SCIENCE CHINA
Information Sciences

• RESEARCH PAPER •

March 2021, Vol. 64 132207:1–132207:10
https://doi.org/10.1007/s11432-019-2944-3

Fault detection for a class of linear systems with integral measurements

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Received 31 October 2019/Revised 9 March 2020/Accepted 30 April 2020/Published online 7 February 2021

Abstract This paper discusses the parity space-based fault detection (FD) method for a class of linear discrete-time systems with integral measurements. The integral measurements are functions of the system states over a given time window. We establish a novel parity relation to tackle integral measurements. The parameters of the FD unit are redesigned such that the generated residual signal is simultaneously decoupled from initial states, robust against disturbances, and sensitive to the faults. We employ the singular value decomposition algorithm to calculate parity space matrices. Finally, an experiment is presented to show the effectiveness of the proposed method.

Keywords fault detection, linear discrete-time systems, integral measurements, parity space, optimization problem

Citation Zhu X Q, Liu Y, Fang J Z, et al. Fault detection for a class of linear systems with integral measurements. Sci China Inf Sci, 2021, 64(3): 132207, https://doi.org/10.1007/s11432-019-2944-3

1 Introduction

In many chemical and nuclear reaction processes, system measurement outputs such as concentration are seldom sampled and can only be obtained with a certain time delay. The sensors may require some time to collect and process the data in a given time window. Therefore, the measurement outputs are proportional to the integral of the system states [1]. Most existing studies have assumed that the measurement outputs only depend on the current state of the system. However, this hypothesis is not always true in practical engineering. This integral measurement phenomenon is frequently encountered when optic, chemical, and nuclear signals are handled, such as undulator field measurements [2], the color of nano-dimensional radiators [3], fission yields measurements, perturbation measurements of uranium metallic core reactors [4], thermal neutron capture cross-sections, and resonance using the neutron activation technique [5]. Ref. [1] established a comprehensive model to formulate the integral measurement phenomenon of the addressed nonlinear system and designed a modified unscented Kalman filter to estimate the system states. Ref. [6] proposed a new hidden belief rule-based model with a power set and considered attribute reliability (PHBRB-r) for hidden fault prediction to integrate the activated rules and generate the final outputs of the PHBRB-r model. Notably, systems with integral measurements are frequently encountered in many practical industrial applications. Moreover, some research results of systems with integral measurements have been published.

In practical industrial applications, fault detection (FD) is a critical problem. The FD technique, which enables the detection of system anomalies and prevention of further catastrophes as early as possible, has received considerable research attention. In the presence of the system model, the model-based FD framework has been well established and excellent results have already been reported on this issue [7–14]. In the event of failure, a system with integral measurements cannot respond in time, which will inevitably cause great losses in terms of personnel and property because of the infrequency of system measurement output. Thus, the FD problem for such systems is critical. Motivated by this, some preliminary research was conducted on systems with integral measurements. In a recent study [15], the problems of state

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estimation and fault reconstruction for linear systems with integral measurements were investigated. A novel observer capable of decoupling the disturbances significantly and reducing the effects of the disturbances that cannot be decoupled on the estimation and reconstruction errors was realized. However, the FD problem for systems with integral measurements has not been sufficiently studied.

Among existing model-based FD approach, a parity space-based scheme, is critical in linear systems. The basic idea is to use the limited time window within the system input and output data to generate the residual. The effects of the initial states can be completely decoupled from the residual, achieving a tradeoff between robustness against disturbances and sensitivity to the faults by selecting appropriate parameters [16–19]. The relationship between the parameters of parity space, observer methods, and factorization approaches has been extensively studied within a unified framework [20]. In [21], the authors proposed a parity space vector machine scheme to tackle the robust FD problem for linear discrete-time systems. To achieve a tradeoff between the false alarm rate and FD rate, an integrated design of the residual generation and residual evaluation based on parity space was studied. For an intrinsically time-varying system, a recursive algorithm was developed so that the method could be readily implemented online [22]. Furthermore, the residual generated by the parity space-based strategy can be directly used to estimate the fault in the least-squares sense, where the estimator design has been expressed as an optimization problem in a matrix quadratic form, and the investigator can compute the stationary point of the quadratic function to solve the optimization problem [23].

