mathrm{T}}(t)P_k\bar{F}_k(t, t-\tau(t))\right). \tag{43}$$

By Assumption 3(ii), (38), and (39), one gets

$$\begin{aligned}
\left|\frac{f_{k,i}(t, t-\tau(t))}{r^{n_k-i+1}}\right| \leqslant{}& \frac{\bar{\theta}_k}{r^{n_k-i+1}}\sum_{p=1}^N\left(\sum_{q=\max\{i+2+n_p-n_k,1\}}^{n_p+1} r^{n_p-q+1}|\eta_{p,q}(t) + e_{p,q}(t)|\right) \\
&+ \frac{\bar{\theta}_k}{r^{n_k-i+1}}\sum_{p=1}^N\left(\sum_{q=\max\{i+2+n_p-n_k,1\}}^{n_p+1} r^{n_p-q+1}|\eta_{p,q}(t-\tau(t)) + e_{p,q}(t-\tau(t))|\right) \\
&+ \frac{\bar{\theta}_k}{r^{n_k-i+1}}\sum_{p=1}^N\left(\left|\sum_{j=1}^{n_p} b_{p,j}\eta_{p,j}(t)\right| + \left|\sum_{j=1}^{n_p} b_{p,j}\eta_{p,j}(t-\tau(t))\right|\right) \\
\leqslant{}& \frac{\bar{\theta}_k}{r^2}\sum_{p=1}^N\left(\sum_{q=\max\{i+2+n_p-n_k,1\}}^{n_p+1}\big(|\eta_{p,q}(t)| + |e_{p,q}(t)| + |\eta_{p,q}(t-\tau(t))|\right. \\
&\left. + |e_{p,q}(t-\tau(t))|\big) + \left|\sum_{j=1}^{n_p} b_{p,j}\eta_{p,j}(t)\right| + \left|\sum_{j=1}^{n_p} b_{p,j}\eta_{p,j}(t-\tau(t))\right|\right) \\
\leqslant{}& \frac{2\bar{\theta}_k}{r^2}\sum_{p=1}^N\left(1 + \max_{j=1,2,\ldots,n_p}\{|b_{p,j}|\}\right)\sqrt{n_p}\left(\|\xi_p(t)\| + \|\xi_p(t-\tau(t))\|\right).
\end{aligned} \tag{44}$$

Then, by (44) and Young's inequality, there exist known positive constants $c_{1,k}^*$ and $c_{2,k}^*$ such that

$$\sum_{k=1}^N 2\xi_k^{\mathrm{T}}(t)P_k\bar{F}_k(t, t-\tau(t)) \leqslant \frac{1}{r^2}\sum_{k=1}^N\left(c_{1,k}^*\|\xi_k(t)\|^2 + c_{2,k}^*\|\xi_k(t-\tau(t))\|^2\right). \tag{45}$$

Substituting (45) into (43), and noting (41), for $t \neq t_m$, $m \in \mathbb{Z}^+$, one can obtain

$$\begin{aligned}
D^+V(t) \leqslant{}& \sum_{k=1}^N\left(-\frac{r - c_{1,k}^*}{r^2}\|\xi_k(t)\|^2 + \frac{c_{2,k}^*}{r^2}\|\xi_k(t-\tau(t))\|^2\right) \\
\leqslant{}& -\sum_{k=1}^N \frac{1}{r^2}\max_{k=1,\ldots,N}\{\lambda_{\max}(P_k)\}\left(\bar{\sigma}_2^*\mu^{2N_0} + 2\Delta_1^*\ln\mu\right)\|\xi_k(t)\|^2
\end{aligned}$$

$$+\sum_{k=1}^{N}\frac{c_{2,k}^{*}}{r^{2}}\|\xi_{k}(t-\tau(t))\|^{2}$$
$$\leqslant -\sigma_{1}^{*}V(t)+\sigma_{2}^{*}V(t-\tau(t)), \tag{46}$$

where $\sigma_1^*=\sigma_2^*\mu^{2N_0}+\frac{2}{r^2}\Delta_1^*\ln\mu$, $\sigma_2^*=\frac{1}{r^2}\bar{\sigma}_2^*$, and $\bar{\sigma}_2^*=\frac{\max_{k=1,\ldots,N}\{c_{2,k}^*\}}{\min_{k=1,\ldots,N}\{\lambda_{\min}(P_k)\}}$.

