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Parametric output regulation using observer-based PI controllers with applications in flexible spacecraft attitude control

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Abstract A parametric multiobjective design approach based on a proportional-integral (PI) controller and a full-state observer is proposed for output regulation in a multivariable linear system. First, a complete parametric form of the observer-based PI control law is established, which yields a closed-loop system with the desired eigenstructure and ensures that the regulated output asymptotically tracks a given constant signal in the presence of constant but unknown disturbances. All design degrees of freedom are preserved and characterized using a set of parameter vectors. Second, a separation principle of eigenvalue sensitivities is proven, and based on this result, the parameters of the closed-loop system are comprehensively optimized to reduce the eigenvalue sensitivity and the control gain, and also to enhance the tolerance to time-varying disturbances. Finally, the proposed method is applied to attitude control of a flexible spacecraft. Moreover, numerical simulations based on practical engineering parameters are performed to verify the superiority of the proposed method over traditional proportional-integral-derivative (PID) control methods.

Keywords parametric control system design, multiobjective design, observer-based control, separation principle, PI regulator, constant signal tracking, disturbance decoupling and attenuation

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1 Introduction

Output regulation is a central issue in control theory that originates from servomechanism and process control. Generally, when a desired output value gradually changes or remains constant and the main task is to reject the disturbance effect, the servo control problem is known as output regulation [1]. To address this problem, in 1968, Johnson [2] designed a proportional-integral (PI) regulator based on the optimal linear-quadratic theory. Under the proposed control law, the system output could asymptotically track constant reference signals in the presence of constant but unknown disturbances. In 1985, in addition to output regulation, eigenstructure assignment was also considered [3] and a new design method was proposed, where the resulting PI control law could also assign the closed-loop poles of the augmented system to the desired locations. The method outlined in [3] was further modified by Saif in 1992 [4, 5], who considered the problem of PI-based output regulation using two types of controllers, one of which was state feedback and the other was based on a Luenberger-type unknown input observer. The design of the state feedback controller was computationally simpler than that proposed in [3]. Notably, the results in [4, 5] only provide a specific solution to the PI regulator design for a given eigenstructure in the augmented system, and the desired closed-loop poles must also be distinct.

Owing to the advantages inherent to completely decoupling the effect of constant disturbances, this type of PI regulation strategy has always garnered attention from scholars and has been developed in different applications. The PI regulation law was combined with an adaptive controller in [6], with the case of time-delay also considered in [7]. The authors in [8] proposed a predictive PI method for linear

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Table 1 Symbols

Symbol	Meaning
$\mathrm{diag}\,(s_1, s_2, \ldots, s_n)$	Diagonal matrix with $s_1, s_2, \ldots, s_n$ as diagonal elements
$\lambda_i(M)$	The i-th eigenvalue of matrix M
$\mathrm{trace}(M)$	Sum of diagonal elements of matrix M
$\mathrm{blockdiag}\,(M_1, M_2, \ldots, M_n)$	Block diagonal matrix with $M_1, M_2, \ldots, M_n$ as diagonal elements
$\mathrm{vec}([\ \eta_1\ \ \eta_2\ \ \cdots\ \ \eta_n\ ])$	$[\ \eta_1^{\mathrm{T}}\ \ \eta_2^{\mathrm{T}}\ \ \cdots\ \ \eta_n^{\mathrm{T}}\ ]^{\mathrm{T}}$
$\mathrm{unvec}([\ \eta_1^{\mathrm{T}}\ \ \eta_2^{\mathrm{T}}\ \ \cdots\ \ \eta_n^{\mathrm{T}}\ ]^{\mathrm{T}})$	$[\ \eta_1\ \ \eta_2\ \ \cdots\ \ \eta_n\ ]$
$A \otimes B$	Kronecker product of A and B
I_n	Identity matrix of n-order
$\mathrm{eig}(M)$	Set of eigenvalues of matrix M

systems under positional and incremental input saturation. Moreover, this PI regulator was generalized to handle linear infinite dimensional systems [9–11], some types of nonlinear systems [12–17], and some practical systems, such as electrical [18–20], vehicle dynamic [21], and optical communication systems [22].

This paper considers attitude control in flexible satellites. The PI regulator proposed in [5] may meet the mission requirements for satellite attitude maneuvering. However, it could not cope with the following several necessary aspects required in many practical applications.

(1) In outer space, unpredictable external disturbances often occur, such as electromagnetic pulses and solar wind. Furthermore, in modeling a satellite attitude system with flexible attachments, high-order unmodeled dynamics are often inevitable. Thus, it is necessary for designs to consider the attenuation of complicated time-varying disturbances.

(2) In long-term satellite operation periods, the effect of parameter perturbations cannot be overlooked. To preserve robust stability, it is also necessary to make closed-loop eigenvalues insensitive to parameter perturbations.

(3) Because of limited and valuable energy resources in satellite and the limitation in the output capacity of the actuator, the control torque must be as small as possible, which is equivalent to minimizing the gains of a controller and/or observer.

In our recent work [23], we proposed a parametric multiobjective design method for the stabilizing controller of a flexible satellite attitude system, which comprehensively considered the above three requirements. In this paper, the idea in [23] is modified and generalized to the output regulator design by adding the integration of the regulation error signal to the control law. The basic steps of the proposed method are to first establish a completely parametric expression of the PI output regulator, and then optimize the parameters to satisfy the above three multiobjective design requirements. The main contributions of this study can be summarized as follows.

(1) All PI regulators that yield a closed-loop system with the desired eigenstructure are obtained. Full degrees of freedom, characterized by a set of parameter vectors, are preserved. Moreover, the restriction of distinct closed-loop poles [4, 5] is removed.

(2) The design degrees of freedom are used to meet the three multiobjective design requirements, namely, lower closed-loop eigenvalue sensitivities, smaller control gains, and greater tolerance to complicated external disturbances.

(3) The separation principle for closed-loop eigenvalue sensitivities is proven, which performs an important role in the design process.

The symbols used in this paper are listed in Table 1. The remainder of this paper is organized as follows. Section 2 provides a description of the problem to be considered. Some preliminary results are presented in Section 3, including the separation principle of closed-loop poles, and closed-loop eigenstructures. A complete parametric form of the observer-based control law is outlined in Section 4. Then, in Section 5, the separation principle of eigenvalue sensitivities is derived, and the parameters are further optimized by minimizing the considered multiobjective design requirements. Finally, in Section 6, the proposed method is applied to attitude control of a flexible spacecraft, which is followed by a brief conclusion.

2 Problem formulation

2.1 The model

Consider the following linear system:

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + D_t d(t) + D_c d_c, \\ y(t) = Cx(t), \\ y_p(t) = C_p y(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^r$ are the state and the control input of the system, respectively; $d(t) \in \mathbb{R}^v$ and $d_c \in \mathbb{R}^w$ are time-varying and constant disturbances, respectively; $y(t) \in \mathbb{R}^m$ is a measured output, $y_p(t) \in \mathbb{R}^p$ is a controlled output which is expected to track a given constant signal y_r; and A, B, C, C_p, D_t, and D_c are known real coefficient matrices with appropriate dimensions satisfying Assumptions 1 and 2.

Assumption 1. $(A,\ B)$ is controllable and $(A,\ C)$ is observable.

Assumption 2. $\operatorname{rank}\left[\begin{smallmatrix} A & B \\ C_pC & 0 \end{smallmatrix}\right] = n + p$.

2.2 The control law

The observer-based PI control law considered in this paper takes the form of

$$\begin{cases} \dot{q}(t) = y_p(t) - y_r, \\ \dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + L\left(C\hat{x}(t) - y(t)\right), \\ u(t) = K_1\hat{x}(t) + K_2 q(t), \end{cases} \tag{2}$$

where $y_r \in \mathbb{R}^p$ is the constant reference signal to be tracked; $q(t) \in \mathbb{R}^p$ and $\hat{x}(t) \in \mathbb{R}^n$ are two newly introduced state variables; and $K_1 \in \mathbb{R}^{r\times n}$, $K_2 \in \mathbb{R}^{r\times p}$, and $L \in \mathbb{R}^{n\times m}$ are gain matrices to be designed. The structure of the control system is shown in Figure 1.