Based on the above discussion, it is natural to study the FD problem for systems with integral measurements using the parity space scheme. The parity space-based approach can decouple the residual signal from the initial states and system inputs, so that the residual signal is only related to disturbances and fault signals. Therefore, we aim to present a parity space-based FD strategy for systems with integral measurements. However, this apparently natural idea would bring in significant challenges, such as (1) how to deal with the integral measurement phenomenon and (2) how to calculate the transfer matrices and augmented vectors to ensure that the parity relation holds with the integral measurements. We therefore aim to resolve these two issues. In this paper, we study the nontrivial FD problem for systems with integral measurements using the parity space approach. Consequently, the parity relation must be reestablished, and the desired parameters should be modified to handle the integral measurements. The main contributions from this perspective are twofold.

(1) We first build a novel parity relation with the integral term to satisfy the parity relation. A residual generator under the novel parity relation constraint is then developed to guarantee that the residual is simultaneously robust against disturbances and sensitive to the faults, where the singular value decomposition (SVD) technique is employed to solve the optimization problem.

(2) The proposed scheme is applied to a three-tank system to demonstrate that the developed method can realize fast FD for systems with integral measurements.

2 Problem statement

Consider the following linear discrete-time system with integral measurements:

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + B_d d(k) + B_f f(k), \\ y(k) = C\displaystyle\sum_{i=0}^{M} x(k-i) + Du(k) + D_d d(k) + D_f f(k), \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$, $y(k) \in \mathbb{R}^q$, $u(k) \in \mathbb{R}^p$, $d(k) \in \mathbb{R}^l$, $f(k) \in \mathbb{R}^m$ are the state, measurement output, control input, unknown input and fault vectors, respectively. A, B, B_d, B_f, C, D, D_d and D_f are known matrices with compatible dimensions. M is an arbitrary nonnegative integer representing the time interval to collect the data. When $M = 0$, the considered system changes to the traditional linear discrete-time one.

The main purpose of this paper is to investigate the FD issue for the system (1) with integral measurements based on parity space approach. Generally speaking, for the typical model-based FD problem, the key process is to design the residual generator and residual evaluation.

However, with the integral measurement phenomenon being considered, the traditional parity space-based FD method is no longer applicable. In the parity space-based strategy, the parity relation is directly dependent on the measurement matrix. When the integral measurement phenomenon is considered, the measurement matrix will become quite different from that of the traditional system owing to the delayed system states in the integral measurements. Consequently, the parity relation needs to be reestablished and the desired parameters should be modified to cope with the integral measurements.

Remark 1. It should be pointed out that there are some differences between the integral measurements and the transmission time-delays. The time-delays mean that the measurement outputs of the system are related to the output values at a previous time while the integral measurements mean that the measurement outputs of the system are in proportion to the integral of the system states. It is worth mentioning that with the integral measurement phenomenon being considered, the traditional FD methods are no longer applicable. So how to solve the FD problem for systems with integral measurements becomes an important yet challenging open problem.

For the system (1) with integral measurements, the novel parity relation can be established as follows:

$$y_s(k) = H_{os}x(k-s-M) + H_{ds}d_s(k) + H_{fs}f_s(k) + H_{us}u_s(k), \tag{2}$$

where s is the so-called parity space order. $y_s(k) \in \mathbb{R}^{q(s+1)}$, $u_s(k) \in \mathbb{R}^{p(s+M+1)}$, $x(k-s-M) \in \mathbb{R}^n$, $d_s(k) \in \mathbb{R}^{l(s+M+1)}$, $f_s(k) \in \mathbb{R}^{m(s+M+1)}$ are augmented vectors. $H_{us} \in \mathbb{R}^{q(s+1)\times(s+M+1)p}$, $H_{os} \in \mathbb{R}^{q(s+1)\times n}$, $H_{ds} \in \mathbb{R}^{q(s+1)\times(s+M+1)l}$, $H_{fs} \in \mathbb{R}^{q(s+1)\times(s+M+1)m}$ are transfer matrices from $u_s(k), x(k-s-M), d_s, f_s$ to y_s, respectively. These augmented vectors and transfer matrices will be designed in Section 3.