For $t=t_m$, $m\in\mathbb{Z}^+$, by Assumption 2, (38), and (40), we deduce

$$\begin{aligned} V(t_m) &\leqslant \sum_{k=1}^{N}\lambda_{\max}(P_k)\mu_k^2\|\xi_k(t_m^-)\|^2 \\ &\leqslant \max_{k=1,2,\ldots,N}\left\{\frac{\mu_k^2\lambda_{\max}(P_k)}{\lambda_{\min}(P_k)}\right\}\sum_{k=1}^{N}\xi_k^{\mathrm{T}}(t_m^-)P_k\xi_k(t_m^-) \\ &\leqslant \mu^2V(t_m^-), \end{aligned} \tag{47}$$

where μ is the same as defined in (17).

Obviously, the remaining proof of Theorem 2 is similar to the proof of Theorem 1, and thus is omitted here.

4.3 Disturbance evaluation of upper TLSNISs

In this subsection, we consider the upper TLSNIS subject to unknown external disturbances; i.e., system (33) satisfies Assumptions 1, 2, 3(ii), and 4.

Similar to Subsection 4.1, by means of observers (37), state transformations (38), and controllers (39), the closed-loop impulsive system can be written as follows:

$$\begin{cases} \dot{\xi}_k(t)=\frac{1}{r}\Xi_k\xi_k(t)+\bar{F}_k(t,t-\tau(t))+\frac{1}{r}\Psi_k(t),\ t\neq t_m,\ t\geqslant t_0, \\ \xi_k(t)=G_k(t^-),\ t=t_m,\ m\in\mathbb{Z}^+, \\ \xi_k(t)=\zeta_k(t)\in\mathbb{PC}_{\bar{\tau}}^{2n_k},\ t\in[t_0-\bar{\tau},t_0],\ k=1,2,\ldots,N, \end{cases} \tag{48}$$

where $\Psi_k(t)=[\mathbf{0}_{n_k}^{\mathrm{T}},d_k^{\mathrm{T}}(t)]^{\mathrm{T}}$, $d_k(t)=\left[\frac{1}{r^{n_k-1}}d_{k,1}(t),\ldots,\frac{1}{r}d_{k,n_k-1}(t),d_{k,n_k}(t)\right]^{\mathrm{T}}$, $k=1,2,\ldots,N$, and other system variables are the same as those in system (40).

The main results of upper TLSNISs in the disturbance case are summarized in the following corollary.

Corollary 2. Consider system (33) under Assumptions 1, 2, 3(ii), and 4. If the static gain r and constants ϑ, N_0 satisfy condition (42) and the following condition:

$$r\geqslant\max\left\{\max_{k=1,\ldots,N}\{c_{1,k}^*\}+\max_{k=1,\ldots,N}\{\lambda_{\max}(P_k)\}\big(\bar{\sigma}_2^*\mu^{2N_0}+2\Delta_1^*\ln\mu\big)+\Delta_2^*,1\right\}, \tag{49}$$

where Δ_2^* is an adjustable positive constant and other parameters are the same as those in (45)–(47), then by decentralized output-feedback controllers (39), all the signals of the resulting closed-loop impulsive system are globally bounded and system states $x_{k,i}(t)$, $i=1,2,\ldots,n_k$, $k=1,2,\ldots,N$ converge to an adjustable neighborhood of the origin.

Proof. Similar to Subsection 4.1, for $t\neq t_m$, $m\in\mathbb{Z}^+$, taking the time derivative of $V(t)$ along the trajectory of system (48), and using (49) as well as Young's inequality, one has

$$\begin{aligned} D^+V(t) &\leqslant \sum_{k=1}^{N}\left(-\frac{r-c_{1,k}^*-\Delta_2^*}{r^2}\|\xi_k(t)\|^2+\frac{c_{2,k}^*}{r^2}\|\xi_k(t-\tau(t))\|^2+\frac{n_k\bar{d}_k^2}{\Delta_2^*}\|P_k\|^2\right) \\ &\leqslant -\sum_{k=1}^{N}\frac{1}{r^2}\max_{k=1,\ldots,N}\{\lambda_{\max}(P_k)\}\big(\bar{\sigma}_2^*\mu^{2N_0}+2\Delta_1^*\ln\mu\big)\|\xi_k(t)\|^2 \\ &\quad+\sum_{k=1}^{N}\frac{c_{2,k}^*}{r^2}\|\xi_k(t-\tau(t))\|^2+\sum_{k=1}^{N}\frac{n_k\bar{d}_k^2}{\Delta_2^*}\|P_k\|^2 \\ &\leqslant -\sigma_1^*V(t)+\sigma_2^*V(t-\tau(t))+\bar{\Psi}^*, \end{aligned} \tag{50}$$