Let

$$X = \begin{bmatrix} x \\ q \\ \hat{x} \end{bmatrix}, \quad y_{rz} = \begin{bmatrix} 0 \\ y_r \\ 0 \end{bmatrix}, \tag{3}$$

then the closed-loop system of (1) with the controller (2) is obtained as

$$\begin{cases} \dot{X} = A_z X + D_{tz} d + D_{cz} d_c - y_{rz}, \\ y_p = C_{pz} X, \end{cases} \tag{4}$$

where

$$A_z = \begin{bmatrix} A & BK_2 & BK_1 \\ C_pC & 0 & 0 \\ -LC & BK_2 & A + BK_1 + LC \end{bmatrix}, \quad C_{pz} = \begin{bmatrix} C_pC & 0_{p\times(n+p)} \end{bmatrix}, \tag{5}$$

and

$$D_{tz} = \begin{bmatrix} D_t \\ 0_{(n+p)\times v} \end{bmatrix}, \quad D_{cz} = \begin{bmatrix} D_c \\ 0_{(n+p)\times w} \end{bmatrix}. \tag{6}$$

2.3 The problem

The problem to be investigated can be stated as follows.

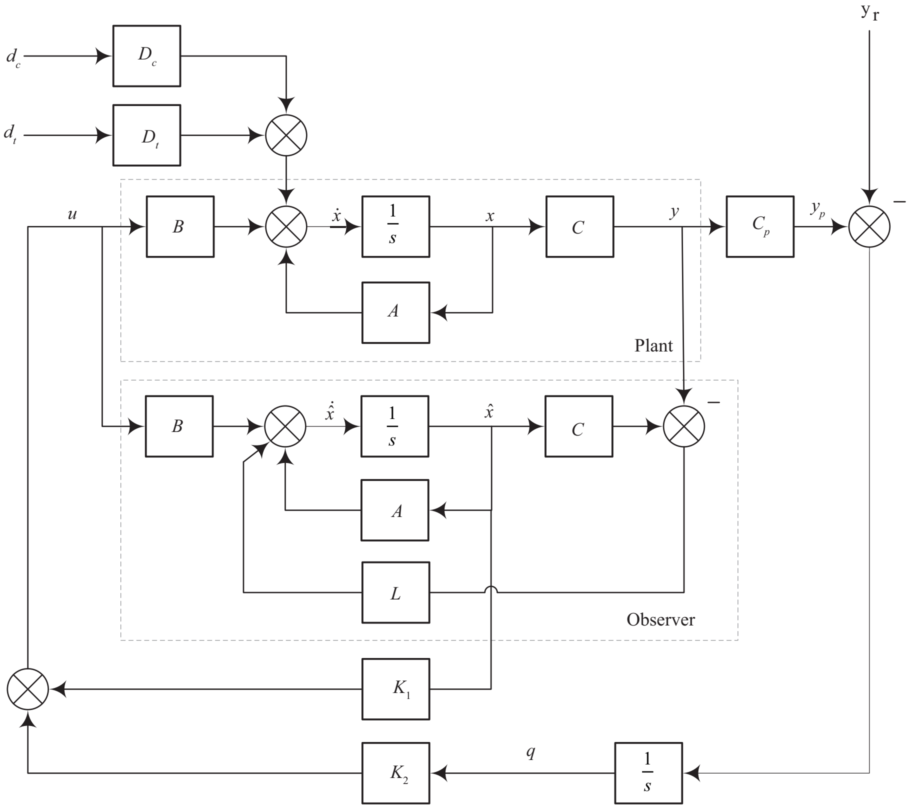

Figure 1 Control system structure.

Problem 1. Suppose that the system (1) satisfies Assumptions 1 and 2. Let $y_r \in \mathbb{R}^p$ be a constant reference signal. Determine the gain matrices K_1, K_2, and L in the control law (2) such that the following design requirements are met simultaneously:

(1) A_z is nondefective, and all its eigenvalues are located in a desired region $\Omega \subset \mathbb{C}^-$;

(2) When $d(t) = 0$, the controlled output y_p asymptotically tracks the given constant reference signal y_r, that is

$$\lim_{t\to\infty} (y_p(t) - y_r) = 0;$$

(3) The controlled output y_p is statically decoupled from the constant disturbance input d_c, that is, the final value $y_p(\infty)$ is independent of the constant disturbance d_c;

(4) The H_2-norm of the transfer function $G_{dy_p}(s)$, from the time-varying disturbance $d(t)$ to the output y_p, given by

$$G_{dy_p}(s) = C_{pz} (sI - A_z)^{-1} D_{tz} \tag{7}$$

is as small as possible;

(5) The closed-loop poles, that is, the eigenvalues of A_z, are as insensitive as possible to parameter perturbations in the system coefficient matrices A, B, and C;

(6) The 2-norm of the gain matrices $K = [\, K_1 \;\; K_2 \,]$ and L are as small as possible.

3 Preliminaries

3.1 Separation principle of closed-loop poles

If we take the following full state feedback control law

$$u(t) = K_1 x(t) + K_2 q(t), \tag{8}$$

and let

$$\eta=\begin{bmatrix}x\\q\end{bmatrix},\quad \tilde{y}_r=\begin{bmatrix}0\\y_r\end{bmatrix},\tag{9}$$

then, when the above controller (8) is applied to system (1), a closed-loop system is obtained as

$$\dot{\eta}=A_c\eta+\tilde{D}_t d(t)+\tilde{D}_c d_c-\tilde{y}_r,\tag{10}$$

where

$$\begin{cases}A_c=\tilde{A}_1+\tilde{B}K,\\ \tilde{A}_1=\begin{bmatrix}A&0\\C_pC&0\end{bmatrix},\ \tilde{B}=\begin{bmatrix}B\\0\end{bmatrix},\ K=[\,K_1\ K_2\,],\end{cases}\tag{11}$$

and

$$\tilde{D}_t=\begin{bmatrix}D_t\\0\end{bmatrix},\quad \tilde{D}_c=\begin{bmatrix}D_c\\0\end{bmatrix}.\tag{12}$$

The expression of A_c given in (11) suggests us to consider the controllability of $(\tilde{A}_1,\ \tilde{B})$.

Lemma 1 ([24])**.** Suppose that system (1) satisfies Assumption 1. Let $\tilde{A}_1$ and $\tilde{B}$ be given by (11). Then $(\tilde{A}_1,\ \tilde{B})$ is controllable if and only if Assumption 2 is satisfied.

On the other hand, the observer (2) can be rewritten as

$$\dot{\hat{x}}(t)=A_o\hat{x}(t)+Bu(t)-Ly(t),\tag{13}$$

where

$$A_o=A+LC.\tag{14}$$

Motivated by the well-known separation principle in the conventional observer-based state feedback control theory, we naturally wonder if the following relation holds:

$$\mathrm{eig}(A_z)=\mathrm{eig}(A_c)\cup\mathrm{eig}(A_o),\tag{15}$$

where A_z is given by (5).

Theorem 1 (Separation principle of closed-loop poles)**.** Let the system be given by (1), and the control law be taken as (2), then the relation (15) holds.

Proof. Let

$$P=\begin{bmatrix}I_{n+p}&0\\P_{21}&-I_n\end{bmatrix},\quad P_{21}=[\,I_n\ 0\,],\tag{16}$$

then, it can be verified that

$$A_\xi=PA_zP^{-1},\tag{17}$$

where

$$A_\xi=\begin{bmatrix}A_c&\tilde{A}_2\\0&A_o\end{bmatrix}\tag{18}$$

with

$$\tilde{A}_2=\begin{bmatrix}-BK_1\\0\end{bmatrix}.\tag{19}$$

It follows from (18) that the eigenvalues of A_ξ consist of those of A_c and A_o. Thus, it follows from (17) and the non-singularity of P that the conclusion holds. Then the proof is completed.