Inspired by [24], the parity relation based residual signal can be defined as

$$\begin{aligned} r(k) &= V_s(y_s(k) - H_{us}u_s(k)) \\ &= V_s(H_{os}x(k-s-M) + H_{ds}d_s(k) + H_{fs}(k)f_s(k)), \end{aligned} \tag{3}$$

where $V_s \neq 0$. $V_s \in \mathbb{R}^{q(s+1)}$ is the parity space matrix, which needs to be designed to balance the robustness of residual to unknown inputs and the sensitivity to faults.

Remark 2. When the integral measurement phenomenon is considered, the relationship between system outputs and system states is changed. Therefore, the transfer matrices and augmented vectors will become quite different from that of the traditional parity space scheme. The parity relation, transfer matrices and augmented vectors need to be redesigned. This also leads to the difference from the classical parity space approach. The detailed process will be given in Section 3.

3 Main results

In the dynamic parity space approach, the first step is to establish the parity relation to design the residual generator. For the residual generator, the problem can be described as finding the transfer matrices $H_{us}, H_{os}, H_{ds}, H_{fs}$ and augmented vectors y_s, u_s, d_s, f_s so that the parity relation still holds.

Theorem 1. For the system (1) with integral measurements, if the parity relation (2) still holds, the transfer matrices $H_{us}, H_{os}, H_{ds}, H_{fs}$ and augmented vectors y_s, u_s, d_s, f_s should be the following forms:

$$H_{os} = \begin{bmatrix} C\sum_{i=0}^{M} A^i \\ CA\sum_{i=0}^{M} A^i \\ \vdots \\ CA^s\sum_{i=0}^{M} A^i \end{bmatrix}, \quad H_{us} = \begin{bmatrix} h_{us}(1) \\ h_{us}(2) \\ \vdots \\ h_{us}(s+1) \end{bmatrix}, \tag{4}$$

where

$$h_{us}(j) = \left[C\sum_{i=0}^{M-2+j} A^iB \ \cdots \ CB \ D \ 0 \ \cdots \ 0 \right],$$

$$y_s(k) = \begin{bmatrix} y(k-s) \\ y(k-s+1) \\ \vdots \\ y(k) \end{bmatrix}, \quad u_s(k) = \begin{bmatrix} u(k-s-M) \\ \vdots \\ u(k-s) \\ \vdots \\ u(k) \end{bmatrix}, \tag{5}$$

$h_{us} \in \mathbb{R}^{q\times(s+M+1)p}$, $h_{ds} \in \mathbb{R}^{q\times(s+M+1)l}$, $h_{fs} \in \mathbb{R}^{q\times(s+M+1)m}$, $j = 1, 2, \ldots, s+1$ represent the jth line of H_{us}, H_{ds} and H_{fs}, respectively. H_{ds}, H_{fs} can be constructed by replacing B, D in H_{us} with B_d, D_d and B_f, D_f, respectively. $d_s(k)$ and $f_s(k)$ are achieved by replacing $u(k)$ with $d(k)$ and $f(k)$ in $u_s(k)$, respectively.

Proof. From (1), it follows that

$$x(k) = A^M x(k-M) + \sum_{i=0}^{M-1} A^i Bu(k-i-1) + \sum_{i=0}^{M-1} A^i B_d d(k-i-1) + \sum_{i=0}^{M-1} A^i B_f f(k-i-1). \tag{6}$$

We then have

$$\begin{aligned}\sum_{i=0}^{M} x(k-i) = &\sum_{i=0}^{M} A^i x(k-M) + \sum_{i=0}^{j}\sum_{j=0}^{M-1} A^i Bu(k-j-1) \\ &+ \sum_{i=0}^{j}\sum_{j=0}^{M-1} A^i B_d d(k-j-1) + \sum_{i=0}^{j}\sum_{j=0}^{M-1} A^i B_f f(k-j-1).\end{aligned} \tag{7}$$

Combing (1) and (7), we can obtain

$$\begin{aligned}y(k) = &C\sum_{i=0}^{M} A^i x(k-M) + C\sum_{i=0}^{j}\sum_{j=0}^{M-1} A^i Bu(k-j-1) + C\sum_{i=0}^{j}\sum_{j=0}^{M-1} A^i B_d d(k-j-1) \\ &+ C\sum_{i=0}^{j}\sum_{j=0}^{M-1} A^i B_f f(k-j-1) + Du(k) + D_d d(k) + D_f f(k).\end{aligned} \tag{8}$$