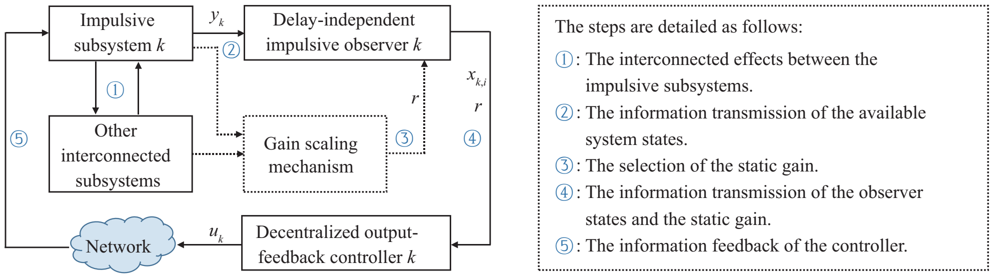

Figure 1 (Color online) The framework of decentralized output-feedback closed-loop control.

where $\bar{\Psi}^* = \sum_{k=1}^{N} \frac{n_k \bar{d}_k^2}{\Delta_2^*} \|P_k\|^2$, and $c_{1,k}^*$, $c_{2,k}^*$, σ_1^*, σ_2^*, $\bar{\sigma}_2^*$, μ are the same as those in (45)–(47).

The remaining proof of Corollary 2 is similar to the proof of Theorem 1, where ω in the comparison system (18) needs to be replaced by $\bar{\Psi}^* + \omega$, and thus is omitted here.

Remark 5. Compared with the LMI-based design approach and iterative design approach, the gain scaling approach has more superiority in the compensation of structural uncertainties and the simplification of design processes. Different from the impulsive observers in [29, 37], the gain-based impulsive observers (2) and (37) are delay-independent and remove the limitation that the structure of system nonlinearities must be known. In Section 3 or 4, we delicately reveal the relationship between the static gain r and the growth rate θ_k or $\bar{\theta}_k$, which plays a crucial role in suppressing the structural uncertainties. From (4) and (39), it can be seen that the designed controllers are delay-independent, disturbance-independent, and observer-based, and thus can be applied to situations involving unmeasured time delays, unknown external disturbances, and unavailable system states.

Remark 6. Notably, existing results of nonlinear systems with triangular structures, such as [11, 12, 24–26, 30], are obtained based on continuous-time dynamics, which is relatively easy to carry out stability analysis of closed-loop systems. However, when impulsive disturbances are involved in the system dynamics, the discontinuity of system states leads to the failure of existing continuous-time-dynamics-based stability analysis methods. In the proof of theorems and corollaries, we divide the analysis process into two parts, i.e., the continuous-time dynamics part and the discrete-time dynamics part. Specifically, in the proof of Theorem 1, we establish the relations (16) and (17) to characterize the continuous-time dynamic property between impulse moments and the discrete-time dynamic property at impulse moments, respectively. Through the above two parts, we effectively analyze the overall dynamic property of the closed-loop impulsive system (9), which is different from the analysis manner of continuous-time systems. More precisely, the analysis process in this paper is more challenging than that in continuous-time systems. Moreover, it also means that the work in this paper is a pioneering attempt to consider impulsive disturbances within the framework of triangular large-scale nonlinear systems.

Remark 7. To achieve a good understanding, we provide a decentralized output-feedback closed-loop control framework, as shown in Figure 1. To the best of our knowledge, the interconnected effects between the impulsive subsystems (i.e., ①) are not considered in existing studies. The information transmission of the available system states (i.e., ②) and the selection of the static gain (i.e., ③) are crucial for the observer to estimate the unavailable system states. Based on the information of the observer states and the static gain, the decentralized output-feedback controller is designed; see ④. In ⑤, the information of the controller is feedback to the plant.