3.2 Closed-loop eigenstructure

The following result further gives the eigenstructure of A_z.

Theorem 2. Suppose that the matrices A_c and A_o given by (11) and (14), respectively, are both nondefective and do not have common eigenvalues. Let

(1) Λ_c be a Jordan form of A_c, and T_c and V_c be the corresponding pair of normalized left and right eigenvector matrices of A_c associated with Λ_c;

(2) Λ_o be a Jordan form of A_o, and T_o and V_o be the corresponding pair of normalized left and right eigenvector matrices of A_o associated with Λ_o.

Then, the matrix A_z given by (5) is also nondefective and has the following Jordan form:

$$\Lambda_z = \text{blockdiag}\left(\Lambda_c, \Lambda_o\right), \tag{20}$$

and the corresponding normalized left and right eigenvector matrices take the forms of

$$T_z^{\mathrm{T}} = \begin{bmatrix} T_c^{\mathrm{T}} - Q_* T_o^{\mathrm{T}} P_{21} & Q_* T_o^{\mathrm{T}} \\ T_o^{\mathrm{T}} P_{21} & -T_o^{\mathrm{T}} \end{bmatrix} \tag{21}$$

and

$$V_z = \begin{bmatrix} V_c & V_c Q_* \\ P_{21} V_c & P_{21} V_c Q_* - V_o \end{bmatrix}, \tag{22}$$

respectively, where P_{21} is given by (16),

$$Q_* = \text{unvec}_{(n+p),n}\left[\Phi^{-1}\text{vec}\left(T_c^{\mathrm{T}} \tilde{A}_2 V_o\right)\right] \tag{23}$$

with $\tilde{A}_2$ being given by (19), and

$$\Phi = \left(\Lambda_o \otimes I_{n+p}\right) - \left(I_n \otimes \Lambda_c\right). \tag{24}$$

Proof. It is known from the assumptions that

$$T_c^{\mathrm{T}} A_c V_c = \Lambda_c, \quad T_c^{\mathrm{T}} V_c = I, \tag{25}$$

and

$$T_o^{\mathrm{T}} A_o V_o = \Lambda_o, \quad T_o^{\mathrm{T}} V_o = I. \tag{26}$$

Thus, in view of (17), (21), and (22), we can verify

$$T_z^{\mathrm{T}} V_z = I, \tag{27}$$

and

$$T_z^{\mathrm{T}} A_z V_z = \begin{bmatrix} \Lambda_c & \Lambda_c Q_* - Q_* \Lambda_o + T_c^{\mathrm{T}} \tilde{A}_2 V_o \\ 0 & \Lambda_o \end{bmatrix}. \tag{28}$$

Since A_c and A_o do not have common eigenvalues, neither do Λ_c and Λ_o. Therefore, it follows from matrix equation theory that there exists a unique solution to the following linear matrix equation with respect to Q:

$$Q\Lambda_o - \Lambda_c Q = T_c^{\mathrm{T}} \tilde{A}_2 V_o. \tag{29}$$

With the help of matrix vectorization operations, it can be verified that Q_* given by (23) is the unique solution of the matrix equation (29). Thus, the relation (28) becomes

$$T_z^{\mathrm{T}} A_z V_z = \Lambda_z. \tag{30}$$

Combining (27) and (30) gives the result. The proof is completed.

According to Theorems 1 and 2, the first design requirement in Problem 1 can be transformed into two eigenstructure assignment subproblems, that is, to find the gain matrices K and L such that $\tilde{A}_1 + \tilde{B}K$ and $A + LC$ possess desired nondefective eigenstructure.

3.3 Eigenvalue sensitivities

The following lemma provides a base for insensitive pole assignment.

Lemma 2 ([25])**.** Let $M(\delta) \in \mathbb{R}^{n\times n}$ be a nondefective matrix associated with a parameter vector δ. Then, for every eigenvalue $\lambda_i(M(0))$, there exist a pair of normalized left and right eigenvectors x_i and y_i satisfying

$$x_i^{\mathrm{T}} M(0) y_i = \lambda_i(M(0)), \quad x_i^{\mathrm{T}} y_i = 1, \tag{31}$$

and the sensitivities of the eigenvalues of the matrix $M(\delta)$ with respect to the parameter vector δ are given by

$$\frac{\partial\lambda_i(M(\delta))}{\partial\delta} \approx x_i^{\mathrm{T}} \frac{\partial M(\delta)}{\partial\delta} y_i. \tag{32}$$

With Lemma 2, we can give a definition of eigenvalue sensitivity function as follows.

Definition 1. Let $M(\delta) \in \mathbb{R}^{n\times n}$ be a nondefective matrix associated with a parameter vector δ. Let $\lambda_i(M(0))$, $i = 1, 2, \ldots, n$, be eigenvalues of $M(0)$ and x_i, y_i, $i = 1, 2, \ldots, n$ given by (31) be the corresponding normalized left and right eigenvectors. Then,

$$\left[\frac{\partial\lambda_i(M(\delta))}{\partial\delta}\right]_0 = x_i^{\mathrm{T}} \frac{\partial M(\delta)}{\partial\delta} y_i \tag{33}$$

is called a sensitivity function of the eigenvalue $\lambda_i(M)$ with respect to the parameter δ.

4 Parameterization of the control law

Clearly, Problem 1 can be solved in two steps. The first step is controller parameterization, that is, to find a complete parametric form of the gain matrices K_1, K_2, and L in the control law (2) such that the design requirements 1–3 in Problem 1 are satisfied. The second step is parameter optimization, that is, to optimize the obtained parameters to meet the design requirements 4–6 in Problem 1.

4.1 Parameterization of K

This subsection considers the parametric design of the gain matrix K. Let

$$\Lambda_c = \mathrm{diag}\left(s_1^c, s_2^c, \ldots, s_{n+p}^c\right) \tag{34}$$

be the desired diagonal Jordan form of $\tilde{A}_1 + \tilde{B}K$, where s_i^c, $i = 1, 2, \ldots, n+p$, are a set of self-conjugate complex numbers with negative real parts. Then, the eigenstructure assignment of A_c can be solved by finding a complete parametric form of K and a nonsingular eigenvector matrix V satisfying

$$A_c V = V\Lambda_c. \tag{35}$$

In view of Assumptions 1 and 2, it follows from Lemma 1 that the matrix pair $(\tilde{A}_1, \tilde{B})$ is controllable. Therefore, according to [26], there exist a pair of polynomial matrices $N(s) \in \mathbb{R}^{(n+p)\times r}[s]$ and $D(s) \in \mathbb{R}^{r\times r}[s]$ satisfying the following right coprime factorization (RCF):

$$(sI - \tilde{A}_1)N(s) = \tilde{B}D(s). \tag{36}$$

Furthermore, according to the eigenstructure assignment result in [26], all the matrices K and V satisfying (35) can be given by

$$\begin{cases} K = WV^{-1}, \\ V = [\, N(s_1^c)f_1 \;\; N(s_2^c)f_2 \;\; \cdots \;\; N(s_{n+p}^c)f_{n+p} \,], \\ W = [\, D(s_1^c)f_1 \;\; D(s_2^c)f_2 \;\; \cdots \;\; D(s_{n+p}^c)f_{n+p} \,], \end{cases} \tag{37}$$

where $f_i \in \mathbb{C}^r$, $i = 1, 2, \ldots, n+p$, are a set of parameters satisfying the following constraints:

Constraint C1. $\det[\, N(s_1^c)f_1 \;\; N(s_2^c)f_2 \;\; \cdots \;\; N(s_{n+p}^c)f_{n+p} \,] \neq 0$.