In order to guarantee the validity of parity relation (2), the initial term of equation (8) should change from time instant $k-s$ to time instant $k-s-M$. Therefore, it is impossible to establish the parity relation directly depending on (8), so we need to redrive $y(k)$. It follows from (8) that

$$\begin{aligned}y(k-s) = &C\sum_{i=0}^{M} A^i x(k-s-M) + C\sum_{i=0}^{j}\sum_{j=0}^{M-1} A^i Bu(k-s-j-1) \\ &+ C\sum_{i=0}^{j}\sum_{j=0}^{M-1} A^i B_d d(k-s-j-1) + C\sum_{i=0}^{j}\sum_{j=0}^{M-1} A^i B_f f(k-s-j-1) \\ &+ Du(k-s) + D_d d(k-s) + D_f f(k-s).\end{aligned} \tag{9}$$

Similarly to (6), $y(k)$ can be rewritten as

$$\begin{aligned}y(k) = &CA^s\sum_{i=0}^{M} A^i x(k-s-M) + C\sum_{i=0}^{j}\sum_{j=0}^{M-1+s} A^i Bu(k-j-1) \\ &+ C\sum_{i=0}^{j}\sum_{j=0}^{M-1+s} A^i B_d d(k-j-1) + C\sum_{i=0}^{j}\sum_{j=0}^{M-1+s} A^i B_f f(k-j-1) \\ &+ Du(k) + D_d d(k) + D_f f(k).\end{aligned} \tag{10}$$

The generalization of (9) and (10) can be written over time and in the order of the latest values. Then the parity relation (2) can be established where H_{ds}, H_{fs} can be obtained by replacing B, D in H_{us} with B_d, D_d and B_f, D_f, respectively. Similarly, $d_s(k)$ and $f_s(k)$ are constructed by replacing $u(k)$ with $d(k)$ and $f(k)$ in $u_s(k)$. Now, the proof is completed.

Remark 3. According to the above calculation and analysis, it should be noticed that $y(k-s), y(k-s+1), \ldots, y(k)$ have the same terms. With the integral measurement phenomenon being taken into consideration, the designed transfer matrices and augmented vectors are different from the traditional parity space approach. In the classical parity space scheme, the first element of the augmented vector u_s is $u(k-s)$. However, resulting from the integral measurement phenomenon, the first element of the augmented vector u_s turns into $u(k-s-M)$. Similarly, the first elements of d_s and f_s become $d(k-s-M)$ and $f(k-s-M)$, respectively. Through continuous iterative calculation and inductive summary, the transfer matrices $H_{us}, H_{os}, H_{ds}, H_{fs}$ and augmented vectors y_s, u_s, d_s, f_s can be obtained so that the parity relation still holds.

To ensure that the residual signal is decoupled from the initial states and system inputs, the parity space matrix V_s should satisfy the following condition:

$$V_s H_{os} = 0,$$

where V_s is the parameter matrix to be designed later. Then we have

$$r(k) = V_s(H_{ds}d_s(k) + H_{fs}(k)f_s(k)). \tag{11}$$

Define $V_s \in \mathbb{P}_s$. $\mathbb{P}_s$ is the set of V_s with the parity space order s. N_s is the basis matrix of parity space $\mathbb{P}_s$. N_s needs to guarantee that $N_s H_{os} = 0$ holds. Let $V_s = P_s N_s$, where P_s is an invertible matrix and V_s is an arbitrary matrix. Set

$$\bar{H}_{ds} = N_s H_{ds}, \quad \bar{H}_{fs} = N_s H_{fs},$$

and then the residual signal $r(k)$ can be rewritten as

$$r(k) = P_s(\bar{H}_{ds}d_s(k) + \bar{H}_{fs}(k)f_s(k)). \tag{12}$$

Furthermore, as mentioned in [24], we have

$$\begin{aligned} \|P_s\bar{H}_{ds}\|_2 &= \sup_{d\neq 0}\frac{\|P_s\bar{H}_{ds}d_s(k)\|}{\|d_s(k)\|}, \\ \|P_s\bar{H}_{fs}\|_2 &= \sup_{f\neq 0}\frac{\|P_s\bar{H}_{fs}f_s(k)\|}{\|f_s(k)\|} \quad \text{or} \\ \tilde{\sigma}(P_s\bar{H}_{fs}) &= \inf_{f\neq 0}\frac{\|P_s\bar{H}_{fs}f_s(k)\|}{\|f_s(k)\|}, \end{aligned} \tag{13}$$