5 Simulation examples

In this section, two simulation examples are given to illustrate the effectiveness of the decentralized output-feedback controllers for lower TLSNISs and upper TLSNISs, respectively.

Example 1. As is well known, chemical reactor systems are very popular in the chemical industry [38]. During the production process, many chemical reactions are subject to discontinuous disturbances (impulsive effects) at certain moments, which may lead to abrupt changes in the system states; see [29]. In this example, we consider the lower TLSNIS consisting of two chemical reactors (i.e., reactor A and reactor B) with delayed recycle streams; see Figure 2 for illustration. Referring to [11, 12], the dynamics

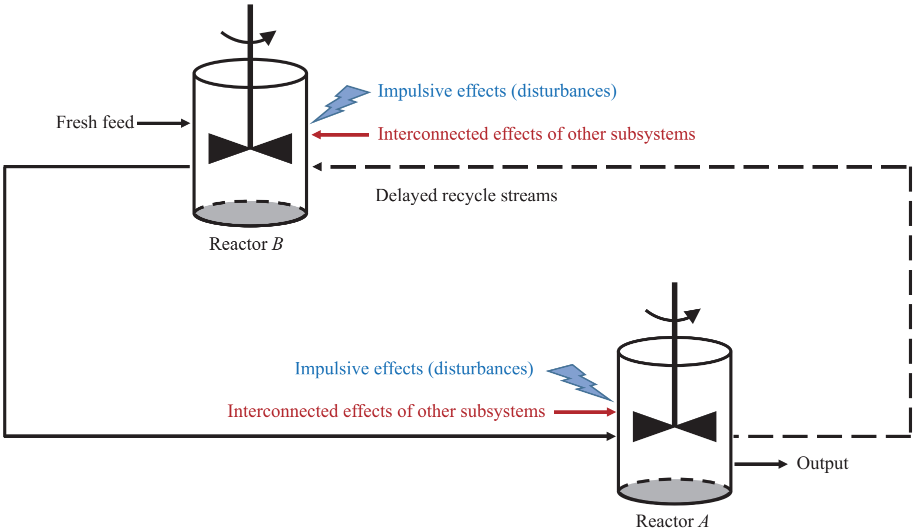

Figure 2 (Color online) Chemical reactor system.

Table 2 Physical meanings of parameters

Parameter	Physical meaning
$x_{k,1}(\cdot), x_{k,2}(\cdot)$	Compositions
$T_{k,1}, T_{k,2}$	Reactor residence times
$C_{k,1}, C_{k,2}$	Reaction constants
$V_{k,1}, V_{k,2}$	Reactor volumes
$f_{k,1}(\cdot), f_{k,2}(\cdot)$	Interconnected functions
R_k	Recycle flow rate
ϱ_k	Feed rate
$\tau(\cdot)$	Time-varying delay
$y_k(\cdot)$	Output of the kth subsystem

of the whole plant can be modeled as follows:

$$\begin{cases} \begin{cases} \dot{x}_{k,1}(t) = \frac{1-R_k}{V_{k,1}} x_{k,2}(t) - \frac{1}{T_{k,1}} x_{k,1}(t) - C_{k,1} x_{k,1}(t) + f_{k,1}(t, t-\tau(t)), \\ \dot{x}_{k,2}(t) = \frac{\varrho_k}{V_{k,2}} u_k(t) - \frac{1}{T_{k,2}} x_{k,2}(t) - C_{k,2} x_{k,2}(t) + f_{k,2}(t, t-\tau(t)), \\ y_k(t) = x_{k,1}(t), \ t \neq t_m, \ t \geqslant 0, \end{cases} \\ x_{k,i}(t) = 1.2 x_{k,i}(t^-), \ t = t_m, \ i = 1,2, \ k = 1,2,3, \end{cases} \tag{51}$$

where the physical meanings of parameters are shown in Table 2.