Constraint C2. $f_i = \bar{f}_l$, if $s_i = \bar{s}_l$ and $\mathrm{Im}\,(s_i) \neq 0$, $i, l = 1, 2, \ldots, n+p$.

4.2 Parameterization of L

Parallel to the parameterization of K, the gain matrix L can be also parameterized in a similar manner. Denote the desired Jordan form of $A_o = A + LC$ by

$$\Lambda_o = \operatorname{diag}\left(s_1^o, s_2^o, \ldots, s_n^o\right), \tag{38}$$

where $s_i^o,\ i = 1, 2, \ldots, n$, are a set of self-conjugate complex numbers with negative real parts. To solve the problem of eigenstructure assignment in A_o, we need to find a complete parametric form of the gain matrix L and a corresponding nonsingular left-eigenvector matrix T satisfying

$$T^{\mathrm{T}} A_o = \Lambda_o T^{\mathrm{T}}. \tag{39}$$

It follows from Assumption 1 that (A, C) is observable, thus $(A^{\mathrm{T}}, C^{\mathrm{T}})$ is controllable. Then, according to [26] again, there exist a pair of polynomial matrices $H(s) \in \mathbb{R}^{n\times m}[s]$ and $L(s) \in \mathbb{R}^{m\times m}[s]$ satisfying the following RCF:

$$(sI - A^{\mathrm{T}})H(s) = C^{\mathrm{T}} L(s), \tag{40}$$

and all the matrices L and T satisfying (39) are given by

$$\begin{cases} L = T^{-\mathrm{T}} G^{\mathrm{T}}, \\ T = \left[\, H(s_1^o)g_1 \ \ H(s_2^o)g_2 \ \cdots \ H(s_n^o)g_n \,\right], \\ G = \left[\, L(s_1^o)g_1 \ \ L(s_2^o)g_2 \ \cdots \ L(s_n^o)g_n \,\right], \end{cases} \tag{41}$$

where $g_i \in \mathbb{C}^m,\ i = 1, 2, \ldots, n$, are parameters satisfying the following constraints.

Constraint C3. $\det\left[\, H(s_1^o)g_1 \ \ H(s_2^o)g_2 \ \cdots \ H(s_n^o)g_n \,\right] \neq 0$.

Constraint C4. $g_i = \bar{g}_l$, if $s_i = \bar{s}_l$ and $\operatorname{Im}(s_i) \neq 0,\ i, l = 1, 2, \ldots, n$.

4.3 Main result

Based on the separation principle of closed-loop poles obtained in Subsection 3.1 and the above parametric expressions of K and L given by (37) and (41), respectively, Theorem 3 can be finally obtained.

Theorem 3. Suppose that system (1) satisfies Assumptions 1 and 2. Let $\Gamma_c = \{s_i^c,\ i = 1, 2, \ldots, n+p\}$ and $\Gamma_o = \{s_j^o,\ j = 1, 2, \ldots, n\}$ be two sets of self-conjugate complex numbers with negative real parts, $\Gamma_c \cap \Gamma_o = \emptyset$, and $y_r \in \mathbb{R}^p$ be a constant reference signal. Then, all the gain matrices K and L which (1) make A_z nondefective and $\operatorname{eig}(A_z) = \Gamma_c \cup \Gamma_o$, and (2) guarantee the second and third design requirements in Problem 1, are given by (37) and (41), respectively, where $f_i \in \mathbb{C}^r,\ i = 1, 2, \ldots, n+p$, and $g_i \in \mathbb{C}^m,\ i = 1, 2, \ldots, n$, are parameters satisfying Constraints C1–C4.

Proof. It follows from Subsections 4.1 and 4.2 that, when K and L are taken as (37) and (41), respectively, Λ_c and Λ_o are the nondefective Jordan matrices of A_c and A_o, respectively. Therefore, the requirement (1) in Theorem 3 holds following from Theorems 1 and 2.

Taking differentials with respect to t on both sides of the first equation in (4), it gives that when $d(t) = 0$,

$$\dot{z} = A_z z, \tag{42}$$

where

$$z = \dot{X}. \tag{43}$$

Considering that both A_c and A_o are stable, we know from Theorem 1 that A_z is also stable. Thus we have

$$\lim_{t\to\infty} z = 0. \tag{44}$$

This implies, in view of (3),

$$\lim_{t\to\infty} \dot{q} = 0. \tag{45}$$

Therefore, it follows from the first equation in (2) that the second and third design requirement in Problem 1 is satisfied. Then the proof is completed.

5 Multiobjective design

The first step to solve Problem 1 is provided in Section 4. In this section we further treat the second step, that is, to optimize the obtained parameters to meet the design requirements 4–6 in Problem 1.

5.1 Disturbance attenuation

For the robustness consideration, the effect of the time-varying disturbance $d(t)$ on the controlled output $y_p(t)$ is expected to be as small as possible, which suggests us to minimize the H_2-norm of the transfer function $G_{dy_p}(s)$ given by (7). With the help of Theorem 2, an explicit expression of $\|G_{dy_p}(s)\|_2$ can be obtained.

Theorem 4. Suppose that the system (1) satisfies Assumptions 1 and 2. Let Λ_c and Λ_o be given by (34) and (38), respectively, with $\mathrm{eig}(\Lambda_c) = \{s_i^c,\ i = 1, 2, \ldots, n+p\}$ and $\mathrm{eig}(\Lambda_o) = \{s_j^o,\ j = 1, 2, \ldots, n\}$ being two sets of self-conjugate complex numbers with negative real parts, and satisfying $\mathrm{eig}(\Lambda_c) \cap \mathrm{eig}(\Lambda_o) = \emptyset$. Then, when K and L are taken as (37) and (41), respectively, and Constraints C1–C4 hold, we have

$$\left\|G_{dy_p}(s)\right\|_2 = \left(\mathrm{trace}\left(\Gamma_2 P_1^* \Gamma_2^{\mathrm{T}}\right)\right)^{\frac{1}{2}} = \left(\mathrm{trace}\left(\Gamma_1^{\mathrm{T}} P_2^* \Gamma_1\right)\right)^{\frac{1}{2}} \tag{46}$$

with

$$\begin{cases} P_1^* = \mathrm{unvec}_{(2n+p),(2n+p)}\left[-\Psi^{-1}\mathrm{vec}(\Gamma_1\Gamma_1^{\mathrm{T}})\right], \\ P_2^* = \mathrm{unvec}_{(2n+p),(2n+p)}\left[-\Psi^{-1}\mathrm{vec}(\Gamma_2^{\mathrm{T}}\Gamma_2)\right], \\ \Gamma_1 = \begin{bmatrix} V^{-1}\tilde{D}_t - Q_* T^{\mathrm{T}} P_{21}\tilde{D}_t \\ T^{\mathrm{T}} P_{21}\tilde{D}_t \end{bmatrix}, \\ \Gamma_2 = [\,\tilde{C}_p V \quad \tilde{C}_p V Q_*\,], \\ Q_* = \mathrm{unvec}_{(n+p),n}\left[\Phi^{-1}\mathrm{vec}(V^{-1}\tilde{A}_2 T^{-\mathrm{T}})\right], \end{cases} \tag{47}$$

where V and T are given by (37) and (41), respectively, $\tilde{D}_t,\ P_{21}$, and Φ are given by (12), (16), and (24), respectively, and

$$\tilde{C}_p = [\,C_p C\ \ 0\,], \tag{48}$$

$$\Psi = (I_{2n+p} \otimes \Lambda_z) + (\Lambda_z \otimes I_{2n+p}) \tag{49}$$

with Λ_z being given by (20).