where $\|P_s\bar{H}_{ds}\|_2$ represents the robustness of residual to unknown inputs; $\|P_s\bar{H}_{fs}\|_2$ and $\tilde{\sigma}(P_s\bar{H}_{fs})$ represent the sensitivity of residual to faults. In order to balance the robustness of residual to unknown inputs and the sensitivity of residual to faults, the problem can be formulated as finding the parity space matrix P_s to satisfy the following optimization problem:

$$\max_{P_s} J = \frac{\|P_s\bar{H}_{fs}\|_2}{\|P_s\bar{H}_{ds}\|_2} \text{ or } \max_{P_s} J_- = \frac{\tilde{\sigma}(P_s\bar{H}_{fs})}{\|P_s\bar{H}_{ds}\|_2}. \tag{14}$$

With the purpose of getting the desired residual signal, the matrix P_s should satisfy the optimization problem (14). Inspired by [24], the optimization problem can be handled via SVD. With the SVD on $\bar{H}_{ds}$, it follows that

$$\begin{aligned} &\bar{H}_{ds} = U\Sigma V^{\mathrm{T}}, \quad UU^{\mathrm{T}} = I, \quad VV^{\mathrm{T}} = I, \\ &\Sigma\ = [S \ \ 0], \quad S = \mathrm{diag}(\sigma_1, \sigma_2, \ldots, \sigma_\gamma), \end{aligned} \tag{15}$$

where U and V are left and right singular matrices respectively. $\sigma_1, \sigma_2, \ldots, \sigma_\gamma$ are singular values of matrix $\bar{H}_{ds}$. Then, the following lemma gives an optimal solution P_s of (14).

Lemma 1 ([24])**.** For the system (1), $P_s = S^{-1}U^{\mathrm{T}}$ solves the optimization problem (14) and the optimization problem can be rewritten as follows:

$$\max_{P_s} J = \|S^{-1}U^{\mathrm{T}}\bar{H}_{fs}\|_2 \text{ or } \max_{P_s} J_- = \tilde{\sigma}(S^{-1}U^{\mathrm{T}}\bar{H}_{fs}), \tag{16}$$

where S and U have been defined in (15).

Remark 4. In order to deal with the optimization problem, Theorem 1 in [24] is applied in this paper. Compared with the classical parity space strategy, although the transfer matrices and augmented vectors are different, the method to settle the optimization problem is the same. The optimization problem can be handled via SVD, so that P_s and V_s can be obtained accordingly. For simplicity, the detailed proof is omitted here. Furthermore, the constraint $V_s H_{os} = 0$ is put forward to guarantee that the influence of initial states and system inputs on the residual can be fully eliminated. It shows that this approach can completely decouple the residual signal from the initial states and system inputs.

The evaluation function is the second stage of parity space FD scheme. Similarly to [8], the evaluation function J_r can be selected with a sliding window as follows:

$$J_r(k) = \frac{1}{N+1}\sum_{i=0}^{N} \|r(k-i)\|^2, \tag{17}$$

where N is the size of the sliding window. It is noted from (12) and (17) that the evaluation function J_r is decided by disturbances and fault signals, and the selection of threshold depends on the disturbance amplitude. With the dramatic fluctuation of disturbances, the threshold value is relatively high. In order to detect a fault effectively, the threshold can be set as the maximum value of the evaluation function without fault. The threshold can be chosen as

$$J_{\text{th}} = \sup J_0, \tag{18}$$

where J_0 is $J_r(k)$ in the fault-free case. Then the following principle of detecting a fault is given:

$$\begin{cases} J_r < J_{\text{th}}, & \text{no alarm,} \\ J_r \geqslant J_{\text{th}}, & \text{fault alarm.} \end{cases} \tag{19}$$

In this section, our attention is concentrated on the design of parity space transfer matrices H_{us}, H_{os}, H_{ds}, H_{fs} and augmented vectors y_s, u_s, d_s, f_s. Moreover, SVD technique is employed to solve the optimization problem and the parity space matrices V_s, P_s can be obtained accordingly. Then, the online FD approach based on parity space for a class of linear discrete-time systems with integral measurements can be concluded into Algorithm 1.