For the chemical reactor system (51), the system parameters are set as $R_k = V_{k,1} = V_{k,2} = \varrho_k = 0.5$, $T_{k,1} = T_{k,2} = 10$, $C_{k,1} = C_{k,2} = 0.1$, $k = 1,2,3$. The interconnected functions are given by $f_{k,1}(t, t-\tau(t)) = 0.22 x_{k,1}(t) + 2^k \cdot 0.01 \cos(x_{4-k,2}(t)) \ln(1 + |x_{4-k,1}(t-\tau(t))|)$ and $f_{k,2}(t, t-\tau(t)) = 0.26 x_{k,2}(t) + 2^k \cdot 0.01(\sin(x_{4-k,1}(t-\tau(t))) + x_{4-k,2}(t) + x_{1,2}(t-\tau(t)))$, $k = 1,2,3$. The unknown time-varying delay is set as $\tau(t) = 2 + 2\sin(t)$.

In this example, the control parameters are selected as $\Delta_1 = 1$, $N_0 = 1$, $a_{k,1} = 1$, $a_{k,2} = 2$, $b_{k,1} = 3$, and $b_{k,2} = 8$, $k = 1,2,3$. Then, we can obtain $\lambda_{\max}(P_k) = 1.1001$, $\lambda_{\max}(P_k) = 2.8993$, $r \geqslant 4.1304$, and $\vartheta \in (1, +\infty)$. Thus, we take $r = 4.2$ and $\vartheta = 1.2$. The impulse time sequence is chosen as $\{t_m\}_{m \in \mathbb{Z}^+} = \{t_{5m-4} = 6m - 5.7, t_{5m-3} = 6m - 4.6, t_{5m-2} = 6m - 3, t_{5m-1} = 6m - 2.1, t_{5m} = 6m\}_{m \in \mathbb{Z}^+}$. Moreover, the initial values are selected as $[x_{1,1}(t), x_{1,2}(t)]^{\mathrm{T}} = [-2, -4]^{\mathrm{T}}$, $[x_{2,1}(t), x_{2,2}(t)]^{\mathrm{T}} = [-1, 2]^{\mathrm{T}}$, $[x_{3,1}(t), x_{3,2}(t)]^{\mathrm{T}} = [-2, 2]^{\mathrm{T}}$, $[\hat{x}_{1,1}(t), \hat{x}_{1,2}(t)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$, $[\hat{x}_{2,1}(t), \hat{x}_{2,2}(t)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$, and $[\hat{x}_{3,1}(t), \hat{x}_{3,2}(t)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$, for $t \in [-4, 0]$.

When there is no control input, i.e., $u(t) = 0$, it is shown in Figure 3 that system (51) is unstable. In contrast, under the controllers designed in Theorem 1, the resulting simulation results are shown in Figures 4 and 5. It can be seen that all the signals of the resulting closed-loop impulsive system are globally bounded and system states converge to the origin. Obviously, the simulation results can

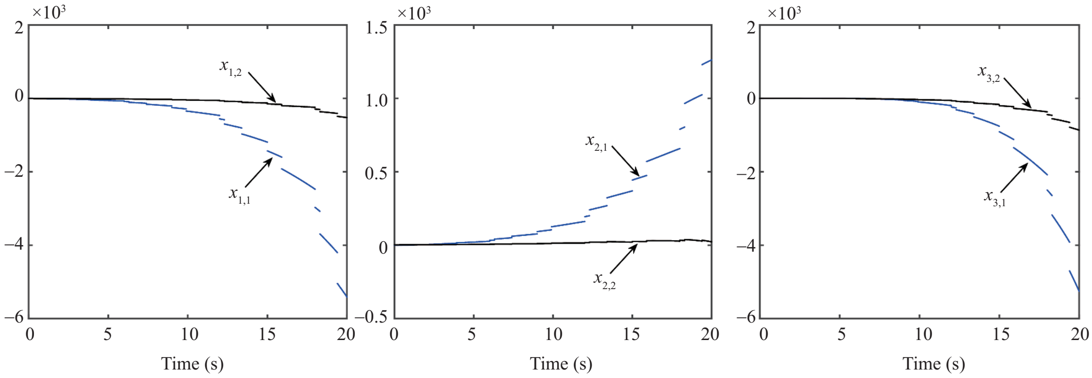

Figure 3 (Color online) The trajectories of $x_{k,i}$, $i = 1, 2$, $k = 1, 2, 3$ without control inputs in Example 1.