Proof. As shown in Subsections 4.1 and 4.2, when K and L are respectively taken as (37) and (41), matrices V^{-1} and V form a pair of normalized left and right eigenvector matrices of A_c, and T^{T} and $T^{-\mathrm{T}}$ form a pair of normalized left and right eigenvector matrices of A_o. Therefore, according to Theorem 2, Λ_z is the Jordan form of A_z, and the corresponding normalized left and right eigenvector matrices are given by

$$V_z = \begin{bmatrix} V & VQ_* \\ P_{21}V & P_{21}VQ_* - T^{-\mathrm{T}} \end{bmatrix}, \quad T_z^{\mathrm{T}} = \begin{bmatrix} V^{-1} - Q_* T^{\mathrm{T}} P_{21} & Q_* T^{\mathrm{T}} \\ T^{\mathrm{T}} P_{21} & -T^{\mathrm{T}} \end{bmatrix}, \tag{50}$$

which means that (27) and (30) hold. Thus, $\|G_{dy_p}(s)\|_2$ can be transformed as

$$\left\|G_{dy_p}(s)\right\|_2 = \left\|C_{pz} V_z (sI - \Lambda_z)^{-1} T_z^{\mathrm{T}} D_{tz}\right\|_2.$$

Since Λ_c and Λ_o are stable, we know from Subsections 4.1 and 4.2 that A_c and A_o are also stable when K and L are taken as (37) and (41), respectively. Then, it follows from Theorem 1 that A_z is stable. Therefore, according to the Lemma 4.1 in [27], there exists unique symmetric positive definite solutions P_1 and P_2 to the following Lyapunov matrix equations

$$\Lambda_z P_1 + P_1 \Lambda_z = -T_z^{\mathrm{T}} D_{tz} D_{tz}^{\mathrm{T}} T_z, \tag{51}$$

$$\Lambda_z P_2 + P_2 \Lambda_z = -V_z^{\mathrm{T}} C_{pz}^{\mathrm{T}} C_{pz} V_z, \tag{52}$$

and $\|G_{dy_p}(s)\|_2$ is given by

$$\left\|G_{dy_p}(s)\right\|_2 = \left(\mathrm{trace}\left(C_{pz} V_z P_1 V_z^{\mathrm{T}} C_{pz}^{\mathrm{T}}\right)\right)^{\frac{1}{2}}$$

$$= \left(\text{trace}\left(D_{tz}^{\mathrm{T}} T_z P_2 T_z^{\mathrm{T}} D_{tz}\right)\right)^{\frac{1}{2}}. \tag{53}$$

With the help of matrix vectorization operations and in view of (50), it can be verified that P_1^* and P_2^* given by (47) are the unique solutions to (51) and (52), respectively. Thus the result (46) can be obtained. The proof is completed.

Based on Theorem 4, we can attenuate the influence of disturbance $d(t)$ on the controlled output $y_p(t)$ by minimizing the following index:

$$J_d\left(f_i, s_i^c,\ i=1,2,\ldots,n+p,\ g_j, s_j^o,\ j=1,2,\ldots,n\right) = \left[\text{trace}\left(\Gamma_2 P_1^* \Gamma_2^{\mathrm{T}}\right)\right]^{\frac{1}{2}}. \tag{54}$$

5.2 Closed-loop eigenvalue sensitivities

In order to handle the fifth design requirement in Problem 1, let us first reveal an important fact about the closed-loop eigenvalue sensitivities.

5.2.1 *Separation principle for eigenvalue sensitivities*

In view of Theorem 1, without loss of generality, we assume

$$\lambda_i(A_z) = \lambda_i(A_c),\ i=1,2,\ldots,n+p, \tag{55}$$

and

$$\lambda_{i+n+p}(A_z) = \lambda_i(A_o),\ i=1,2,\ldots,n. \tag{56}$$

The sensitivities of these closed-loop eigenvalues are given in the following result.

Theorem 5 (Separation principle for eigenvalue sensitivities)**.** Let A_c and A_o be given by (11) and (14), respectively, which are both nondefective and do not have common eigenvalues. Then, we have

$$\left[\frac{\partial \lambda_i(A_z)}{\partial \delta}\right]_0 = \left[\frac{\partial \lambda_i(A_c)}{\partial \delta}\right]_0,\ i=1,2,\ldots,n+p, \tag{57}$$

and

$$\left[\frac{\partial \lambda_{i+n+p}(A_z)}{\partial \delta}\right]_0 = \left[\frac{\partial \lambda_i(A_o)}{\partial \delta}\right]_0,\ i=1,2,\ldots,n, \tag{58}$$

where A_z is given by (5).

Proof. Let Λ_c and Λ_o denote the Jordan matrices of A_c and A_o, respectively;

$$T_c = [\,t_1^c\ t_2^c\ \cdots\ t_{n+p}^c\,] \text{ and } V_c = [\,v_1^c\ v_2^c\ \cdots\ v_{n+p}^c\,] \tag{59}$$

be the corresponding pair of normalized left and right eigenvector matrices of the matrix A_c; and

$$T_o = [\,t_1^o\ t_2^o\ \cdots\ t_n^o\,] \text{ and } V_o = [\,v_1^o\ v_2^o\ \cdots\ v_n^o\,] \tag{60}$$

be the corresponding pair of normalized left and right eigenvector matrices of the matrix A_o. let $[M]_{ij}$ denote the i-th row and j-th column element of a matrix M, then, in view of (59) and (60), we have

$$\left[\frac{\partial \lambda_i(A_c)}{\partial \delta}\right]_0 = (t_i^c)^{\mathrm{T}} \frac{\partial A_c}{\partial \delta} v_i^c = \left[T_c^{\mathrm{T}} \frac{\partial A_c}{\partial \delta} V_c\right]_{ii},\ i=1,2,\ldots,n+p, \tag{61}$$

and

$$\left[\frac{\partial \lambda_i(A_o)}{\partial \delta}\right]_0 = (t_i^o)^{\mathrm{T}} \frac{\partial A_o}{\partial \delta} v_i^o = \left[T_o^{\mathrm{T}} \frac{\partial A_o}{\partial \delta} V_o\right]_{ii},\ i=1,2,\ldots,n. \tag{62}$$

According to Theorem 2, when A_c and A_o are both nondefective and do not have common eigenvalues, the matrix Λ_z given by (20) forms the Jordan matrix of A_z, and T_z and V_z given by (21) and (22), respectively, form the corresponding pair normalized left and right eigenvector matrices. Thus, similar to (61) and (62), we have

$$\left[\frac{\partial \lambda_i(A_z)}{\partial \delta}\right]_0 = \left[T_z^{\mathrm{T}} \frac{\partial A_z}{\partial \delta} V_z\right]_{ii},\ i=1,2,\ldots,2n+p, \tag{63}$$

which, in view of (17), can be rewritten as

$$\left[\frac{\partial\lambda_i(A_z)}{\partial\delta}\right]_0 = \left[T_z^{\mathrm{T}} P^{-1}\frac{\partial A_\xi}{\partial\delta} P V_z\right]_{ii},\ i = 1, 2, \ldots, 2n+p, \tag{64}$$

where A_ξ and P are given by (18) and (16), respectively. It follows from (21) and (22) that Eq. (64) can be further transformed into

$$\left[\frac{\partial\lambda_i(A_z)}{\partial\delta}\right]_0 = \begin{bmatrix} T_c^{\mathrm{T}}\dfrac{\partial A_c}{\partial\delta}V_c & * \\ 0 & V_o^{\mathrm{T}}\dfrac{\partial A_o}{\partial\delta}T_o \end{bmatrix}_{ii},\quad i = 1, 2, \ldots, 2n+p. \tag{65}$$

Combining (61), (62) and (65) produces (57) and (58). Then the proof is completed.