Algorithm 1 Online fault detection algorithm

1: Set $x(1) = [0\ 0\ 0]^{\mathrm{T}}$, $u(1) = [0\ 0\ 0]^{\mathrm{T}}$ as initial values of the system state and control input.
2: Calculate $H_{os}, H_{us}, H_{ds}, H_{fs}$ and y_s, u_s, d_s, f_s by using (4) and (5).
3: Calculate P_s and V_s according to Lemma 1 and SVD technique to ensure the solvability of (14).
4: Update $r(k)$ by using (12).
5: Update $J_r(k)$ by using (17) and compare with J_{th}. As a result, the occurrence of a fault can be detected by applying (19).
6: Let $k = k + 1$, and go to step 2 until the end of the process.

4 Experiment

In this section, the proposed scheme will be applied to a three-tank system to demonstrate its effectiveness and usefulness. TTS-20 laboratory setup of the three-tank system will be used to complete this experiment in this paper. The experimental setup of the three-tank system is described in Figure 1.

The system model can be described as

$$\begin{aligned}
&A\frac{\mathrm{d}h_1}{\mathrm{d}t} = Q_1 - Q_{13}, \\
&A\frac{\mathrm{d}h_3}{\mathrm{d}t} = Q_{13} - Q_{32}, \\
&A\frac{\mathrm{d}h_2}{\mathrm{d}t} = Q_2 + Q_{32} - Q_{20}, \\
&Q_{13} = az_1 S_n \mathrm{sgn}(h_1 - h_3)\sqrt{(2g)|h_1 - h_3|}, \\
&Q_{32} = az_3 S_n \mathrm{sgn}(h_3 - h_2)\sqrt{(2g)|h_3 - h_2|},
\end{aligned}$$

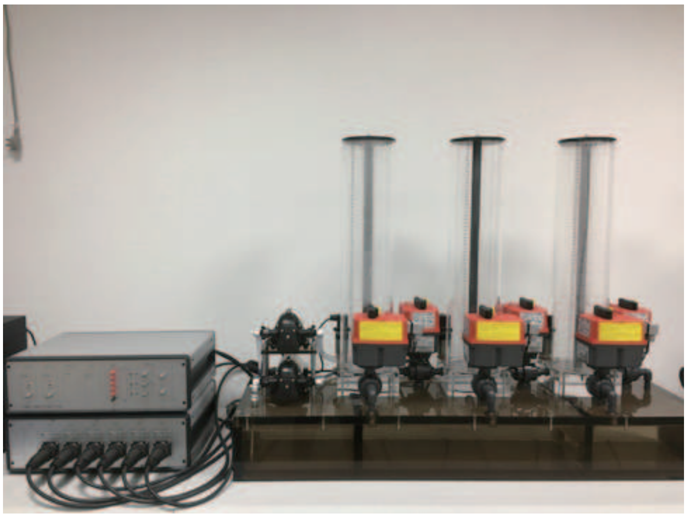

Figure 1 (Color online) The experimental setup of the three-tank system.

$$Q_{20} = az_2 S_n \sqrt{(2gh_2)},$$

where Q_{ij} is the flow rate from the ith tank to the jth tank. Q_{ij} can be obtained through the generalized Torricelli-rule. The parameters of the system are given in Table 1.

Table 1 Parameters of the three-tank system

Symbol	Value	Description
A	0.0154 m^2	Surface area of the tanks
S_n	5×10^{-5} m^2	Surface area of the pipes
$Q_{1\max}$	100 ml/s	Max flow rate of pump 1
$Q_{2\max}$	100 ml/s	Max flow rate of pump 2
$H_{\max}$	0.62 m	Max height of tanks
az_1	0.46	Fluid constants for pipe 1
az_2	0.48	Fluid constants for pipe 2
az_3	0.58	Fluid constants for pipe 3

Let

$$h = \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}, \quad A(h) = \frac{1}{A}\begin{bmatrix} -Q_{13} \\ Q_{32} - Q_{20} \\ Q_{13} - Q_{32} \end{bmatrix}, \quad B = \frac{1}{A}\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}, \quad y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}.$$