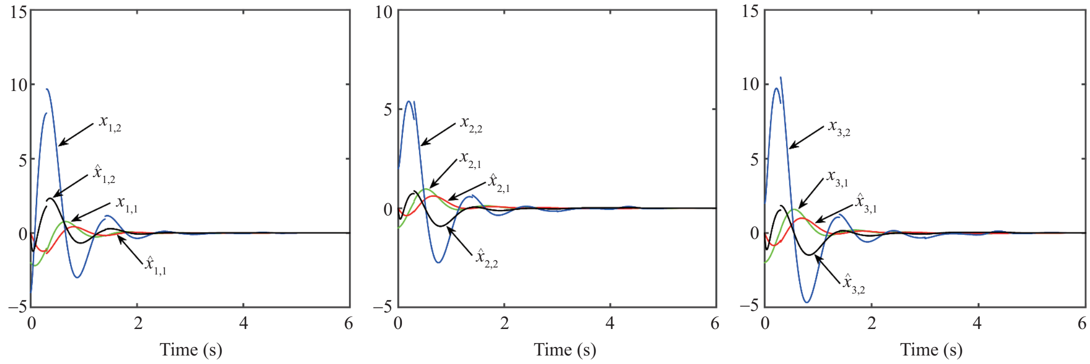

Figure 4 (Color online) The trajectories of $x_{k,i}$ and $\hat{x}_{k,i}$, $i = 1, 2$, $k = 1, 2, 3$ with control inputs in Example 1.

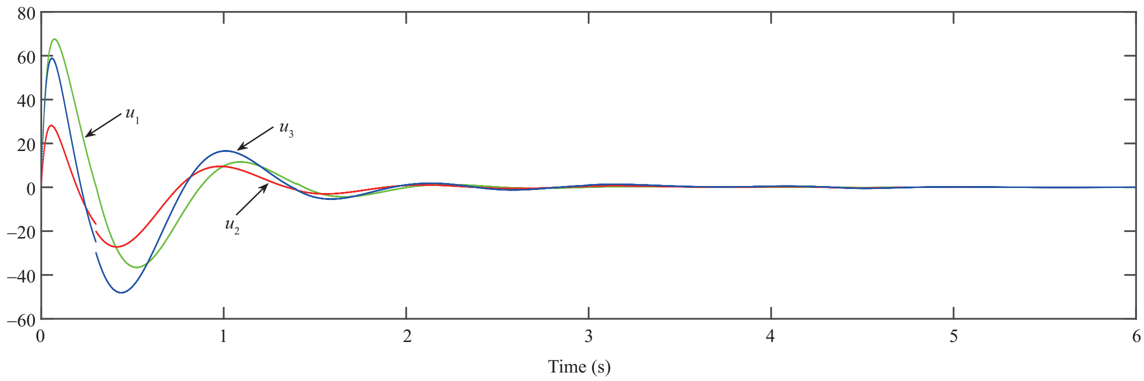

Figure 5 (Color online) The trajectories of u_k, $k = 1, 2, 3$ in Example 1.

demonstrate the effectiveness of the designed controllers.

Example 2. Consider the following upper TLSNIS with time-varying delays:

$$
\begin{array}{cc}
\text{Subsystem 1} & \text{Subsystem 2} \\
\left\{\begin{array}{l}
\left\{\begin{array}{l}
\dot{x}_{1,1}(t) = x_{1,2}(t) + \bar{\theta}_1 \mathrm{e}^{-t} u_2(t) \\
\qquad\qquad + \bar{\theta}_1 \arctan(x_{2,2}(t)) u_1(t - \tau(t)), \\
\dot{x}_{1,2}(t) = u_1(t), \\
y_1(t) = x_{1,1}(t),\ t \neq t_m,\ t \geqslant 0,
\end{array}\right. \\
x_{1,1}(t) = 1.2\sin(t^-) x_{1,1}(t^-),\ t = t_m, \\
x_{1,2}(t) = 1.2\sin(t^-) x_{1,2}(t^-),\ t = t_m,
\end{array}\right.
&
\left\{\begin{array}{l}
\left\{\begin{array}{l}
\dot{x}_{2,1}(t) = x_{2,2}(t) + \bar{\theta}_2 \tanh(t) u_1(t) \\
\qquad\qquad + \bar{\theta}_2 \arctan(x_{1,2}(t)) u_2(t - \tau(t)), \\
\dot{x}_{2,2}(t) = u_2(t), \\
y_2(t) = x_{2,1}(t),\ t \neq t_m,\ t \geqslant 0,
\end{array}\right. \\
x_{2,1}(t) = -1.2\mathrm{sign}(\cos(t^-)) x_{2,1}(t^-),\ t = t_m, \\
x_{2,2}(t) = -1.2\mathrm{sign}(\cos(t^-)) x_{2,2}(t^-),\ t = t_m,
\end{array}\right.
\end{array}
\tag{52}
$$