Remark 1. It follows from the separation principle of closed-loop eigenvalues (Theorem 1) that the closed-loop eigenvalues are composed of those of the PI control system and those of the observer system, while the above result further states another important fact: as an eigenvalue of the overall closed-loop system, an eigenvalue of the PI control system has a sensitivity defined with the overall system, which is equal to its sensitivity defined with the PI control system; meanwhile, as also an eigenvalue of the overall closed-loop system, an eigenvalue of the observer system has a sensitivity defined with the overall system, which is equal to its sensitivity defined with the observer system. This is why the result is called the separation principle of eigenvalue sensitivities. This result is actually a generalization of the one introduced in [28]. In fact, the result reduces to the one in [28] when the integral part in the controller is removed.

5.2.2 *Explicit expressions*

Assume that the coefficients A, B, and C in (1) are perturbed by

$$\Delta A(\delta) = \sum_{i=1}^{l} A_i\delta_i,\quad \Delta B(\delta) = \sum_{i=1}^{l} B_i\delta_i,\quad \Delta C(\delta) = \sum_{i=1}^{l} C_i\delta_i, \tag{66}$$

where $\delta = [\,\delta_1\ \delta_2\ \cdots\ \delta_l\,]^{\mathrm{T}}$, and the perturbed coefficients are described by

$$A(\delta) = A_0 + \Delta A(\delta),\quad B(\delta) = B_0 + \Delta B(\delta),\quad C(\delta) = C_0 + \Delta C(\delta). \tag{67}$$

Theorem 5 allows us to only treat $[\frac{\partial\lambda_i(A_c)}{\partial\delta}]_0$ and $[\frac{\partial\lambda_i(A_o)}{\partial\delta}]_0$ instead of $[\frac{\partial\lambda_i(A_z)}{\partial\delta}]_0$.

Firstly, let us find the explicit form of $[\frac{\partial\lambda_i(A_c)}{\partial\delta}]_0$. It is known from Subsection 4.1 that when K is taken as (37), the matrices V^{-1} and V are a pair of normalized left and right eigenvector matrices of A_c. Therefore, according to Definition 1, we have

$$\left[\frac{\partial\lambda_i(A_c)}{\partial\delta_j}\right]_0 = \left[V^{-1}\frac{\partial A_c}{\partial\delta_j}V\right]_{ii} = e_i^{\mathrm{T}}V^{-1}\frac{\partial A_c}{\partial\delta_j}Ve_i,\quad i = 1, 2, \ldots, n+p,\ j = 1, 2, \ldots, l, \tag{68}$$

where e_i represents a vector with the i-th element being 1, and the remaining elements being 0. Substituting (11) into (68) yields

$$\left[\frac{\partial\lambda_i(A_c)}{\partial\delta_j}\right]_0 = e_i^{\mathrm{T}}V^{-1}\Omega_j Ve_i,\quad i = 1, 2, \ldots, n+p,\ j = 1, 2, \ldots, l, \tag{69}$$

where

$$\Omega_j = \begin{bmatrix} A_j + B_jK_1 & B_jK_2 \\ C_pC_j & 0 \end{bmatrix}. \tag{70}$$

Secondly, let us consider $[\frac{\partial\lambda_i(A_o)}{\partial\delta}]_0$. Similarly, as shown in Subsection 4.2, when L is taken as (41), the matrices T^{T} and $T^{-\mathrm{T}}$ become a pair of normalized left and right eigenvector matrices of A_o. Thus we know

$$\left[\frac{\partial\lambda_i(A_o)}{\partial\delta_j}\right]_0 = \left[T^{\mathrm{T}}\frac{\partial A_o}{\partial\delta_j}T^{-\mathrm{T}}\right]_{ii} = e_i^{\mathrm{T}}T^{\mathrm{T}}\frac{\partial A_o}{\partial\delta_j}T^{-\mathrm{T}}e_i,\quad i = 1, 2, \ldots, n,\ j = 1, 2, \ldots, l. \tag{71}$$

Substituting (14) into (71), in view of (67) and (66), gives

$$\left[\frac{\partial\lambda_i(A_o)}{\partial\delta_j}\right]_0 = e_i^{\mathrm{T}}T^{\mathrm{T}}\left(A_j + LC_j\right)T^{-\mathrm{T}}e_i, \quad i = 1,2,\ldots,n,\ j = 1,2,\ldots,l. \tag{72}$$

Based on the expressions (69) and (72), in order to attenuate the influence of the parameter perturbation δ on the eigenvalues of A_c and A_o, we can minimize the following two indices:

$$J_{rc}\left(f_i, s_i^c,\ i = 1,2,\ldots,n+p\right) = \sum_{i=1}^{n+p}\sum_{j=1}^{l}\left|e_i^{\mathrm{T}}V^{-1}\Omega_j V e_i\right|, \tag{73}$$

and

$$J_{ro}\left(g_i, s_i^o,\ i = 1,2,\ldots,n\right) = \sum_{i=1}^{n}\sum_{j=1}^{l}\left|e_i^{\mathrm{T}}T^{\mathrm{T}}\left(A_j + LC_j\right)T^{-\mathrm{T}}e_i\right|. \tag{74}$$

5.3 The algorithm

To reduce the amplitude of the control inputs, the following two indices also need to be minimized:

$$J_K\left(f_i, s_i^c,\ i = 1,2,\ldots,n+p\right) = \|[\,K_1\ \ K_2\,]\|_2 = \|K\|_2, \tag{75}$$

and

$$J_L\left(g_i, s_i^o,\ i = 1,2,\ldots,n\right) = \|L\|_2, \tag{76}$$

where K and L are given by (37) and (41), respectively, and Constraints C1–C4 hold.

With all above explicit expressions for indices (54), (73)–(76), a procedure for solving Problem 1 can be stated as follows:

(1) Establish the complete parametric forms (37) and (41) of the gain matrices $K_1,\ K_2$, and L in the proposed observer-based control law (2).

(2) Define an index function as

$$\begin{aligned} &J\left(f_i, s_i^c,\ i = 1,2,\ldots,n+p,\ g_j, s_j^o,\ j = 1,2,\ldots,n\right) \\ &= \alpha_{ro}J_{ro} + \alpha_{rc}J_{rc} + \alpha_d J_d + \alpha_K J_K + \alpha_L J_L, \end{aligned} \tag{77}$$

where J_d, J_{rc}, J_{ro}, J_K and J_L are given by (54), (73)–(76), respectively, and α_d, α_{rc}, α_{ro}, α_K, and α_L are proper weighting factors.

(3) Determine the ranges of the closed-loop poles $\mathcal{S}_c$ and $\mathcal{S}_o$ according to practical requirements.

(4) Solve the following optimization problem:

$$\begin{aligned} &\min\ J\left(f_i, s_i^c,\ i = 1,2,\ldots,n+p,\ g_j, s_j^o,\ j = 1,2,\ldots,n\right), \\ &\text{s.t. Constraints C1–C4}, \\ &\quad\left(s_1^c, s_2^c, \ldots, s_{n+p}^c\right) \in \mathcal{S}_c,\ \left(s_1^o, s_2^o, \ldots, s_n^o\right) \in \mathcal{S}_o, \end{aligned} \tag{78}$$

and obtain a sub-optimal solution $f_i^*, s_i^{c*},\ i = 1,2,\ldots,n+p,\ g_j^*, s_j^{o*},\ j = 1,2,\ldots,n$.

(5) Substitute the optimal solution $f_i^*, s_i^{c*},\ i = 1,2,\ldots,n+p,\ g_j^*, s_j^{o*},\ j = 1,2,\ldots,n$, into (37) and (41) to obtain the gain matrices K^* and L^*, and further partition K^* as

$$K^* = [\,K_1^*\ \ K_2^*\,],\quad K_1^* \in \mathbb{R}^{r\times n},\ K_2^* \in \mathbb{R}^{r\times p}.$$

Then, by replacing K_1, K_2, and L in (2) with K_1^*, K_2^*, and L^*, respectively, the designed observer-based control law is obtained.

Remark 2. The nonlinear programming (78) can be solved offline by some nonlinear optimization algorithms. The optimization toolbox in MATLAB is a ready-used choice.