Then the system can be rewritten as

$$\frac{\mathrm{d}h_1}{\mathrm{d}t} = A(h) + BQ,$$
$$y = [h_1 \ h_2]^{\mathrm{T}}.$$

For the three-tank system, consider the equilibrium point $h_e = [h_{1e} \ h_{2e} \ h_{3e}]^{\mathrm{T}}$. The discrete linearized system with a sampling period $T_s = 1$ s is described as

$$\delta h(k+1) = A\delta h(k) + BQ(k),$$
$$\delta y(k) = C\delta h(k),$$

where $\delta y = y - Ch_e, \delta h = h - h_e$. Let $h_e = [0.31825 \ 0.23145 \ 0.15175]^{\mathrm{T}}$, and then the parameters of the system can be obtained as

$$A = \begin{bmatrix} 0.9889 & 0.0001 & 0.0110 \\ 0.0001 & 0.9774 & 0.0119 \\ 0.0110 & 0.0119 & 0.9770 \end{bmatrix}, \quad B = \begin{bmatrix} 64.5993 & 0.0015 \\ 0.0015 & 64.2236 \\ 0.3604 & 0.3910 \end{bmatrix},$$

$$B_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad D_d = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

In this system, the system state vector $h(k) = [h_1(k)\ h_2(k)\ h_3(k)]^{\mathrm{T}}$ represents the liquid level of the three tanks; $y(k) = [h_1(k)\ h_2(k)]^{\mathrm{T}}$ is the measurement output and represents the liquid heights of Tanks 1 and 2, respectively; $u(k) = [Q_1(k)\ Q_2(k)]^{\mathrm{T}}$ is the liquid inflow. The closed-loop control input is designed as $u(k) = Q_p y(k) + Q_i y(k-1)$, where

$$Q_p = \begin{bmatrix} -0.0013 & -0.0004 \\ -0.0004 & -0.0012 \end{bmatrix}, \quad Q_i = \begin{bmatrix} -0.0022 & -0.0007 \\ -0.0006 & -0.0020 \end{bmatrix}.$$

The integral measurement phenomenon is added artificially, and the measurement of the system can be rewritten as $\delta y(k) = C\sum_{i=0}^{M}\delta h(k-i)$. The time interval to collect the data is set as $M = 3$ and the parity space order can be selected as $s = 4$. With the given parameters, the following parity space matrices P_s and V_s satisfying the optimization problem can be obtained:

$$P_s = \begin{bmatrix} -0.0146 & -0.0149 & 0.0188 & -0.0479 & 0.0547 & -0.0835 & 0.0926 \\ 0.3836 & 0.4572 & 0.5040 & 0.4693 & 0.5103 & -0.0897 & -0.1081 \\ -0.5299 & 0.3493 & -0.1725 & 0.8078 & -0.3661 & -0.0404 & 0.6052 \\ 0.4122 & -0.7313 & -0.7174 & 0.5665 & 0.6121 & 0.2028 & 0.2061 \\ -0.3387 & -0.9143 & 09812 & 0.2975 & -0.1052 & 0.2793 & 0.0685 \\ 0.365 & 0.3171 & 0.1682 & -0.4262 & 0.0525 & 1.152 & 0.8623 \\ -1.1128 & 0.2150 & -0.1439 & -0.1561 & 0.9406 & 0.4617 & -0.3313 \end{bmatrix},$$

$$V_s = \begin{bmatrix} 0.0015 & -0.4880 & -0.0711 & 0.8433 & 0.0102 & -0.1377 & 0.0240 & -0.1194 & 0.0374 & -0.1019 \\ -0.4499 & 0.0628 & 0.0044 & 0.0291 & 0.8426 & -0.0029 & -0.1839 & -0.0334 & -0.2096 & -0.0626 \\ -0.0033 & -0.3759 & 0.1620 & -0.1061 & -0.0228 & 0.8606 & -0.0546 & -0.1707 & -0.0853 & -0.2007 \\ -0.4492 & 0.1534 & 0.2134 & 0.0713 & -0.1852 & -0.0070 & 0.7476 & -0.0818 & -0.3173 & -0.1532 \\ -0.0079 & -0.2686 & 0.3845 & -0.0578 & -0.0543 & -0.1406 & -0.1297 & 0.7804 & -0.2025 & -0.2949 \\ -0.4484 & 0.2408 & 0.4153 & 0.1120 & -0.2121 & -0.0111 & -0.3186 & -0.1286 & 0.5786 & -0.2408 \\ -0.0124 & -0.1661 & 0.5968 & -0.0115 & -0.0844 & -0.1418 & -0.2014 & -0.2662 & -0.3144 & 0.6152 \end{bmatrix}.$$