where $\bar{\theta}_1 = \bar{\theta}_2 = 0.05$, and the unknown time-varying delay is set as $\tau(t) = 3 + 3\cos(t)$.

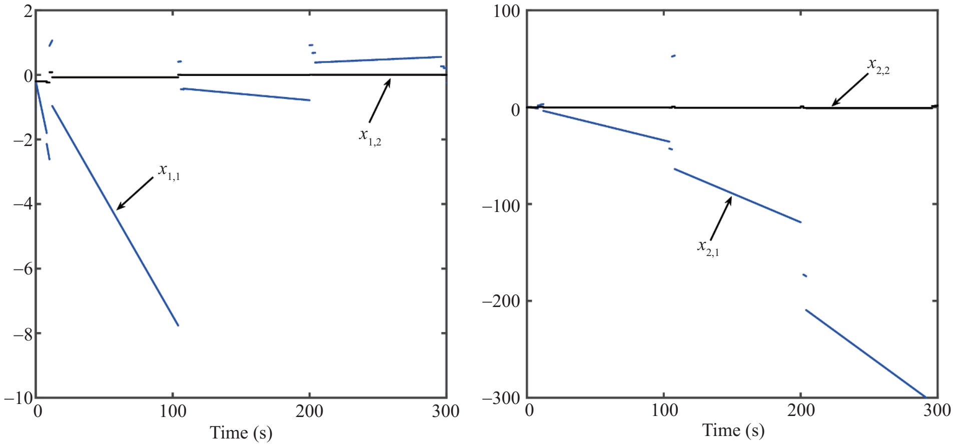

Figure 6 (Color online) The trajectories of $x_{k,i}$, $i = 1, 2$, $k = 1, 2$ without control inputs in Example 2.

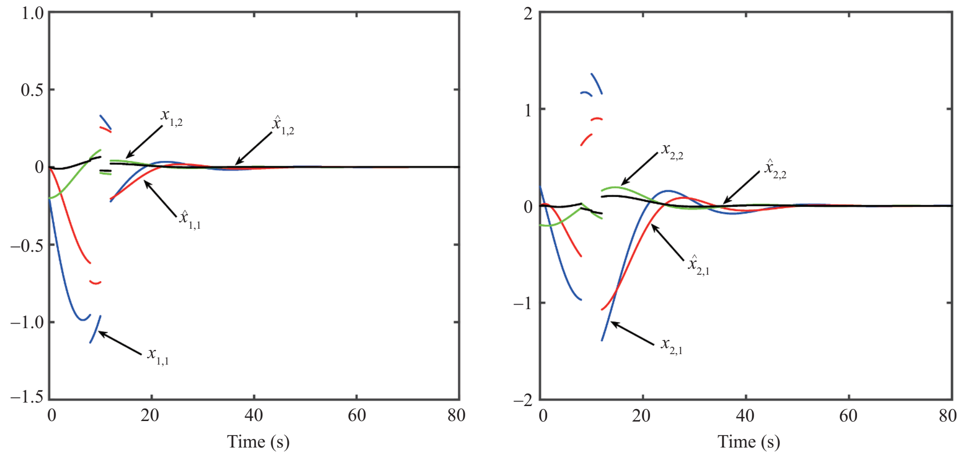

Figure 7 (Color online) The trajectories of $x_{k,i}$ and $\hat{x}_{k,i}$, $i = 1, 2$, $k = 1, 2$ with control inputs in Example 2.