6 Flexible satellite attitude control

6.1 The system model

Consider the pitch channel of the attitude system of a flexible satellite, whose model can be arranged into the state space model described by (1) [23, 29], with

$$x(t) = \begin{bmatrix} \theta & q_y & \dot{\theta} & \dot{q}_y \end{bmatrix}^{\mathrm{T}},$$

$$\begin{cases} A = A_0 + A_1\delta_1 + A_2\delta_2, \\ A_0 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -0.0111 & 0 & -0.0001 \\ 0 & -2.1111 & 0 & -0.0223 \end{bmatrix}, \\ A_1 = [\,0 \ \zeta \ 0 \ 0\,], \\ A_2 = [\,0 \ 0 \ 0 \ \zeta\,], \\ \zeta = [\,0 \ 0 \ 0.0109 \ 2.0667\,]^{\mathrm{T}}, \end{cases}$$

$$B = D_c = \begin{bmatrix} 0 \\ 0 \\ 0.0001 \\ 0.0124 \end{bmatrix}, \ D_t = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.0001 & 0.0124 \\ 0.0124 & 2.3452 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, \ C_p = [\,1 \ 0\,],$$

and

$$d(t) = \begin{bmatrix} 0.1\sin(2\pi t) & 0.1q_y^{(3)} \end{bmatrix}^{\mathrm{T}}, \ d_c = 0.12,$$

where θ is the attitude angle, and q_y is the flexible mode. The first component of $d(t)$ is the time-varying part of the disturbance torque, and the second component is the high-order unmodeled dynamics; d_c is the constant part of the disturbance torque. The regulation requirement is to realize the asymptotically tracking of $y_p = \theta$ to $y_r = -0.3^\circ$.

6.2 Control system design

It can be easily verified that the system satisfies Assumptions 1 and 2. Then, the polynomial matrices $H(s)$ and $L(s)$ satisfying RCF (40) are [23]

$$H(s) = \begin{bmatrix} 1 & 0 \\ 0 & 0.0111s \\ 0 & -s^2 - 0.0223s - 2.1111 \\ 0 & 0.0001s+0.0111 \end{bmatrix}, \tag{79}$$

and

$$L(s) = \begin{bmatrix} s & 0 \\ -1 & -s^3 - 0.0223s^2 - 2.1111s \end{bmatrix}, \tag{80}$$

respectively, and $N(s)$ and $D(s)$ satisfying RCF (36) can be given by

$$N(s) = \begin{bmatrix} -0.0001s^3 - 1.0766 \times 10^{-6}s^2 - 0.0001s \\ -0.0124s^3 \\ -0.0001s^4 - 1.0766 \times 10^{-6}s^3 - 0.0001s^2 \\ -0.0124s^4 \\ -0.0001s^2 - 1.0766 \times 10^{-6}s - 0.0001 \end{bmatrix}, \tag{81}$$

and

$$D(s) = -s^5 - 0.0223s^4 - 2.1111s^3, \tag{82}$$

respectively.

According to the practical requirements, determine the desired regions of closed-loop poles

$$\begin{cases} s_1^c \in [-0.0552, -0.0408], \\ s_{2,3}^c = -\alpha_2^c \pm \alpha_3^c\,\mathrm{i},\ \alpha_2^c \in [0.0684, 0.0926],\ \alpha_3^c \in [1.1514, 1.5578], \\ s_{4,5}^c = -\alpha_4^c \pm \alpha_5^c\,\mathrm{i},\ \alpha_4^c \in [0.0992, 0.1343],\ \alpha_5^c \in [0.0883, 0.1194], \end{cases} \tag{83}$$

and

$$\begin{cases} s_{1,2}^o = -\alpha_1^o \pm \alpha_2^o\,\mathrm{i},\ \alpha_1^o \in [0.1416, 0.1915],\ \alpha_2^o \in [0.2226, 0.3011], \\ s_3^o \in [-4.2748, -3.1596],\ s_4^o \in [-0.1159, -0.0857]. \end{cases} \tag{84}$$

Setting the weighting factors as

$$\alpha_{ro} = \alpha_{rc} = 10^{-4},\ \alpha_d = 100,\ \alpha_K = \alpha_L = 10^{-3}, \tag{85}$$

we can obtain a sub-optimal solution to the optimization problem (78) as follows:

$$\begin{cases} s_1^{c*} = -0.0459,\ s_{2,3}^{c*} = -0.0712 \pm 1.2135\,\mathrm{i},\ s_{4,5}^{c*} = -0.0996 \pm 0.1008\,\mathrm{i}, \\ s_{1,2}^{o*} = -0.1538 \pm 0.2679\,\mathrm{i},\ s_3^{o*} = -3.5340,\ s_4^{o*} = -0.1005, \\ g_{1,2}^* = \begin{bmatrix} -67.618 \mp 65.707\,\mathrm{i} \\ 94.905 \mp 32.796\,\mathrm{i} \end{bmatrix},\ g_3^* = \begin{bmatrix} 1.5340 \\ -10.036 \end{bmatrix},\ g_4^* = \begin{bmatrix} -297.75 \\ -7.9399 \end{bmatrix}, \\ f_1^* = 1,\ f_{2,3}^* = 1 \pm 1\,\mathrm{i},\ f_{4,5}^* = 1 \pm 1\,\mathrm{i}, \end{cases} \tag{86}$$

which gives the index value $J^* = 13.2877$. In this case the corresponding gain matrices are

$$K^* = [\,-423.76\ 50.279\ -3575.2\ 3.2751\ -13.321\,], \tag{87}$$

and

$$L^* = \begin{bmatrix} -0.10654 & -1.6039 \\ -0.29415 & -85.828 \\ 6.2805 \times 10^{-4} & -3.8135 \\ 0.31620 & -694.12 \end{bmatrix}. \tag{88}$$

Remark 3. Since the system is single-input, the expression of K depends only on $s_i^c, i = 1,2,3,4,5$ and is independent of $f_i, i = 1,2,3,4,5$ (see [26]). In fact, we can also prove that the index (77) is also independent of $f_i, i = 1,2,3,4,5$. Therefore, here we simply set these parameters to 1 or $1 \pm 1\,\mathrm{i}$.

Partitioning K^* as

$$K^* = [\,K_1^*\ K_2^*\,],\quad K_1^* \in \mathbb{R}^{1\times 4},\ K_2^* \in \mathbb{R},$$

and replacing K_1, K_2, and L in (2) with K_1^*, K_2^*, and L^*, respectively, gives the final designed controller.

For exactly the same system, three control laws are presented in [23], namely, an observer-based controller, a dynamic compensator, and a PID controller with structural filters. Since the first two control methods aim at the stabilization problem, they failed in the asymptotically tracking task in this paper. Therefore, for comparison, we only consider the third method in [23], that is, the traditional PID controller, which takes the structure in Figure 2, where

$$G(s) = \frac{60s + 1}{1.5625s^2 + 3.5s + 1}, \tag{89}$$

$$K_p = K_d = 15,\ K_i = 0.03. \tag{90}$$

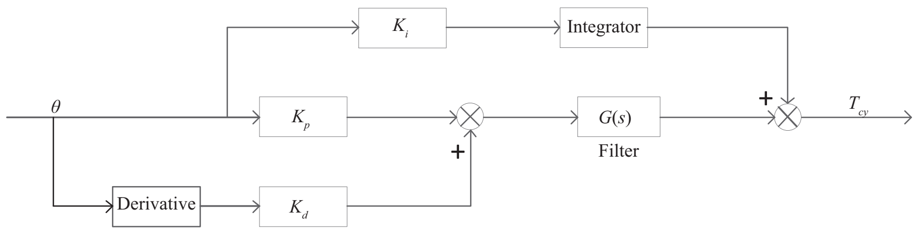

Figure 2 Structure of classic PID controller.