Run the system for 100 s. Two fault signals (leakage fault and sensor fault) are designed at the 20th second and the 40th second, respectively. First, the leakage fault in Tank 1 is considered. The leakage valve of Tank 1 is opened 15° artificially at the 20th second to indicate the leakage fault in Tank 1. Then the parameter matrices of fault can be defined as

$$B_f = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad D_f = \begin{bmatrix} 0 \\ 0 \end{bmatrix}.$$

The sliding window is set as $N = 20$. The threshold can be defined through (18). Algorithm 1 is applied to realize online FD and the experimental results are shown in Figure 2.

The sensor fault in Tank 2 can be described as that there is a fault occurred when the sensor is used to measure the liquid level of Tank 2. The additive fault signal is defined as $f = 0.05$ m at the 40th second and disappears at the 60th second. Then the parameter matrices of fault can be defined as

$$B_f = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad D_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

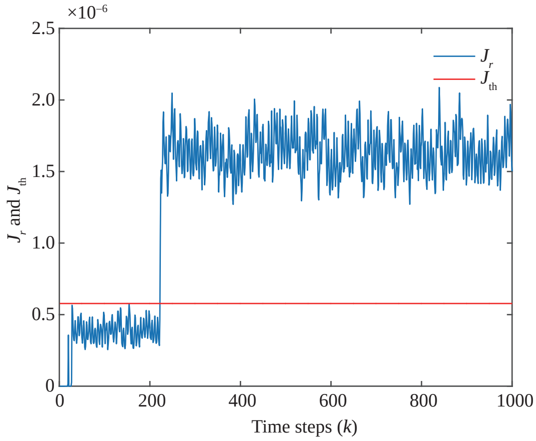

Figure 2 (Color online) Detection results for the leakage fault.

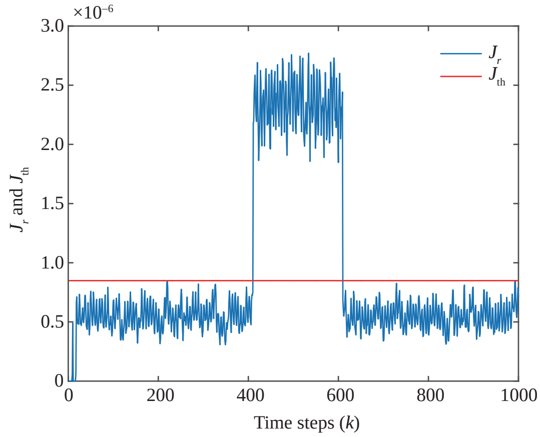

Figure 3 (Color online) Detection results for the sensor fault.

The threshold can be got through (18) with the sliding window $N = 10$. The experimental results are shown in Figure 3.

It can be seen from Figures 2 and 3 that the proposed method can achieve a good FD performance. J_r exceeds the threshold $J_{\rm th}$ immediately when a fault occurs in the system. When the fault signal disappears, J_r returns below $J_{\rm th}$. According to the experimental results, it can be seen easily that the proposed FD scheme can effectively realize the fast FD for linear discrete-time systems with integral measurements.

5 Conclusion

In this paper, the parity space-based FD problem was addressed for a class of discrete-time linear systems with integral measurements. A novel parity relation was established in the presence of integral measurements to ensure that the residual is decoupled from the initial states and system inputs. The SVD technique was proposed to achieve a balance between the robustness of the residual against disturbances and the sensitivity of the residual to the faults. Finally, an experiment was conducted to demonstrate the effectiveness of the proposed scheme. Fault isolation and fault-tolerant control for systems with integral measurements would be significant research interests in our future work. We also aim to apply these research results to a reduced-order aircraft system [25] or other practical systems in the future.

Acknowledgements This work was supported in part by National Natural Science Foundation of China (Grant Nos. 61873149, 61733009, 61703244), and Research Fund for the Taishan Scholar Project of Shandong Province of China.

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