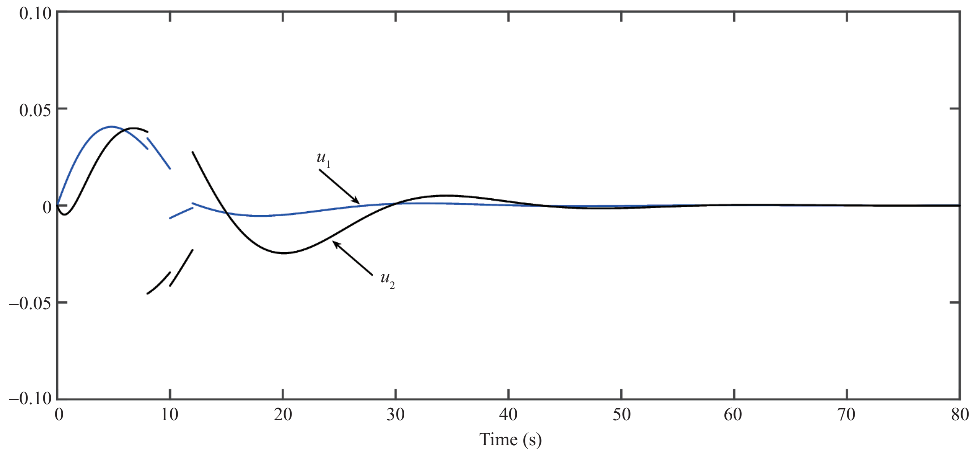

Figure 8 (Color online) The trajectories of u_k, $k = 1, 2$ in Example 2.

In this example, the control parameters are selected as $\Delta_1^* = 1$, $N_0 = 1$, $a_{k,1} = 1.1$, $a_{k,2} = 1.9$, $b_{k,1} = 3.5$, and $b_{k,2} = 5.5$, $k = 1, 2$. Then, we deduce $\lambda_{\min}(P_k) = 1.1007$, $\lambda_{\max}(P_k) = 2.7976$, and $r \geqslant 5.5829$. Hence, we take $r = 5.6$ and $\vartheta = 32$. The impulse time sequence is chosen as $\{t_m\}_{m\in\mathbb{Z}^+} = \{t_{3m-2} = 96m - 88, t_{3m-1} = 96m - 86, t_{3m} = 96m - 84\}_{m\in\mathbb{Z}^+}$. Moreover, the initial values are selected as $[x_{1,1}(t), x_{1,2}(t)]^{\mathrm{T}} = [-0.2, -0.2]^{\mathrm{T}}$, $[x_{2,1}(t), x_{2,2}(t)]^{\mathrm{T}} = [0.2, -0.2]^{\mathrm{T}}$, $[\hat{x}_{1,1}(t), \hat{x}_{1,2}(t)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$, and $[\hat{x}_{2,1}(t), \hat{x}_{2,2}(t)]^{\mathrm{T}} = [0, 0]^{\mathrm{T}}$, for $t \in [-6, 0]$.

As shown in Figure 6, system (52) is unstable without control inputs. By applying the controllers designed in Theorem 2, the simulation results of the resulting closed-loop impulsive system are presented in Figures 7 and 8. Obviously, it is shown that the simulation results can verify the effectiveness of the designed controllers.

6 Conclusion

In this paper, we have proposed a gain scaling approach for TLSNISs with time-varying delays. Novel delay-independent impulsive observers are constructed to estimate unavailable states. The undesirable effects of time-varying delays and impulsive disturbances are eliminated by using the comparison principle and average impulsive interval technique. It was shown that under the designed decentralized output-feedback controllers, the system states converge exponentially to the origin in the disturbance-free case. Moreover, by strengthening the gain scaling mechanism, the improved controllers can tolerate unknown external disturbances and thus enhance their potential applications.

The following aspects can be explored in future work. (1) It should be noted that nonlinear interconnected functions are still conservative in terms of growth rate, despite having structural uncertainties. Consequently, extending the results of this study to scenarios involving unknown (or time-varying) growth rates would certainly be an interesting topic for further study. (2) Event-triggered control schemes have attracted extensive attention because of their efficient utilization of communication resources. Hence, the design of event-triggered decentralized output-feedback controllers for TLSNISs is a promising topic. (3) Extending the results of this study to impulsive networked systems with triangular structures would also be of great significance.

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