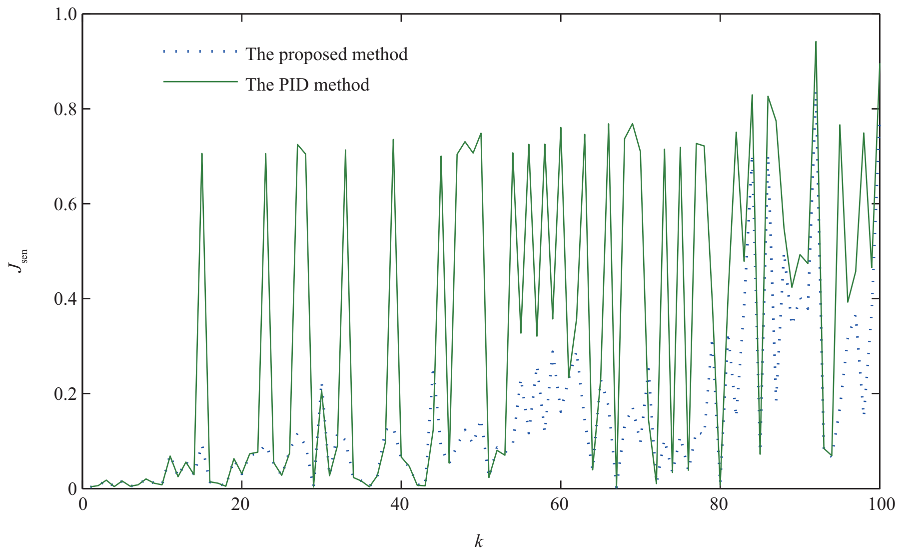

Figure 3 (Color online) The index values $J_{\rm sen}$ under the two control methods.

6.3 Numerical simulation

6.3.1 *Verification of closed-loop pole sensitivities*

In order to quantify the degree to which the closed-loop poles are affected by parameter perturbations, the following index is introduced:

$$J_{\rm sen} = \frac{1}{n}\sqrt{\sum_{i=1}^{n}\left(s_i^{\rm per} - s_i^{\rm nom}\right)^2},$$

where $s_i^{\rm nom}$, $i = 1, 2, \ldots, n$, are the nominal closed-loop poles, $s_i^{\rm per}$, $i = 1, 2, \ldots, n$, represent the closed-loop poles when the coefficient matrix A is perturbed, and n is the number of closed-loop poles, which equals 7 or 9 corresponding to the PID controller or the proposed method. Obviously, the smaller the value of $J_{\rm sen}$, the less sensitive the closed-loop poles are to parameter perturbations.

In order to avoid the contingency of the experimental results, we generate 100 sets of random parameter perturbations as follows:

$$\begin{cases} \delta_{1k} = \mathrm{WGN}\left(2\times 10^{-6}k, 0\right), \\ \delta_{2k} = \mathrm{WGN}\left(2\times 10^{-8}k, 0\right), \end{cases} \quad k = 1, 2, \ldots, 100, \tag{91}$$

where $\mathrm{WGN}(\sigma^2, \mu)$ generates a white Gaussian noise with variance σ^2 and mean μ. For each of these 100 cases, we calculate the index values corresponding to the two control methods, and connect the scattered points into two curves, as shown in Figure 3, which apparently reflects the superiority of the proposed method.

6.3.2 *Simulation results*

Same as [23, 29], the initial values of the attitude angle and its estimation are taken as $\theta_0 = \hat{\theta}_0 = 0.06°$, and those of the attitude angular velocity and its estimation are $\omega_0 = \hat{\omega}_0 = -0.003°/s$. The initial values

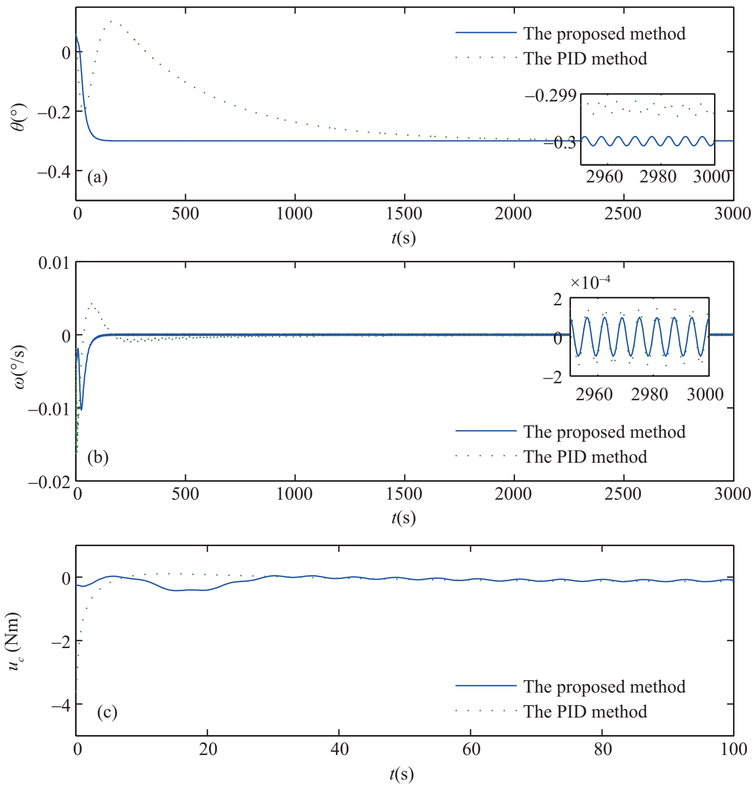

Figure 4 (Color online) Dynamic responses. (a) Pitch angle; (b) pitch anglular velocity; (c) control input.

of the remaining state variables are set to be zeros. The specific values of the parameter perturbations are chosen to be

$$\delta_1 = -0.7766, \quad \delta_2 = -6.8048 \times 10^{-4},$$

which corresponds to the case of $k = 15$ in the 100 sets of random parameter perturbations (91). At this point, the index $J_{\rm sen}$ corresponding to the proposed method and the PID method are

$$J_{\rm sen}^{\rm OB} = 0.0928 \text{ and } J_{\rm sen}^{\rm PID} = 0.7061,$$

respectively. The version of MATLAB software used in the simulation is 2014a. The numerical simulation algorithm is the well-known Runge-Kutta method, which corresponds to the "ode4" function in MATLAB. The fixed-step size is 0.5 s, and the simulation results are shown in Figure 4.

It is shown in Figures 4(a) and (b) that the proposed method has a higher steady-state accuracy than the PID control method. As for transient performance, the proposed method is far superior to the PID method. The convergence time of the proposed method is about 150 s, while that of the PID method is about 2200 s.

As shown in Figure 4(c), the peak value of the control torque of the proposed method is within 0.5 Nm, while that of the PID controller is about 4 Nm.

7 Conclusion

A parametric multiobjective design approach is proposed for output regulation in a multivariable linear system based on a PI controller and full-state observer. The superiority of the proposed method is primarily reflected in the following characteristics.

(1) The complete parametric expressions of the gains of both PI controller and observer are established, and all design degrees of freedom are obtained, which are characterized by a set of parameter vectors. Moreover, the restriction of distinct closed-loop poles, which is often assumed as in [4, 5], is removed.

(2) The design degrees of freedom are used to ensure that the closed-loop system has lower eigenvalue sensitivities, smaller control gains and greater tolerance to time-varying disturbances. The separation principle for eigenvalue sensitivities is proven, which is crucial in the multiobjective design process.

(3) The proposed method is successfully applied to attitude control of a flexible satellite. Its superiority over the traditional PID controller in [23] is well demonstrated using the simulation results.

The above facts clearly indicate the merits of the proposed method in both theory and application. More successful practical applications of the proposed method are definitely expected, particularly in cases where the disturbances are primarily constant or gradually time-varying in nature, which often occurs in industrial systems.

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